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A Descriptor Approach to Robust Leader-Following Output Consensus of Uncertain Multi-Agent Systems with Delay

Ala Shariati, Mahdi Tavakoli

Abstract— **In this technical note, a descriptor approach to leader-following output consensus of multi-agent systems with both stationary and dynamic leaders is given in the presence of transmission delay and model uncertainty. The proposed method can deal with stable and unstable agents described by general linear models. To this end, a new proportional-derivative-integral (PID) consensus protocol for the closed-loop multi-agent system is proposed under a directed graph. Applying this consensus protocol to the multi-agent system leads to a time-delay closed-loop equation of neutral type. To deal with the resulting neutral system, a descriptor model transformation is used to derive delay-dependent sufficient conditions for the existence of the consensus protocol in terms of certain linear matrix inequalities (LMI). The application of the proposed method is illustrated in a teleoperation system. Simulation results are given to show the effectiveness of the proposed approach.**

Index Terms—**Multi-agent systems, descriptor systems, robust control, time-delay.**

I. Introduction

Cooperative control of multi-agent systems has attracted extensive attention during the last two decades [1]-[6]. Consensus problem as an essential task in cooperative control attempts that all the agents' states reach an agreement under the agents' interactions. In this area, leader-following problem of time-delay multi-agent systems have also been an active research field in recent years. Meng *et al.* [7] provided results for the analysis of both leaderless and leader-following consensus algorithms for first-order and second-order agents in the presence of communication and input delays. A leader-following consensus analysis was studied in [8] for time-varying delayed first-order systems with a static leader. Event-based leader-following consensus of general linear multi-agent systems with time-delay was presented in [9] and [10].

It is worth noting that an extensive number of the studies presented for leader-following multi-agent systems are limited to the integrator behavior agents [8], [11], [12], double integrator dynamics for the agents [13], [14], [15] or both integral and double integral behavior agents [7], [16] in the presence of stationary or dynamic ramp type leaders. Furthermore, many researchers have focused on the study of the leader-following multi-agent systems with general linear agents such as [9], [10], [17]. In these and similar papers, the leader dynamics are usually assumed to have the same dynamics as the agents whereas in many practical problems, the control objective of a leader-following consensus problem with general linear agents is to follow a stationary or dynamic leader. This subject is one of the most challenging topics in the area of leader-following multi-agent systems especially in the presence of time-delay. Furthermore, most of the existing works presented for leader-following consensus of time-delay multi-agent systems have focused on the analysis problem rather than designing a consensus controller. Few studies such as [10] address how to design a leader-following consensus protocol for time-delay multi-agent systems with general linear agents. To the best of the authors' knowledge, no analysis or design study on robust leader-following output consensus for uncertain time-delay multi-agent systems with a general state-space equation has been presented in the literature. Considering the aforementioned facts, in this paper, we present a novel descriptor approach to robust leader-following output consensus for uncertain general linear multi-agent systems in the presence of transmission delay for both stationary and dynamic leaders. The main contributions of this paper are summarized as follows:

- An uncertain multi-agent system with general linear model in this paper. The considered model, which includes both uncertainty and time-delay at the same time and is allowed to be stable and unstable, is one of the most general linear models assumed for the linear multi-agent systems in the literature.
- A new proportional-integral-derivative (PID) consensus protocol is proposed in this paper. Applying the proposed consensus protocol in the closed-loop multi-agent system leads to a time-delay multi-agent system of neutral type [18] that is one of the challenging topics in the area of time-delay systems.
- To deal with the neutral type multi-agent system, a descriptor transformation is used on the closed-loop equation of the multi-agent system. Then, new sufficient conditions for the design of the proposed consensus protocol are provided.
- The proposed PID consensus protocol provides leader-following output consensus for both stable and unstable uncertain general linear multi-agent systems in the presence of transmission delay for both stationary and dynamic leaders.

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This paper is organized as follows. Problem statement and preliminary results as well as the network topology of the multi-agent system are given in Section II. Moreover, the new consensus protocol is stated in the same section. In Section III, the robust design conditions are provided in terms of certain linear matrix inequalities. Illustrative examples are provided in Section IV to show the effectiveness of the proposed methods. Finally, the concluding remarks are given in Section V.

II. Problem Statement and Preliminary Results

A. Graph Theory

Let $G = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a directed graph (digraph) where, $\mathcal{V} = \{1, 2,\ldots, N\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ are the set of vertices and the set of edges, respectively. In the digraph G, the i-th vertex represents the i-th agent. A directed edge from i to j is denoted as an ordered pair $(i, j) \in \mathcal{E}$, which means that the information flow from agent i to agent j. The adjacency matrix $\mathcal{A}= [a_{ij}] \in \mathcal{R}^{N\times N}$ models the communication topology among the agents. $a_{ij} > 0$ if the j-th agent is a neighbor of the i-th agent; otherwise, $a_{ij} = 0$. The degree matrix of digraph G is denoted by $\mathcal{D} = \mathrm{diag}\ \{d_1, d_2,\ldots, d_N\}$, where the diagonal element is represented as $d_j = \sum_{j=1}^{N} a_{ij}$ and the Laplacian matrix of the digraph G is defined as $L_s = \mathcal{D} - \mathcal{A}$. A directed path is a sequence of edges that connects a sequence of vertices in a digraph, in which the edges all are directed in the same direction.

Assume $\tilde{G}$ is a graph with N follower nodes and a leader node 0. A diagonal matrix $M \in \mathcal{R}^{N\times N}$ where $M = \mathrm{diag}\ \{m_1, m_2,\ldots, m_N\}$ is the leader adjacency matrix with $m_i \geq 0$. If the leader information is accessible for the i-th agent, then $m_i > 0$; otherwise, $m_i = 0$. If there is a path in $\tilde{G}$ from every node i in G to node 0, then the node 0 is globally reachable in $\tilde{G}$. The following assumption is needed throughout the paper.

Assumption 1: The graph G is fixed and directed.

B. Problem Formulation

Consider a group of N uncertain n-th order agents represented by the following linear differential equation for each agent:

$$\begin{aligned} \dot{x}_i(t) &= \big(A + \Delta A(t)\big)x_i(t) + \big(B + \Delta B(t)\big)u_i(t) \\ y_i(t) &= Cx_i(t), \quad i = 1, 2, \ldots, N \end{aligned} \tag{1}$$

where $x_i(t) \in \mathcal{R}^n$, $u_i(t) \in \mathcal{R}^m$, $y_i(t) \in \mathcal{R}^r$ are respectively the agent i's state, the agent i's input and the agent i's output which can only use the local information from its neighbor agents. Moreover, the constant matrices $A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times m}$ and $C \in \mathcal{R}^{r\times n}$ are the nominal parts and $\Delta A(t)$ and $\Delta B(t)$ are real matrix functions representing time-varying parameter uncertainties. Throughout the paper, the notations ΔA and ΔB are used instead of $\Delta A(t)$ and $\Delta B(t)$ in some of the equations for the sake of brevity. These uncertainties are the result of model linearization and unmodeled dynamics and are assumed to be of the form

$$\Delta A(t) = D_a F_a(t) E_a, \quad \Delta B(t) = D_b F_b(t) E_b \tag{2}$$

where $F_a(t)$ and $F_b(t)$ are unknown real time-varying matrices with Lebesgue measurable elements satisfying

$$F_a^T(t) F_a(t) \leq I, \quad F_b^T(t) F_b(t) \leq I \quad \forall t \tag{3}$$

and D_a, E_a, D_b, E_b are real known constant matrices that represent how the uncertain parameters in $F_a(t)$ and $F_b(t)$ enter the nominal matrices A and B. Moreover, the stationary and dynamic leaders are given in (4) and (5), respectively.

$$y_0(t) := \begin{cases} a_0, & t \geq 0 \\ 0, & t < 0 \end{cases} \tag{4}$$

$$y_0(t) := \begin{cases} r_0 t, & t \geq 0 \\ 0, & t < 0 \end{cases} \tag{5}$$

where $y_0(t) \in \mathcal{R}^r$ is the output of the leader and $a_0, r_0 \in \mathcal{R}$. The leaders introduced in (4) and (5) refer to stationary and dynamic leaders with constant velocity, respectively.

The control objective is to design a network based control input $u_i(t)$, $i = 1,\ldots, N$, such that $\lim_{t\to\infty} \|y_i(t) - y_0(t)\| = 0$ for $i = 1,\ldots, N$. The topology of the network considered in this paper is given by Fig. 1.

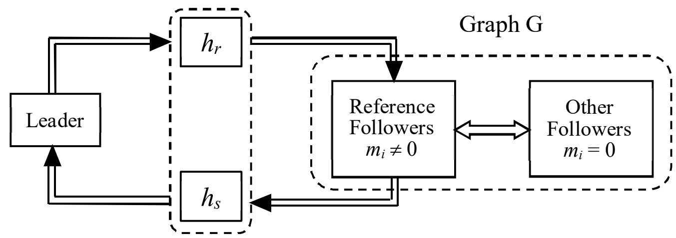

Fig. 1. Network topology of the directed multi-agent system.

It is assumed that N' number of the agents ($0 < N' \leq N$) in the network topology shown in Fig. 1 have access to the leader, and vice versa; these are called "reference followers". Therefore, by reachability characteristics of the followers for the leader and applying appropriate feedback control to each agent, the rest of the followers will reach consensus with the leader output.

Let h_r be the delay of transmitting data from the leader to the reference followers and h_s be the delay of data transmission from the reference followers back to the leader. It is supposed that the transmission delays between the follower agents are negligible. Moreover, it is supposed that all the controller computations are carried out in the leader and sent to the followers through the network. Consequently, both transmission delays h_s and h_r affect the control input $u_i(t)$. By the above structure, in addition to the reachability of the followers for the leader, the leader node should be globally reachable in the graph $\tilde{G}$. This means that the digraph $\tilde{G}$ is strongly connected. Throughout the paper, the following assumptions hold for the network topology shown in Fig. 1.

Assumption 2: The pair (A, B) is stabilizable and the pair (A, C) is detectable.

Assumption 3: The digraph $\tilde{G}$ is strongly connected.

C. Consensus Protocol

To deal with the problem of leader-following output consensus of general linear multi-agent systems in the presence of uncertainty and network transmission delay, we propose a new proportional-integral-derivative (PID) consensus protocol as follows:

$$\begin{aligned} u_i(t) &= K\sum_{j=1}^{N} a_{ij}\left(y_i(t-h)-y_j(t-h)\right) + K_I\sum_{j=1}^{N} a_{ij}\int_0^{t-h}\left(y_i(\alpha)\right.\\ &\left.-y_j(\alpha)\right)d\alpha + K_D\sum_{j=1}^{N} a_{ij}\left(\dot{y}_i(t-h)-\dot{y}_j(t-h)\right)\\ &+m_iK_i'\left(y_i(t-h)-y_0(t-h_r)\right)+m_iK_{Ii}'\left(\int_0^{t-h}y_i(\alpha)\,d\alpha\right.\\ &\left.-\int_0^{t-h_r}y_0(\alpha)\,d\alpha\right)+m_iK_{Di}'\left(\dot{y}_i(t-h)-\dot{y}_0(t-h_r)\right) \end{aligned} \tag{6}$$

in which $h = h_r + h_s$, $0 \le h_r < \overline{h}_r$, $0 \le h_s < \overline{h}_s$ and $0 \le h < \overline{h}$ where $\overline{h} = \overline{h}_r + \overline{h}_s$. Obviously, for the reference followers that are a neighbor of the leader, the condition $m_i \neq 0$ holds and for the rest of the followers, $m_i = 0$. Thus, the closed-loop system equation of the multi-agent system is given as

$$\begin{aligned} \dot{x}_i(t) &= A_\Delta x_i(t) + B_\Delta KC\sum_{j=1}^{N} a_{ij}\left(x_i(t-h)-x_j(t-h)\right)\\ &+B_\Delta K_I C\sum_{j=1}^{N} a_{ij}\int_0^{t-h}\left(x_i(\alpha)-x_j(\alpha)\right)d\alpha\\ &+B_\Delta K_D C\sum_{j=1}^{N} a_{ij}\left(\dot{x}_i(t-h)-\dot{x}_j(t-h)\right)\\ &+m_iB_\Delta K_i'\left(Cx_i(t-h)-y_0(t-h_r)\right)\\ &+m_iB_\Delta K_{Ii}'\left(C\int_0^{t-h}x_i(\alpha)\,d\alpha-\int_0^{t-h_r}y_0(\alpha)\,d\alpha\right)\\ &+m_iB_\Delta K_{Di}'\left(C\dot{x}_i(t-h)-\dot{y}_0(t-h_r)\right) \end{aligned} \tag{7}$$

with $A_\Delta \triangleq A + \Delta A$ and $B_\Delta \triangleq B + \Delta B$.

Remark 1. As we know, it is well known that a PID controller designed for a stabilization problem can be further used for tracking problem of the same closed-loop system. Considering these facts, it suffices to find a stability condition for the closed-loop system (7) independent of the leader $y_0(t)$ to achieve leader-following output consensus of the uncertain multi-agent system for the leaders (4) and (5) in the presence of network transmission delay.

Remark 2. It is worthwhile mentioning that the design method for consensus protocol (4) that will be discussed in the next section can be further applied for designing a proportional-integral-double integral (PII^2) consensus protocol. Therefore, the leader-following output consensus for multi-agent system (1) in the presence of the dynamic leader in (5) is guaranteed. To achieve this goal, it suffices to augment the agents' model with an integrator. Consequently, the design of the PID consensus protocol for the augmented agents' model is equivalent to designing a PII^2 controller for the original multi-agent system. The details have been given in [19].

As seen in (6), the closed-loop system has a neutral type time-delay equation [18] due to the terms $B_\Delta K_D C\sum_{j=1}^{N} a_{ij}\left(\dot{x}_i(t-h)-\dot{x}_j(t-h)\right)$ and $m_iB_\Delta K_{Di}'C\dot{x}_i(t-h)$. To deal with this closed-loop system of neutral type, we define

$$\xi_i(t) = \int_0^t x_i(\alpha)\,d\alpha,\ \dot{x}_i(t) = \theta_i(t) \tag{8}$$

Considering Remark 1 and using a descriptor transformation, the descriptor form of the closed-loop system equation of the multi-agent system is represented as

$$\begin{aligned} \dot{\xi}(t) &= x(t)\\ \dot{x}(t) &= \theta(t)\\ 0 &= -\theta(t) + (I\otimes A_\Delta)x(t) + \left(L_s\otimes(B_\Delta KC)\right)x(t-h)\\ &+\left(L_s\otimes(B_\Delta K_I C)\right)\xi(t-h) + \left(L_s\otimes(B_\Delta K_D C)\right)\theta(t-h)\\ &+\left(\overline{M}\otimes B_\Delta\right)K'(I\otimes C)x(t-h) + \left(\overline{M}\otimes B_\Delta\right)K_I'(I\otimes C).\\ &\xi(t-h) + \left(\overline{M}\otimes B_\Delta\right)K_D'(I\otimes C)\theta(t-h) = \Theta(t) \end{aligned} \tag{9}$$

or

$$E\dot{\overline{x}}(t) = \overline{A}\overline{x}(t) + \overline{A}_h\overline{x}(t-h) \tag{10}$$

where $\otimes$ is the Kronecker product [20] and L_s is the Laplacian matrix. Moreover,

$$E = \begin{bmatrix} I & 0 & 0\\ 0 & I & 0\\ 0 & 0 & 0 \end{bmatrix},\ \overline{A} = \begin{bmatrix} 0 & I & 0\\ 0 & 0 & I\\ \overline{A}_{31} & \overline{A}_{32} & \overline{A}_{33} \end{bmatrix},\ \overline{A}_h = \begin{bmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ \overline{A}_{h31} & \overline{A}_{h32} & \overline{A}_{h33} \end{bmatrix}$$

$\overline{A}_{31} = 0,\ \overline{A}_{32} = I\otimes A_\Delta,\ \overline{A}_{33} = -I,\ \overline{A}_{h31} = L_s\otimes B_\Delta K_I C +$
$\left(\overline{M}\otimes B_\Delta\right)K_I'(I\otimes C),\ \overline{A}_{h32} = L_s\otimes B_\Delta KC + \left(\overline{M}\otimes B_\Delta\right)K'(I\otimes C)$
$\overline{A}_{h33} = L_s\otimes(B+\Delta B)K_D C + \left(\overline{M}\otimes(B+\Delta B)\right)K_D'(I\otimes C)$

$$\overline{x}(t) = \begin{bmatrix} \xi(t)\\ x(t)\\ \theta(t) \end{bmatrix},\ \tilde{x}(t) = \begin{bmatrix} \xi(t)\\ x(t) \end{bmatrix},\ \xi(t) = \begin{bmatrix} \xi_1\\ \vdots\\ \xi_N \end{bmatrix},\ x(t) = \begin{bmatrix} x_1\\ \vdots\\ x_N \end{bmatrix},\ \theta(t) = \begin{bmatrix} \theta_1\\ \vdots\\ \theta_N \end{bmatrix}$$

$K' = diag(K_1', \ldots, K_N'),\ K_I' = diag(K_{I1}', \ldots, K_{IN})$
$K_I' = diag(K_{D1}', \ldots, K_{DN}),\ \overline{M} = diag(m_1\otimes I_n, \ldots, m_N\otimes I_n).$

Now, we present the following lemmas, which will be used for obtaining the main results of the paper.

Lemma 1 [21]: Let H, L and $F(t)$ be real matrices of appropriate dimensions with $F(t)$ being a matrix function. Then, for any $\sigma > 0$ and $F^T(t)F(t) \le I$, we have

$$LF(t)H + H^T F^T(t)L^T \le \frac{1}{\sigma^2}LL^T + \sigma^2 H^T H \tag{11}$$

Lemma 2 [22]: If $W > 0$, there exist W^{-1}. Thus,

$$-SW^{-1}S \le -\left(S^T + S - W\right) \tag{12}$$

III. Robust Controller Design

In this section, we give the design conditions for robust leader-following output consensus of the multi-agent system

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(10). A Lyapunov-Krasovskii functional for system (10) has the form

$$V = V_1 + V_2 + V_3 \tag{13}$$

where

$$V_1 = \bar{x}^T(t) E\bar{P}\,\bar{x}(t), V_2 = \int_{-h}^{0}\int_{t+\beta}^{t} \dot{\bar{x}}^T(\alpha) Z\dot{\bar{x}}(\alpha)\,d\alpha\,d\beta$$
$$V_3 = \int_{t-h}^{t} \bar{x}^T(\alpha) Q\bar{x}(\alpha)\,d\alpha + \int_{t-h}^{t} \dot{\theta}^T(\alpha) R\dot{\theta}(\alpha)\,d\alpha \tag{14}$$

in which $\bar{P} = \begin{bmatrix} \bar{P}_1 & 0 \\ \bar{P}_2 & \bar{P}_3 \end{bmatrix}$ and $\bar{P}_1 \in \mathcal{R}^{2nN\times 2nN}$, $\bar{P}_3 \in \mathcal{R}^{nN\times nN}$, $Q \in \mathcal{R}^{3nN\times 3nN}$, $R \in \mathcal{R}^{nN\times nN}$, $Z \in \mathcal{R}^{3nN\times 3nN}$ are symmetric positive definite matrices. Therefore, by the definition of E and $\tilde{x}(t)$ in (10) and knowing that $\bar{P}_1 = \bar{P}_1^T > 0$, we have $\bar{x}^T E\bar{P}\,\bar{x} = \tilde{x}^T \bar{P}_1 \tilde{x} = \tilde{x}^T \bar{P}_1^T \tilde{x} = \bar{x}^T (E\bar{P})^T \bar{x}$. Consequently, the functionals V_1, V_2, V_3 are admissible.

Remark 3: To enable the application of Lyapunov method for the stability of the system (8), the difference operator $\mathcal{D}$: $\mathcal{C}\,[-\bar{h}, 0] \rightarrow \mathcal{R}^n$, given by $\mathcal{D}(x_t) = x(t) - \big((L_s \otimes B_\Delta K_D C) + (\bar{M} \otimes B_\Delta) K'_D .(I\otimes C)\big) x(t-h)$ should be stable independent of delay with respect to all delays [18]. Therefore a sufficient condition for the stability of $\mathcal{D}$ is given as

$$\Big|\big(L_s \otimes ((B+\Delta B)K_D C)\big) + \big(\bar{M} \otimes (B+\Delta B)\big) K'_D (I\otimes C)\Big| < I \tag{15}$$

where $|\,.\,|$ is any matrix norm. Now, considering Remark 2, we state the following theorem which gives a sufficient condition for robust stability design of leader-following output consensus for the uncertain multi-agent system (10).

Theorem 1: Consider the multi-agent system (10) with communication delay $0 \le h < \bar{h}$. Suppose Assumptions 1-3 hold. Then, leader-following output consensus is asymptotically achieved if there exist scalars $\sigma_i, \alpha_i, \upsilon > 0, \lambda, \varepsilon_i, i = 1,2,3$, positive definite symmetric matrices $\bar{L}$, $M \in \mathcal{R}^{3nN\times 3nN}$, $R \in \mathcal{R}^{nN\times nN}$, and matrices $K, K_D, K_I, K'_i, K'_{Di}, K'_{Ii} \in \mathcal{R}^{m\times r}$, $i = 1,\dots,N$ satisfying the following LMIs:

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} \\ \Xi_{12}^T & \Xi_{22} \end{bmatrix} < 0 \tag{16}$$

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}^T & \Sigma_{22} \end{bmatrix} < 0 \tag{17}$$

$$\begin{bmatrix} M & -\lambda I \\ -\lambda I & Z \end{bmatrix} > 0 \tag{18}$$

in which

$$\Sigma_{11} = \begin{bmatrix} I & A_{h33} \\ * & I \end{bmatrix},\ \Sigma_{22} = -diag\big((1/\upsilon^2)I,\ \ \upsilon^2 I\big),$$

$$\Sigma_{12} = \begin{bmatrix} 0 & 0 & I\otimes E_b K_D C & \bar{M}\otimes D_b \\ (L_s \otimes D_b)^T & \big((I\otimes E_b)K'_D(I\otimes C)\big)^T & 0 & 0 \end{bmatrix}$$

$$\Xi_{11} = \begin{bmatrix} \phi_{2n} & \lambda I + \bar{A}_{hn} & 0 & \bar{L}\Omega_{1n}^T & \bar{L}\Omega_{1n}^T & \bar{L}\begin{bmatrix}0\\I\\0\end{bmatrix} & \bar{L}\begin{bmatrix}0\\0\\I\end{bmatrix} & \bar{L} \\ * & -Q & 0 & \Omega_{2n}^T & \Omega_{2n}^T & 0 & 0 & 0 \\ * & * & -R & \Omega_{3n}^T & \Omega_{3n}^T & 0 & 0 & 0 \\ * & * & * & -2I+R & 0 & 0 & 0 & 0 \\ * & * & * & * & \Theta_3 & 0 & 0 & 0 \\ * & * & * & * & * & \Theta_1 & 0 & 0 \\ * & * & * & * & * & * & \Theta_2 & 0 \\ * & * & * & * & * & * & * & -2I+Q \end{bmatrix}$$

$$\Xi_{12} = \begin{bmatrix} \Gamma_1 & \Lambda_1 & \Gamma_2 & \Lambda_2 & \Gamma_3 & \Lambda_3 \end{bmatrix}$$

$$\Xi_{22} = -diag\big((1/\sigma_1^2)I,\ \sigma_1^2 I,\ (1/\sigma_2^2)I,\ \sigma_2^2 I,\ (1/\sigma_3^2)I,\ \sigma_3^2 I\big)$$

$$\phi_{2n} = \bar{L}\begin{bmatrix} 0 & I & 0 \\ 0 & 0 & I \\ 0 & I\otimes A & -I \end{bmatrix}^T + \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & I \\ 0 & I\otimes A & -I \end{bmatrix}\bar{L}^T + \bar{h}M - 2\lambda I$$

$$\Theta_1 = -2I + \bar{h}Z_1,\ \Theta_2 = -2I + \bar{h}Z_2,\ \Theta_3 = -2I + \bar{h}Z_3,$$

$$\Gamma_1^T = \varphi(I\otimes D_a) = [\varphi_{ij}],\ for\ i = 1,\dots,8\ and\ j = 1,\dots,3.$$

$$\varphi_{11} = \begin{bmatrix} 0 & 0 & (I\otimes D_a)^T \end{bmatrix}, \varphi_{24} = \varphi_{35} = (I\otimes D_a)^T, otherwise\ \varphi_{ij} = 0.$$

$$\Gamma_2^T = \varphi(L_s \otimes D_b),\ \Gamma_3^T = \varphi(M \otimes D_b)$$

$$\Lambda_1^T = \theta(I\otimes E_a, \bar{L}) = \begin{bmatrix} [0 & I\otimes E_a & 0]\bar{L} & 0 & & 0 \\ [0 & 0 & I\otimes E_a]\bar{L} & 0 & \dots & 0 \\ [0 & 0 & I\otimes E_a]\bar{L} & \underbrace{0 \quad\quad 0}_{7} \end{bmatrix}$$

$$\Lambda_2^T = \delta(j,k,l) = \begin{bmatrix} 0 & [j & k & l] & 0 & 0 & & 0 \\ 0 & [0 & j & k] & l & 0 & \dots & 0 \\ 0 & [0 & j & k] & l & \underbrace{0 \quad\quad 0}_{5} \end{bmatrix}$$

$$\Lambda_3^T = \mu(m,n,p) = \begin{bmatrix} 0 & [0 & m & n] & 0 & 0 & & 0 \\ 0 & [0 & m & n] & p & 0 & \dots & 0 \\ 0 & [0 & m & n] & p & \underbrace{0 \quad\quad 0}_{5} \end{bmatrix}$$

$$m = (I\otimes E_b)K'_I(I\otimes C),\ n = (I\otimes E_b)K'(I\otimes C),\ j = I\otimes E_b K_I C,$$
$$k = I\otimes E_b KC,\ l = I\otimes E_b K_D C,\ p = (I\otimes E_b)K'_D(I\otimes C)$$

$$\bar{A}_{hn} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ A_{h31} & A_{h32} & A_{h33} \end{bmatrix}, A_{h31} = L_s \otimes BK_I C + (\bar{M}\otimes B)K'_I(I\otimes C)$$

$$A_{h32} = L_s \otimes BKC + (\bar{M}\otimes B)K'(I\otimes C),\ A_{h33} = L_s \otimes BK_D C + (\bar{M}\otimes B)K'_D(I\otimes C), \Omega_{1n} = \begin{bmatrix} 0 & 0 & (I\otimes A) \end{bmatrix},\ \Omega_{2n} = \begin{bmatrix} 0 & A_{h31} & A_{h32} \end{bmatrix},$$

$$\Omega_{3n} = A_{h33},\ \bar{L} = Hdiag(\hat{L}, \hat{L}, \hat{L}),\ \hat{L} = diag(\underbrace{L, L, \dots, L}_{N}),$$

$$H = [h_{ij}],\ i,j = 1,2,3,\ h_{ii} = \alpha_i I_N,\ h_{12} = -\varepsilon_1 I,\ h_{13} = -\varepsilon_2 I,\ h_{23} = -\varepsilon_3 I,$$
$$otherwise,\ h_{ij} = 0,\ K' = diag(K'_1,\dots,K'_N),\ K'_D = diag(K'_{D1},\dots,K_{DN}),$$
$$K'_I = diag(K'_{I1},\dots,K_{IN}).$$

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Proof: Note that

$$V_1 = \bar{x}^T(t) E\bar{P}\bar{x}(t) = \begin{bmatrix} \xi^T(t) & x^T(t) \end{bmatrix} \bar{P}_1 \begin{bmatrix} \xi(t) \\ x(t) \end{bmatrix} \tag{19}$$

Differentiating V_1 with respect to t results into

$$\dot{V}_1 = 2\begin{bmatrix} \xi^T(t) & x^T(t) \end{bmatrix} \bar{P}_1 \begin{bmatrix} \dot{\xi}^T(t) & \dot{x}^T(t) \end{bmatrix}^T$$
$$= 2\begin{bmatrix} \xi^T(t) & x^T(t) & \theta^T(t) \end{bmatrix} \bar{P}^T \begin{bmatrix} \dot{\xi}(t) \\ \dot{x}(t) \\ 0 \end{bmatrix} = 2\bar{x}(t)\bar{P}^T \begin{bmatrix} x(t) \\ \theta(t) \\ \Theta(t) \end{bmatrix} \tag{20}$$

where $\Theta(t)$ is substituted from (9). Obtaining the time-derivative of V_2 and V_3, using definite integral formula known as Leibniz-Newton formula as well as the integral inequality in [23], an upper bound for $\dot{V}$ is found. Then, using Schur complement, partitioning the nominal and uncertain parts and considering the definition in (2) as well as applying suitable congruence transformations, the matrix inequalities (16)-(18) are obtained. For the sake of brevity, the detailed proof is omitted. ■

Remark 4: Theorem 1 obtains robust stabilization conditions for the uncertain multi-agent system (10). The set of controller gains K, K_D, K_I, K'_i, K'_{Di}, K'_{Ii} that make the LMI conditions (16)-(18) feasible guarantee the robust leader-following output consensus of the uncertain time-delay multi-agent system (10). The significant advantage of this theorem is in giving a novel set of stabilization LMI conditions for the system (10) that gives the controller gains of the proposed consensus protocol (6).

A. Special Case 1: First-Order Systems

For a first-order system ($n = 1$), it is possible to derive a less conservative set of conditions compared to the conditions (16)-(18). Consequently, the system matrices A, ΔA, B, ΔB and C are represented by the scalars a, Δa, b, Δb and c, respectively. Moreover, $a_\Delta \triangleq a + \Delta a$, and $b_\Delta \triangleq b + \Delta b$. These conditions are given in the following corollary.

Corollary 1: For the multi-agent system (10) with first-order agents and delay $0 \le h < \bar{h}$, under Assumptions 1-3, the leader-following output consensus is asymptotically achieved if there exist scalars $\upsilon, \sigma_i, \alpha_i > 0, \varepsilon_i, i = 1,2,3$, positive definite symmetric matrices $\bar{L}$, $M \in \mathcal{R}^{3N\times 3N}$, $R \in \mathcal{R}^{N\times N}$, and variables V, V_D, V_I, V'_i, V'_{Di}, $V'_{Ii} \in \mathcal{R}$ satisfying the following LMIs:

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} \\ \Xi_{12}^T & \Xi_{22} \end{bmatrix} < 0 \tag{21}$$

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}^T & \Sigma_{22} \end{bmatrix} < 0 \tag{22}$$

$$\begin{bmatrix} M & S \\ S^T & 2\bar{L} - U \end{bmatrix} > 0 \tag{23}$$

in which

$$\Xi_{11} = \begin{bmatrix} \phi_{2n} & -S + \bar{A}_{hn} & 0 & \Omega_{1n}^T & \bar{h}\Omega_{1n}^T & \bar{h}\bar{L}\begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix} & \bar{h}\bar{L}\begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix} \\ * & -T & 0 & \Omega_{2n}^T & \bar{h}\Omega_{2n}^T & 0 & 0 \\ * & * & -2\hat{L} + W & \Omega_{3n}^T & \bar{h}\Omega_{3n}^T & 0 & 0 \\ * & * & * & -W & 0 & 0 & 0 \\ * & * & * & * & -\bar{h}U_3 & 0 & 0 \\ * & * & * & * & * & -\bar{h}U_1 & 0 \\ * & * & * & * & * & * & -\bar{h}U_2 \end{bmatrix}$$

$$\Xi_{12} = \begin{bmatrix} \Gamma_1 & \Lambda_1 & \Gamma_2 & \Lambda_2 & \Gamma_3 & \Lambda_3 \end{bmatrix}, \Gamma_1^T = \varphi(I \otimes D_a),$$
$$\Gamma_2^T = \varphi(L_s \otimes D_b), \Gamma_3^T = \varphi(M \otimes D_b), \Lambda_1^T = \theta(I \otimes E_a, \bar{L})$$
$$\Lambda_2^T = \delta\left(I \otimes (E_b c V_I), I \otimes (E_b c V), I \otimes (E_b c V_D)\right) diag(0,\ H,\ \underbrace{0,..,0}_{5})$$
$$\Lambda_3^T = \mu\left((I \otimes E_b c)V', (I \otimes E_b c)V', (I \otimes E_b c)V'_D\right) diag(0,\ H,\ \underbrace{0,..,0}_{5})$$
$$\Xi_{22} = -diag\left((1/\sigma_1^2)I,\ \sigma_1^2 I,\ (1/\sigma_2^2)I,\ \sigma_2^2 I,\ (1/\sigma_3^2)I,\ \sigma_3^2 I\right)$$
$$\phi_{2n} = \bar{L}\begin{bmatrix} 0 & I & 0 \\ 0 & 0 & I \\ 0 & I\otimes A & -I \end{bmatrix}^T + \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & I \\ 0 & I\otimes A & -I \end{bmatrix}\bar{L}^T + \bar{h}M + S + S^T + T$$
$$\bar{A}_{hn} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \bar{A}_{h31} & \bar{A}_{h32} & \bar{A}_{h33} \end{bmatrix} H,\ \bar{A}_{h31} = L_s \otimes (bcV_I) + (\bar{M} \otimes bc)V'_I$$
$$\bar{A}_{h32} = L_s \otimes (bcV) + (\bar{M} \otimes bc)V',\ \bar{A}_{h33} = L_s \otimes (bcV_D) + (\bar{M} \otimes bc)V'_D$$
$$\Omega_{1n} = \begin{bmatrix} 0 & 0 & (I \otimes A) \end{bmatrix}\bar{L},\ \Omega_{3n} = A_{h33},\ \Omega_{2n} = \begin{bmatrix} 0 & A_{h31} & A_{h32} \end{bmatrix} H$$
$$\Sigma_{11} = \begin{bmatrix} 2\hat{L} - I & A_{h33} \\ A_{h33}^T & 2\hat{L} - I \end{bmatrix},\ \Sigma_{22} = -diag\left((1/\upsilon^2)I,\ \upsilon^2 I\right),$$
$$\Sigma_{12} = \begin{bmatrix} 0 & 0 & I \otimes (E_b c V_D) & \bar{M} \otimes D_b \\ (L_s \otimes D_b)^T & ((I \otimes E_b c)V'_D)^T & 0 & 0 \end{bmatrix},$$
$$\bar{L} = Hdiag(\hat{L},\ \hat{L},\ \hat{L}),\ \hat{L} = diag(\underbrace{L,\ L,\ ...,\ L}_{N}),\ U = diag(U_1,\ U_2,\ U_3)$$
$$V' = diag(V'_1,\ldots,V'_N),\ V'_I = diag(V'_{I1},\ldots,V'_{IN}),\ V'_D = diag(V'_{D1},\ldots,V'_{DN}).$$

Moreover, H is as defined in Theorem 1. The controller gains are given as $K = VL^{-1}$, $K_I = V_I L^{-1}$, $K_D = V_D L^{-1}$, $K' = V'\hat{L}^{-1}$, $K'_I = V'_I \hat{L}^{-1}$ and $K'_D = V'_D \hat{L}^{-1}$.

Proof: The proof is omitted since it can be established using the proof of Theorem 1. ■

Remark 5: As mentioned earlier, the LMI conditions (21)-(23) are less conservative than the set of LMIs (16)-(18). In the proof of Theorem 1, there was a necessity to define the matrix variable Y as $Y = -\lambda\bar{L}^{-1}$. This definition enabled us to present a set of LMI conditions rather than bilinear matrix inequalities (BMIs) that are classified in the category of Non-deterministic Polynomial-time hard (NP-hard) problems. In Corollary 1, since the agents are first-order, the aforementioned assumption for the matrix variable Y is not necessary. Consequently, a set

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of less conservative LMI conditions in (21)-(23) is obtained compared to the LMIs presented in (16)-(18).

B. Special case 2: PD Controller

In case the design of a PD controller is considered, one can set $K_I = K_{Ii} = 0$ and derive a set of simpler stability analysis and design conditions than the general form. This special case will lead to a consensus protocol which is a sub-case of the consensus protocol (6). Then, the following corollary is obtained.

Corollary 2: Under Assumptions 1-3, for a given delay $0 \le h < \bar{h}$, the leader-following output consensus for the multi-agent system (1) with PD type of the consensus protocol (6) ($K_I = K_{Ii} = 0$) is asymptotically achieved if there exist scalars $\sigma_i, \alpha_i, \upsilon > 0, \lambda, \varepsilon_i, i = 1,2,3$, positive definite symmetric matrices $\bar{L}$, $M \in \mathcal{R}^{3nN \times 3nN}$, $R \in \mathcal{R}^{nN \times nN}$ and matrices K, K_D, K_i', $K_{Di}' \in \mathcal{R}^{m \times r}$, $i = 1,\dots,N$ satisfying the following LMIs:

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} \\ \Xi_{12}^T & \Xi_{22} \end{bmatrix} < 0 \tag{24}$$

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}^T & \Sigma_{22} \end{bmatrix} < 0 \tag{25}$$

$$\begin{bmatrix} M & -\lambda I \\ -\lambda I & Z \end{bmatrix} > 0 \tag{26}$$

in which

$$\Gamma_1^T = \varphi(I \otimes D_a)\, diag\big(\bar{L} \;\; \underbrace{0,\dots,0}_{7}\big),\; \Gamma_2^T = \varphi(L_s \otimes D_b)\, diag\big(\bar{L} \;\; \underbrace{0,\dots,0}_{7}\big)$$

$$\Gamma_3^T = \varphi(M \otimes D_b)\, diag\big(\bar{L} \;\; \underbrace{0,\dots,0}_{7}\big)$$

$$\Lambda_1^T = \begin{bmatrix} [I \otimes E_a \;\; 0] & 0 & & 0 \\ [I \otimes E_a \;\; 0] & 0 & \dots & 0 \\ [I \otimes E_a \;\; 0] & \underbrace{0} & & \underbrace{0}_{6} \end{bmatrix},\; \Lambda_2^T = \begin{bmatrix} 0 & [k \;\; l] & 0 & 0 & & 0 \\ 0 & [0 \;\; k] & l & 0 & \dots & 0 \\ 0 & [0 \;\; k] & l & \underbrace{0} & & \underbrace{0}_{4} \end{bmatrix}$$

$$\Lambda_3^T = \begin{bmatrix} 0 & [0 \;\; n] & 0 & 0 & & 0 \\ 0 & [0 \;\; n] & p & 0 & \dots & 0 \\ 0 & [0 \;\; n] & p & \underbrace{0} & & \underbrace{0}_{4} \end{bmatrix},\; \bar{A}_{hn} = \begin{bmatrix} 0 & 0 \\ A_{h31} & A_{h32} \end{bmatrix}$$

$$\Xi_{11} =$$

$$\begin{bmatrix} \phi_{2n} & \lambda I + \bar{A}_{hn} & 0 & \bar{L}\Omega_{1n}^T & \bar{L}\Omega_{1n}^T & [0 \;\; I]^T & \bar{L} \\ * & -Q & 0 & \Omega_{2n}^T & \Omega_{2n}^T & 0 & 0 \\ * & * & -R & \Omega_{3n}^T & \Omega_{3n}^T & 0 & 0 \\ * & * & * & -2I + R & 0 & 0 & 0 \\ * & * & * & * & -2I + \bar{h}Z_2 & 0 & 0 \\ * & * & * & * & * & -2I + \bar{h}Z_1 & 0 \\ * & * & * & * & * & * & -2I + Q \end{bmatrix}$$

$$\Xi_{12} = [\Gamma_1 \quad \Lambda_1 \quad \Gamma_2 \quad \Lambda_2 \quad \Gamma_3 \quad \Lambda_3]$$

$$\Xi_{22} = -diag\big((1/\sigma_1^2)I,\; \sigma_1^2 I,\; (1/\sigma_2^2)I,\; \sigma_2^2 I,\; (1/\sigma_3^2)I,\; \sigma_3^2 I\big)$$

$$\phi_{2n} = \bar{L}\begin{bmatrix} 0 & I \\ I \otimes A & -I \end{bmatrix}^T + \begin{bmatrix} 0 & I \\ I \otimes A & -I \end{bmatrix}\bar{L}^T + \bar{h}M - 2\lambda I$$

$$A_{h31} = L_s \otimes BKC + (\bar{M} \otimes B)K'(I \otimes C),\; \Omega_{1n} = [0 \;\; (I \otimes A)],$$

$$A_{h32} = L_s \otimes BK_D C + (\bar{M} \otimes B)K_D'(I \otimes C),\; \Omega_{2n} = [0 \;\; A_{h31}],$$

$$\Omega_{3n} = A_{h32},\; \Sigma_{22} = -diag\big((1/\upsilon^2)I,\; \upsilon^2 I\big),$$

$$\Sigma_{11} = \begin{bmatrix} I & A_{h32} \\ A_{h32}^T & I \end{bmatrix},\; \Sigma_{12} = \begin{bmatrix} 0 & 0 & k & \bar{M} \otimes D_b \\ (L_s \otimes D_b)^T & p^T & 0 & 0 \end{bmatrix},$$

$$n = (I \otimes E_b)K'(I \otimes C),\; p = (I \otimes E_b)K_D'(I \otimes C),\; k = I \otimes E_b KC,$$

$$l = I \otimes E_b K_D C,\; \bar{L} = H diag(\hat{L}, \hat{L}, \hat{L}),\; \hat{L} = diag(\underbrace{L, L, \dots, L}_{N})$$

$K' = diag(K_1', \dots, K_N')$, $K_D' = diag(K_{D1}', \dots, K_{DN})$. Moreover, H is as defined in Theorem 1

Proof: The proof is omitted here as it can be easily derived by following the proof of Theorem 1. ■

Remark 6: Multi-agent systems with integrator agents, i.e., $\dot{x}_i(t) = (b + \Delta b(t))u_i(t)$, are categorized in the special case 2. Using the idea of the special case 1, a set of less conservative LMI conditions compared to the LMI conditions (24)-(26) can be provided that are omitted here for the sake of space limitation.

Remark 7: To the best knowledge of the authors, all the stability conditions presented in the literature for the leader-following multi-agent systems, which are more complicated than the conventional consensus problems, are sufficient conditions similar to the ones presented in this paper. Therefore, Theorem 1 and Corollaries 1 & 2 are not necessarily more conservative than the other results presented in the literature for leader-following multi-agent systems.

IV. Simulation Results

A. Example 1

Consider four uncertain time-delayed unstable agents that communicate through a directed topology with the following transfer function as

$$G(s) = \frac{0.4 + \Delta b}{s - (0.02 + \Delta a)} e^{-0.1s} \tag{27}$$

in which Δa and Δb are the system parametric uncertainties. In a canonical representation form of the system (27), Δa and Δb are the same uncertainties considered in Corollary 1 where a = 0.02, b = 0.4, c = 1 and h = 0.1s. Considering the definitions (2) and (3), we assume that D_a = 0.05, E_a = 0.1, D_b = 0.1, E_b = 1. Moreover, the communication topology and the leader adjacency matrix are represented by the Laplacian matrix as $L_S = [l_{ij}]$ for i,j = 1,…,4 where l_{ii}=1 for i =1,…,4, $l_{1,2} = l_{2,3} = l_{3,4} = l_{4,1} = -1$, otherwise $l_{i,j} = 0$ and the matrix M = diag{0, 1, 1, 0} respectively. We set $\varepsilon_{1,2,3}$=0.2, α_1=1.5, α_2 =0.75, α_3 =1.5, $\sigma_1 = \sigma_2 = \sigma_3$ =1. Using Corollary 1 and the LMI Toolbox in Matlab, the designed PID controller gains are obtained as K = −5.15, K_I = −0.3, K_D = −0.23, K' = diag (0, −10.2, −10.13, 0), K_I' = diag (0,

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–2.3, –2.44, 0), K'_D = diag (0, –0.18, –0.17, 0). Fig. 2 displays the simulation results of the closed-loop multi-agent system. As shown in Fig. 1, all the uncertain unstable agents has been stabilized and followed the leader set-point using the consensus protocol (6).

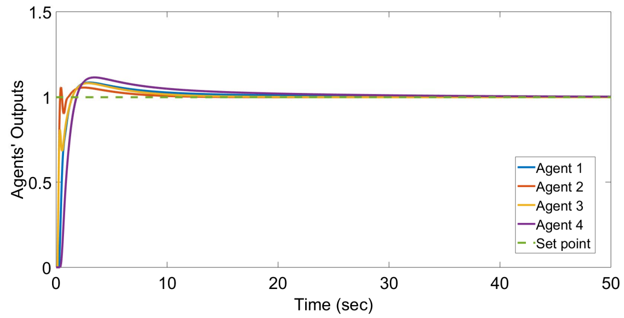

Fig. 2. Controlled outputs of the follower agents (solid) and output of the leader (dashed)

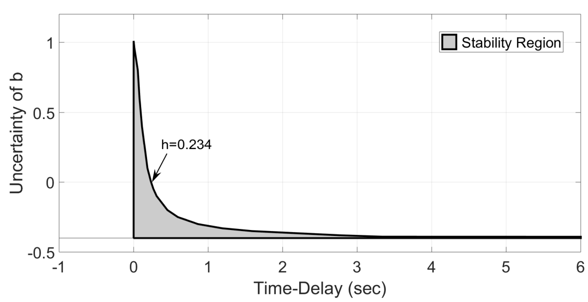

Fig 3. Stability region in presence of Δb and h variations when Δa = 0

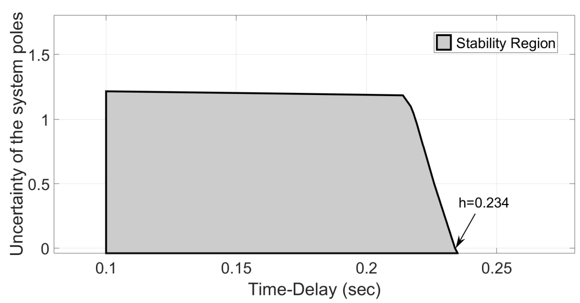

Fig 4. Stability region in presence of Δa and h variations when Δb = 0

Now, we perform some simulations to investigate the robust stability of the closed-loop system in the presence of the system parameter uncertainties. Since the uncertainty in gain and the poles of a system affects the stability of the closed-loop system, we investigate the performance of the closed-loop system in presence of these uncertainties. To this aim, we firstly set Δa=0. Then, simulating the closed-loop system in the presence of the variations of Δb and h, the stability region for the closed-loop multi-agent system is obtained. This is shown as the colored area in Fig. 3. It is seen that for h=0, the closed-loop multi-agent system is stable for $-0.4 \le \Delta b \le 1.01$. To obtain the stability region of the closed-loop multi-agent system in the presence of the uncertainty in Δa and h, we firstly set Δb = 0. Then, we obtain the stability region for $-2a \le \Delta a \le \Delta a|_{h=0.1}$ in which $a = \lambda$ = 0.02. The colored area in Fig. 4 shows the stability region of the closed-loop multi-agent system in the presence of the uncertainty in the pole of the system (27) and the delay h. As seen in Fig. 4, increasing the maximum time-delay h, the upper limit of the uncertainty of the system pole decreases. Moreover, for Δa = Δb = 0, the maximum time-delay tolerated by the closed-loop system is obtained as h = 0.234s.

B. Example 2: Teleoperation system

In this example, a teleoperation system is considered that is composed of a local central control system and three remote manipulators. It is assumed that the remote manipulators are in free space. Suppose that the central control information is accessible for the manipulator 1. The communication topology and the leader adjacency matrix are represented by the Laplacian matrix L_S and M, respectively. $L_S = [l_{ij}]$ for $i,j = 1,\ldots,3$ in which l_{ij}=2 for $i=j$ and $l_{ij} = -1$ for $i \neq j$. Moreover, the matrix $M = \text{diag}\{1, 0, 0\}$. The control objective is to design a feedback control system to provide a leader following formation control for the introduced multi-agent system with 3 agents. As we know, a 3-DOF manipulator has nonlinear dynamics that is shown by the Euler-Lagrange equations of motion in joint space [24]. Using feedback linearization, we may write the manipulator equation in a decoupled canonical form that is always controllable from u_i [24] as

$$\dot{\delta}_i = \begin{bmatrix} 0 & I_p \\ 0 & 0 \end{bmatrix} \delta_i + \begin{bmatrix} 0 \\ I_p \end{bmatrix} u_i(t-h), \qquad p = 3 \tag{28}$$

where $\delta_i = col\,[\,\xi_i,\ \dot{\xi}_i\,]$ and $\xi_i(t) = q_{di}(t) - q_i(t)$ is the joint tracking error. Therefore, tending $\xi_i(t)$ to zero leads the joint variables $q_i(t)$ to track their desired value $q_{di}(t)$. Fig. 5 shows the block diagram of the closed-loop system for each feedback linearized agent i. Now, we are in a position to apply Corollary 2 and present a leader following output consensus for the teleoperation system with three robot manipulators. To this end, we consider the joint tracking errors as the output of each manipulator as y_i = $[I_p\ 0]\delta_i$. Setting $\bar{h} = 200ms$ and using Corollary 2 as well as solving the LMI conditions in (24)-(26) by the LMI toolbox in Matlab, we obtain the following PD controller gains as K = 1.32, K_D = 1.29, K' = diag (1.02, 0, 0), K'_D = diag (0.98, 0, 0). Since the output of each feedback linearized agent is the vector of joint tracking errors, we need to solve a leader-following output consensus problem with the set-point of zero. This set-point is applied to the first feedback linearized manipulator while the other feedback linearized manipulators track this set-point via the communication topology shown by the adjacency matrix $\mathcal{A}$. Consequently, the joint variables $q_i(t)$ tend to the desired values $q_{di}(t)$. Setting the joint desired values as q_{di} = [π/2, π/3, -π/3] for i=1, 2, 3 and simulating the closed-loop multi-agent system by Simulink, the simulation results of the teleoperation system with the initial conditions q_1 = [2π/3, -π/6, -π/5], q_2 = [π/4, π/6, -π/3], q_3 = [π/3, π/5, -π/4] for joint variables are displayed in Figs. 6 and 7. As seen in Fig. 6, all the joint variables reach a consensus and track the desired values $q_{di}(t)$ for i=1, 2, 3. Moreover, Fig. 7 shows the joint torques τ_i of the joint variables that stay bounded. Additionally, the maximum tolerable round-trip transmission delay between the leader and the first reference follower (first follower) in the closed-loop multi-agent system is obtained as $\bar{h}_{\max} = 288ms$.

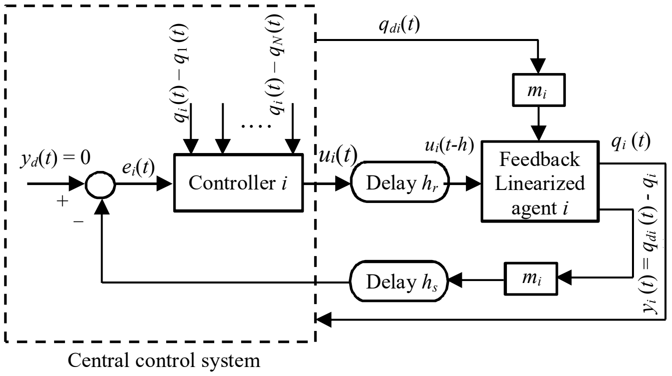

Fig. 5. Closed-loop system of the feedback linearized agent i

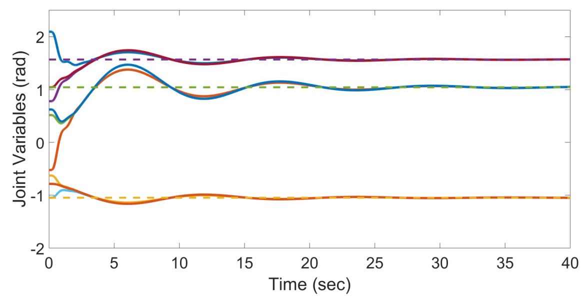

Fig. 6. Tracking of the joint desired values (dashed) by the joint variables (solid)

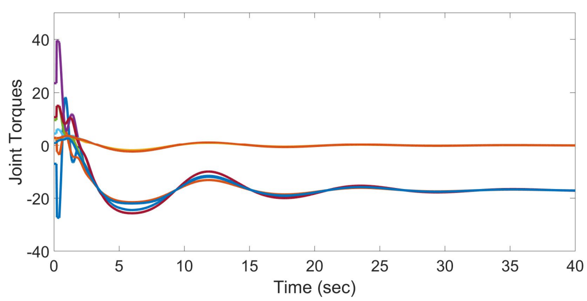

Fig. 7. Joint torques of the manipulators (τ_i)

V. Conclusion

The problem of leader-following output consensus of uncertain multi-agent systems with general linear agents and transmission delay has been studied in this paper in the presence of stationary and dynamic leaders. The proposed method can be used for both stable and unstable follower agents under a directed graph. To this end, we proposed a new Proportional-Derivative-Integral (PID) consensus protocol for the closed-loop system. A Lyapunov-Krasovskii functional is used to derive the design conditions in terms of certain linear matrix inequalities (LMIs). Augmenting each agent model with an integrator and designing the PID consensus protocol for the augmented model is equivalent to designing a proportional-integral-double integral (PII^2) controller for the original multi-agent system. Thus, the presented stability LMI conditions can be used for a dynamic leader as well. Finally, we showed the application of our method in a multi-lateral teleoperation system. The simulation results confirm the effectiveness of our method.

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