

# Adaptive Control for State Synchronization of Nonlinear Haptic Telerobotic Systems with Asymmetric Varying Time Delays

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**Abstract** In this paper, we introduce a new adaptive controller design scheme for nonlinear telerobotic systems with varying time delays where the delays and their variation rates are unknown. The designed controller has the ability to synchronize the state behaviors of the local and the remote robots. In this paper, asymptotic stability in the presence of varying time delays is of interest. Using the proposed controller, asymptotic stability of the bilateral telerobotic system subject to any bounded yet unknown varying delay with a bounded yet unknown rate of change can be guaranteed. Besides the varying time delay, the proposed adaptive controller has the ability to adapt to the parameter variations in the local

and the remote robots' dynamics. It is shown that position and velocity errors between the local and the remote manipulators converge to the zero asymptotically, thus ensuring teleoperation transparency. Experimental and simulation results with a pair of PHANToM haptic devices and a pair of planar manipulators under varying time delays in the communication channel demonstrate the effectiveness of the proposed scheme.

**Keywords** Teleoperation · State synchronization · Varying time delay · Lyapunov–Krasovskii function · Adaptive control

## 1 Introduction

Using a telerobotic system, a human operator can carry out tasks in a remote environment. Different applications of telerobotic systems vary from tele-surgery to space manipulation. Teleoperation performance is greatly enhanced if haptic feedback about interaction occurring between the remote robot and the remote environment is provided to the human operator through the local robot [1]. Such systems are called *bilateral* as information flows in two directions between the operator and the remote environment [2]. On the other hand, in telerobotic applications with a distance between the local and the remote robots, there will be

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a time delay in the communication channel of the system [3–5]; this time delay in the closed-loop system can destabilize the telerobotic system [5–7].

Control schemes have been developed to compensate for the time delay, most of which are based on the passivity theory. Passivity based control schemes [12–15] are inspired from energy interaction between interconnected systems [16]. Anderson and Spong [8] proposed scattering schemes based on the passivity theory. A similar passivity-based scheme is the wave variable formulation for two port networks proposed by Niemeyer and Slotine [9, 10]. These passivity based approaches can guarantee the passivity of bilateral teleoperation systems only for constant time delays [17].

In most passivity based bilateral teleoperation architectures, only velocity and force information is transmitted between the local and the remote sides [18]. This means that only force and velocity tracking can be ensured in such architectures, leaving the possibility that any initial position mismatch between the local and the remote robots would lead to a position drift between the robots. To solve this problem, one can transmit position information along with the velocity information through the communication channel [19–21].

The scattering and the wave variable approaches are the best known methods in the passivity based control framework, and have been the subject of recent studies concerning teleoperation under varying delays. An extension of the scattering approach to the case of varying time delays is reported in [11], in which a small positive gain is added in the communication channel to dissipate the extra energy generated due to the distorted scattered signals caused by varying time delay. The gain should be less than  $1 - \dot{T}$ , where  $T$  is the instantaneous value of the varying time delay, such that communication channel remains passive. Also, an extended version of the wave variable approach with varying time delay was reported in [22], in which besides the wave variables, extra variables are transmitted in the communication channel to preserve passivity.

Important schemes in passivity based control of manipulators in the presence of variable time delays include damping injection controllers usually

referred to as P+d and PD+d [23]. The physical interpretation of damping injection controllers is that the interconnection between the remote and the local manipulators includes virtual dampers and springs. In damping injection controllers, a control gain affecting the velocity signals should have the exact value of  $\sqrt{1 - \dot{T}}$ . Although these controllers guarantee asymptotic stability of velocities and position errors and are robust to time varying delays, their stability condition is not delay-independent and they are sensitive to rapid changes in delays. This is due to the variable gain  $\sqrt{1 - \dot{T}}$ , which depends on the time derivative of the delay [23].

Another interesting and recent topic in passivity-based analysis of telerobotic systems is the synchronization-based approach [24–26]. In synchronization-based schemes which its applications on bilateral teleoperation were studied in [24], all states including positions and velocities of local and the remote robots act synchronously.

In this paper, a new controller is proposed to guarantee asymptotic stability of the bilateral teleoperation system. The delay-independent proposed controller is an extension of the controller in [26] to the case of time varying delays and is able to synchronize the behavior of the local and the remote robots in the presence of unknown varying time delays in communication channel. In this paper, synchronization means asymptotic state (joint angle) tracking between the local and the remote robots. The proposed adaptive controller is able to guarantee state synchronization between the local and the remote robots in the presence of unknown varying time delays with unknown rates of variation. To use this controller, there is no need to know the robots parameters exactly; only estimations of the robots parameters are used in the controller. The controller parameters will change adaptively to guarantee the zero convergence of the tracking errors under robot parameter variations.

This paper is organized as follows. Section 2 concerns that tele-manipulator dynamic model while the controller design is presented in Section 3. In Sections 4 and 5, simulation and experimental results demonstrate the efficiency of the proposed controller followed by the conclusions presented in Section 6.

## 2 Tele-Manipulator Dynamic Model

The local and the remote manipulators can be modeled by the following nonlinear equations:

$$\begin{aligned}
 M_l(q_l)\ddot{q}_l + C_l(q_l, \dot{q}_l)\dot{q}_l + G_l(q_l) &= \tau_h - \tau_l \\
 M_r(q_r)\ddot{q}_r + C_r(q_r, \dot{q}_r)\dot{q}_r + G_r(q_r) &= \tau_r - \tau_e \quad (1)
 \end{aligned}$$

where  $q_i, \dot{q}_i$  and  $\ddot{q}_i$  for  $i \in \{r, l\}$  are the joint positions, velocities and accelerations of the local and the remote robots, respectively. Also,  $M_i(q_i), C_i(q_i, \dot{q}_i)$  and  $G_i(q_i)$  are the inertia matrix, the Coriolis and centrifugal term and the gravitational force respectively,  $\tau_l$  and  $\tau_r$  are control torques for the local and the remote robots, and  $\tau_h$  and  $\tau_e$  are applied torques from the human operator and the environment sides respectively.

Some important properties of the above nonlinear dynamic model are [27, 28]:

1. For a manipulator with revolute joints, the inertia matrix  $M_i(q_i)$  is symmetric positive definite and has the following upper and lower bounds:

$$0 < \lambda_{\min}(M_i) I \leq M_i(q_i) \leq \lambda_{\max}(M_i) I \leq \infty$$

where  $I$  stands for identity matrix

2. For a manipulator, the relation between the Coriolis/centrifugal and the inertia matrices is as follows:

$$\dot{M}_i(q_i) = C_i(q_i, \dot{q}_i) + C_i^T(q_i, \dot{q}_i)$$

3. For a manipulator with revolute joints, there exists a positive number  $\eta$  bounding the Coriolis/centrifugal terms as follows:

$$|C_i(q_i, \dot{q}_i)\dot{q}_i| \leq \eta \dot{q}_i^2$$

4. The nonlinear manipulator dynamics could be linearly parameterized as follows [28]:

$$\begin{aligned}
 M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) \\
 = Y_i(q_i, \dot{q}_i, \ddot{q}_i)\theta_i
 \end{aligned}$$

where  $Y_i$  is a matrix of known functions of the generalized coordinates and their higher derivatives and  $\theta_i$  is a vector of the manipulator dynamic parameters.

## 3 Control Design

In this part, the proposed controller to cope with varying time delays in a telerobotic system is presented. Since the model and consequently the dynamic equation of the system is uncertain. So, the estimates of the robots' dynamics are employed in the controllers  $\tau_l$  and  $\tau_r$ .

The controllers  $\tau_l$  and  $\tau_r$  in Eq. 1, are defined as follows

$$\begin{aligned}
 \tau_l &= -\hat{M}_l(q_l)\dot{e}_{pl} - \hat{C}_l(q_l, \dot{q}_l)e_{pl} - \hat{G}_l(q_l) + \bar{\tau}_l \\
 \tau_r &= \hat{M}_r(q_r)\dot{e}_{pr} + \hat{C}_r(q_r, \dot{q}_r)e_{pr} + \hat{G}_r(q_r) - \bar{\tau}_r \quad (2)
 \end{aligned}$$

where  $\hat{\cdot}$  represents estimates of the remote and the local manipulators parameters and  $\bar{\tau}_i$  for  $i \in \{l, r\}$  are the new control signals. Also,  $e_{pl}$  and  $e_{pr}$ , which are position errors in local and remote sides, are defined as

$$\begin{aligned}
 e_{pl} &\triangleq q_r(t - T_2(t)) - q_l(t) \\
 e_{pr} &\triangleq q_l(t - T_1(t)) - q_r(t) \quad (3)
 \end{aligned}$$

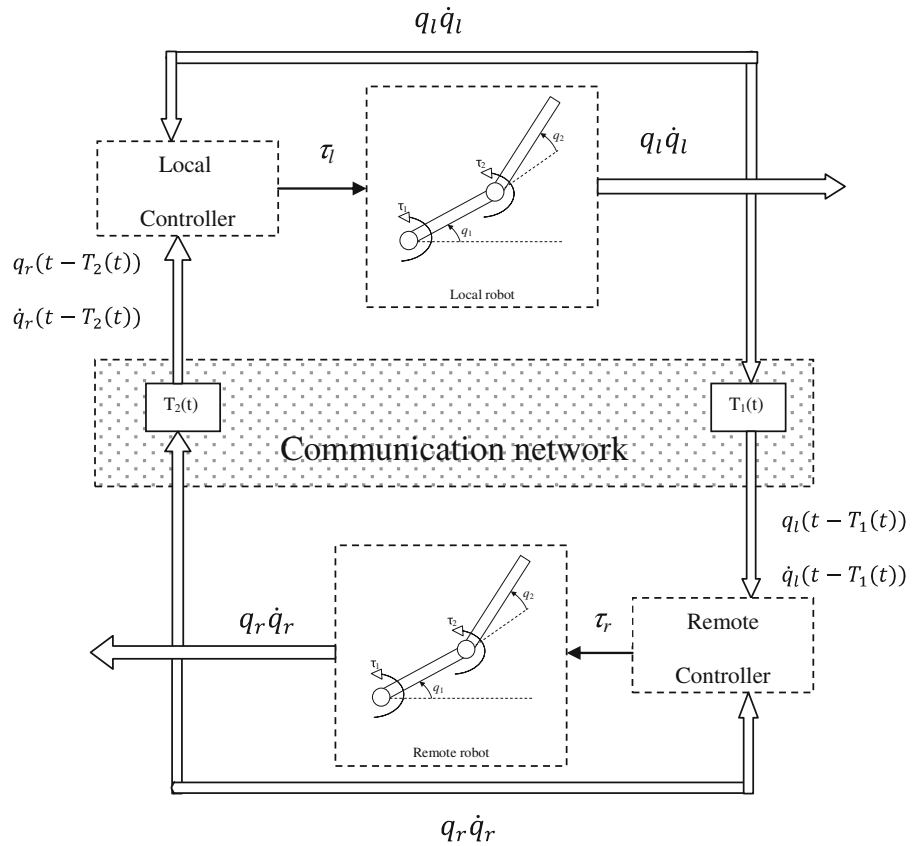
where  $T_1(t)$  is the delay in the feedforward path and  $T_2(t)$  is the delay in the feedback path. The overall scheme of teleoperation with varying time delay is shown in Fig. 1.

We proposed to define the new control signals, i.e.,  $\bar{\tau}_l$  and  $\bar{\tau}_r$ , as follows:

$$\begin{aligned}
 \bar{\tau}_l &= \begin{cases} K_l \varepsilon_l - \frac{1}{2} \dot{e}_{pl} - \frac{1}{2} e_{vl} \\ - \frac{e_{vl}^T (e_{pl} + \dot{e}_{pl} - e_{vl})}{2 \|\varepsilon_l\|_2^2} \varepsilon_l, & \|\varepsilon_l\|_2 \neq 0 \\ 0, & \|\varepsilon_l\|_2 = 0 \end{cases} \\
 \bar{\tau}_r &= \begin{cases} K_r \varepsilon_r - \frac{1}{2} \dot{e}_{pr} - \frac{1}{2} e_{vr} \\ - \frac{e_{vr}^T (e_{pr} + \dot{e}_{pr} - e_{vr})}{2 \|\varepsilon_r\|_2^2} \varepsilon_r, & \|\varepsilon_r\|_2 \neq 0 \\ 0, & \|\varepsilon_r\|_2 = 0 \end{cases} \quad (4)
 \end{aligned}$$

where  $K_i$  for  $i \in \{l, r\}$  is a positive definite matrix and  $\|\cdot\|_2$  denotes Euclidean norm. Also,  $\dot{e}_{pi}$  is the time derivative of position error  $e_{pi}$ , and  $e_{vi}$

**Fig. 1** The closed-loop teleoperation system under varying time delays



(velocity error) and  $\varepsilon_i$  for  $i \in \{l, r\}$  are defined as follows:

$$\begin{aligned} \varepsilon_i &\triangleq \dot{q}_i - e_{pi} & i \in \{l, r\} \\ e_{vl} &\triangleq \dot{q}_r(t - T_2(t)) - \dot{q}_l \\ e_{vr} &\triangleq \dot{q}_l(t - T_1(t)) - \dot{q}_r \end{aligned} \quad (5)$$

Note that because of the variation of time delays, the velocity error  $e_{vi}$  and the derivative of the position error  $\dot{e}_{pi}$  are not the same.

Combining Eqs. 2 and 1, the closed-loop system equations are found:

$$\begin{aligned} &M_l(q_l) \dot{e}_l + C_l(q_l, \dot{q}_l) \varepsilon_l \\ &= -\tilde{M}_l(q_l) \dot{e}_{pl} - \tilde{C}_l(q_l, \dot{q}_l) e_{pl} - \tilde{G}_l(q_l) - \bar{\tau}_l + \tau_h \\ &M_r(q_r) \dot{e}_r + C_r(q_r, \dot{q}_r) \varepsilon_r \\ &= -\tilde{M}_r(q_r) \dot{e}_{pr} - \tilde{C}_r(q_r, \dot{q}_r) e_{pr} - \tilde{G}_r(q_r) - \bar{\tau}_r - \tau_e \end{aligned} \quad (6)$$

where  $\sim$  represents the estimation error in the manipulator parameters, e.g.,  $\tilde{M}_i = M_i - \hat{M}_i$ ,  $\tilde{C}_i = C_i - \hat{C}_i$ , and  $\tilde{G}_i = G_i - \hat{G}_i$  for  $i \in \{l, r\}$ .

Using the fact that the equations of robot motions are linear in their parameters (Property IV), let us define the regressor matrix  $Y_i$  and the parameter vector  $\theta$  such that the nominal robot dynamics can be written as

$$M(q) \dot{e} + C(q, \dot{q}) e + G(q) = -Y(q, \dot{q}, e, \dot{e}) \theta \quad (7)$$

Equation 7 can be achieved from Property IV via replacing  $\ddot{q}$  with  $\dot{e}$  and the exterior  $\dot{q}$  in  $C(q, \dot{q}) \dot{q}$  with  $e$  and then negating  $Y$ .

Using the above linearity property, we have

$$\begin{aligned} &Y_i(q_i, \dot{q}_i, e_{pi}, \dot{e}_{pi}) \tilde{\theta}_i \\ &= -\tilde{M}_i(q_i) \dot{e}_{pi} - \tilde{C}_i(q_i, \dot{q}_i) e_{pi} - \tilde{G}_i(q_i), \quad i \in \{l, r\} \end{aligned} \quad (8)$$

where  $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$  and the regressor  $Y_i(q_i, \dot{q}_i, e_{pi}, \dot{e}_{pi})$  is a matrix whose elements are known functions of the generalized coordinates, derivatives of generalized coordinates, position errors and velocity errors. It is possible to find the following closed-loop dynamical equations from the above.

$$\begin{aligned}
 &M_l(q_l) \dot{e}_l + C_l(q_l, \dot{q}_l) e_l \\
 &= Y_l(q_l, \dot{q}_l, e_{pl}, \dot{e}_{pl}) \tilde{\theta}_l - \bar{\tau}_l + \tau_h \\
 &M_r(q_r) \dot{e}_r + C_r(q_r, \dot{q}_r) e_r \\
 &= Y_r(q_r, \dot{q}_r, e_{pr}, \dot{e}_{pr}) \tilde{\theta}_r - \bar{\tau}_r - \tau_e
 \end{aligned} \tag{9}$$

Now, we introduce the following adaptive update law for manipulators parameter estimation to be used in conjunction with the controllers (2):

$$\dot{\hat{\theta}}_i = \Gamma Y_i^T \varepsilon_i \quad i \in \{l, r\} \tag{10}$$

In the following, we analyze the stability of the system in the sense of Lyapunov.

*Theory 1* In free motion ( $\tau_h = \tau_e = 0$ ), the bilateral tele-manipulator (1) with the controller (2)–(5) is asymptotically stable in the sense of Lyapunov. Also,  $q_i$  converges to a constant value, and the position error  $e_{pi}$  and the velocity error  $e_{vi}$  converge to zero for any bounded varying time delay with a bounded time derivative. Here,  $i \in \{l, r\}$ .

*Proof* To study the asymptotic stability in the sense of Lyapunov under varying time delays in the communication channel, we use the following Lyapunov–Krasovskii functional:

$$\begin{aligned}
 V = &\frac{1}{2} \int_{t-T_1(t)}^t \dot{q}_l^T \dot{q}_l dt + \frac{1}{2} \int_{t-T_2(t)}^t \dot{q}_r^T \dot{q}_r dt \\
 &+ \frac{1}{2} \sum_{i \in \{r,l\}} \left[ \varepsilon_i^T M_i \varepsilon_i + \tilde{\theta}_i^T \Gamma^{-1} \tilde{\theta}_i + \frac{1}{2} e_{pi}^T e_{pi} \right]
 \end{aligned} \tag{11}$$

where  $\Gamma$  is a positive definite matrix. The time derivative of  $V$  is

$$\begin{aligned}
 \dot{V} = &\sum_{i \in \{r,l\}} \left[ \frac{1}{2} \varepsilon_i^T \dot{M} \varepsilon_i + \varepsilon_i^T M \dot{\varepsilon}_i + \tilde{\theta}_i^T \Gamma^{-1} \dot{\tilde{\theta}}_i + \frac{1}{2} e_{pi}^T \dot{e}_{pi} \right] \\
 &+ \frac{1}{2} \dot{q}_l(t)^T \dot{q}_l(t) - \frac{1}{2} (1 - \dot{T}_1) \dot{q}_l(t - T_1(t))^T \dot{q}_l \\
 &\times (t - T_1(t)) + \frac{1}{2} \dot{q}_r(t)^T \dot{q}_r(t) \\
 &- \frac{1}{2} (1 - \dot{T}_2) \dot{q}_r(t - T_2(t))^T \dot{q}_r(t - T_2(t))
 \end{aligned} \tag{12}$$

Using Eq. 9, we can simplify  $\dot{V}$  as

$$\begin{aligned}
 \dot{V} = &\sum_{i \in \{r,l\}} \left[ \frac{1}{2} \varepsilon_i^T \dot{M} \varepsilon_i + \varepsilon_i^T \{-C_i \varepsilon_i + Y_i \tilde{\theta}_i - \bar{\tau}_i\} \right. \\
 &\left. + \tilde{\theta}_i^T \Gamma^{-1} \dot{\tilde{\theta}}_i + \frac{1}{2} e_{pi}^T \dot{e}_{pi} \right] + \frac{1}{2} \dot{q}_l(t)^T \dot{q}_l(t) \\
 &- \frac{1}{2} (1 - \dot{T}_1) \dot{q}_l(t - T_1(t))^T \dot{q}_l(t - T_1(t)) \\
 &+ \frac{1}{2} \dot{q}_r(t)^T \dot{q}_r(t) - \frac{1}{2} (1 - \dot{T}_2) \dot{q}_r(t - T_2(t))^T \\
 &\times \dot{q}_r(t - T_2(t))
 \end{aligned} \tag{13}$$

Using the following skew-symmetry property, which is equivalent to the property II,

$$x^T (\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)) x = 0 \quad \forall x \in R^n \tag{14}$$

and after some simplifications, we get:

$$\begin{aligned}
 &\frac{1}{2} \varepsilon_i^T \dot{M} \varepsilon_i + \varepsilon_i^T \{-C_i \varepsilon_i + Y_i \tilde{\theta}_i - \bar{\tau}_i\} + \tilde{\theta}_i^T \Gamma^{-1} \dot{\tilde{\theta}}_i \\
 &= \tilde{\theta}_i^T \left\{ Y_i^T \varepsilon_i + \Gamma^{-1} \dot{\tilde{\theta}}_i \right\} - \varepsilon_i^T \bar{\tau}_i
 \end{aligned} \tag{15}$$

To simplify the right-hand side of Eq. 15, we introduce the following adaptive rule

$$\dot{\tilde{\theta}}_i = -\Gamma Y_i^T \varepsilon_i \tag{16}$$

With the assumption that the variation of unknown parameters  $\theta$  is slow, we get  $\dot{\tilde{\theta}} = -\dot{\hat{\theta}}$ , and

the above adaptive rule for parameter updates becomes

$$\hat{\theta}_i = \Gamma Y_i^T \varepsilon_i \tag{17}$$

which is same as Eq. 10.

Using the above, it is possible to simplify  $\dot{V}$ , as follows:

$$\begin{aligned} \dot{V} = & \sum_{i \in \{r,l\}} \left[ -\varepsilon_i^T \bar{\tau}_i + \frac{1}{2} e_{pi}^T \dot{e}_{pi} \right] + \frac{1}{2} \dot{q}_l(t)^T \dot{q}_l(t) \\ & - \frac{1}{2} (1 - \dot{T}_1) \dot{q}_l(t - T_1(t))^T \dot{q}_l(t - T_1(t)) \\ & + \frac{1}{2} \dot{q}_r(t)^T \dot{q}_r(t) - \frac{1}{2} (1 - \dot{T}_2) \\ & \times \dot{q}_r(t - T_2(t))^T \dot{q}_r(t - T_2(t)) \end{aligned} \tag{18}$$

Using the definition of  $\bar{\tau}_i$  in Eq. 4 and after some manipulations, we get

$$\begin{aligned} \dot{V} = & \sum_{i \in \{r,l\}} \left[ -\varepsilon_i^T K_i \varepsilon_i + \frac{1}{2} \varepsilon_i^T \dot{e}_{pi} + \frac{1}{2} \varepsilon_i^T e_{vi} \right. \\ & \left. + \frac{1}{2} e_{vi}^T (e_{pi} + \dot{e}_{pi} - e_{vi}) + \frac{1}{2} e_{pi}^T \dot{e}_{pi} \right] \\ & + \frac{1}{2} \dot{q}_l(t)^T \dot{q}_l(t) - \frac{1}{2} (1 - \dot{T}_1) \dot{q}_l(t - T_1(t))^T \\ & \times \dot{q}_l(t - T_1(t)) + \frac{1}{2} \dot{q}_r(t)^T \dot{q}_r(t) \\ & - \frac{1}{2} (1 - \dot{T}_2) \dot{q}_r(t - T_2(t))^T \dot{q}_r(t - T_2(t)) \end{aligned} \tag{19}$$

Applying the following relationships

$$\begin{aligned} & \frac{1}{2} \dot{q}_l(t)^T \dot{q}_l(t) - \frac{1}{2} \dot{q}_r(t - T_2(t))^T \dot{q}_r(t - T_2(t)) \\ & = -\frac{1}{2} e_{vl}^T e_{vl} - \dot{q}_l(t)^T e_{vl} \end{aligned} \tag{20}$$

$$\begin{aligned} & \frac{1}{2} \dot{q}_r(t)^T \dot{q}_r(t) - \frac{1}{2} \dot{q}_l(t - T_1(t))^T \dot{q}_l(t - T_1(t)) \\ & = -\frac{1}{2} e_{vr}^T e_{vr} - \dot{q}_r(t)^T e_{vr} \end{aligned} \tag{21}$$

it is found that

$$\begin{aligned} \dot{V} = & \sum_{i \in \{r,l\}} \left[ -\varepsilon_i^T K_i \varepsilon_i + \frac{1}{2} \varepsilon_i^T \dot{e}_{pi} + \frac{1}{2} \varepsilon_i^T e_{vi} \right. \\ & \left. + \frac{1}{2} e_{vi}^T (e_{pi} + \dot{e}_{pi} - e_{vi}) + \frac{1}{2} e_{pi}^T \dot{e}_{pi} \right] \\ & - \frac{1}{2} e_{vl}^T e_{vl} - \dot{q}_l(t)^T e_{vl} \\ & + \frac{1}{2} \dot{T}_1 \dot{q}_l(t - T_1(t))^T \dot{q}_l(t - T_1(t)) \\ & - \frac{1}{2} e_{vr}^T e_{vr} - \dot{q}_r(t)^T e_{vr} \\ & + \frac{1}{2} \dot{T}_2 \dot{q}_r(t - T_2(t))^T \dot{q}_r(t - T_2(t)) \end{aligned} \tag{22}$$

Considering the time derivatives of the position errors,  $\dot{e}_{pl}$  and  $\dot{e}_{pr}$ , as follows

$$\begin{aligned} \frac{d(e_{pr})}{dt} &= \frac{d}{dt} (q_l(t - T_1(t)) - q_r(t)) \\ &= \left( 1 - \frac{d}{dt} (T_1(t)) \right) \dot{q}_l(t - T_1(t)) - \dot{q}_r(t) \\ &= \dot{q}_l(t - T_1(t)) - \dot{q}_r(t) \\ &\quad - \frac{d}{dt} (T_1(t)) \dot{q}_l(t - T_1(t)) \\ &= e_{vr} - \dot{T}_1 \dot{q}_l(t - T_1(t)) \\ \frac{d(e_{pl})}{dt} &= \frac{d}{dt} (q_r(t - T_2(t)) - q_l(t)) \\ &= \left( 1 - \frac{d}{dt} (T_2(t)) \right) \dot{q}_r(t - T_2(t)) - \dot{q}_l(t) \\ &= \dot{q}_r(t - T_2(t)) - \dot{q}_l(t) \\ &\quad - \frac{d}{dt} (T_2(t)) \dot{q}_r(t - T_2(t)) \\ &= e_{vl} - \dot{T}_2 \dot{q}_r(t - T_2(t)) \end{aligned} \tag{23}$$

we get the following relationships between  $\dot{e}_{pi}$  and  $e_{vi}$

$$\dot{e}_{pl} = e_{vl} - \dot{T}_2 \dot{q}_r(t - T_2(t)) \tag{24}$$

$$\dot{e}_{pr} = e_{vr} - \dot{T}_1 \dot{q}_l(t - T_1(t)) \tag{25}$$

Applying Eqs. 24 and 25 to Eq. 22,  $\dot{V}$  could be simplified as becomes

$$\begin{aligned} \dot{V} = \sum_{i \in \{r,l\}} & \left[ -\varepsilon_i^T K_i \varepsilon_i + \frac{1}{2} \varepsilon_i^T \dot{e}_{pi} + \frac{1}{2} \varepsilon_i^T e_{vi} \right. \\ & + \frac{1}{2} e_{vi}^T (e_{pi} + \dot{e}_{pi} - e_{vi}) + \frac{1}{2} e_{pi}^T \dot{e}_{pi} \\ & \left. - \frac{1}{2} e_{vi}^T e_{vi} - \dot{q}_i(t)^T e_{vi} \right] \\ & + \frac{1}{2} \dot{q}_l(t - T_1(t))^T (e_{vr} - \dot{e}_{pr}) \\ & + \frac{1}{2} \dot{q}_r(t - T_2(t))^T (e_{vl} - \dot{e}_{pl}) \end{aligned} \tag{26}$$

Considering that

$$\dot{q}_l(t - T_1(t)) = e_{vr} + \dot{q}_r \tag{27}$$

$$\dot{q}_r(t - T_2(t)) = e_{vl} + \dot{q}_l \tag{28}$$

$\dot{V}$  is further simplified to

$$\begin{aligned} \dot{V} = \sum_{i \in \{r,l\}} & \left[ -\varepsilon_i^T K_i \varepsilon_i + \frac{1}{2} \varepsilon_i^T \dot{e}_{pi} + \frac{1}{2} \varepsilon_i^T e_{vi} \right. \\ & + \frac{1}{2} e_{vi}^T (e_{pi} + \dot{e}_{pi} - e_{vi}) + \frac{1}{2} e_{pi}^T \dot{e}_{pi} \\ & - \frac{1}{2} e_{vi}^T e_{vi} - \dot{q}_i(t)^T e_{vi} \\ & \left. + \frac{1}{2} (e_{vi} + \dot{q}_i)^T (e_{vi} - \dot{e}_{pi}) \right] \end{aligned} \tag{29}$$

More simplification gives

$$\begin{aligned} \dot{V} = \sum_{i \in \{r,l\}} & \left[ -\varepsilon_i^T K_i \varepsilon_i - \frac{1}{2} e_{vi}^T e_{vi} \right. \\ & + \frac{1}{2} \{ \varepsilon_i - \dot{q}_i(t) + e_{pi} \}^T \dot{e}_{pi} \\ & \left. + \frac{1}{2} \{ \varepsilon_i - \dot{q}_i(t) + e_{pi} \}^T e_{vi} \right] \end{aligned} \tag{30}$$

Using the definition of  $\varepsilon_i$  in Eq. 5, negative semi-definiteness of  $\dot{V}$  is seen as

$$\dot{V} = - \sum_{i \in \{r,l\}} \left[ \varepsilon_i^T K_i \varepsilon_i + \frac{1}{2} e_{vi}^T e_{vi} \right] \tag{31}$$

Integrating Eq. 31, it is easy to see that  $V(t) - V(0) = \int_0^t \dot{V} = - \left( \int_0^t (\varepsilon_i^T K_i \varepsilon_i + \frac{1}{2} e_{vi}^T e_{vi}) \right) \leq 0$ , i.e.,  $V(t) \leq V(0)$ . Using the fact that  $V(t) \geq 0$ ,  $V(t) \leq V(0)$  and  $\dot{V}(t) \leq 0$ , it is possible to say that  $V(t)$  is *positive bounded decreasing* function. Thus, it is concluded that all terms in  $V(t)$  are bounded.

Now let us proceed to the analysis of transparency of the system by proving  $\lim_{t \rightarrow \infty} e_{vi}(t) = \lim_{t \rightarrow \infty} e_{pi}(t) = 0$ . We will also show that  $\lim_{t \rightarrow \infty} q_i(t)$  is bounded to establish closed-loop stability.

Previously, it is shown that  $V(t)$  is bounded, so all terms in  $V(t)$  including  $\varepsilon_i$ ,  $\dot{q}_i$ ,  $e_i$  and  $\tilde{\theta}_i \in \mathcal{L}_\infty$ . Using  $V(t) \geq 0$  and integrating Eq. 31,  $\int_0^t (\varepsilon_i^T K_i \varepsilon_i) + \frac{1}{2} \int_0^t (e_{vi}^T e_{vi}) = V(0) - V(t) \leq V(0)$ , it follows that  $\varepsilon_i, e_{vi} \in \mathcal{L}_2$ . It is easy to see from Eq. 5 that, since  $\dot{q}_i \in \mathcal{L}_\infty$ , we have  $e_{vi} \in \mathcal{L}_\infty$ . Combining these with Eqs. 24 and 25 and the assumption that  $\dot{T}_i$  is bounded, it is seen that  $\dot{e}_{pi} \in \mathcal{L}_\infty$ . All these bounded signals result in the boundedness of the regressor matrix  $Y_i$ , i.e.,  $Y_i \in \mathcal{L}_\infty$ . Using the boundedness of  $\varepsilon_i, \tilde{\theta}_i, \bar{\tau}_i$  and  $Y_i$  and Properties I and III in Eq. 9, it is seen that  $\dot{e}_i \in \mathcal{L}_\infty$ . Using Barbalat's lemma (see Appendix), given that  $\varepsilon_i \in \mathcal{L}_2$  and  $\dot{e}_i \in \mathcal{L}_\infty$ , it is concluded that  $\lim_{t \rightarrow \infty} \varepsilon_i = 0$ . Using  $\dot{e}_i = \ddot{q}_i - \dot{e}_{pi}$ , it is determined that  $\ddot{q}_i \in \mathcal{L}_\infty$ . Invoking the time derivative of  $e_{vi}$ , e.g.,  $\dot{e}_{vl} = (1 - \dot{T}_2) \ddot{q}_r(t - T_2) - \ddot{q}_l$ , it is concluded that  $\dot{e}_{vi} \in \mathcal{L}_\infty$ . Therefore, using Barbalat's lemma again, since  $e_{vi} \in \mathcal{L}_2$  and  $\dot{e}_{vi} \in \mathcal{L}_\infty$ , it is resulted that  $\lim_{t \rightarrow \infty} e_{vi} = 0$ . Replacing  $e_{pl} = q_r(t - T_2(t)) - q_l(t)$  in  $\varepsilon_l = \dot{q}_l - e_{pl}$  and using the fact that  $\varepsilon_l \rightarrow 0$ , stability of the system  $\dot{q}_l(t) + q_l(t) = q_r(t - T_2(t))$  can be analyzed with calculating the response of  $q_l(t)$  to the  $q_r(t - T_2(t))$ . Homogenous response of stable differential equation  $\dot{q}_l(t) + q_l(t) = q_r(t - T_2(t))$  is  $q_l(t) = e^{-t} \int e^t q_r(t - T_2(t))$ . Similar result could be achieved for  $q_r(t)$  as  $q_r(t) = e^{-t} \int e^t q_l(t - T_1(t))$ . If  $\int e^t q_r(t - T_2(t))$  is bounded then  $q_l \rightarrow 0$ , which implies that  $q_r \rightarrow 0, e_{pr}$  and  $e_{pl} \rightarrow 0$ . If

$\int e^t q_r(t - T_2(t))$  be unbounded then  $q_l(t)$  would be indeterminate which could be evaluated using Hopital's rule as  $\lim_{t \rightarrow \infty} q_r(t) = \frac{\frac{d}{dt}(\int e^t q_l(t - T_1(t)))}{\frac{d}{dt}(e^t)} = q_l(t - T_1(t))$ ,  $e_{pr} \rightarrow 0$ . Similar results could be achieved for  $e_{pl} \rightarrow 0$ . Using the definition of  $\varepsilon_i$  in Eq. 5 and using the fact that  $\lim_{t \rightarrow \infty} \varepsilon_i(t) = 0$  and  $\lim_{t \rightarrow \infty} e_{pi}(t) = 0$ , it is easy to see from Eq. 5 that  $\lim_{t \rightarrow \infty} \dot{q}_i(t) = 0$  and, in other words,  $\lim_{t \rightarrow \infty} q_i(t) = \text{Constant}$ .

Thus, it was proved that

$$\begin{aligned} \lim_{t \rightarrow \infty} q_i(t) &= \text{Constant and } \lim_{t \rightarrow \infty} e_{vi} \\ &= \lim_{t \rightarrow \infty} e_{pi} = 0 \quad i \in \{l, r\} \end{aligned} \quad (32)$$

□

Thus, in free motion of the bilateral telemanipulation system (1), state synchronization is satisfied under time varying communication delays. Also, the closed-loop telemanipulator is input-to-state stable from the human and environment input forces to the local and remote manipulator states.

### 4 Simulation Results

To verify the theoretical results of this paper, the local and remote manipulators are considered to be a pair of two-degree-of-freedom serial robots with revolute joints. The local and remote manipulator dynamics (1) have the following elements of inertia, Coriolis/centrifugal and gravity matrices:

$$\begin{aligned} M_i(q_i) &= \begin{bmatrix} M_{i11} & M_{i12} \\ M_{i21} & M_{i22} \end{bmatrix}, \quad C_i(q_i, \dot{q}_i) \\ &= \begin{bmatrix} C_{i11} & C_{i12} \\ C_{i21} & C_{i22} \end{bmatrix} \text{ and } G_i(q_i) = \begin{bmatrix} G_{i1} \\ G_{i2} \end{bmatrix} \end{aligned}$$

where for  $i \in \{l, r\}$ ,  $M_{i11} = l_2^2 m_{i2} + l_1^2 (m_{i1} + m_{i2}) + 2l_1 l_2 m_{i2} \cos(q_{i2})$ ,  $M_{i12} = M_{i21} = l_1^2 m_{i2} + l_1 l_2 m_{i2} \cos(q_{i2})$ ,  $M_{i22} = l_2^2 m_{i2}$ ,  $C_{i11} = -2l_1 l_2 m_{i2} \sin(q_{i2}) \dot{q}_{i2}$ ,  $C_{i12} = -l_1 l_2 m_{i2} \sin(q_{i2}) \dot{q}_{i2}$ ,  $C_{i21} = l_1 l_2 m_{i2} \sin(q_{i2}) \dot{q}_{i1}$ ,  $C_{i22} = 0$ ,  $G_{i1} = g l_2 m_{i2} \cos(q_{i1} + q_{i2}) + l_1 (m_{i1} + m_{i2}) \cos(q_{i1})$ ,  $G_{i2} = g l_2 m_{i2} \cos(q_{i1} + q_{i2})$ . Here,  $q_{i1}$  and  $q_{i2}$  are the positions of the first and the second revolute joints,  $l_{i1}$  and  $l_{i2}$  are the link lengths and  $m_{i1}$  and

$m_{i2}$  are the masses of the first and the second links for each robot. For both manipulators, we used the same linear parameterization (see Property IV) as in [26]:

$$\begin{aligned} Y_i(q_i, \dot{q}_i, e_{pi}, \dot{e}_{pi}) &= \begin{bmatrix} Y_{i11} & Y_{i12} & Y_{i13} & Y_{i14} & Y_{i15} \\ Y_{i21} & Y_{i22} & Y_{i23} & Y_{i24} & Y_{i25} \end{bmatrix}, \\ \hat{\theta}_i &= [\hat{\theta}_{i1} \quad \hat{\theta}_{i2} \quad \hat{\theta}_{i3} \quad \hat{\theta}_{i4} \quad \hat{\theta}_{i5}] \end{aligned}$$

where,  $Y_{i11} = -\dot{e}_{pi1}$ ,  $Y_{i12} = -2\dot{e}_{pi1} \cos(q_{i2}) - \dot{e}_{pi2} \cos(q_{i2}) + \dot{q}_{i2} e_{pi2} \sin(q_{i2}) + 2e_{pi1} \dot{q}_{i2} \sin(q_{i2})$ ,  $Y_{i13} = -\dot{e}_{pi2}$ ,  $Y_{i14} = -g \cos(q_{i1} + q_{i2})$ ,  $Y_{i15} = -g \cos(q_{i1})$ ,  $Y_{i21} = 0$ ,  $Y_{i22} = -\dot{e}_{pi1} \cos(q_{i2}) - \dot{q}_{i1} e_{pi1} \sin(q_{i2})$ ,  $Y_{i23} = -\dot{e}_{pi2} - \dot{e}_{pi2}$ ,  $Y_{i24} = -g \cos(q_{i1} + q_{i2})$ ,  $Y_{i25} = 0$  and  $\hat{\theta}_{i1} = l_2^2 \hat{m}_{i2} + l_1^2 (\hat{m}_{i1} + \hat{m}_{i2})$ ,  $\hat{\theta}_{i2} = \hat{l}_1 \hat{l}_2 \hat{m}_{i2}$ ,  $\hat{\theta}_{i3} = \hat{l}_2^2 \hat{m}_{i2}$ ,  $\hat{\theta}_{i4} = \hat{l}_2 \hat{m}_{i2}$ ,  $\hat{\theta}_{i5} = \hat{l}_1 (\hat{m}_{i1} + \hat{m}_{i2})$ ,  $i \in \{l, r\}$ .

Using the above definitions for the elements of the matrix  $\hat{\theta}_i$ , it is possible to estimate matrices  $M_i$ ,  $C_i$  and  $G_i$  based on the elements of  $\hat{\theta}_i$  that will be estimated online.

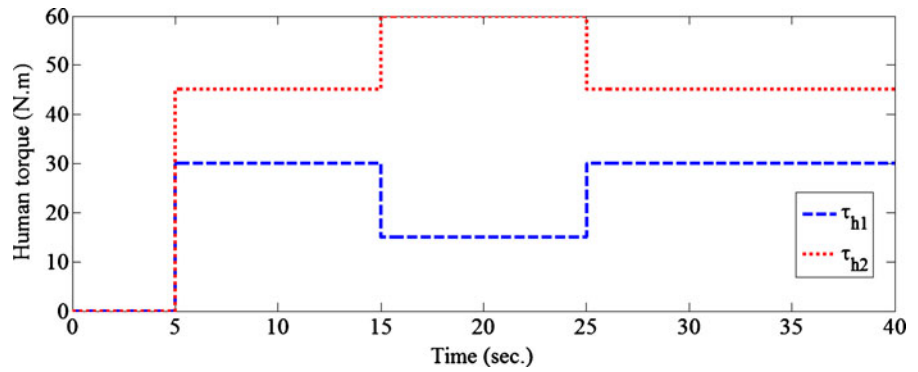
In simulations, the physical parameters of the manipulators are set to  $m_{l1} = 4 \text{ kg}$ ,  $m_{l2} = 0.5 \text{ kg}$ ,  $l_{l1} = 50 \text{ cm}$ ,  $l_{l2} = 50 \text{ cm}$ ,  $m_{r1} = 3.4 \text{ kg}$ ,  $m_{r2} = 0.25 \text{ kg}$ ,  $l_{r1} = 50 \text{ cm}$ ,  $l_{r2} = 50 \text{ cm}$  and the controller gain  $K_i$  is set to  $3I$ . In the following, three simulation scenarios are considered involving constant time delays, random time delays and sinusoidal time delays (scenarios A, B and C, respectively). A human torque, which is shown in Fig. 2, is applied to the local manipulator and the tracking performance of the first and the second joints of the local and remote manipulators are considered.

#### 4.1 Simulation with Constant Time Delays

In Fig. 3, simulation results for a constant time delay similar to that used in [26],  $T_1 = 0.4$  and  $T_2 = 0.4$  seconds, in terms of joint positions of the remote manipulator and delayed joint positions of the local manipulator in the presences of the exerted human torque are shown. Comparing the results, it can be seen that the results of the proposed scheme is exactly the same as that of [26]. This similarity is because of the fact that in this simulation time delays are constant.



**Fig. 2** Exerted human torque



#### 4.2 Simulation with Random Time Delays

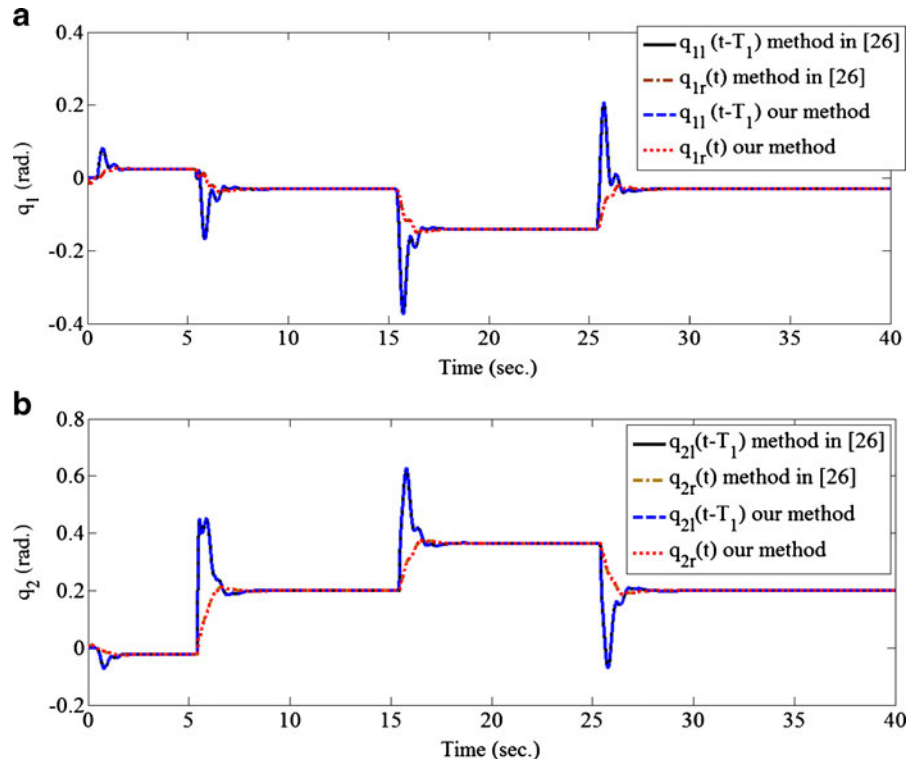
In this part, simulation results of the proposed controller compared with [26] for random time delays with Gaussian distribution with mean 0.48 second and standard deviation of 0.022, are shown. In this simulation, again the positions of the first and the second joints of the remote manipulator compared with delayed joint positions of the local manipulator in the presences of exerted human torque are shown. As it can be seen

from Fig. 4, the proposed scheme has better tracking performance and less fluctuations and settling time than the controller [26].

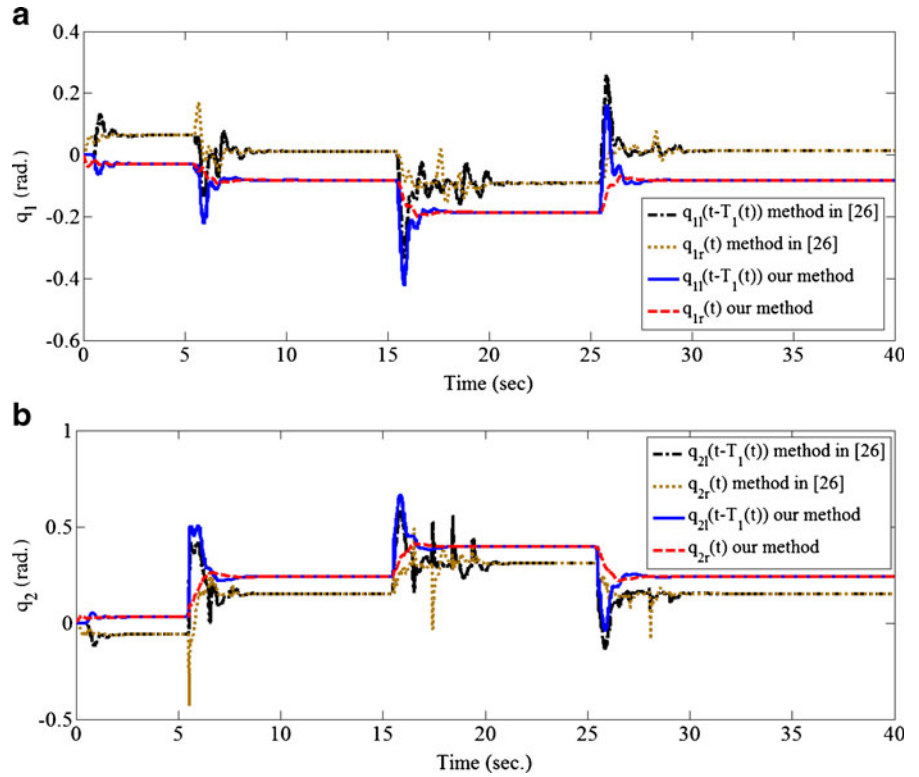
#### 4.3 Simulation with Sinusoidal Time Delays

Let us verify the telemanipulator’s free motion tracking performance under sinusoidal time varying delays. The feedforward and feedback delays in the communication channel are assumed to

**Fig. 3 a** Positions of the first joints of the local and remote manipulator in telemanipulation with constant time delay. **b** Positions of the second joints of the local and remote manipulator in telemanipulation with constant time delay



**Fig. 4** **a** Positions of the first joints of the local and remote manipulators in telemanipulation with random time delay. **b** Positions of the second joints of the local and remote manipulators in telemanipulation with random time delay



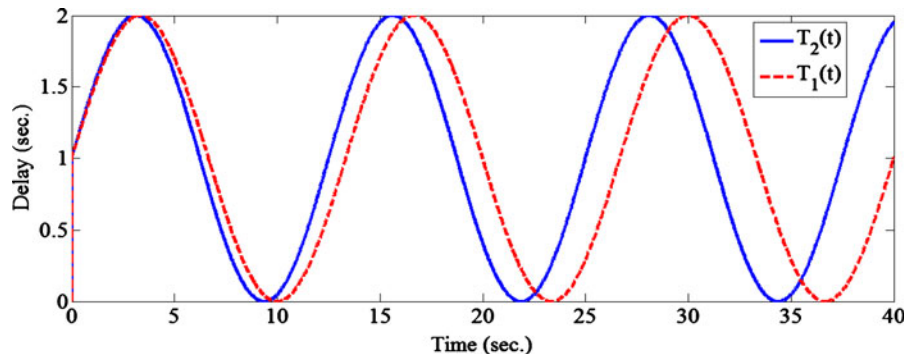
be changing as sinusoids with a mean of 1 second and frequencies of 0.5027 and 0.4714 rad/sec – see Fig. 5.

Joint positions of the local and the remote manipulators in the presences of the exerted human torque of Fig. 2 are shown in Fig. 6. It is remarkable that state synchronization of the bilateral teleoperation system is satisfied in the presence of the fast varying communication delays. In Fig. 7, tracking errors in the first and the second joints

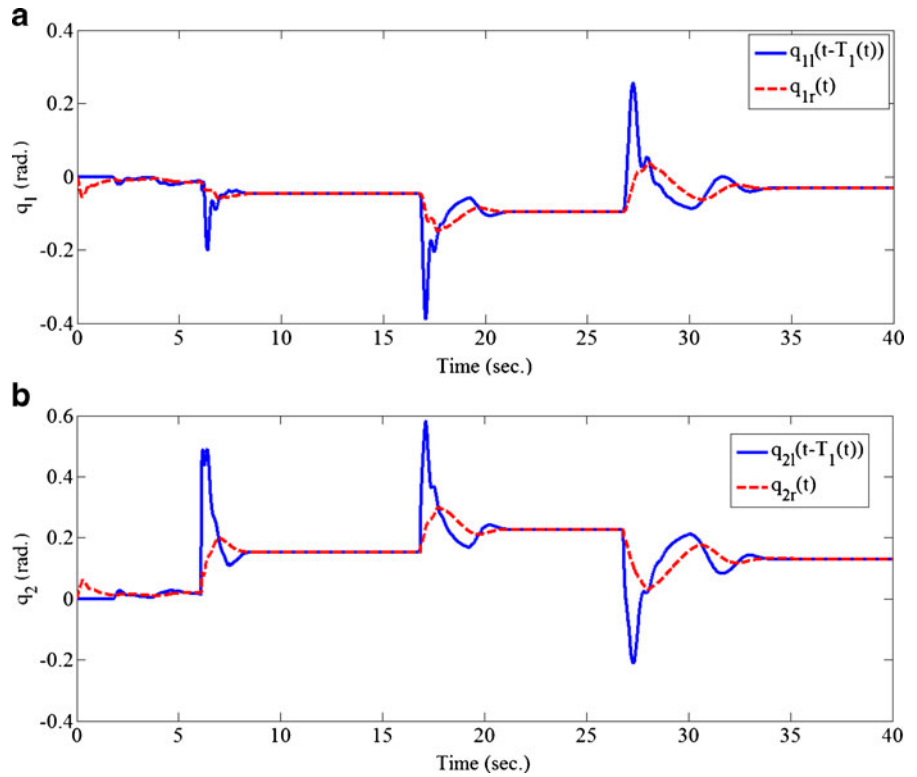
of the local and remote manipulators are shown, which are asymptotically converging to zero as predicted by the theory.

As shown in Figs. 3, 4 and 6, after a command is being applied to the local robot, a couple of seconds of settling time is needed to achieve synchronization between the local and the remote robots. This settling time depends on the values of the constant time delay (Fig. 3) or the varying time delay (Figs. 4 and 6). Considering these figures,

**Fig. 5** Time varying delays in communication channel



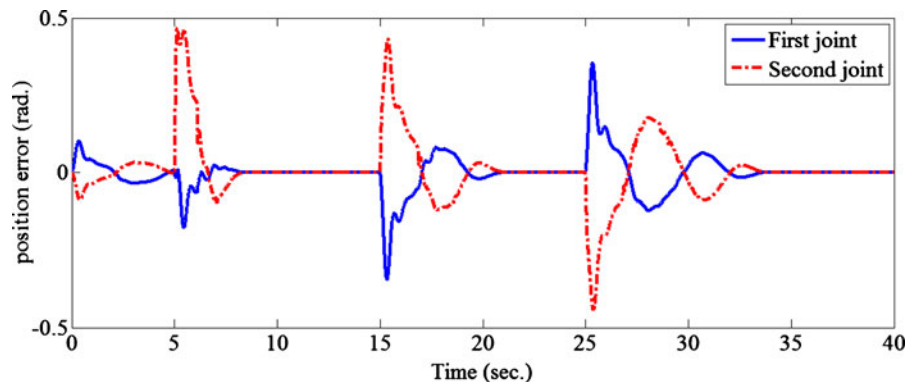
**Fig. 6** **a** Positions of the first joints of the local and remote manipulators in telemanipulation with sinusoidal time varying delay. **b** Positions of the second joints of the local and remote manipulators in telemanipulation with sinusoidal time varying delay



the proposed adaptive controller is able to handle time varying delays and guarantee asymptotic synchronization between the local and the remote robots as opposed to prior art [26], which is only meant for constant time delays.

If we apply the controller in [26] to the same local and remote robots with the same sinusoidal delays in the communication channel, instability happens in the local and the remote manipulators as shown in Fig. 8.

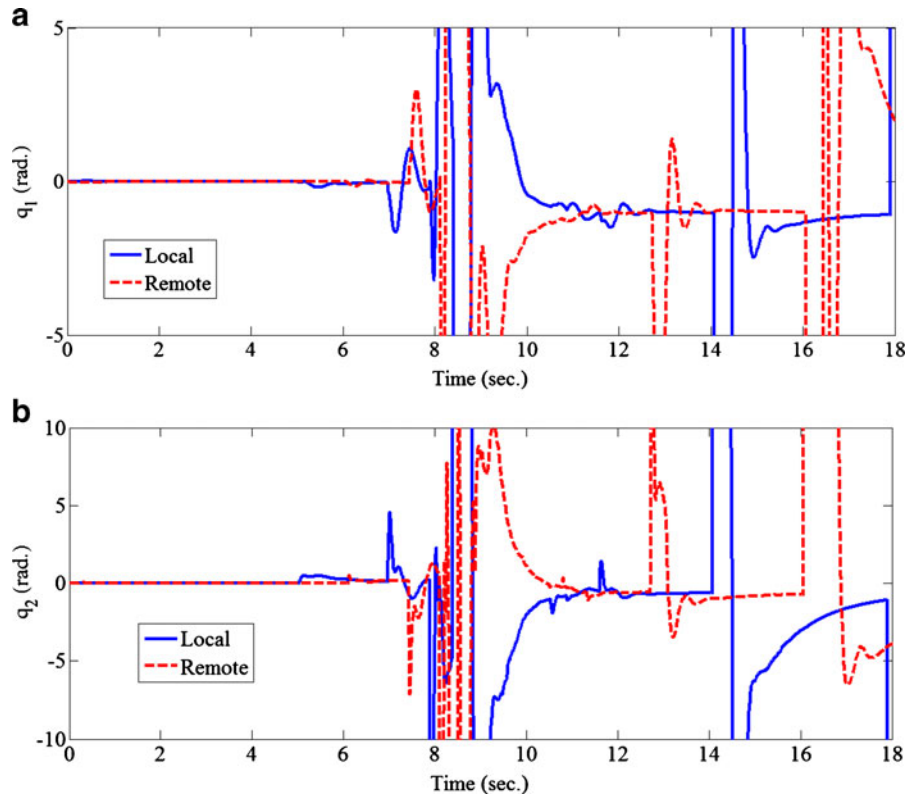
**Fig. 7** Tracking errors of the positions of the first and second joints between the local and the remote manipulators



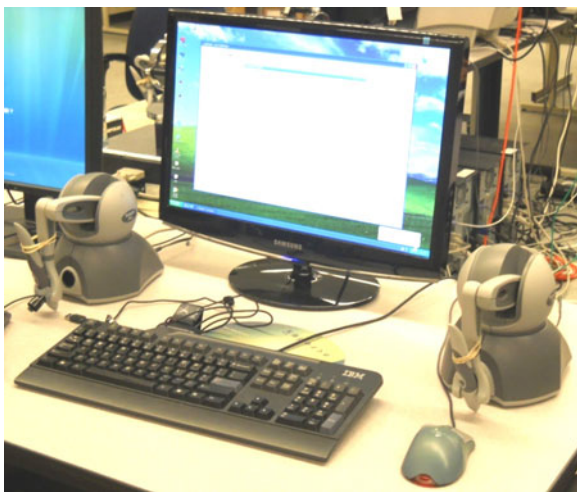
### 5 Experimental Results

To verify the theoretical results of this paper, the local and the remote manipulators are considered to be two PHANToM Omni robots (Sensable Technologies, Inc., Wilmington, MA) as shown in Fig. 9. The utilized PHANToM robots are three degree of freedom robots that map the generalized joint angles of the robot ( $q_1, q_2$  and  $q_3$ ) to the Cartesian position ( $X, Y$  and  $Z$ ) of

**Fig. 8** **a** Positions of the first joints of the local and remote manipulators with controller in [26] with sinusoidal time varying delay. **b** Positions of the second joints of the local and remote manipulators with controller in [26] with sinusoidal time varying delay



the gimbal. These local and remote robots are connected to the computer using 1394 ports. As the human operator moves the local robot, the



**Fig. 9** Teleoperation system with two PHANToM Haptic devices

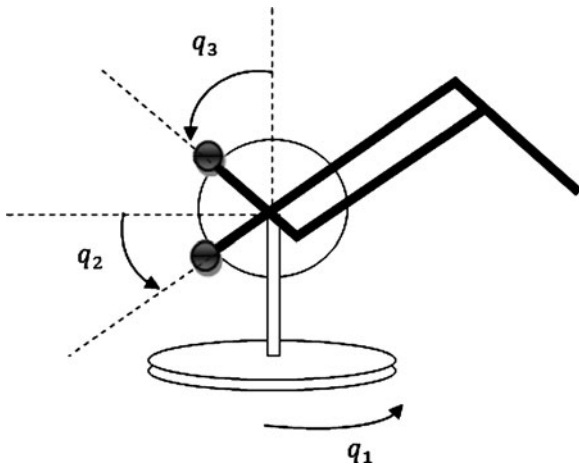
remote robot which is in free motion follows the state of the local robot. Controllers for the local and the remote robots are implemented based on Eqs. 2–5. To artificially create varying time delays between the local and the remote robots, a first-input, first-output circular buffer is used for each robot – changing the length of these buffers create time-varying time delays in the communication channel.

The schematics of PHANToM robot with its corresponding joint angles are shown in Fig. 10.

The local and remote PHANToM dynamics (1) have the following elements of inertia, Coriolis/centrifugal and gravity matrices:

$$M_i(q_i) = \begin{bmatrix} M_{i11} & M_{i12} & M_{i13} \\ M_{i21} & M_{i22} & M_{i23} \\ M_{i31} & M_{i32} & M_{i33} \end{bmatrix},$$

$$C_i(q_i, \dot{q}_i) = \begin{bmatrix} C_{i11} & C_{i12} & C_{i13} \\ C_{i21} & C_{i22} & C_{i23} \\ C_{i31} & C_{i32} & C_{i33} \end{bmatrix} \text{ and } G_i(q_i) = \begin{bmatrix} G_{i1} \\ G_{i2} \\ G_{i3} \end{bmatrix}$$



**Fig. 10** Schematics of PHANToM robot with its corresponding joint angles

where for  $i \in \{l, r\}$ ,  $M_i(q_i)$ ,  $C_i(q_i, \dot{q}_i)$  and  $G_i(q_i)$  defined in terms of kinematic and inertial properties of the PHANToM [29].

The linear parameterization of the PHANToM (see Property IV) is reported in [30]. We use a similar parameterization, Eq. 7, as:

$$Y_i(q_i, \dot{q}_i, e_{pi}, \dot{e}_{pi}) = \begin{bmatrix} Y_{i11} & Y_{i12} & Y_{i13} & Y_{i14} & Y_{i15} & Y_{i16} & Y_{i17} & Y_{i18} \\ Y_{i21} & Y_{i22} & Y_{i23} & Y_{i24} & Y_{i25} & Y_{i26} & Y_{i27} & Y_{i28} \\ Y_{i31} & Y_{i32} & Y_{i33} & Y_{i34} & Y_{i35} & Y_{i36} & Y_{i37} & Y_{i38} \end{bmatrix}$$

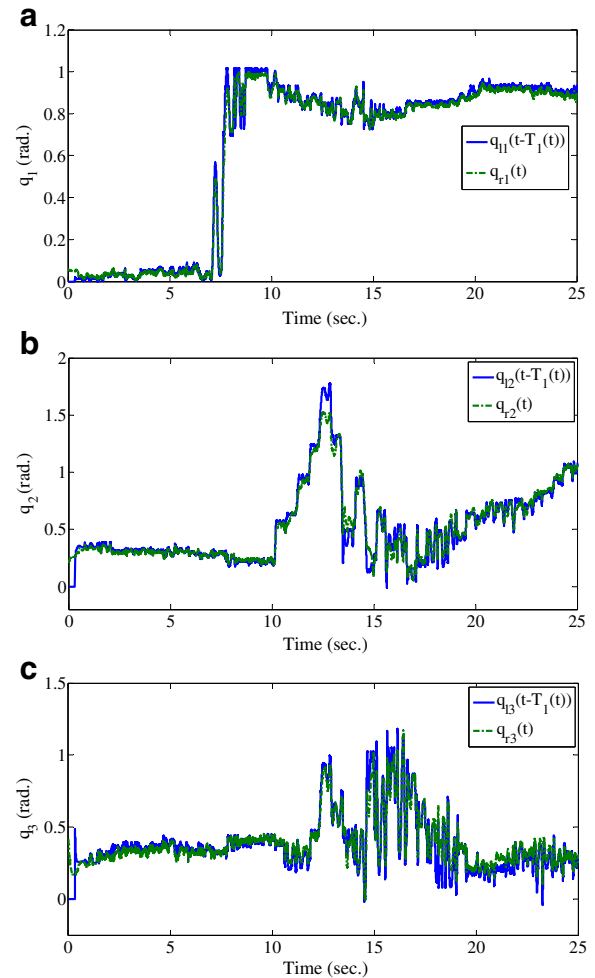
$$\hat{\pi}_i = [\hat{\pi}_{i1} \hat{\pi}_{i2} \hat{\pi}_{i3} \hat{\pi}_{i4} \hat{\pi}_{i5} \hat{\pi}_{i6} \hat{\pi}_{i7} \hat{\pi}_{i8}]$$

where,  $Y_{i11} = \dot{e}_{pi1}$ ,  $Y_{i12} = \dot{e}_{pi1}c_{i2,2} - 2e_{pi1}\dot{q}_{i2}s_{i2}c_{i2} - \dot{q}_{i1}e_{pi2}s_{i2,2}$ ,  $Y_{i13} = \dot{e}_{pi1}c_{i2,3} - 2e_{pi1}\dot{q}_{i3}s_{i3}c_{i3} - \dot{q}_{i1}e_{pi3}s_{i2,3}$ ,  $Y_{i14} = \dot{e}_{pi1}c_{i2}s_{i3} - \frac{1}{2}(e_{pi1}\dot{q}_{i2} + \dot{q}_{i1}e_{pi2})s_{i2}s_{i3} + \frac{1}{2}(e_{pi1}\dot{q}_{i3} + \dot{q}_{i1}e_{pi3})c_{i2}c_{i3}$ ,  $Y_{i15} = 0, Y_{i16} = 0, Y_{i17} = 0, Y_{i18} = 0, Y_{i21} = 0, Y_{i22} = \dot{q}_{i1}e_{pi1}s_{i2,2}$ ,  $Y_{i23} = 0, Y_{i24} = -\frac{1}{2}\dot{e}_{pi3}s_{i2,3} + \frac{1}{2}\dot{q}_{i1}e_{pi1}s_{i2}s_{i3} + \frac{1}{2}\dot{q}_{i3}e_{pi3}c_{i2,3}$ ,  $Y_{i25} = \dot{e}_{pi2}$ ,  $Y_{i26} = 0, Y_{i27} = c_{i2}$ ,  $Y_{i28} = 0, Y_{i31} = 0, Y_{i32} = 0, Y_{i33} = \dot{q}_{i1}e_{pi1}s_{i2,3}$ ,  $Y_{i34} = -\frac{1}{2}\dot{e}_{pi2}s_{i2,3} - \frac{1}{2}\dot{q}_{i1}e_{pi1}c_{i2}c_{i3} + \frac{1}{2}\dot{q}_{i3}e_{pi3}c_{i2,3}$ ,  $Y_{i35} = 0, Y_{i36} = \dot{e}_{pi3}$ ,  $Y_{i37} = 0, Y_{i38} = s_{i3}$  and  $\hat{\pi}_i$  is same as reported in [30]. In the elements of matrix  $Y_i$ ,  $s_{im}$ ,  $c_{im}$ ,  $s_{imn}$ ,  $c_{imn}$ ,  $s_{i2,m}$  and  $c_{i2,m}$ ,  $m, n = 1, 2, 3$ , stands for  $\sin(q_{im})$ ,  $\cos(q_{im})$ ,  $\sin(q_{im} - q_{in})$ ,  $\cos(q_{im} - q_{in})$ ,  $\sin(2q_{im})$ ,  $\cos(2q_{im})$  respectively.

Let us verify the PHANToM's free motion tracking performance under sinusoidal time varying delays. The feedforward and feedback delays

in the communication channel are assumed to be changing as sinusoids with a mean of 0.2 s. Joint positions of the local and the remote PHANToM robots are shown in Fig. 11. It is remarkable that state synchronization of the bilateral teleoperation system is satisfied in the presence of the varying communication delays.

To compare the experimental tracking performance of the proposed controller with the controller in [26], experimental results corresponding



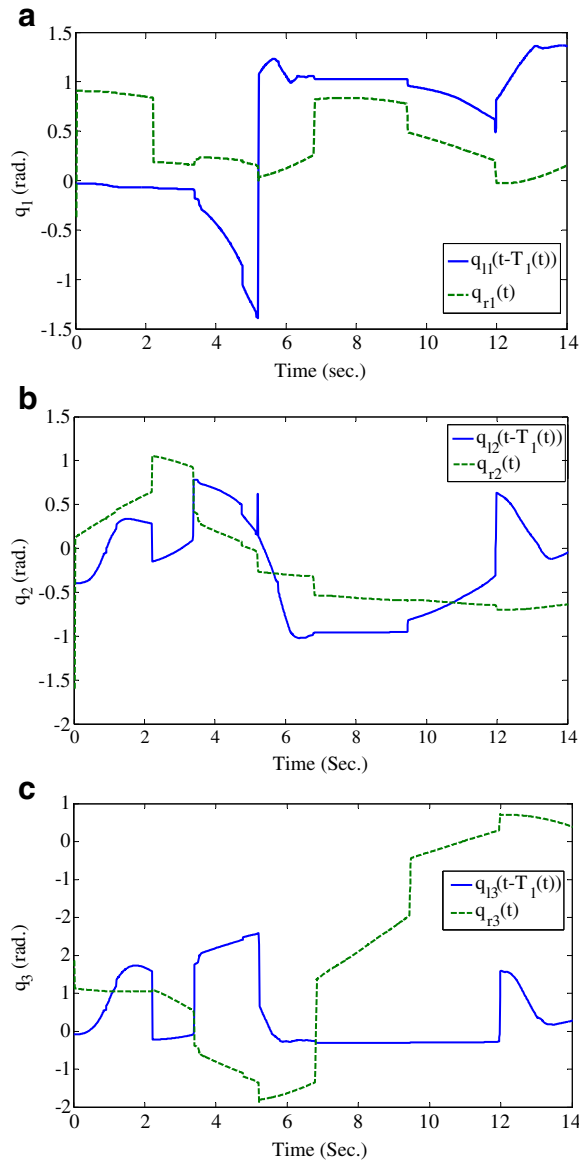
**Fig. 11** **a** Positions of the first joints of the local and remote PHANToM robots in telemanipulation with sinusoidal time varying delay. **b** Positions of the second joints of the local and remote PHANToM robots in telemanipulation with sinusoidal time varying delay. **c** Positions of the third joints of the local and remote PHANToM robots in telemanipulation with sinusoidal time varying delay

to the controller in [26] being used with the same two PHANTOM robots in the presence of varying time delays are demonstrated in Fig. 12. It is evident from Fig. 12 that instability happens in

the teleoperation in the presence of time varying delay when the controller in [26] is used.

## 6 Conclusion and Future Work

In this paper, a new state synchronizing controller for bilateral teleoperation systems with varying time delays in the communication channel is proposed. Lyapunov stability of the closed-loop system in the presence of time varying delays is established. Besides, it is proved and also shown via simulations that using the proposed controller, asymptotic synchronization between the local and the remote robots occurs. The proposed controller entails adaptive tuning rules in the local and the remote sides to estimate the unknown/uncertain dynamic parameters of the manipulators. Thus, only the estimated values of the robots' parameters are needed in the controller when providing for the asymptotic state synchronization between the local and the remote robots under varying time delays. As future work, state synchronization under varying time delays during contact-motion telemanipulation with consideration for force tracking to obtain full transparency will be studied.



**Fig. 12** **a** First joint position tracking between the local and the remote robots during teleoperation using the controller in [26]. **b** Second joint position tracking between the local and remote robots during teleoperation using the controller in [26]. **c** Third joint position tracking between the local and remote robots during teleoperation using the controller in [26]

## Appendix

1. If  $f(t) \in \mathcal{L}_2$ , then there exist a positive constant  $M$  such that  $\int |f(t)|^2 dt < M$ . This implies that  $f$  is Riemann integrable.
2. If  $\dot{f}(t) \in \mathcal{L}_\infty$ , then  $f$  is uniformly continuous.

Barbalat's lemma says that if function  $f : \mathbb{R}^+ \rightarrow \mathbb{R}^+$  is *uniformly continuous* and be Riemann integrable, then  $\lim_{t \rightarrow \infty} f(t) = 0$ .

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