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Discrete-time Control Barrier Function: High-order case and Adaptive case

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***Abstract*—This paper proposes the novel concepts of high order discrete-time control barrier function and adaptive discrete-time control barrier function. The high order discrete-time control barrier function is used to guarantee forward invariance of a safe set for discrete-time systems of high relative degree. An optimization problem is then established unifying high order discrete-time control barrier functions with discrete-time control Lyapunov functions to yield a safe controller. To improve the feasibility of such optimization problems, the adaptive discrete-time control barrier function is designed, which can relax constraints on system control input through time-varying penalty functions. The effectiveness of the proposed methods in dealing with high relative degree constraints and improving feasibility is verified on the discrete-time system of a three-link manipulator.**

***Index Terms*—Control barrier functions, discrete-time systems, optimization problem.**

I. INTRODUCTION

SAFETY is often important for dynamical systems in addition to achieving control objectives, which is explained in terms that "bad" things do not happen [1], for example, protecting humans from harm in human–robot interaction [2]. It is known that Lyapunov functions and control Lyapunov functions (CLFs) are commonly used for stabilizing the closed-loop dynamics of dynamical systems [3]. For example, exponentially stabilizing control Lyapunov function can exponentially stabilize the periodic orbits of hybrid zero dynamics [4]. Therefore, the control objectives can be achieved by making use of control Lyapunov functions. Motivated by barrier certificates [5] for safety verification and the idea of control Lyapunov functions, the concept of control barrier functions (CBFs) is proposed by [6] to ensure safety through feedback design. The recent formulation of CBFs is given in [7] to satisfy safety conditions specified as forward invariance of a safe set, introducing the reciprocal control barrier functions and zeroing control barrier functions. In this case, CBFs and CLFs are unified together to form quadratic programs to mediate the possible conflict between safety and control objectives for safety-critical systems. In particular, a nominal controller can be added to the quadratic programs to achieve the performance objectives [8], which can be specified as any well-behaved controller such as [9].

Control barrier functions have gained great attention in recent years and have been applied in many fields, such as adaptive cruise control [10], robotic systems [11], [12], etc. However, the CBFs proposed in [7] can only be applied to dynamical systems of relative degree one. [11] introduced a backstepping method to address constraints of high relative degree and applied it to the case of relative degree two. [13] and [14] specially modified zeroing control barrier functions to ensure constraint satisfaction for Euler-Lagrange systems and mechanical systems of relative degree two, respectively. Based on the pole placement control method in linear control theory, [15] designed exponential control barrier functions to handle arbitrary high relative degree constraints. A more general barrier function for high relative degree systems, namely high order control barrier function, is introduced in [16], [17], which does not limit the types of class $\mathcal{K}$ functions compared with exponential control barrier functions. Moreover, another concept of high-order (zeroing) control barrier function is proposed in [18], where extended class $\mathcal{K}$ functions are incorporated and the general definition of relative degree is relaxed. Considering the optimization problem unifying CBFs and CLFs, [19] shows how adaptive control barrier function can adapt to time-varying control bounds and noise in dynamical systems. The two types of adaptive CBF, i.e., parameter-adaptive CBF and relaxation-adaptive CBF, can address the feasibility problem of quadratic programs.

Noting that, all the above-mentioned concepts about CBF are designed for continuous-time systems. Given that most controllers in practices are implemented in a digital way, the discrete-time CBF has also achieved more and more attention. [20] first introduced the notion of discrete-time control barrier functions and discrete-time exponential control barrier functions based on their continuous-time counterparts, and unified discrete-time CBFs with discrete-time CLFs [21] to form nonlinear programming problems. [22] also combined discrete-time CBFs with model predictive control into an optimization problem. Based on discrete-time exponential CBFs, Gaussian process regression was used to identify the disturbance model in the presence of stochastic disturbances, so as to establish robust discrete-time CBFs [23]. [24] derived the discrete-time (zeroing) barrier function from zeroing barrier function of continuous-time systems, so that the solution of discrete-time systems can be studied even if it is outside the invariant set. In correspondence with the concept of relative degree in continuous-time systems, there is also a similar definition in discrete-time systems [25], [26]. However, the existing

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discrete-time CBF methods are unable to deal with high relative degree constraints of discrete-time systems.

In this paper, we generalize the concept of discrete-time (zeroing) barrier function [24] to high order discrete-time control barrier function for systems with high relative degree. The necessity and ability of high order discrete-time CBF to handle high relative degree constraints are illustrated by a simple numerical example of a linear system with relative degree two. Then, the optimization problem unifying high order discrete-time CBF and discrete-time CLF is formulated. Considering that the input is often bounded when designing a controller [27], control input bounds are added in the optimization problem. However, as the environment changes over time, control bounds may tighten and conflict with the CBF constraints, making the optimization problem infeasible. To improve the feasibility of the algorithm, the adaptive discrete-time control barrier function is proposed. Time-varying penalty functions are designed in discrete-time auxiliary systems and applied to class $\mathcal{K}$ functions to provide the "adaptivity". These concepts are validated on the discrete-time model of a three-link manipulator by handling the joint constraints of relative degree two and changing the control input bounds.

The main contributions of this paper are as follows:

- High order discrete-time CBF is proposed, which guarantees constraint satisfaction of discrete-time systems with high relative degree. The high order discrete-time CBF is combined with discrete-time CLF into an optimization problem to yield a safe controller.
- The adaptive discrete-time CBF is constructed to improve the feasibility of the above-mentioned optimization problem when external environment of the system changes.
- Based on the two novel concepts, the optimization-based control design is verified through simulations on the discrete-time model of a three-link manipulator to deal with constraints on joint angles.

The rest of the paper is organized as follows. Section II briefly reviews the concepts of control Lyapunov functions and control barrier functions for discrete-time systems. Section III introduces the notion of high order discrete-time control barrier function for arbitrary high relative degree systems and combines high order discrete-time CBF with discrete-time CLF to form an optimization problem. Section IV proposes the adaptive discrete-time control barrier function to improve the optimization problem feasibility. Section V summarizes the optimization-based control design and shows the numerical simulation verifications of proposed methods on a three-link manipulator. Finally, a conclusion is drawn in Section VI.

II. Preliminaries

In this section, the concepts of discrete-time control Lyapunov functions (CLFs) and discrete-time control barrier functions (CBFs) are revisited.

Consider the discrete-time system in the form of

$$x(k+1) = f(x(k), u(k)), \tag{1}$$

where $x(k) \in \mathcal{D} \subset \mathbb{R}^n$ is the state of the system at time step $k \in N$, $f : \mathcal{D} \to \mathcal{D} \subset \mathbb{R}^n$ is a continuous function, and the control input $u(k) \in \mathbb{U}$ is applied to the system.

A. Control Lyapunov functions for discrete-time systems

The use of Lyapunov stability theory in nonlinear control design has been applied to discrete-time domain, which leads to the notion of discrete-time control Lyapunov function.

Definition 1: (Discrete-Time Exponentially stabilizing Control Lyapunov Function [20]) For the control system (1), $V : \mathcal{D} \to \mathbb{R}^n$ is a discrete-time Exponential Control Lyapunov Function if there exist positive constants c_1, c_2 and c_3 such that

$$c_1\|x(k)\|^2 \le V(x(k)) \le c_2\|x(k)\|^2, \tag{2}$$

$$\Delta V(x(k), u(k)) + c_3\|x(k)\|^2 \le 0, \tag{3}$$

for all $x(k) \in \mathcal{D}$, where $\Delta V(x(k), u(k)) = V(x(k+1)) - V(x(k))$.

The existence of a discrete-time Exponential Control Lyapunov Function makes the system (1) exponentially stable, and yields a set of controllers $u(k)$ that are supposed to satisfy (3).

Inspired by the quadratic program introduced in [10], an optimization problem is induced taking the discrete-time CLF condtion (3) as a constraint. The min-norm controller $u^*(k)$ for the control system (1) is presented as following:

$$\begin{aligned} u^*(k) &= \underset{u(k)\in\mathbb{U}}{argmin}\ \frac{1}{2}\|u(k)\|^2, \\ s.t. &\quad \Delta V(x(k), u(k)) + c_3\|x(k)\|^2 \le 0. \end{aligned} \tag{4}$$

B. Control barrier functions for discrete-time systems

In addition to stabilization of the dynamical system through a discrete-time CLF, safety is also crucial. Therefore, in some practical application scenarios, we want to drive the system state x not to leave a safe region which is denoted as a set $\mathcal{C}$,

$$\mathcal{C} := \{x(k) \in \mathcal{D} \mid h(x(k)) \ge 0\}, \tag{5}$$

$$Int(\mathcal{C}) := \{x(k) \in \mathcal{D} \mid h(x(k)) > 0\}, \tag{6}$$

$$\partial\mathcal{C} := \{x(k) \in \mathcal{D} \mid h(x(k)) = 0\}, \tag{7}$$

where $h : \mathbb{R}^n \to \mathbb{R}$ is a continuous function.

Before introducing barrier functions to ensure safety, some necessary concepts are stated as below.

Definition 2: The set $\mathcal{C}$ is forward invariant for system (1), if $x(k) \in \mathcal{C}, \forall k \in N$ for every $x(0) \in \mathcal{C}$.

Definition 3: (Class $\mathcal{K}$ function [28]) A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ function if it is strictly increasing and $\alpha(0) = 0$.

In order to satisfy the safety conditions, i.e., forward invariance of the safe set $\mathcal{C}$, formulations of barrier functions for discrete-time systems are established. The following definition of discrete-time barrier function can be regarded as the extension of zeroing barrier function [7] in continuous-time systems.

Definition 4: (Discrete-Time Barrier Function [24]) Considering the following discrete-time system with no input

$$x(k+1) = f(x(k)), \tag{8}$$

let $\mathcal{C} \subseteq \mathcal{D} \subset \mathbb{R}^n$ be defined by (5)-(7) for a continuous function $h : \mathbb{R}^n \to \mathbb{R}$. h is a discrete-time barrier function if there exists class $\mathcal{K}$ function α satisfying $\alpha(r) < r$ for all $r > 0$ such that

$$\Delta h(x(k)) \geq -\alpha(h(x(k))), \quad \forall x(k) \in \mathcal{D}, \tag{9}$$

where $\Delta h(x(k)) = h(x(k+1)) - h(x(k))$ and r is the argument of α.

Remark 1: From [24], the necessary and sufficient condition for the invariance of the set (Theorem 1 in [24]) is obtained, that is, $\mathcal{C}$ is forward invariant if and only if h is a discrete-time barrier function. Then a controller can be designed for discrete-time system (1) to ensure the set invariance by directly extending discrete-time barrier functions to discrete-time control barrier functions.

Definition 5: (Discrete-Time Control Barrier Function) For the discrete-time system (1), let $\mathcal{C} \subseteq \mathcal{D} \subset \mathbb{R}^n$ be defined by (5)-(7) for a continuous function $h : \mathbb{R}^n \to \mathbb{R}$. h is a discrete-time control barrier function if there exists class $\mathcal{K}$ function α satisfying $\alpha(r) < r$ for all $r > 0$ such that

$$\Delta h(x(k), u(k)) \geq -\alpha(h(x(k))), \quad \forall x(k) \in \mathcal{D}, \tag{10}$$

where $\Delta h(x(k), u(k)) = h(x(k+1)) - h(x(k))$ and r is the argument of α.

Lemma 1: Given a set $\mathcal{C} \subseteq \mathcal{D} \subset \mathbb{R}^n$ defined by (5)-(7) for a continuous function $h : \mathbb{R}^n \to \mathbb{R}$, if h is a discrete-time control barrier function on $\mathcal{D}$, any discrete-time controller $u(k)$ satisfying (10) will render the set $\mathcal{C}$ forward invariant.

Proof: The proof is similar to the proof of Theorem 1 in [24]. ■

Remark 2: Note that in (10), the class $\mathcal{K}$ function α can be selected as a linear function with coefficient $\gamma, 0 < \gamma < 1$. In this way, it can be derived that $h(x(k)) \geq (1-\gamma)^k h(x(0))$, $\forall k \in N$. For $0 < \gamma \leq 1$, forward invariance of the set is still guaranteed (this is obviously true when $\gamma = 1$), and the discrete-time CBF turns into the discrete-time exponential CBF [20].

C. Optimization problem unifying discrete-time CLF and discrete-time CBF

Referring to (4), add the discrete-time CBF condition (10) as well as the constraint on control input $u(k)$ to form a new optimization problem

$$\begin{aligned} u^*(k) = & \underset{U(k)}{argmin} \ \frac{1}{2}\|u(k)\|^2 + p \cdot \delta^2, \\ & U(k) = \begin{bmatrix} u(k) \\ \delta \end{bmatrix} \\ s.t. \quad & \Delta V(x(k), u(k)) + c_3\|x(k)\|^2 \leq \delta, \\ & \Delta h(x(k), u(k)) + \alpha(h(x(k))) \geq 0, \\ & u_{min} \leq u(k) \leq u_{max}, \end{aligned} \tag{11}$$

where δ is a relaxation parameter on CLF constraint and u_{min} / u_{max} represent the lower/upper bound on control input $u(k)$. δ works when the discrete-time CLF constraints conflict with the discrete-time CBF constraints.

Remark 3: (Feasibility analysis) The optimization problem is always feasible without the constraint of the control input, because the existence of the relaxation parameter δ makes the safety constraint satisfied preferentially when CLF and CBF constraints conflict. However, taking the control input constraint into account, the feasibility is not always guaranteed, which remains a challenge we need to address in Section IV.

III. High order discrete-time control barrier functions

In this section, the definition of relative degree for discrete-time systems is introduced and the notion of high order discrete-time control barrier function is proposed to deal with systems of high relative degree.

A. High relative degree systems

The concept of relative degree for discrete-time systems is given below, followed by a simple example of system with relative degree two.

Definition 6: (Relative degree [25] [26]) The output $y(k) = g(x(k))$ of system (1) is said to have relative degree r iff

$$\begin{aligned} &y(k+r) = g_r(x(k), u(k)), \\ &y(k+i) = g_i(x(k)), \quad \forall 0 \leq i < r, \end{aligned} \tag{12}$$

which implies that r is the steps of delay in the output $y(k)$ in order for the control input $u(k)$ to appear.

For systems of high relative degree, it means that the output $y(k)$ has relative degree $r > 1$. The constraint $y(k) \geq 0$ is also said to be of relative degree r in this paper.

Example 1: (Relative degree 2 system [25]) Let us consider a second order discrete linear time-invariant (LTI) system

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ &= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ b \end{bmatrix} u(k), \\ y(k) &= cx_1(k) = Cx(k), \end{aligned} \tag{13}$$

with the system state $x(k) := [x_1(k), \ x_2(k)]^T$. Assuming that $CB = 0$ and $CAB \neq 0$, the output $y(k)$ only related with $x_1(k)$ is of relative degree 2.

Suppose that the output $y(k)$ should not exceed a specific value M, i.e., $y(k)$ needs to satisfy the constraint

$$M - y(k) \geq 0. \tag{14}$$

Now, we treat the set $\mathcal{C} = \{x(k) \mid M - y(k) \geq 0\}$ as a "safe set" and attempt to design a controller to make it forward invariant utilizing the discrete-time CBF formulated in Definition 5.

According to (10), we have

$$-c[a_{11}x_1(k) + a_{12}x_2(k)] + cx_1(k) \geq -\alpha(M - cx_1(k)). \tag{15}$$

Obviously, the control input does not appear in the above inequality, which indicates that controllers cannot be obtained through discrete-time CBF in the face of high relative degree constraints.

B. High order discrete-time control barrier function

Next, the high order discrete-time control barrier function is specially designed for high relative degree cases. For the discrete-time system (1), a series of functions are defined based on the original function $h : \mathbb{R}^n \to \mathbb{R}$ with relative degree m:

$$\begin{aligned} \psi_0(x_k) :=& h(x_k), \\ \psi_1(x_k) :=& \Delta\psi_0(x_k, u_k) + \alpha_1(\psi_0(x_k)), \\ &\vdots \\ \psi_m(x_k) :=& \Delta\psi_{m-1}(x_k, u_k) + \alpha_m(\psi_{m-1}(x_k)), \end{aligned} \tag{16}$$

where $x(k)$ and $u(k)$ at time step k are replaced by the simplified symbol x_k and u_k; $\Delta\psi_i(x_k, u_k) := \psi_i(x_{k+1}) - \psi_i(x_k), i = 0, 1, ..., m-1$; class $\mathcal{K}$ functions $\alpha_j(\cdot)$ satisfy $\alpha_j(r) < r$ for $j = 1, 2, ..., m$.

These functions yield a series of sets $\mathcal{C}_i, i = 0, 1, ..., m-1$, as

$$\begin{aligned} \mathcal{C}_0 :=& \{x_k \in \mathcal{D} \mid \psi_0(x_k) \geq 0\}, \\ \mathcal{C}_1 :=& \{x_k \in \mathcal{D} \mid \psi_1(x_k) \geq 0\}, \\ &\vdots \\ \mathcal{C}_{m-1} :=& \{x_k \in \mathcal{D} \mid \psi_{m-1}(x_k) \geq 0\}. \end{aligned} \tag{17}$$

Definition 7: (High Order Discrete-Time Control Barrier Function) For the discrete-time system (1), the continuous function $h : \mathbb{R}^n \to \mathbb{R}$ is a high order discrete-time control barrier function of relative degree m if there exist $\psi_i(x_k), i \in \{0, 1, ..., m\}$ defined by (16) and $\mathcal{C}_i, i \in \{0, 1, ..., m-1\}$ defined by (17) such that

$$\psi_m(x_k) \geq 0, \tag{18}$$

for all $x_k \in \bigcap_{i=0}^{m-1} \mathcal{C}_i$.

Theorem 1: Given a series of sets $\mathcal{C}_i, i \in \{0, 1, ..., m-1\}$ defined by (17) and a continuous function $h : \mathbb{R}^n \to \mathbb{R}$. If h is a high order discrete-time control barrier function of relative degree m defined on $\bigcap_{i=0}^{m-1} \mathcal{C}_i$, any discrete-time controller $u(k)$ ensuring (18) will render the set $\bigcap_{i=0}^{m-1} \mathcal{C}_i$ forward invariant.

Proof: If h is a high order discrete-time CBF, condition (18) holds, that is,

$$\begin{aligned} \psi_m(x_k) &= \Delta\psi_{m-1}(x_k, u_k) + \alpha_m(\psi_{m-1}(x_k)) \\ &\geq 0, \ \forall k \in N. \end{aligned}$$

According to Lemma 1, it is obvious that the corresponding set $\mathcal{C}_{m-1}$ is forward invariant, i.e., $x_k \in \mathcal{C}_{m-1}, \forall k \in N$ if $x_0 \in \mathcal{C}_{m-1}$. It implies that

$$\begin{aligned} \psi_{m-1}(x_k) &= \Delta\psi_{m-2}(x_k, u_k) + \alpha_{m-1}(\psi_{m-2}(x_k)) \\ &\geq 0, \ \forall k \in N, \end{aligned}$$

which verifies the forward invariance of set $\mathcal{C}_{m-2}$ through Lemma 1 again.

By iterative induction, it can be proved that if $x_0 \in \mathcal{C}_i$, then $x_k \in \mathcal{C}_i, \forall k \in N, \forall i \in \{0, 1, ..., m-1\}$. Therefore, the set $\bigcap_{i=0}^{m-1} \mathcal{C}_i$ is forward invariant. ■

Remark 4: In a sense, the above concept can be seen as a generalization of high order CBF from continuous-time systems [16] to discrete-time systems. It should be noted that, due to the characteristics of discrete-time CBF, the class $\mathcal{K}$ functions $\alpha_j(\cdot)$ are especially required to meet certain conditions ($\alpha_j(r) < r, j = 1, 2, ..., m$) compared with the continuous-time case.

C. Optimization problem unifying discrete-time CLF and high order discrete-time CBF

Similar to Section II-C, an optimization problem can be formed unifying discrete-time CLF and high order discrete-time CBF. In this optimization problem framework, the controller is designed in a minimally invasive way based on an existing nominal controller u_{norm} [1]. The discrete-time CBF works as a "safety filter" [12] for it modifies the system behavior when the desired action resulted from u_{norm} conflicts with the safety constraints.

$$\begin{aligned} u_k^* =& \underset{U_k}{argmin} \ \frac{1}{2}\|u_k - u_{norm_k}\|^2 + p \cdot \delta^2, \\ & U_k = \begin{bmatrix} u_k \\ \delta \end{bmatrix} \\ s.t. \quad & \Delta V(x_k, u_k) + c_3\|x_k\|^2 \leq \delta, \\ & \psi_m(x_k) \geq 0, \\ & u_{min} \leq u_k \leq u_{max}. \end{aligned} \tag{19}$$

Remark 5: Compared with continuous-time cases, the above optimization problem is not necessarily quadratic for general nonlinear discrete-time systems. However, if the discrete-time exponential CBF [20] is applied, i.e., class $\mathcal{K}$ functions are linear functions with positive coefficients, the nonlinear program (19) turns into a quadratically constrained quadratic program for a linear system or nonlinear control affine system with Linear and/or Quadratic Lyapunov and Barrier functions.

IV. Adaptive discrete-time control barrier functions

On the basis of high order discrete-time control barrier function, the feasibility of corresponding optimization problem is further studied in this section. Motivated by the adaptive idea of continuous-time systems in [19], the process of constructing adaptive discrete-time control barrier functions is presented.

A. Example of relative degree 2 system revisited

Considering the Example 1 again, high order discrete-time CBF is utilized to satisfy the system safety constraint $M - cx_{1_k} \geq 0$. For simplicity, let us set $c = 1$ and select the class $\mathcal{K}$ functions to be linear functions. A series of functions are derived as

$$\begin{aligned} \psi_0(x_k) =& M - x_{1_k}, \\ \psi_1(x_k) =& \Delta\psi_0(x_k, u_k) + \gamma_1\psi_0(x_k), \\ \psi_2(x_k) =& \Delta\psi_1(x_k, u_k) + \gamma_2\psi_1(x_k), \end{aligned} \tag{20}$$

where $\gamma_1, \gamma_2 \in (0, 1)$ are constants. According to the system state equations and condition (18), it can be easily obtained that

$$\begin{aligned} ba_{12}u_k \leq \ & [(a_{11} - 1)x_{1_k} + a_{12}x_{2_k}](1 - a_{11} - \gamma_1 - \gamma_2) + \\ & (M - x_{1_k})\gamma_1\gamma_2 - a_{12}[a_{21}x_{1_k} + (a_{22} - 1)x_{2_k}]. \end{aligned} \tag{21}$$

Suppose inequality (21) is a discrete-time CBF constraint in the optimization problem (19), then it corresponds to an upper (lower) bound constraint on control input u_k if $ba_{12} > 0$ ($ba_{12} < 0$), which may contradict with the predefined bound $[u_{min}, u_{max}]$. Consequently, we need to tune the parameters γ_1, γ_2 carefully to ensure feasibility of the optimization problem at each time step k. However, the environment and safety region of the system are likely to change over time in the real world modifying constraint (21) on u_k or affecting the capability of actuator, in which case the optimization problem may become infeasible for fixed γ_1, γ_2.

B. Adaptive discrete-time control barrier function

To improve the feasibility of the optimization problem when external environment changes, the definition of adaptive discrete-time control barrier function is formulated.

For discrete-time systems with safety constraints of relative degree m, just as the safety region may change over time, $\gamma_i, i = 1, 2, ..., m$ are set to be time-varying as well. These parameters work as penalty functions applied to class $\mathcal{K}$ functions and $(m-1)$ auxiliary systems are respectively constructed for $\gamma_i, i = 1, 2, ..., m-1$. The i-th auxiliary system is in the form of

$$\begin{aligned}\gamma_{i,1}(k+1) :=& \gamma_{i,1}(k) + \gamma_{i,2}(k),\\ \gamma_{i,2}(k+1) :=& \gamma_{i,2}(k) + \gamma_{i,3}(k),\\ &\vdots\\ \gamma_{i,m-i}(k+1) :=& \gamma_{i,m-i}(k) + v_i(k),\end{aligned} \tag{22}$$

for $i = 1, 2, ..., m-1$, where $(\gamma_{i,1}(k), \gamma_{i,2}(k), ..., \gamma_{i,m-i}(k))$ are the auxiliary state variables and $v_i(k)$ is the virtual control input. Among these state variables, $\gamma_{i,1}(k) := \gamma_i(k)$ is the required penalty term, while other variables have no special meaning.

Remark 6: Note that the auxiliary systems are only constructed for the first $(m-1)$ penalty functions, while γ_m is determined as a decision variable in the optimization problem. Besides, it doesn't matter what form the system is as long as $\gamma_i, i = 1, 2, ..., m-1$ is of relative degree $m-i$ for the i-th system. The linear form shown in (22) is just for simplicity.

With these time-varying penalty functions, (16) is modified as

$$\begin{aligned}\psi_0(x_k) :=& h(x_k),\\ \psi_1(x_k, \Gamma_k) :=& \Delta\psi_0(x_k, u_k) + \gamma_{i_k}\alpha_1(\psi_0(x_k)),\\ &\vdots\\ \psi_m(x_k, \Gamma_k) :=& \Delta\psi_{m-1}(x_k, \Gamma_k, u_k) + \gamma_{m_k}\alpha_m(\psi_{m-1}(x_k, \Gamma_k)),\end{aligned} \tag{23}$$

where γ_{i_k} represents $\gamma_i(k), i = 1, 2, ..., m$ at time step k, $\Gamma_k := (\gamma_{1_k}, \gamma_{2_k}, ..., \gamma_{m_k})$ and the other symbols inherit the previous definitions. The relevant sets are redefined as

$$\begin{aligned}\mathcal{C}_0 :=& \{x_k \in \mathcal{D} \mid \psi_0(x_k) \geq 0\},\\ \mathcal{C}_i :=& \{x_k \in \mathcal{D} \mid \psi_i(x_k, \Gamma_k) \geq 0\},\end{aligned} \tag{24}$$

for $i = 1, 2, ..., m-1$.

Definition 8: (Adaptive Discrete-Time Control Barrier Function) For the discrete-time systems (1) and (22) with $\psi_i(x_k, \Gamma_k), i \in \{1, 2, ..., m\}$ defined by (23) and $\mathcal{C}_i, i \in \{0, 1, ..., m-1\}$ defined by (24), the continuous function $h : \mathbb{R}^n \to \mathbb{R}$ is an adaptive discrete-time control barrier function of relative degree m, if $\gamma_i, \forall i \in \{1, 2, ..., m-1\}$ is a high order discrete-time control barrier function of relative degree $m-i$ satisfying $0 < \gamma_{i_k} < 1$ such that

$$\psi_m(x_k, \Gamma_k) \geq 0, \tag{25}$$

for all $x_k \in \bigcap_{i=0}^{m-1} \mathcal{C}_i$ and $\gamma_{m_k} \in (0, 1)$.

Theorem 2: Given a series of sets $\mathcal{C}_i, i \in \{0, 1, ..., m-1\}$ defined by (24) and a continuous function $h : \mathbb{R}^n \to \mathbb{R}$. If h is an adaptive discrete-time control barrier function of relative degree m defined on $\bigcap_{i=0}^{m-1} \mathcal{C}_i$, any discrete-time controller $u(k)$ ensuring (25) will render the set $\bigcap_{i=0}^{m-1} \mathcal{C}_i$ forward invariant.

Proof: If h is an adaptive discrete-time CBF, condition (25) holds, that is,

$$\begin{aligned}\psi_m(x_k, \Gamma_k) =& \Delta\psi_{m-1}(x_k, \Gamma_k, u_k)\\ &+ \gamma_{m_k}\alpha_m(\psi_{m-1}(x_k, \Gamma_k)) \geq 0,\end{aligned}$$

for all $k \in N$. According to Theorem 1, it is obvious that the corresponding set $\mathcal{C}_{m-1}$ is forward invariant for $\gamma_{m_k} \in (0, 1)$, which implies that if $x_0 \in \mathcal{C}_{m-1}$,

$$\begin{aligned}\psi_{m-1}(x_k, \Gamma_k) =& \Delta\psi_{m-2}(x_k, \Gamma_k, u_k)\\ &+ \gamma_{m-1_k}\alpha_{m-1}(\psi_{m-2}(x_k, \Gamma_k)) \geq 0,\end{aligned}$$

for all $k \in N$. Since $\gamma_{m-1_k} \in (0, 1)$, the forward invariance of set $\mathcal{C}_{m-2}$ is proved by Theorem 1 again.

Under the condition of $\gamma_{i_k} \in (0, 1), i \in \{1, 2, ..., m\}$, it can be iteratively proved that if $x_0 \in \mathcal{C}_i$, then $x_k \in \mathcal{C}_i, \forall k \in N, \forall i \in \{0, 1, ..., m-1\}$. Therefore, the set $\bigcap_{i=0}^{m-1} \mathcal{C}_i$ is forward invariant. ■

Remark 7: The conflict between the CBF constraint on control input u and the predefined control bounds can be avoided by adaptively adjusting $\gamma_i, i = 1, 2, ..., m$, where the variation of $\gamma_i, i = 1, 2, ..., m-1$ is caused by the virtual control input $v := (v_1, v_2, ..., v_{m-1})$. Therefore, the "adaptivity" of discrete-time control barrier functions comes from the input v of auxiliary systems.

However, constraints are imposed on the virtual input v because the state variables $\gamma_i, i = 1, 2, ..., m-1$ in auxiliary systems should be limited to the interval (0,1), which may have some impact on the adaptive effect. If the parameters of the auxiliary system are properly chosen, this impact can be ignored.

V. Application to three-link manipulator control

In this section, the proposed high order discrete-time CBF and adaptive discrete-time CBF are applied to the control of a three-link manipulator.

A. Optimization-based control design

The control design of the proposed novel concepts on discrete-time systems is elaborated before introducing the application to the manipulator.

Generally, for the high order discrete-time CBF, the optimal control framework (19) is employed to achieve safety-critical control. In (19), the CBF constraint can guarantee forward invariance of a safe set, i.e., ensure system safety, the CLF constraint can be used if convergence to a state is desired (performance/stability objectives), and the existing nominal controller u_{norm} performs predetermined control tasks. For the discrete-time system (1), the optimization problem is solved at each time step and the resulting optimal control input is applied at the current step. Then, update the state with dynamics (1) and repeat the process.

As for the adaptive case, replace the high order discrete-time CBF constraint (18) with the adaptive discrete-time CBF condition (25) and add the constraints on penalty functions $\gamma_i, i = 1, 2, ..., m$ in the optimization problem (19), which leads to improved feasibility.

According to the above analysis, the control design is summarized as below:

1) Given a discrete-time control system (1), specify its control tasks and safety conditions.
2) Find a suitable nominal controller u_{norm} or utilize the discrete-time CLF to implement the control tasks.
3) Define the safe sets according to safety conditions, and then derive the corresponding (high order/adaptive) discrete-time CBF constraints.
4) Combining the u_{norm}, CLF constraints, and CBF constraints, construct the optimal control problem based on (19) to yield a safe controller.

It is worth noting that both the nominal controller and the discrete-time CLF can be used for achieving control tasks, and how they are used depends on the actual situation. Next, the safety-critical control of the discrete-time three-link manipulator is analyzed.

B. Discrete-time model of an n-link manipulator

The discrete-time dynamical model of an n-link manipulator can be obtained by applying numerical discretization technique to the minimization of the Lagrange action functional [29], and an implicit form is shown in [30]:

$$B(q_{k+1})\dot{q}_{k+1} - B(q_k)\dot{q}_k - f(q_k, \dot{q}_k)T = Tu_k, \tag{26}$$

where $q_k, \dot{q}_k \in \mathbb{R}^n$ are the vectors of generalized joint angle and velocity at time step k respectively, $B(q_k)$ is the inertia matrix, $f(q_k, \dot{q}_k)$ represents centrifugal, Coriolis and gravitational torques, u_k denotes the control input vector and T denotes the sampling time.

Then, the explicit form is derived as

$$\begin{aligned} q_{k+1} =& q_k + a_k T\dot{q}_k, \\ \dot{q}_{k+1} =& B^{-1}(q_k + a_k T\dot{q}_k)[B(q_k)\dot{q}_k + f(q_k, \dot{q}_k)T] + \\ & B^{-1}(q_k + a_k T\dot{q}_k)Tu_k, \end{aligned} \tag{27}$$

where we approximate that $\dot{q}_{k+1} = a_k\dot{q}_k$, and a_k represents the change of the slope of the robot joint trajectories at time step k, which is estimated by the actual trajectory to be followed. The resulting model is in state space form of

$$\begin{aligned} x_1(k+1) =& x_1(k) + a_k T x_2(k), \\ x_2(k+1) =& B^{-1}(x_1(k) + a_k T x_2(k)) \cdot \\ & [B(x_1(k))x_2(k) + f(x_1(k), x_2(k))T] + \\ & B^{-1}(x_1(k) + a_k T x_2(k))Tu(k), \end{aligned} \tag{28}$$

where $x_1(k) = q_k$, $x_2(k) = \dot{q}_k$ and $u(k) = u_k$.

C. Problem setup

Given the discrete-time model of a three-link manipulator in the Appendix of [29], the sampling time is $T = 0.01s$. The aim is to drive the joint angle $q = [q_1, q_2, q_3]^T$ to track specific sinusoidal trajectory $r = [\frac{2\pi}{3}sin(t_k), \frac{\pi}{2}sin(0.65t_k) + \frac{\pi}{3}, \frac{\pi}{2}cos(1.5t_k)]^T$rad. This is easy to implement with a nominal PD controller u_{norm} in ideal conditions. In practice, suppose that the joint angles should be constrained for safe operation: $q_{1_{max}} = \frac{3\pi}{5}$rad, $q_{1_{min}} = -\pi$rad, $q_{2_{max}} = \frac{3\pi}{4}$rad, $q_{2_{min}} = -\frac{\pi}{8}$rad, $q_{3_{max}} = \pi$rad, $q_{3_{min}} = -\frac{5\pi}{12}$rad, and the system has limited control input: $u_{1_{max}} = 800$Nm, $u_{1_{min}} = -1000$Nm, $u_{2_{max}} = 1400$Nm, $u_{2_{min}} = -1600$Nm, $u_{3_{max}} = 1000$Nm, $u_{3_{min}} = -800$Nm. The desired trajectory r apparently violates the joint constraints.

D. Simulation results with high order discrete-time CBF

In order to keep the actual trajectory within the safety boundary, constraints on the joint angle can be converted into $h_i = q_{i_{max}} - q_i \geq 0$ and $b_i = q_i - q_{i_{min}} \geq 0$ for $i = 1, 2, 3$, which are of relative degree two for the discrete-time model in (28). Considering the desired trajectory, we only set

$$\begin{aligned} h_1 &= q_{1_{max}} - q_1, \\ h_2 &= q_{2_{max}} - q_2, \\ b_2 &= q_2 - q_{2_{min}}, \\ b_3 &= q_3 - q_{3_{min}}. \end{aligned} \tag{29}$$

Safe sets are defined on the basis of the above safety conditions, such as $\mathcal{S}_1 := \{q_{1_k} \in \mathbb{R}^3 \mid q_{1_{max}} - q_{1_k} \geq 0\}$ for the upper bound $q_{1_{max}}$ on joint q_1.

According to the concept of high order discrete-time CBF, all the class $\mathcal{K}$ functions are chosen to be linear functions with positive coefficients set as $\gamma_{h_{11}} = 0.9, \gamma_{h_{12}} = 0.1, \gamma_{h_{21}} = 0.9, \gamma_{h_{22}} = 0.1, \gamma_{b_{21}} = 0.2, \gamma_{b_{22}} = 0.8, \gamma_{b_{31}} = 0.2, \gamma_{b_{32}} = 0.8$. For example, define

$$\begin{aligned} \psi_{h_{10}}(x_k) =& q_{1_{max}} - q_{1_k}, \\ \psi_{h_{11}}(x_k) =& \Delta\psi_0(x_k, u_k) + \gamma_{h_{11}}\psi_0(x_k), \\ \psi_{h_{12}}(x_k) =& \Delta\psi_1(x_k, u_k) + \gamma_{h_{12}}\psi_1(x_k), \end{aligned} \tag{30}$$

where $\psi_{h_{12}}(x_k) \geq 0$ is the corresponding CBF condition for h_1.

Then, an optimization problem can be constructed as presented in (19). The nominal PD controller with $K_P = diag(5000, 6000, 4000)$, $K_D = diag(300, 500, 100)$ allows the system to track the desired trajectory but may not be safe. Only high order discrete-time CBF constraints and control input constraints are added, and the discrete-time CLF constraint is ignored. The optimization problem is solved using the solver “fmincon” in MATLAB R2019a. In fact, it is a quadratic programming problem.

The simulation results are shown in Fig. 1 and Fig. 2. Figure 1 depicts the joint angle trajectory. The black dashed line and the black dash-dotted line represent the upper and lower constraints on joint angle, respectively. It can be clearly seen that the joint angles are constrained within the safety boundary while tracking the desired trajectory.

In Fig. 2, the control input u_k is within the control boundary for each time step k. For an upper (lower) constraint on q_k, the related high order discrete-time CBF constraint results in an upper (lower) bound on u_k most of the time, so that as the CBF constraint works, u_k is limited to a smaller (larger) value compared with u_{norm}. However, when the desired trajectory is near the sinusoidal peak (trough), the corresponding control will appear an overshooting, that is, u_k tends to approach the nominal control u_{norm} again. This is due to the fact that a_k changes from a positive number to a negative number at this specific moment, which affects the constraints on u_k imposed by high order discrete-time CBF.

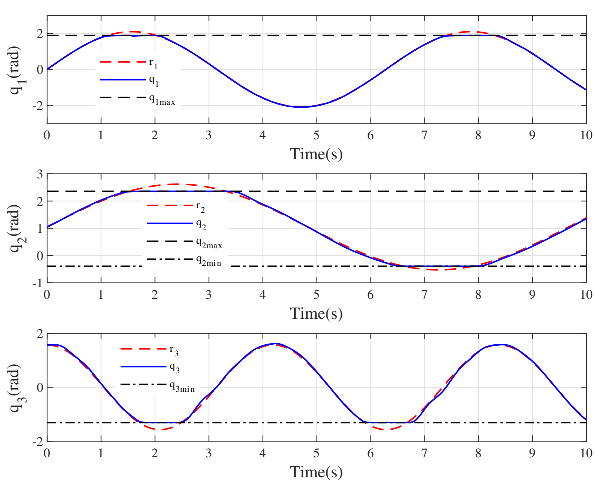

Fig. 1. Joint angle trajectory in simulation with high order discrete-time CBF. The blue solid line represents the actual trajectory for each joint while the red dashed line represents the desired trajectory. The black dashed line and the black dash-dotted line indicate the upper and lower bound on the joint angle respectively.

E. Simulation results with adaptive discrete-time CBF

Next, this section will demonstrate how to improve the feasibility utilizing adaptive discrete-time CBFs. Let us modify the settings for previous problem in Section V-C slightly. Just take the upper bound constraint on joint q_1 into account and remove the constraints on the other two joints. It is assumed that the change of environment causes fluctuation of output capacity of the actuator. During the simulation, the control input u_1 is restricted with a fixed upper bound $u_{1_{max}} = 400$Nm, while the lower bound $u_{1_{min}}$ is tightened to verify the proposed method.

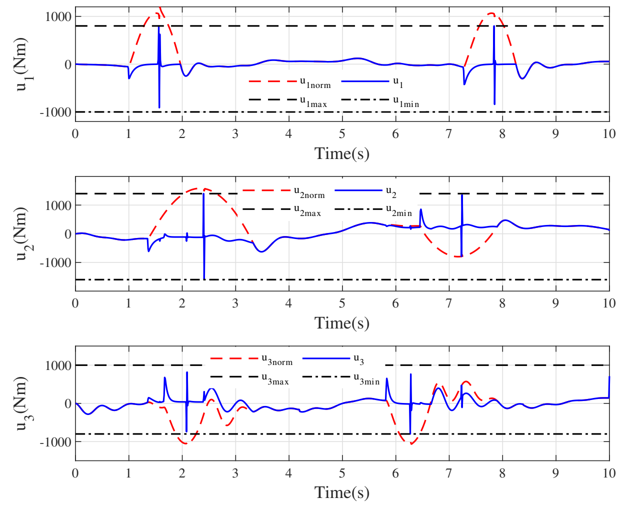

Fig. 2. Control input u in simulation with high order discrete-time CBF. The blue solid line represents the actual control input for each joint while the red dashed line represents the nominal control. The black dashed line and the black dash-dotted line indicate the upper and lower bound on control input respectively.

An optimization problem similar to (19) is constructed based on adaptive discrete-time CBF and discrete-time CLF as below:

$$\begin{aligned}
u_k^* =& \underset{U_k}{argmin}\ P_1\|u_k - u_{norm_k}\|^2 + P_2 v_k^2 && \text{(31a)}\\
&+ P_3\delta^2 + P_4(\gamma_{2_k} - \gamma_2^*)^2, &&\\
&U_k = \begin{bmatrix} u_k \\ v_k \\ \delta \\ \gamma_{2_k} \end{bmatrix} \in \mathbb{R}^6 &&\\
s.t.\quad & \psi_2(x_k, \Gamma_k) \geq 0, && \text{(31b)}\\
& \Delta H_{1,2}(\gamma_{1_k}, v_k) + \beta_{1,2} \cdot H_{1,2}(\gamma_{1_k}) \geq 0, && \text{(31c)}\\
& \Delta V(\gamma_{1_k}, v_k) + c_3\|\gamma_{1_k}\|^2 \leq \delta, && \text{(31d)}\\
& 0 < \gamma_{2_k} < 1, && \text{(31e)}\\
& u_{min} \leq u_k \leq u_{max}. && \text{(31f)}
\end{aligned}$$

(31b) represents the adaptive discrete-time CBF condition for $h_1 = q_{1_{max}} - q_1 = \frac{3\pi}{5} - q_1$, where class $\mathcal{K}$ functions are linear functions with positive coefficients $\gamma_{1_k}, \gamma_{2_k}$. The auxiliary system is chosen as

$$\gamma_{1_{k+1}} = \gamma_{1_k} + v_k.$$

(31c) guarantees $0 < \gamma_{1_k} < 1$ by constructing discrete-time CBFs

$$\begin{aligned}
H_1 &= \gamma_1 - \gamma_{1_{min}},\\
H_2 &= \gamma_{1_{max}} - \gamma_1,
\end{aligned}$$

where $\gamma_{1_{min}}$ and $\gamma_{1_{max}}$ are selected as close as possible to 0 and 1 respectively, along with $\beta_1 = 0.2, \beta_2 = 0.5$. (31d) is the discrete-time CLF condition for $V = (\gamma_{1_k} - \gamma_1^*)^2$, stabilizing γ_{1_k} to desired value $\gamma_1^* = 0.8$ with $c_3 = 1$ and a relaxation parameter δ. The cost function (31a) contains the term $(\gamma_{2_k} - \gamma_2^*)^2$ that stabilizes γ_{2_k} to desired value $\gamma_2^* = 0.05$ in the process of optimization. The weights are set as $P_1 = e^{-8}, P_2 = P_3 = P_4 = 1$. This nonlinear programming is solved using "fmincon" in MATLAB R2019a.

As shown by the cyan line in Fig. 3, if the high order discrete-time CBF is used to deal with such a problem with fixed $\gamma_1 = 0.8$ and $\gamma_2 = 0.05$ for $u_{1_{min}} = -400$Nm, the optimization problem becomes infeasible at some step k. It is because the high order discrete-time CBF constraint on u_k conflicts with the lower bound $u_{1_{min}}$. Adaptive discrete-time CBF can significantly improve the feasibility. Figure 3 shows that the optimization problem (31) is always feasible when the lower bound $u_{1_{min}}$ is tightened at $(-400, -300, -200)$Nm in turn. Figure 4 plots γ_1, γ_2 and v at each time step k. After the CBF constraint becomes active, these parameters vary with time, providing relaxation on u and thus avoiding conflicts between constraints.

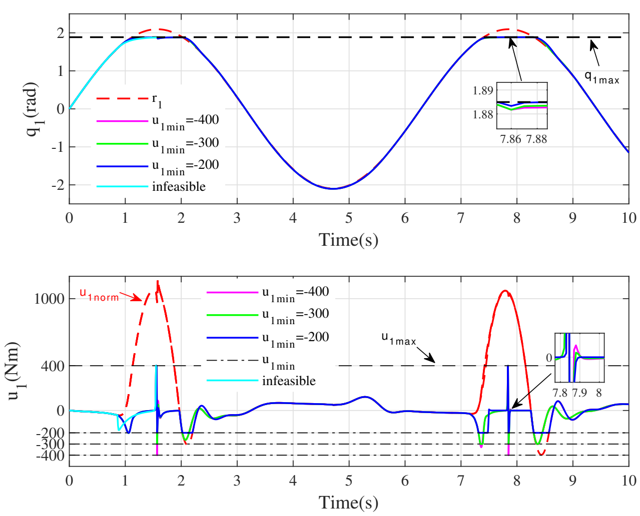

Fig. 3. Joint angle and control input for q_1 in simulation with adaptive discrete-time CBF. For lower control bound $u_{1_{min}}$ with different values, solid lines depict the actual joint trajectory and control input; red dashed lines represent the desired trajectory and nominal control; black dashed lines and black dash-dotted lines represent the limits on joint angle and control input. The cyan solid lines indicate the infeasibility of non-adaptive case.

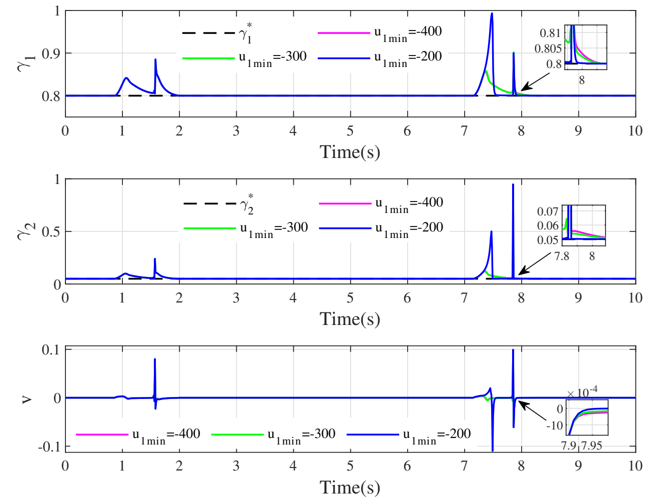

Fig. 4. Variation of γ_1, γ_2 and virtual control input v for the auxiliary system when the lower control bound $u_{1_{min}}$ changes with different values. The black dashed lines indicate the desired γ_1^*, γ_2^*.

VI. Conclusion

In this paper, high order discrete-time control barrier function and adaptive discrete-time control barrier function are formulated to achieve set invariance. We generalize high order discrete-time CBF for high relative degree systems and combine it with discrete-time CLF into an optimization problem. The adaptive discrete-time control barrier function is proposed to improve the feasibility of optimization problems. Finally, numerical simulation on the three-link manipulator is carried out to verify the effectiveness of the proposed methods by applying constraints on joint angles. In future work, we will consider the robustness of high order discrete-time CBF to external disturbances.

References

[1] A. D. Ames, S. Coogan, M. Egerstedt, G. Notomista, K. Sreenath and P. Tabuada, "Control Barrier Functions: Theory and Applications," in 2019 18th European Control Conference (ECC), 2019, pp. 3420-3431.

[2] W. He, C. Xue, X. Yu, Z. Li and C. Yang, "Admittance-Based Controller Design for Physical Human–Robot Interaction in the Constrained Task Space," IEEE Transactions on Automation Science and Engineering, vol. 17, no. 4, pp. 1937-1949, 2020.

[3] R. A. Freeman and P. V. Kokotović, Robust Nonlinear Control Design: State-Space and Lyapunov Techniques, Birkhäuser, Boston, 1996.

[4] A. D. Ames, K. Galloway and J.W. Grizzle, "Control Lyapunov Functions and Hybrid Zero Dynamics," in 51st IEEE Conference on Decision and Control (CDC), 2012, pp. 6837–6842.

[5] S. Prajna and A. Jadbabaie, "Safety Verification of Hybrid Systems Using Barrier Certificates," in International Workshop on Hybrid Systems: Computation and Control, 2004, pp. 477–492.

[6] P. Wieland and F. Allgöwer, "Constructive Safety Using Control Barrier Functions," IFAC Proceedings Volumes, vol. 40, no. 12, pp. 462–467, 2007.

[7] A. D. Ames, X. Xu, J. W. Grizzle and P. Tabuada. "Control Barrier Function Based Quadratic Programs for Safety Critical Systems," IEEE Transactions on Automatic Control, vol. 62, no. 8, pp. 3861–3876, 2017.

[8] X. Xu, J. W. Grizzle, P. Tabuada and A. D. Ames, "Correctness Guarantees for the Composition of Lane Keeping and Adaptive Cruise Control," IEEE Transactions on Automation Science and Engineering, vol. 15, no. 3, pp. 1216–1229, 2018.

[9] Z. Liu, Z. Han, Z. Zhao and W. He, "Modeling and Adaptive Control for a Spatial Flexible Spacecraft with Unknown Actuator Failures," Science China Information Sciences, vol. 64, pp. 1-16(Art. no. 152208), 2021.

[10] A. D. Ames, J. W. Grizzle and P. Tabuada, "Control Barrier Function Based Quadratic Programs with Application to Adaptive Cruise Control," in 53rd IEEE Conference on Decision and Control (CDC), 2014, pp. 6271–6278.

[11] S.-C. Hsu, X. Xu and A. D. Ames, "Control Barrier Functions Based Quadratic Programs with Application to Bipedal Robotic Walking," in American Control Conference (ACC), 2015, pp. 4542–4548.

[12] F. Ferraguti, M. Bertuletti, C. T. Landi, M. Bonfè, C. Fantuzzi and C. Secchi, "A Control Barrier Function Approach for Maximizing Performance While Fulfilling to ISO/TS 15066 Regulations," IEEE Robotics and Automation Letters, vol. 5, no. 4, pp. 5921-5928, 2020.

[13] W. S. Cortez and D. V. Dimarogonas, "Correct-by-Design Control Barrier Functions for Euler-Lagrange Systems with Input Constraints," in American Control Conference (ACC), 2020, pp. 950-955.

[14] W. S. Cortez, D. Oetomo, C. Manzie and P. Choong, "Control Barrier Functions for Mechanical Systems: Theory and Application to Robotic

Grasping," IEEE Transactions on Control Systems Technology, vol. 29, no. 2, pp. 530-545, 2021.
[15] Q. Nguyen and K. Sreenath, "Exponential Control Barrier Functions for Enforcing High Relative-Degree Safety-Critical Constraints," in American Control Conference (ACC), 2016, pp. 322-328.
[16] W. Xiao and C. Belta, "Control Barrier Functions for Systems with High Relative Degree," in 58th IEEE Conference on Decision and Control (CDC), 2019, pp. 27–34.
[17] W. Xiao and C. Belta, "High Order Control Barrier Functions," IEEE Transactions on Automatic Control, to be published. Doi: 10.1109/TAC.2021.3105491.
[18] X. Tan, W. S. Cortez and D. V. Dimarogonas, "High-order Barrier Functions: Robustness, Safety and Performance-Critical Control," IEEE Transactions on Automatic Control, to be published. Doi: 10.1109/TAC.2021.3089639.
[19] W. Xiao, C. Belta and C. G. Cassandras, "Adaptive Control Barrier Functions," IEEE Transactions on Automatic Control, to be published. Doi: 10.1109/TAC.2021.3074895.
[20] A. Agrawal and K. Sreenath. "Discrete Control Barrier Functions for Safety-Critical Control of Discrete Systems with Application to Bipedal Robot Navigation," in Robotics: Science and Systems, 2017.
[21] J. W. Grizzle and J.-M. Kang, "Discrete-Time Control Design with Positive Semi-Definite Lyapunov Functions," Systems & Control Letters, vol. 43, no. 4, pp. 287–292, 2001.
[22] J. Zeng, B. Zhang and K. Sreenath, "Safety-Critical Model Predictive Control with Discrete-Time Control Barrier Function," in American Control Conference (ACC), 2021, pp. 3882-3889.
[23] R. Takano, H. Oyama and M. Yamakita, "Application of Robust Control Barrier Function with Stochastic Disturbance Model for Discrete Time Systems," IFAC-PapersOnLine, vol. 51, no. 31, pp. 46-51, 2018.
[24] M. Ahmadi, A. Singletary, J. W. Burdick and A. D. Ames, "Safe Policy Synthesis in Multi-Agent POMDPs via Discrete-Time Barrier Functions," in 58th IEEE Conference on Decision and Control (CDC), 2019, pp. 4797-4803.
[25] S. Chakrabarty, B. Bandyopadhyay, J. A. Moreno and L. Fridman, "Discrete Sliding Mode Control for Systems with Arbitrary Relative Degree Output" in 2016 14th International Workshop on Variable Structure Systems (VSS), 2016, pp. 160-165.
[26] M. Sun and D. Wang, "Initial Shift Issues on Discrete-time Iterative Learning Control with System Relative Degree," IEEE Transactions on Automatic Control, vol. 48, no. 1, pp. 144-148, 2003.
[27] W. He, X. Mu, L. Zhang and Y. Zou, "Modeling and Trajectory Tracking Control for Flapping-wing Micro Aerial Vehicles," IEEE/CAA Journal of Automatica Sinica, vol. 8, no. 1, pp. 148-156, 2021.
[28] H. K. Khalil, Nonlinear Systems, Prentice Hall, 3rd Edtion, 2002.
[29] S. Nicosia, P. Tomei and A. Tornambe, "Discrete Dynamics of Robot Arms," IFAC Proceedings Volumes, vol. 19, no. 14, pp. 89-94, 1986.
[30] F. Sun and Z. Sun, "Stable Sampled-data Adaptive Control of Robot Arms Using Neural Networks," Journal of Intelligent and Robotic Systems, vol. 20, no. 2, pp. 131–155, 1997.

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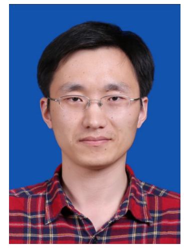

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