An Observer-Based Responsive Variable Impedance Control for Dual-User Haptic Training System

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Abstract— This paper proposes a variable impedance control architecture to facilitate eye surgery training in a dual-user haptic system. In this system, an expert surgeon (the trainer) and a novice surgeon (the trainee) collaborate on a surgical procedure using their own haptic devices. The mechanical impedance parameters of the trainer's haptic device remain constant during the operation, whereas those of the trainee vary with his/her proficiency level. The trainee's relative proficiency might be objectively quantified in real-time based on position error between the trainer and the trainee. The proposed architecture enables the trainer to intervene in the training process as needed to ensure the trainee is following the right course of action and to avoid the trainee's from potential tissue injuries. The stability of the overall nonlinear closed-loop system has been investigated using the input-to-state stability (ISS) criterion. High-gain observer with unknown inputs is considered in this work to estimate the interaction forces. Simulation and experimental results under different scenarios confirm the effectiveness of the proposed control methods.

Index Terms—Compliance and Impedance Control, Physically Assistive Device, Dual-user Haptic System, Surgery Training, Input-to-State Stability (ISS)

I. INTRODUCTION

Medical errors are the third leading cause of death after cancer and heart failure [1]. Although eye surgery errors rarely result in death, they can have severe consequences. Residents and fellows who are getting their first experiences are more likely to make mistakes and cause complications like retinal detachment or lesions [2]. The "see one, do one, teach one" principle [3] is believed to underlie most traditional eye surgery training methods, which deprive skilled surgeons of control over trainees, resulting in accidental tissue damage. Haptic feedback and teleoperation technologies can prevent such complications.

A dual-user haptic teleoperation system is one of the most promising architecture for integrating an expert surgeon into the training procedure in a hands-on and physical manner [4].

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^{††}M. Tavakoli is with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB T6G 1H9, Canada. mahdi.tavakoli@ualberta.ca Thus, the expert surgeon can interfere at any time to prevent surgical complications, while trainees have their own freedom of movement. Haptic training systems may replace conventional training methods by providing continuous data flow between the trainer and the trainee.

Numerous control schemes have been proposed in the literature to have a stable dual-user haptic interface operation such as H_{∞} control [5], six channel control [6], Impedancebased control [7], S-shaped function [8], and fuzzy logic based virtual fixture approach [9]. Furthermore, variable impedance control for surgical skill transfer was developed in [10] incorporating an automatic approach for tuning the impedance gains, and furthermore, by pushing a mechanical pedal [11]. Even though all the above control methodologies have attempted to facilitate training differently, there are still critical research gaps. In some studies, the control system is constructed solely using users' position information. However, force information is just as important as position information when interacting with an unknown environment. On the other hand, various studies have investigated the stability of the proposed controllers without considering nonlinearities, unmodeled dynamics, etc. while, there has been no research on varying control schemes, especially the impedance parameters, which this paper attempts to address.

The most notable aspect of the present research is to introduce a haptic-enabled training system with responsive variable impedance controllers. The haptic consoles of the trainer and the trainee are synchronized with one another in terms of their positions. In this way, the trainer is aware of the maneuvers of the trainee at every stage of the surgery. This also eliminates the need for the trainer to perform every trivial task individually, and the trainer only plays a supervisory role throughout the procedure. Whenever the trainee's movement poses a risk to the patient's tissue, the trainer has the opportunity to intervene in the operation and steer the trainee's haptic device toward the right path. Meanwhile, the system automatically increases the impedance parameter values of the trainee's haptic device, making his/her console stiffer and preventing the trainee from further performing the wrong maneuver. This smooth increase in the impedance parameters is based on the magnitude of the position error between the two users.

The proposed control scheme uses high-gain observers [12] to obtain user-device interaction forces, thus having a minimal implementation cost. Furthermore, due to its adaptive and robust characteristics, the controller can achieve accurate tracking even when there exist uncertainties and un-



Fig. 1: Proposed dual-user haptic training system.

modeled nonlinear dynamics in the system. Finally, the inputto-state stability (ISS) approach [13] is used to investigate the stability of the interconnected system composed of the proposed controller and the trainee and the trainer's haptic devices. The remainder of this paper is organized as follows. Section II provides a thorough description of the haptic training system. Section III discusses the proposed variable impedance control architecture. Detailed stability analysis of the proposed training system is outlined in Section IV. The paper is followed by simulation and experimental results in Section V, while the concluding remarks are given in Section VI.

II. HAPTIC TRAINING SYSTEM

A. System Description

Figure 1 illustrates the proposed impedance control architecture for a dual-user haptic training system. The system consists of eight different blocks: haptic consoles #1 and #2 correspond to the trainer and the trainee, respectively. The trainee's haptic device is connected directly to the real or virtual environment. Each haptic device has its own impedance controller. The only difference between the two sides' controllers is that the trainee's control parameters are adaptively adjusted via the controller adjustment unit during the training process while the trainer's parameters remain constant.

The main application of this architecture is in intraocular eve surgery training. As previously stated, the complication rate in these delicate surgeries is much higher in residents and fellows due to their lack of skills. The proposed training framework allows the trainee to get acquainted with the proper method of force exertion and tool maneuvers based on the expert's knowledge in the training process and involves him/her in critical surgery moments. The training process starts with the trainee taking control of the haptic console#2. While the trainee's maneuvers are trusted by the trainer, the trainee can continue with the surgery. Whenever a trainee exceeds a predefined position error threshold, the impedance parameter of his/her haptic device increases accordingly. Therefore, the trainee feels that the device becomes stiffer and harder to move. At this stage, the trainer steers his/her own haptic device and guides the trainee on the correct path. By this means, potential complications in the traditional

training methods due to the improper involvement of the trainer in the training process would be significantly reduced.

B. System Dynamics

A dual-user haptic training system can be considered as an n-DOF robot with the following dynamics [14]

$$M_{i}(q_{i})\ddot{q}_{i} + C_{i}(q_{i},\dot{q}_{i})\dot{q}_{i} + g_{i}(q_{i}) + F_{qi} = u_{i} + J_{i}^{T}(q)f_{hi} \quad (1)$$

where $q_i \in \mathbb{R}^{n \times 1}$ denote joint positions, $M_i(q_i) \in \mathbb{R}^{n \times n}$ denote the inertia matrices, $C_i(q_i, \dot{q}_i) \in \mathbb{R}^{n \times n}$ denote the centrifugal and Coriolis matrices, $g_i(q_i) \in \mathbb{R}^{n \times 1}$ denote the gravity vectors, F_{qi} is model uncertainty and/or exogenous disturbance vectors. Furthermore, $u_i \in \mathbb{R}^{n \times 1}$ are the control torque vectors, and $f_{hi} \in \mathbb{R}^{n \times 1}$ are interaction forces all for i = m, s where the subscript *i* denotes the trainer haptic console #1 as i = m and the trainee haptic console #2 as i = s. The dynamics equation (1) has the following properties [14]: *Property 1.1*: The inertia matrix $M_i(q_i)$ is a symmetric positive-definite matrix.

Property 1.2: The matrix $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew symmetric, i.e., $v^T (\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)) v = 0 \quad \forall v \in \mathbb{R}^n$. Note that this is correct for any v, provided that $C_i(q_i, \dot{q}_i)$ is in Christoffel form.

Property 1.3: The left hand side of (1) may be represented as a linear regression form of the physical parameters as $M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + g_i(q_i) = Y_i(q_i,\dot{q}_i,\ddot{q}_i)\theta_i$. The regressor and physical parameters vector are indicated by Y_i , and θ_i , respectively. Furthermore, this formulation can be extended to the Slotine-Li regressor form as

$$M_{i}(q_{i}) \varphi_{1,i} + C_{i}(q_{i},\dot{q}_{i}) \varphi_{2,i} + g_{i}(q_{i}) = Y_{i}(\varphi_{1,i}, \varphi_{2,i}, q_{i}, \dot{q}_{i}) \theta_{i}$$
(2)

III. THE RESPONSIVE VARIABLE IMPEDANCE CONTROL

Using variable impedance coefficients responsive to the evaluated performance of participants is a potential method for skill transfer from trainer to trainee. To reach this goal, a performance metrics must be developed to evaluate the trainee's performance. In this paper, an evaluation criterion is considered based on the trainee's and trainer's motions. Thus, the adjustment parameter based on this criteria will adjust the impedance coefficients and formulated as:

$$e = (q_s - q_m), \quad \alpha(e) = \begin{cases} 0 & ||e|| < \overline{\rho} \\ \sigma & ||e|| \ge \overline{\rho} \end{cases}$$
(3)

where q_m and q_s are the joint position vector of the trainer's and the trainee's device, respectively, σ is an arbitrary parameter that is chosen to tackle the undesirable motions of the trainee, and ρ is a threshold that determines admissible range of trainee-trainer position deviations. By this means, the adjustment parameter α_f is defined as the smoothed form of the varying parameter. Furthermore, the proposed training scheme uses a reference impedance model, in which the impedance coefficients are changed responsive to the adjustment parameter α_f . Therefore, the adjusted desired impedance coefficients at the trainee side are determined by

$$\left\{ \begin{aligned} M_{ds} & B_{ds} & K_{ds} \\ \end{bmatrix} = \\ \alpha_f \times \left\{ 0 & B_{fs} & K_{fs} \\ \end{bmatrix} + \left\{ M_{fs} & B_{fs} & K_{fs} \\ \end{bmatrix},$$
 (4)

where $M_{fs} \in \mathbb{R}^{n \times n}$, $B_{fs} \in \mathbb{R}^{n \times n}$ and $K_{fs} \in \mathbb{R}^{n \times n}$ are the fixed mass, damping coefficient, and stiffness coefficients.

The reference impedance model is defined as

$$M_{di}\left(\ddot{q}_{refi}-\ddot{q}_{di}\right)+B_{di}\left(\dot{q}_{refi}-\dot{q}_{di}\right)+K_{di}\left(q_{refi}-q_{di}\right)=\hat{\tau}_{hi} \quad (5)$$

where $M_{di} \in \mathbb{R}^{n \times n}$, $B_{di} \in \mathbb{R}^{n \times n}$ and $K_{di} \in \mathbb{R}^{n \times n}$ indicate each haptic console desired mass, damping coefficient, and spring stiffness. Furthermore, for each haptic console, q_{di} , $\hat{\tau}_{hi}$ is defined as the position of the other side and the estimated interaction force of the operator which interacted with haptic console. To satisfy $q_{refi} \rightarrow q_{di}$, sliding surfaces are considered as

$$s_i = \dot{\tilde{q}}_i + \gamma \tilde{q}_i \tag{6}$$

where $\tilde{q}_i = q_i - q_{refi}$ is a haptic console position error with respect to its reference impedance models responses (5). The reference velocities are also expressed as q_{ri} , where

$$\dot{q}_{ri} = \dot{q}_{refi} - \gamma \tilde{q}_i. \tag{7}$$

The trajectories of the system are forced towards the sliding surface by applying the following force:

$$\tau_i = \hat{M}_i(q_i)\ddot{q}_{ri} + \hat{C}_i(q_i,\dot{q}_i)\dot{q}_{ri} + \hat{G}_i(q_i) - \hat{\tau}_{hi} - \kappa_i \tanh(s_i/\varepsilon)$$
(8)

where $\hat{M}_i(q_i)$, $\hat{C}_i(q_i, \dot{q}_i)$, and $\hat{G}_i(q_i)$ are dynamic matrices estimation. Slotine-Li regressor-based control inputs that apply motor torques to the robot's joint space are given by

$$\tau_i = Y_{q_i} \left(\ddot{q}_{ri} , \dot{q}_{ri}, q_i, \dot{q}_i \right) \hat{\theta}_i - \hat{\tau}_{hi} - \kappa_i \tanh(s_i/\varepsilon).$$
(9)

Substitute the control laws in the system dynamics to get the closed-loop system dynamics as follows:

$$M_i(q_i)\dot{s}_i = -C_i(q_i, \dot{q}_i)s_i + Y_{q_i}\tilde{\theta}_i - \kappa_i \tanh(s_i/\varepsilon) + D_{qi}, \quad (10)$$

where $\tilde{\theta}_i = \hat{\theta}_i - \theta_i$ is the error vector of the haptic console parameters. Furthermore, D_{qi} is denote different types of uncertainty and interaction force misestimation as $D_{qi} = -F_{qi} + (\tau_{hi} - \hat{\tau}_{hi})$.

As required by the control input (8), the trainee's and trainer's forces should be measured or estimated. A highgain observer estimates the interaction force dynamically by taking into account the haptic device's position, velocity, and torque. For the haptic console #i, set the state vector as $[\eta_1^T \quad \eta_2^T]^T = [q_i^T \quad \dot{q}_i^T \quad]^T$ and the formulation of the proposed observer as

$$\begin{split} \dot{\hat{\eta}}_{1} &= \hat{\eta}_{2} + \frac{c_{1}}{\varepsilon} \left(\eta_{1} - \hat{\eta}_{1} \right) \\ \dot{\hat{\eta}}_{2} &= \hat{M}_{i}^{-1} \left(\eta_{1} \right) \left[-\hat{C} \left(\hat{\eta}_{1}, \hat{\eta}_{2} \right) \hat{\eta}_{2} - \hat{G} \left(\hat{\eta}_{1} \right) + \tau_{i} \right] + \frac{\bar{c}_{2}}{\varepsilon^{2}} \left(\eta_{1} - \hat{\eta}_{1} \right) \\ \hat{\tau}_{hi} &= \frac{\bar{c}_{2}}{\varepsilon^{2}} \hat{M}_{i} \left(\eta_{1} \right) \left(\eta_{1} - \hat{\eta}_{1} \right) \end{split}$$

$$(11)$$

where ε is a small positive constant and the positive values \bar{c}_1 and \bar{c}_2 are designed in a way that confines a quadratic equation with left eigenvalues $\lambda^2 + \bar{c}_1 \lambda + \bar{c}_2 = 0$.

IV. STABILITY ANALYSIS

In this section, the stability analysis of the presented impedance control design for a dual-user haptic system is explored using the ISS stability method. Proposition 1 will investigate ISS stability of the haptic console *#i*. Furthermore, in Theorem 1, the stability analysis of the whole system will be examined.

Lemma 1: The high-gain observer (11) is ISS with respect to the input $[q_i^T, \dot{q}_i^T]^T$ and the observation error as the state variable.

The proof is given in Appendix.

Lemma 2: The system (5) is ISS with respect to the state $[e_i^T, \dot{e}_i]^T$ and the input $[q_{di}^T, \dot{q}_{di}^T, \hat{\tau}_{hi}^T]^T$.

The proof is given in Appendix.

Proposition 1: Each haptic console (#*i*) closed-loop subsystem (10) is ISS with respect to the state $[s_i^T, \tilde{\theta}_i^T, \zeta_i, \rho_i]^T$ and the input $\hat{\tau}_{hi}^T$.

Proof: Consider the following Lyapunov function candidate:

$$V_{i} = \frac{1}{2} \left[s_{i}^{T} M_{i} s_{i} + \tilde{\theta}_{i}^{T} \Gamma^{-1} \tilde{\theta}_{i} + \varepsilon \zeta_{i}^{T} P \zeta_{i} + \rho_{i}^{T} P_{m} \rho_{i} \right].$$
(12)

Differentiate V_i with respect to time:

$$\dot{V}_i = \underbrace{s_i^T M_i \dot{s}_i + \frac{1}{2} s_i^T \dot{M}_i s_i}_{V_1} + \underbrace{\tilde{\theta}_i^T \Psi_i^{-1} \dot{\tilde{\theta}}_i}_{V_2} + \underbrace{\varepsilon \zeta_i^T P \dot{\zeta}_i + \rho_i^T P_m \dot{\rho}_i}_{V_3}.$$

Consider the relation $s_i = \dot{q}_i - \dot{q}_{ri}$, with some simplifications:

$$V_{1} = s_{i}^{T} \left[Y_{qi} \hat{\theta}_{i} - Y_{qi} \theta_{i} \right] - s_{i}^{T} \kappa_{i} \tanh \frac{s_{i}}{\varepsilon} + s_{i}^{T} D_{qi} + \frac{1}{2} s_{i}^{T} \left[\dot{M}_{i} \left(q_{i} \right) - 2C_{i} \left(\dot{q}_{i}, q_{i} \right) \right] s_{i}$$

$$(13)$$

then, by considering Property 1.2

$$V_1 = s_i^T Y_{qi} \tilde{\theta}_i - s_i^T \kappa_i \tanh \frac{s_i}{\varepsilon} + s_i^T D_{qi}.$$
(14)

It is assumed that the dynamic parameters have reached to steady state, and as a result, $\tilde{\theta} = \dot{\theta}$. This assumption, with consideration that the adaptation laws for parameters are $\hat{\theta}_i = -\Psi_i Y_{q,i} s_i$, simplifies the second part of the Lyapunov function as

$$V_{2} = \tilde{\theta}_{i}^{T} \Psi_{i}^{-1} \tilde{\theta}_{i} = \tilde{\theta}_{i} \Psi_{i}^{-1} \hat{\theta}_{i}$$

$$= \tilde{\theta}_{i}^{T} \Psi_{i}^{-1} \left[-\Psi_{i} Y_{qi} s_{i} \right] = -\tilde{\theta}_{i}^{T} Y_{qi} s_{i}.$$
 (15)

Therefore, the Lyapunov function derivative may be simplified to

$$\dot{V}_{i} = -s_{i}^{T} \kappa_{i} \tanh \frac{s_{i}}{\varepsilon} + s_{i}^{T} D_{qi} + \varepsilon \zeta_{i}^{T} P \dot{\zeta}_{i} + \rho_{i}^{T} P_{m} \dot{\rho}_{i}.$$
(16)

In the presence of bounded disturbances and model uncertainties, robust stability can be achieved by choosing control laws with positive constant parameter κ_i that satisfy the $\kappa_i \geq ||D_{qi}||_{\infty} + \mu_i$ inequality, where μ_i is positive constant. Thus, using the results of the *Lemma 1* and *Lemma 2*, the Lyapunov function derivative simplified to

$$\begin{split} \dot{V}_{i} &\leq -\mu_{i} \|s_{i}\| + \left(-\lambda_{\min}\left(Q\right) + 2\varepsilon^{2}L \|PB\|\right) \|\zeta_{i}\|^{2} \\ &+ \left[-\lambda_{\min}\left\{Q_{m}\right\} + \left\{\frac{c_{1}}{2}\right\} \|P_{m}\|^{2}\right] \|\rho_{i}\|^{2} + \left\{\frac{1}{2c_{1}}\right\} \|B\|^{2} \|\hat{\tau}_{hi}\|^{2}. \end{split}$$



(a) The operator's real and estimated (b) Joint Position and impedance mode hand forces

Fig. 2: Simulation Results

It is necessary to substitute the estimation of the interaction force to calculate the upper bound of the last term, which is related to the reference impedance model. Since the estimation force itself is derived from system state variables, the relevant relations need to be modified by

$$\hat{\tau}_{h_i} = \frac{\bar{c}_2}{\varepsilon^2} \hat{M}_i \left(\eta_{i1} - \hat{\eta}_{i1} \right), \qquad (17)$$

in which $\eta_{i1} - \hat{\eta}_{i1}$ equals to $F_m E_{om}^{-1} \zeta_i$. Where F_m, E_{om} is defined as

$$F_m = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}^T, \quad E_{om} = \begin{bmatrix} 1 & 0 \\ 0 & \varepsilon \end{bmatrix}.$$
(18)

Hence, the estimation force is derived from

$$\hat{\tau}_{h_i} = \frac{\bar{c}_2}{\epsilon^2} \hat{M}_i \left(F_m E_{om}^{-1} \zeta_i \right) \tag{19}$$

and corollary, the upper limit of the Lyapunov's derivative is simplified to

$$\begin{split} \dot{V} &\leq -\mu_{i} \left\| s_{i} \right\| + \left[-\lambda_{\min} \left\{ Q_{m} \right\} + \left\{ \frac{c_{1}}{2} \right\} \left\| P_{m} \right\|^{2} \right] \left\| \rho_{i} \right\|^{2} \\ & \left(-\lambda_{\min} \left(Q \right) + 2\varepsilon^{2}L \left\| PB \right\| + \left\{ \frac{1}{2c_{1}} \right\} \left\| B \right\|^{2} \left\| \frac{\bar{c}_{2}}{\varepsilon^{2}} \hat{M}_{i} \left(F_{m} E_{om}^{-1} \right) \right\|^{2} \right) \left\| \zeta_{i} \right\|^{2}. \end{split}$$

In the case where the Lyapunov function is negative-definite, the stability of each haptic console is guaranteed.

Theorem 1: The overall dual-user haptic training system (10) is ISS with respect to the state $[s_m^T, s_s^T, \tilde{\theta}_m^T, \tilde{\theta}_s^T, \zeta_m, \zeta_s, \rho_m, \rho_s]^T$, and the input $[\hat{\tau}_{hm}^T, \hat{\tau}_{hs}^T]^T$. *Proof*: The following Lyapunov candidate is considered:

$$V = \frac{1}{2} \begin{bmatrix} s_m^T M_m s_m + s_s^T M_s s_s + \theta_m^T \Gamma^{-1} \tilde{\theta_m} + \tilde{\theta_s}^T \Gamma^{-1} \tilde{\theta_s} \\ + \varepsilon \zeta_m^T P \zeta_m + \varepsilon \zeta_s^T P \zeta_s + \rho_m^T P_m \rho_m + \rho_s^T P_m \rho_s \end{bmatrix},$$
(20)

similar to the previous procedure, the Lyapunov function derivative is obtained as

$$\begin{split} \dot{V} &\leq -\mu \, \|s_m\|_1 - \mu \, \|s_s\|_1 \\ &+ \left(-\lambda_{\min} \left(\mathcal{Q} \right) + 2\varepsilon^2 L \, \|PB\| + \frac{1}{2c_1} \|B\|^2 \, \left\| \frac{\bar{c}_2}{\varepsilon^2} M_m \left(F_m E_{om}^{-1} \right) \right\|^2 \right) \|\zeta_m\|^2 \\ &+ \left(-\lambda_{\min} \left(\mathcal{Q} \right) + 2\varepsilon^2 L \, \|PB\| + \frac{1}{2c_1} \|B\|^2 \, \left\| \frac{\bar{c}_2}{\varepsilon^2} M_s \left(F_m E_{om}^{-1} \right) \right\|^2 \right) \|\zeta_s\|^2 \\ &+ \left[-\lambda_{\min} \left\{ \mathcal{Q}_m \right\} + \frac{c_1}{2} \|P_m\|^2 \right] \left(\|\rho_m\|^2 + \|\rho_s\|^2 \right). \end{split}$$

In order to ensure the overall system stability, the following conditions must be satisfied:

$$\lambda_{\min}(Q) \ge 2\varepsilon^{2}L \|PB\| \frac{1}{2c_{1}} \|B\|^{2} \left\| \frac{\bar{c}_{2}}{\varepsilon^{2}} \hat{M}_{i} \left(F_{m} E_{om}^{-1} \right) \right\|^{2}$$

$$\lambda_{\min} \{Q_{m}\} \ge \frac{c_{1}}{2} \|P_{m}\|^{2}.$$
(21)

Since the proper choice of the \bar{c}_1 , \bar{c}_2 , ε is directly affecting the positive-definite symmetrical matrices P, and Q in the Lyapunov eauation (26), these parameters must be selected carefully by the designer, both for achieving the appropriate behavior from the high gain observer as well as forming a stable system. Meanwhile, L indicates a limit bound for Δ in (28). If the robot dynamics were better estimated, the estimated parameters would be closer to reality, which would lead to a lower parameter L and a wider stability area. Furthermore, c_1 is an arbitrary positive parameter that is introduced in (33) from the Young inequalities prerequisite and it must be selected carefully due to the fact that it affects (21) both directly and inversely in two stable conditions. Hence, from (12) and (20) under the appropriate selection of (21), the overall nonlinear closed-loop system in combination with the high gain observer is ISS with respect to the state $[s_m^T, s_s^T, \tilde{\theta}_m^T, \tilde{\theta}_s^T, \zeta_m, \zeta_s, \rho_m, \rho_s]^T$, and the input $[\hat{\tau}_{hm}^T, \hat{\tau}_{hs}^T]^T$.

V. SIMULATION AND EXPERIMENTAL RESULTS

A. Simulation Results

This section is intended to explore the effects of the responsive dual-user variable impedance structure on the trainee's performance in a surgical training task. Simulations of the proposed control scheme has been performed by using the dynamic model of custom-made haptic interfaces, called ARASH:ASiST, designed for eye surgery training at Advanced Robotics and Automated Systems Lab. The dynamic model of the haptic devices can be found in [15].

In the simulations, no path following task is considered for the haptic consoles, but rather exogenous forces are applied by the operators on their haptic devices to cause the movement. The exogenous forces applied by the trainee and the trainer are depicted in Fig. 2a. The force signals are square waves passed through a low-pass filter $\frac{1}{s+1}$ with different amplitudes for the trainer and trainee to resemble the difference in forces applied by expert and novice



Fig. 3: Left: Joint Position for $\alpha_f = 0$ and $\alpha_f \neq 0$, Right: Joint Position for $\alpha_f \neq 0$ and different σ

TABLE I: The impedance parameters of each haptic device

User	Inertia (gr)	Damping (N.s/m)	Stiffness (N/m)
Novice	$0.05I_{3\times 3}$	$20(1+\alpha_f)I_{3\times 3}$	$20(1+\alpha_f)I_{3\times 3}$
Expert	0.05 <i>I</i> _{3×3}	20 <i>I</i> _{3×3}	20 <i>I</i> _{3×3}

surgeons in a real training scenario. The solid blue and green lines indicate the trainer's and the trainee's force signals, respectively, while the dashed red and black lines illustrate the estimated forces using the high-gain observer. The trainee exerted larger forces on his/her haptic device compared to that of the trainer in the time interval [20, 30] s, almost equal force in $[0, 10] \ s \cup [30, 50] \ s$, and smaller forces in [10, 20] s. The parameters of the high-gain observer are set to $\bar{c}_1 = 0.957$, and $\bar{c}_2 = 2.7225$, while by solving the corresponding Lyapunov equation with an arbitrary PD matrix Q, $\varepsilon = 0.07$ is calculated as an upper bound. As can be seen in 2a, the observer performs well in accurately estimating the interaction forces. As for selecting the control parameters, conditions (21) should be satisfied. A series of experiments is performed for tuning the parameters. In order to meet the stability condition, it is sufficient to choose $\gamma = \text{diag}(4.6, 3.45, 4.6), \kappa_i = \text{diag}(7.6, 14.45, 7.8).$

In both haptic devices, parameters of impedance coefficients were chosen in accordance with (33), as given in Table I. Furthermore, each of these parameters is chosen based on the specific reliable region within which the impedance parameters can alter while the system remains stable. The stability conditions can be satisfied from the absolute stability theorem [16]. Fig.2b demonstrates the joint position and the corresponding reference impedance model in the first degree of freedom of the haptic devices. Solid blue and red lines indicate the trainer's and trainee's positions, respectively. Impedance models are shown in dashed green and yellow lines as well. As can be seen from the figure, the two haptic devices follow each other perfectly when equal forces are exerted, and they are extremely close in other cases. Furthermore, with a suitable transient, the control efforts shown in Fig. 2c have reasonable amplitudes. This is due to the initial values of the observers and exhibits the transient behavior of the system, which dies down shortly.

It can be observed from Fig. 2a that as the trainee applied different amounts of forces compared to those of the trainer, the haptic position of the trainee differs from that of the trainer. If the magnitude of this error crosses a certain threshold, as it does from t = 10 to t = 30, then the stiffness



Fig. 4: Experimental setup.

and damping of the trainee's haptic device increase. This trend is better illustrated in Fig. 2d, in which the solid blue line depicts the parameter α_f , which directly influences the impedance parameters. During this time period, the dashed red line represents the error between the two haptic consoles, which is considered to be above the safe threshold. It is not necessary to adjust the impedance of the trainee's haptic device outside of this time period. As a result, trainees are allowed to explore the task and maneuver freely without any interference from the trainer as long as the trainer trusts in their movements. Depending on the specific task and expert opinion, the safe error bond can be adjusted before training.

Furthermore, a series of simulations are conducted to evaluate the functionality of the proposed responsive variable impedance control scheme in two different cases. The first case involves the examination of a situation in which the impedance parameters of the trainee's haptic device are not adjusted in real time, i.e. $\alpha_f = 0$. That causes less adaptation between the positions of the two haptic operators during the task than when $\alpha_f \neq 0$. Referring to Figure 3 which is the result of controller parameter selection $\sigma = 0.1$ and $\overline{\rho} = 0.05$ degree, the results are in line with our expectations. The second case investigates how different values of the σ parameter affect the adaptation rate of the trainee's maneuvers. The parameter is set to two different values, $\sigma = 0.1$ and $\sigma = 0.5$. As can be seen from the Figure 3, increasing the σ parameter will force the trainee to follow the correct path of the trainer and operates in the closer vicinity of the trainer's path. There are some regions in the eye globe where any deviations from the correct path by the trainees can harm the tissue and cause severe complications for the patient. In these areas, this type of collaboration is very promising.

B. Experimental Results

Figure 4 depicts the experimental setup used to implement the proposed control framework. It consists of two identical haptic consoles designed specifically for eye surgery training. MATLAB/Simulink Real-Time is used on a central computer to run the controller code. Both haptic devices are connected to this PC via a UDP (User Datagram Protocol) channel. The data can be exchanged between the consoles as well with the frequency of one KHz. Similar to the simulation scenario, one of these haptic consoles is controlled by the trainer, and the other by the trainee.

The control parameters tuned in the simulations are used



(b) Applied Control Efforts

Fig. 6: Implementation Results for second scenario.

in practice to satisfy the stability conditions. The sampling rate for the closed-loop system is 1 kHz, which is high enough to provide smooth and delicate robot movements. To elaborate more on the robust performance of the proposed controller in practice, two different scenarios have been considered in the implementation phase. The first scenario emulates the case where the trainee is at the beginning of the training procedure, and the trainer is to conduct the entire procedure while the trainee puts his/her hand without any resistance on the haptic device. The purpose of this step is to familiarize the trainee with primitive motor control maneuvers. As illustrated in Fig. 5a, the trainee's slight pressure over the haptic device does not hinder the trainer from teaching delicate maneuvers. However, the adjustment parameters, as depicted in Fig. 5c, still got changed due to the difference between users' hand motions. The employment of a chattering-free control law by using *tanh* function, results in no fluctuation in the error signal about the sliding surface, as seen in Fig. 5b.

The second scenario in the implementation phase is associated with the performance of the proposed control structure against impulsive forces when $\alpha_f \neq 0$ and $\alpha_f = 0$. For this purpose, a considerable amount of force was applied to the system by the trainee from t = 25 to t = 30, with and without the adjustment parameter. This scenario considers unintentional and abrupt motions of the trainee's hand during the operation, which thanks to using the proposed adjustment parameter results in a more subtle and smooth change in the trainee's movement following an abrupt error. According to Fig. 6a, the abrupt force applied in the presence of the adjustment parameter scenario (solid black line) is significantly large. This applied force is measured from the well-tuned force observer, whose accurate performance is verified in the simulation results.

By analyzing the experimental results from Fig. 6a specif-

ically from t = 25 to t = 30, it is well implied that varying the impedance coefficients based on error allows the trainee to behave more in accordance with the trainer's behavior. Fig. 6b shows the control effort generated by the control architecture in this case. As shown in this figure, more force (blue solid line) is applied by the controller of the proposed structure during the sudden interaction force, specifically from t = 25 to t = 30. Due to differences between the trainer and the trainee hand motion, the adjustment parameter changes accordingly over time and suitably changes the impedance coefficients as illustrated in Fig. 6c. Furthermore, this figure shows that changing the coefficients significantly reduces the position errors compared to that of fixed coefficients.

VI. CONCLUSIONS

This study investigates a variable impedance control architecture for a surgical training haptic system. The overall system's stability is determined by input-to-state stability (ISS) analysis. In addition, a high-gain observer with unknown inputs is incorporated into the closed loop structure to estimate the interaction forces between the user, the device, and the environment, which eliminates the requirement for high-cost force sensors. The proposed architecture is evaluated in detail using a variety of simulation and experimental scenarios. Further research will be conducted on the user's task-specific training with the proposed responsive variable impedance control compared to the benchmark controllers.

APPENDIX

A. Proof of Lemma 1

The state space formulation of the high-gain force observer may be written as

$$e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} \eta_1 - \hat{\eta}_1 \\ \eta_2 - \hat{\eta}_2 \end{bmatrix}$$
(22)

$$\dot{e} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} e + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \Delta - \begin{bmatrix} \bar{c}_1/\varepsilon & 0 \\ \bar{c}_2/\varepsilon^2 & 0 \end{bmatrix} e \qquad (23)$$

while Δ is defined as

$$\Delta = M^{-1}(\eta_1) \left(-C(\eta_1, \eta_2) \eta_2 - g(\eta_1) + \tau_i \right) - \hat{M}^{-1}(\eta_1) \left(-\hat{C}(\hat{\eta}_1, \hat{\eta}_2) \hat{\eta}_2 - \hat{g}(\hat{\eta}_1) + \tau_i \right).$$
(24)

Let us define $\zeta = \begin{bmatrix} 1 & 0 \\ 0 & \varepsilon \end{bmatrix} e = E_{om}e$, and rewrite (23) into

$$\varepsilon \dot{\zeta} = \underbrace{\begin{bmatrix} -\bar{c}_1 & 1\\ -\bar{c}_2 & 0 \end{bmatrix}}_{A} \zeta + \varepsilon^2 \underbrace{\begin{bmatrix} 0\\ 1 \end{bmatrix}}_{B} \Delta.$$
(25)

If matrix A is Hurwitz then the error equation is stable and will converge to zero. This claim can be proven by the following Lyapunov candidate

$$V = \varepsilon \zeta^T P \zeta, \tag{26}$$

where *P* is the positive–definite symmetrical solution of the Lyapunov equation $A^TP + PA + Q = 0$, in which, *Q* is any arbitrary positive-definite matrix. Differentiate the Lyapunov candidate with respect to time

$$\dot{V} = \zeta^T \left(A^T P + P A \right) \zeta + \varepsilon^2 B^T P \zeta \Delta + \varepsilon^2 \zeta^T P B \Delta.$$
(27)

Substitute $A^T P + PA$ with -Q to find

$$\begin{split} \dot{V} &= -\zeta^{T} Q \zeta + 2\varepsilon^{2} \zeta^{T} P B \Delta \\ &\leq -\zeta^{T} Q \zeta + 2\varepsilon^{2} \| P B \| . \| \zeta \| . |\Delta| \\ \Delta &\leq L \zeta \leq -\lambda_{\min} \left(Q \right) \| \zeta \|^{2} + 2\varepsilon^{2} L \| P B \| \| \zeta \|^{2} . \end{split}$$

$$(28)$$

B. Proof of Lemma 2

The following equation may be derived from (5) by rewriting the impedance dynamics in error terms:

$$M_{di}\ddot{e}_i + B_{di}\dot{e}_i + K_{di}e_i = \hat{\tau}_{hi}.$$
 (29)

The reference error model may be written in the state space

$$\dot{\rho} = A\rho + B\hat{\tau}_{hi} \tag{30}$$

where the state vector ρ and the matrices *A* and *B* are defined as:

$$A = \begin{bmatrix} 0 & I \\ -M_{di}^{-1}K_{di} & -M_{di}^{-1}B_{di} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ -M_{di}^{-1} \end{bmatrix}, \rho = \begin{bmatrix} \rho_1 \\ \rho_2 \end{bmatrix} = \begin{bmatrix} e_i \\ \dot{e}_i \end{bmatrix}.$$
(31)

By considering that matrix A is Hurwitz, for each given positive matrix Q_m , a given positive matrix P_m exists that the Lyapunov equation $A^T P_m + P_m A = -Q_m$ is satisfied. Based on the aforementioned relation, the stability of the impedance model is proved. To this end, consider the following Lyapunov candidate

$$V = \rho^T P_m \rho. \tag{32}$$

Differentiate the Lyapunov candidate with respect to time $\dot{V} =$

$$\rho^{T} P_{m} \dot{\rho} + \dot{\rho}^{T} P_{m} \rho = \rho^{T} P_{m} [A \rho + B \hat{\tau}_{hi}] + [A \rho + B \hat{\tau}_{hi}]^{T} P_{m} \rho$$
$$= -\rho^{T} Q_{m} \rho + \rho^{T} P_{m} [B \hat{\tau}_{hi} + B \hat{\tau}_{hi}]^{T} P_{m} \rho$$

where by utilizing *Lemma 1.1* and Young inequalities we have

$$\dot{V} \leq -\left[\lambda_{\min}\left\{Q_{m}\right\} - \frac{c_{1}}{2}\|P_{m}\|^{2}\right]\|\rho\|^{2} + \frac{1}{2c_{1}}\|B\|^{2}\|\hat{\tau}_{hi}\|^{2},$$
(33)

which indicates that the reference model system is also ISS by choosing appropriate control parameters.

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