Wireless Multicast Using Relays: Incentive Mechanism and Analysis

Bo Hu, *Student Member, IEEE*, H. Vicky Zhao, *Member, IEEE,* and Hai Jiang, *Member, IEEE*

***Abstract*— In wireless multicast systems, cooperative multicast, in which successful users help to relay received packets to unsuccessful users, has been shown to be effective in combating channel fading and improving the system performance. However, this mechanism requires users' voluntary contributions, which cannot be guaranteed since users are selfish and only care about their own performance. Furthermore, users may have heterogeneous costs (which are their private information) to forward packets, and they may lie about their costs if cheating can improve their utilities. To address these problems, in this work, we model the interaction among users in the wireless multicast system as a multi-seller multi-buyer payment-based game, where users pay to receive relay service and get paid if they forward packets to others. A simplified case with homogeneous users that have the same cost to forward packets is investigated first. Then for the case with heterogeneous users, to encourage users to tell their true costs, we use the second-price sealed-bid auction, which is a truth-telling auction, since bidding the true cost is a weakly dominant strategy. To analyze the multi-seller multi-buyer payment-based game, we observe that under different selected prices, the game can converge to different equilibria, resulting in different user free-riding probabilities and system throughput. We also study the price selection problem and derive the optimal price that maximizes the system throughput. Simulation results show the effectiveness of our proposed mechanism.**

I. INTRODUCTION

In the past decade, with the emergence of high-speed broadband wireless networks and the increasing popularity of advanced mobile devices such as smart phones and iPads, the demand for wireless multimedia broadcast/multicast service keeps increasing, e.g., Internet Protocol Television over WiMAX, 3$\sim$4 G wireless networks and beyond. In these applications, a base station (BS) or access point (AP) broadcasts multimedia data to a group of users. However, due to channel fading, it is very challenging to design a multicast system that provides reliable quality of service. To address this issue, cooperative wireless multicast mechanisms [1]–[6] have been proposed, where after the BS broadcasts its data, users who receive the packets correctly will serve as relays, and forward the received packets to other users. In those works, it is assumed that users are altruistic and serve as relays if requested, even though they gain nothing during this process but simply deplete the battery power by forwarding packets. However, this assumption cannot be guaranteed since users are usually selfish and only care about their own performance. Therefore, it is important to design incentive mechanisms to stimulate user cooperation in wireless multicast systems.

In the literature, game theoretic methods [7] are adopted to study user behavior in various wireless networks, and different incentive mechanisms have been proposed to stimulate user cooperation. For example, in [8]–[13], punishment-based schemes in repeated games are proposed, where it is assumed that users stay in the game for a long time. Since punishment is used, such as in the worst behavior tit-for-tat strategy [13], users that deviate from cooperation will be punished with a long term utility loss. Thus, cooperation becomes the dominant strategy that everyone will choose. Payment-based schemes are proposed in [14]–[19], where virtual currency circulates in the network. Users need to pay to receive others' help, and users that help others will get paid to compensate their costs. Reputation-based indirect reciprocity schemes are proposed in [20]–[23], where users help others to accumulate good reputations, and users with good reputations have a larger probability to receive others' help.

However, most of those incentive mechanisms cannot be directly applied to wireless multicast networks. First, in wireless multicast, users may frequently join and leave the multicast service, which makes the repeated game based methods impractical. Second, payment and reputation based methods are usually designed for unicast scenarios. However, in wireless multicast, all users receive the same packets. Due to the broadcast nature of wireless communications, the relay either bought by a user or requested by a user with good reputation can be overheard by all others. Thus, users tend to free ride rather than either pay for the relay service or help others to accumulate good reputation for reciprocity. Furthermore, it is usually assumed (e.g., in [6]) that users have the same cost for forwarding packets. However, in practice, users may use different mobile devices, whose costs to forward packets are different. Since each user's cost is his/her private information, he/she may lie about it if cheating can increase his/her utility. For example, a user can claim a high cost in order to request a high payment for providing relay service. Thus, the cheating behavior caused by heterogeneous costs should be addressed.

In this work, we address these challenges and design an incentive mechanism to stimulate user cooperation in wireless multicast. In this system, data traffic is divided into segments. For the transmission of each segment, we consider a two-portion wireless multicast. In the first portion, the BS broadcasts a segment to a group of users who are close to each other. The users who receive the segment correctly are successful users and they can choose whether to relay

Manuscript received July 8, 2012; revised October 4, 2012 and November 11, 2012; accepted December 10, 2012. The associate editor coordinating the review of this paper and approving it for publication was S. Zhong. This work was supported by the Natural Science and Engineering Research Council (NSERC) of Canada. The authors are with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, Alberta, Canada T6G 2V4 (email: {bhu2, hvzhao, hai1}@ualberta.ca).

the segment to unsuccessful users in the second portion. To stimulate user cooperation, we propose a multi-seller multi-buyer payment-based game. In this game, each successful user decides whether to provide relay service in the sellers' game, and each unsuccessful user decides whether to purchase relay service in the buyers' game. Once a transaction happens, fees are collected from all buyers and paid to the relay node. The major contributions of this work can be summarized as follows:

- We model users' interaction in the wireless multicast system as a multi-seller multi-buyer payment-based game. Unlike the repeated game in [13], our game is a one-shot game for one segment transmission. Thus, it also works in dynamic wireless networks, where users can frequently join and leave this system. We also model the buyers' game as an evolutionary game, and derive the evolutionarily stable strategy (ESS), which is a stable equilibrium. That is, even if some players may deviate from it at some time, they will still move back to this equilibrium, since using the ESS gives a higher utility.
- We also consider the case that users have heterogeneous costs of providing relay service. To address the heterogeneous cost issue and encourage users to tell their true costs, we formulate the sellers' game as a second-price sealed-bid auction game. It is a truth-telling auction, where each user bidding his/her real cost is a weakly dominant strategy.
- In this work, we observe that in our payment-based mechanism, under different selected prices, the buyers' game can converge to different equilibria, where unsuccessful users have different probabilities to free ride (i.e., not buy but overhear the relay bought by others), resulting in different system throughput. From the system designer's point of view, we aim at selecting the optimal price to maximize the system throughput. For the simple scenario where users have homogeneous cost of forwarding segments, we derive the closed-form optimal price, under which unsuccessful users cannot free ride, and they will share the cost of the relay and pay together to afford the relay service, while the system throughput is maximized at the same time. For the scenario with heterogeneous costs, we propose an efficient algorithm to find the optimal price, under which unsuccessful users have very low probability to free ride and the system throughput is also maximized.

The rest of the paper is organized as follows. Section II describes the wireless multicast system model, and introduces the multi-seller multi-buyer payment-based game model. Section III and IV analyze the Nash equilibrium when users have the same and different costs of forwarding segments, respectively. Simulation results are shown in Section V, and conclusion is drawn in Section VII.

II. System Model

In this section, we will introduce the cooperative wireless multicast system and the multi-seller multi-buyer payment-based game model. Table I lists the frequently used notations in this paper.

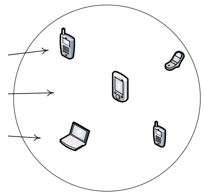


Fig. 1: System model.

A. Wireless Multicast Using Relays

A base station (BS) provides multicast service to a group of users, who are close to each other in a circular area as shown in Fig. 1. We consider a dynamic network, where users frequently join and leave the multicast service. Let $N(t)$, or in its short form N (for presentation simplicity), be the number of users at time t. The data traffic is divided into segments with equal number of bits per segment. For the transmission of each segment, we consider a two-portion wireless multicast as shown in Fig. 2. In the broadcast portion, the BS broadcasts a segment. Then, the users who decode the segment correctly are successful users, and they decide whether to provide relay service to unsuccessful users. In this work, we consider a simple scenario where at most one successful user forwards the segment in the relay portion[1]. At the beginning of the relay portion, there are some information exchanges among the users, to be detailed in Section II-B. Similar to the work in [5], we assume that all communications in the relay portion, including information exchanges and segment relaying, are on a different frequency band from the band used by the BS. Therefore, when the BS finishes broadcast of one segment, it can start broadcasting the next segment immediately.

In this work, it is assumed that the distances from the BS to the users are much larger than those between users. Therefore, each user has the same probability, denoted by p_1, to receive a segment from the BS successfully. Since users are close to each other, we assume that all information exchanges and segment relaying in the relay portion are received correctly with probability 1 by all users.

To evaluate the system performance, we define the relay portion throughput, T_R, as the average percentage of unsuccessful users who receive the segment correctly in the relay portion.

B. Payment-Based Game Formulation

In cooperative multicast systems, relays use their own power to forward segments and help others, but they cannot benefit during this process. To stimulate user cooperation, in this work, we model users' interaction as a multi-seller multi-buyer payment-based game, where each successful user decides whether to sell relay service, and each unsuccessful user decides whether to purchase it. To implement the billing process, we assume that there exists a trusted local agent, who

[1] In this work, for simplicity, unsuccessful users decode the received signal from the relay without combining the previously received signal from the BS. Since combining can improve the system performance, it can be seen that our work can provide a performance lower bound.

TABLE I: Frequently used notations

$b^r(N_s)$	the reserve bid of the auction game with N_s bidders	b^w	the payment that the winner of the auction gets paid if he/she relays a segment
b_i	the bid sent by user i	c_i	user i's cost to forward a segment
$[L, H]$	the range of users' cost to forward a segment	g	the utility gain of correctly receiving a segment
N	the number of users in the multicast service	N_s, N_b	the number of sellers, buyers
N_{su}	the number of successful users after the broadcast from the BS	p_1	the probability of correctly receiving a segment from the BS
p_s	the probability that a user who does not participate in the auction game is a successful user	r^*	the optimal price that maximizes the relay portion throughput
r	the price selected by the local agent	T_R	the relay portion throughput
$\bar{U}_B, \bar{V}_B$	the average utility gain to be a buyer in homogeneous, heterogeneous user case	$\bar{U}_{NB}, \bar{V}_{NB}$	the average utility gain not to be a buyer in homogeneous, heterogeneous user case
$\breve{x}_f, \breve{x}_h$	the root of $f(x) = 0$, $h(x) = 0$	$\tilde{x}_f, \tilde{x}_h$	the root of $f'(x) = 0$, $h'(x) = 0$
x^*	Evolutionarily Stable Strategy (ESS)	$\phi(\cdot)$	the probability distribution function of cost to relay a segment in heterogeneous user case

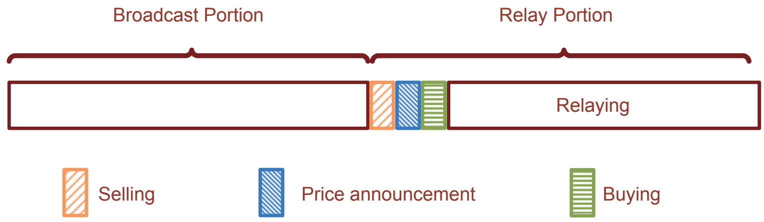


Fig. 2: The procedure of a segment transmission.

listens to the data transmission in the relay portion, charges fees from buyers, and pays the relay. Our multi-seller multi-buyer cooperative multicast game is a 4-stage Stackelberg game as described in details below.

Stage 1: The Sellers' Game. After the broadcast portion, suppose that there are N_{su} successful users. Each of them decides whether to sell relay service. Let $\{\mathbf{S}, \mathbf{NS}\}$ denote their strategy set, including being a seller ($\mathbf{S}$) and not being a seller ($\mathbf{NS}$). Suppose that N_s $(\leq N_{su})$ successful users decide to be sellers. They will send feedbacks to the local agent attached with their IDs and the cyclic redundancy check (CRC)[2] bits of the received segment. Note that messages from sellers in Stage 1 are encrypted and then sent simultaneously[3] in the selling part at the beginning of the relay portion as shown in Fig. 2. Thus, only the local agent can decrypt them, which can prevent a successful user from observing other sellers' messages and adjusting his/her own decision. Then, the local agent knows the N_s sellers.

Stage 2: The Price Setting Game. If $N > N_s > 0$,[4] the local agent selects a seller to provide relay service if there are more than one sellers, and selects a relay price r that will be charged to each buyer for the relay service, and then broadcasts to all users the number N_s of sellers, the user ID of the selected seller, and the relay price r, in the price announcement part in Fig. 2. Thus, everyone will receive the broadcasted information. Details of seller and price selections are given in subsequent sections.

Stage 3: The Buyers' Game. In Stage 3, each unsuccessful user decides whether to purchase the relay service at price r. Let $\{\mathbf{B}, \mathbf{NB}\}$ denote their strategy set, including being a buyer ($\mathbf{B}$) and not being a buyer ($\mathbf{NB}$). All buyers broadcast their IDs simultaneously using CDMA technology in the buying part in Fig. 2. Thus, all others, including the selected seller, hear the buyers' messages and know the number of buyers, denoted N_b $(\leq (N - N_s))$, and thus know the total payment, $N_b r$, that the buyers provide for the relay service.

Stage 4: The Transaction Game. For the selected seller, if forwarding the segment is profitable, i.e., the selected seller can gain a non-negative net utility, then he/she will broadcast the segment to all users in the relaying part in Fig. 2 and every unsuccessful user can hear it; otherwise, the selected seller will not broadcast. After the relaying, the local agent charges from the buyers and pays to the relay node.

This game is repeated for the transmission of all segments.

C. Utility Functions

For each user i, let g denote the utility gain of receiving a segment correctly, and c_i denote his/her cost to forward one segment. In the 4-stage Stackelberg game, if user i is a successful user, his/her utility function is his/her received payment minus c_i if he/she is the selected seller and forwards the segment, and 0 otherwise. If user i is an unsuccessful user, his/her utility function is $(g-r)I_{relay}$ if he/she is a buyer, and

[2]Letting sellers attach the CRC bits of the received segment can prevent an unsuccessful user from pretending to be a successful user.

[3]This can be achieved by code-division multiple access (CDMA) technology. Each node is assigned a unique code. The code is used to spread the node's message. The local agent monitors codes of all the users.

[4]If $N_s = N$, all users are successful after the broadcast portion. If $N_s = 0$, there is no seller. In either scenario, there is no need for the following stages and the game ends.

gI_{relay} otherwise (i.e., user i is a free-rider).[5] Here I_{relay} is a binary value: $I_{relay} = 1$ if there is relay service in the relay portion, and $I_{relay} = 0$ otherwise. Note that we ignore the cost of information exchanges in the selling and buying parts, since the amount of related information exchanges is small.

III. Game Analysis with Homogeneous Users

We start with a simple scenario where the users are homogeneous, i.e., they have the same cost of providing relay service with $c_i = c$ being a positive constant. In addition, when there are more than one sellers, the local agent will randomly select one to forward the segment, and all sellers have the same probability to be selected.

We use backward induction to find the subgame perfect Nash Equilibrium (SPNE) of the game. Typically, backward induction first analyzes the last stage of the game, moves up stage by stage, and studies the first stage the last. However, the result of the transaction game can simplify the sellers' game and we can easily find the sellers' optimal strategy that belongs to the SPNE. Thus, we will study the sellers' game after the transaction game. The result of the analysis for the sellers' game can help reduce the number of possible outcomes of the sellers' game, and simplify the analysis of the price setting game and the buyers' game.

A. The Transaction Game

After buyers broadcast their decisions, the selected seller knows the amount of payment buyers offer, $N_b r$. The selected seller will forward the segment if $N_b r \geq c$, and will not forward otherwise. Therefore, the transaction game ensures that the selected seller will always receive a non-negative net utility gain. After the relaying, the local agent charges price r from each buyer and pays $N_b r$ to the relay node.

B. The Sellers' Game

Since the selected seller will make a non-negative net utility gain in the transaction game, the sellers' game has an obvious solution belonging to the SPNE, i.e., all successful users take strategy $\mathbf{S}$ and become sellers. This is because by taking the strategy $\mathbf{S}$ and being a seller, a successful user's utility gain in the relay portion is no less than zero, while by taking the strategy $\mathbf{NS}$ and not being a seller, his/her utility gain in the relay portion is zero. Therefore, $\mathbf{S}$ is a weakly dominant strategy over $\mathbf{NS}$, and every successful user should choose it. Thus, after the sellers' game we have $N_s = N_{su}$, i.e., the number of sellers equals the number of successful users. As the local agent will announce (in the price setting game) the number N_s of sellers, all unsuccessful users will know the value of $N_{su} = N_s$ before the buyers' game.

[5]Recall that we assume the relay channels are perfect. When the relay channels are imperfect, similar to [6,13], we can assume that an unsuccessful user can receive a segment correctly from a relay with the same probability p_2, since the probability is usually high and close to 1. Then we only need to replace g in the utility functions of unsuccessful users by $p_2 g$, and all subsequent derivations will keep the same as the case with perfect relay channels.

C. The Buyers' Game

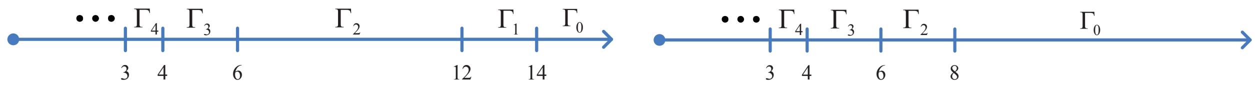
Given the relay price r decided by the local agent and the number N_{su} of sellers, unsuccessful users decide whether to purchase the relay service. Recall that when the total payment from all buyers $N_b r$ is no less than the cost c, the selected seller will relay the segment. Due to the broadcast nature of wireless communications, unsuccessful users who do not pay may overhear the segment forwarded by the relay and enjoy a free ride. Here, unsuccessful users face a dilemma: everyone wants to free ride the relay service bought by others and pay nothing, while there will be no relay service if there are not sufficient buyers, and every unsuccessful user will gain nothing. To solve this problem, we model the buyers' game as an evolutionary game [24], and derive the *Evolutionarily Stable Strategy* (ESS), which is a stable Nash Equilibrium. This means that, even if some players deviate from the ESS, they will still come back to the ESS, since using the ESS gives a higher payoff.

To derive the ESS, evolutionary game theory provides a useful tool, called *replicator dynamics*. In our game, each unsuccessful user has two strategies: $\mathbf{B}$ or $\mathbf{NB}$. For all unsuccessful users, let x be the population share playing strategy $\mathbf{B}$, where $x \in [0, 1]$, and the rest $(1 - x)$ population share plays strategy $\mathbf{NB}$. By replicator dynamics, we have the following differential equation:

$$\begin{aligned} \dot{x} &= \eta(\bar{U}_B(x) - \bar{U}(x))x \\ &= \eta[\bar{U}_B(x) - x\bar{U}_B(x) - (1 - x)\bar{U}_{NB}(x)]x \\ &= \eta x(1 - x)f(x), \end{aligned} \quad (1)$$

where $\dot{x}$ is the population increase of strategy $\mathbf{B}$, η is a constant step size, $\bar{U}_B(x)$ is the average payoff of using pure strategy $\mathbf{B}$, $\bar{U}_{NB}(x)$ is the average payoff of using pure strategy $\mathbf{NB}$, $\bar{U}(x) = x\bar{U}_B(x) + (1 - x)\bar{U}_{NB}(x)$ denotes the average payoff of the population, and $f(x) = \bar{U}_B(x) - \bar{U}_{NB}(x)$. The intuition behind this differential equation is that if using pure strategy $\mathbf{B}$ introduces a higher payoff than the average payoff of the entire population, the population share of pure strategy $\mathbf{B}$ should increase. At the stable state x, this differential equation should be equal to 0. Similar to [25], [26], the population share x can be interpreted as a mixed strategy, which denotes the probability that players adopt pure strategy $\mathbf{B}$. Since any unsuccessful user gets the same gain g if he/she correctly receives a segment, all unsuccessful users are symmetric and should have the same mixed strategy x, denoted x^*, when they reach the ESS. For presentation simplicity, we say ESS is x^*. In the following, given the number N_{su} of sellers, and for any relay price r selected by the local agent, we derive $\bar{U}_B(x)$ and $\bar{U}_{NB}(x)$ for unsuccessful user i, and then find the ESS x^*.[6]

[6]In games with incomplete information, each user has private information, which is unknown to the others. Replicator dynamics can help solve games with incomplete information (e.g. [25]), where the game is repeated for multiple shots, and users learn from the interactions with others, adjust their strategies towards a higher payoff, and finally may reach the ESS. Unlike the game with incomplete information, our game is a one-shot game with complete information, where each user's gain g, the number of unsuccessful users $(N - N_{su})$, and the relay cost c are all public information. Thus, similar to [26], the ESS can be derived directly by solving (1), and there is no learning process involved in our game analysis.



(a) (b)

Fig. 3: Examples of price intervals. (a) $g = 14$ and $c = 12$. (b) $g = 8$, $c = 12$ and Γ_1 does not exist in this case.

1) Analysis of $\bar{U}_B(x)$ and $\bar{U}_{NB}(x)$: Given N_{su} sellers, for unsuccessful user i, let $\mathcal{X}_{-i}$ denote the set of all other unsuccessful users. So $|\mathcal{X}_{-i}| = l \triangleq N - N_{su} - 1$. Recall that each unsuccessful user purchases the relay service with probability x. Therefore, the number of buyers in $\mathcal{X}_{-i}$, denoted k, follows Binomial distribution $B(l, x)$.

In this context, if user i decides to be a buyer, the total number of buyers is $(k+1)$, and thus, the total payment from all buyers is $(k+1)r$. If $(k+1)r \geq c$, this payment can afford the relay service, and user i receives the segment correctly and pays the price r; otherwise, there is no relay service and user i's utility in the relay portion is 0. Therefore, in the relay portion, user i's average utility of strategy $\mathbf{B}$ is

$$\bar{U}_B(x) = (g-r)\sum_{k=0}^{l}\binom{l}{k}x^k(1-x)^{l-k}I\left[(k+1)r \geq c\right], \quad (2)$$

where $I[\cdot]$ is an indicator function. If user i decides not to be a buyer, the total number of buyers is k, and the total payment is kr. If $kr \geq c$, this payment can still afford a relay. After the relay portion, user i can overhear the relay and receive the segment correctly. Therefore, the average utility of the strategy $\mathbf{NB}$ is

$$\bar{U}_{NB}(x) = g\sum_{k=0}^{l}\binom{l}{k}x^k(1-x)^{l-k}I[kr \geq c]. \quad (3)$$

Then, we have $f(x) = \bar{U}_B(x) - \bar{U}_{NB}(x)$ as

$$\begin{aligned} f(x) =& \bar{U}_B(x) - \bar{U}_{NB}(x) \\ =& \sum_{k=0}^{l}\binom{l}{k}x^k(1-x)^{(l-k)}\Big\{\Big(I[(k+1)r \geq c] \\ &- I[kr \geq c]\Big)g - I[(k+1)r \geq c]r\Big\} \\ =& g\binom{l}{k^*}x^{k^*}(1-x)^{(l-k^*)} - r\sum_{k=k^*}^{l}\binom{l}{k}x^k(1-x)^{(l-k)}, \end{aligned} \quad (4)$$

where $k^* = \lceil c/r \rceil - 1$, and $\lceil \cdot \rceil$ is the ceiling function. Here, $\lceil c/r \rceil$ is the minimal number of buyers required to afford the relay service at price r.

2) The ESS Solution: From (1), at the stable state $\dot{x} = 0$, there are three possible solutions: $x = 0$, $x = 1$, and x that satisfies $f(x) = \bar{U}_B(x) - \bar{U}_{NB}(x) = 0$. In our game, the relay price r plays an important role in the unsuccessful users' decision-making process, and all those three solutions can be ESS x^*, which will be discussed as follows. The analysis results are summarized in Theorem 1 following the analysis.

Define $\underline{j} = \lfloor c/g \rfloor + 1 \geq 1$, where $\lfloor \cdot \rfloor$ is the floor function. We partition the price range $[0, +\infty)$ into the following subintervals:

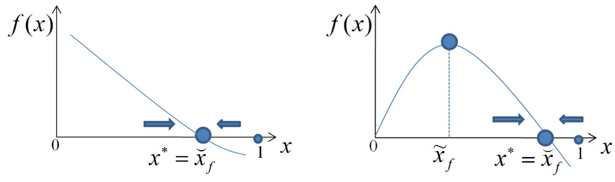


Fig. 4: (a) $r \in \Gamma_j$, where $j = \underline{j} = 1$. (b) $r \in \Gamma_j$, where $1 < j < N - N_{su}$.

$$\Gamma_0 = [g, \infty), \Gamma_{\underline{j}} = \left[\frac{c}{\underline{j}}, g\right), \text{ and}$$
$$\Gamma_j = \left[\frac{c}{j}, \frac{c}{j-1}\right) \text{ for } j > \underline{j}. \quad (5)$$

When the price r is in range Γ_j with $j \geq \underline{j}$, at least j buyers are needed to afford the relay service, and Γ_0 is the range of the price that equals or exceeds users' utility gain of receiving a segment correctly. Fig. 3 shows examples of the price intervals when g and c take different values.

• Case 1, $r \in \Gamma_0$, i.e., $r \geq g$: From (4), for all $x \in [0, 1]$, we have

$$\begin{aligned} f(x) =& \binom{l}{k^*}x^{k^*}(1-x)^{(l-k^*)}g - \sum_{k=k^*}^{l}\binom{l}{k}x^k(1-x)^{(l-k)}r \\ \leq& \binom{l}{k^*}x^{k^*}(1-x)^{(l-k^*)}(g-r) \leq 0. \end{aligned} \quad (6)$$

Thus, the strategy $\mathbf{NB}$ always outperforms $\mathbf{B}$ and users will converge to $x^* = 0$, which is the ESS. This is because given $r \geq g$, the price is too high when compared to the utility gain from receiving the relay service. Thus, nobody will buy.

• Case 2, $r \in \Gamma_j$ with $j \in \{\underline{j}, \underline{j}+1, ..., N - N_{su} - 1\}$: In this case, we analyze the ESS when $j = \underline{j} = 1$ (which happens only when $c < g$) and when $1 < j < N - N_{su}$ separately.

When $j = \underline{j} = 1$, i.e., $r \in \Gamma_1 = [c, g)$, one buyer is sufficient to buy the relay service and $k^* = \lceil c/r \rceil - 1 = 0$ in (4). Therefore, $f(x)$ in (4) can be simplified as $f(x) = (1-x)^{N-N_s-1}g - r$ with $f(0) = g - r > 0$ and $f(1) = -r < 0$. In addition, $f'(x) = -g(N - N_s - 1)(1-x)^{N-N_s-2} < 0$ and thus, $f(x)$ is a decreasing function for $x \in (0, 1)$, as shown in Fig. 4a. Thus, $f(x) = 0$ has a single root $\breve{x}_f = (1 - \sqrt[l]{\frac{r}{g}}) \in (0, 1)$, which is the ESS. To understand this, let x deviate from $\breve{x}_f$. If $x \in [0, \breve{x}_f)$, we have $f(x) > 0$, which means strategy $\mathbf{B}$ can give a higher utility than $\mathbf{NB}$. Therefore, users will increase the probability of using $\mathbf{B}$ and x will move towards $\breve{x}_f$. Similarly, if $x \in (\breve{x}_f, 1]$, we have $f(x) < 0$, which means strategy $\mathbf{B}$ will give a lower utility. Thus, users will reduce the probability of using strategy $\mathbf{B}$ and adjust their strategy towards $\breve{x}_f$. Thus, $x^* = \breve{x}_f$ is the ESS.

When $r \in \Gamma_j$ with $1 < j < N - N_{su}$, i.e., $r \in [\frac{c}{j}, \frac{c}{j-1})$, at least j buyers are required to afford the relay service, and $k^* = \lceil c/r \rceil - 1 > 0$. From (4), $f(0) = 0$ and $f(1) = -r < 0$. In addition, we prove in Appendix I that $f'(x) = 0$ has a single root $\tilde{x}_f$ in the range $(0, 1)$, where $f'(x) > 0$ when $x \in (0, \tilde{x}_f)$ and $f'(x) < 0$ when $x \in (\tilde{x}_f, 1)$, as shown in Fig. 4b. Therefore, $f(\tilde{x}_f) > 0$, and $f(x) = 0$ has a single root $\breve{x}_f$ in the range $(\tilde{x}_f, 1)$. Same as the analysis in Fig. 4a, $x^* = \breve{x}_f$ is the ESS.

• Case 3, $r \in \Gamma_{N-N_{su}} = [\frac{c}{N-N_{su}}, \frac{c}{N-N_{su}-1})$: In this price range, the relay price requires at least $(N-N_{su})$ buyers, while there are $(N-N_{su})$ unsuccessful users and thus at most $(N-N_{su})$ buyers. Therefore, there is no chance to free ride, and all unsuccessful users will buy with $x^* = 1$. Mathematically, when $r \in \Gamma_{N-N_{su}}$, $k^* = \lceil c/r \rceil - 1 = N - N_{su} - 1 = l$ and $f(x)$ in (4) can be simplified as $f(x) = (g-r)x^{N-N_{su}-1} \geq 0$ for all $x \in [0, 1]$. Therefore, the strategy $\mathbf{B}$ always outperforms strategy $\mathbf{NB}$, and $x^* = 1$.

• Case 4, $r \in \Gamma_j$ with $j \geq N - N_{su} + 1$: In these price ranges, at least $j > N - N_{su}$ buyers are required to afford the relay service, while there are only $(N - N_{su})$ unsuccessful users. Therefore, there are not sufficient buyers to afford the relay service, and the game ends.

In summary, we have the following theorem.

Theorem 1: Given N_{su} sellers in stage 1 and the relay price r,

- Case 1, when $r \geq g$ (i.e., $r \in \Gamma_0$), $x^* = 0$ is the ESS and no one buys;
- Case 2, when $\frac{c}{N-N_{su}-1} \leq r < g$ (i.e., $r \in \Gamma_j \cup \Gamma_{j+1} \cup ... \cup \Gamma_{N-N_{su}-1}$), for $x \in (0, 1)$, $f(x) = 0$ has a single root $\breve{x}_f$, which is the ESS, i.e., $x^* = \breve{x}_f$;
- Case 3, when $\frac{c}{N-N_{su}} \leq r < \frac{c}{N-N_{su}-1}$ (i.e., $r \in \Gamma_{N-N_{su}}$), $x^* = 1$ is the ESS and all unsuccessful users buy;
- Case 4, when $r < \frac{c}{N-N_{su}}$ (i.e., $r \in \Gamma_{N-N_{su}+1} \cup \Gamma_{N-N_{su}+2} \cup ...$), there are not sufficient buyers, and the game ends.

Note that in the above discussion and Theorem 1, we assume that $\frac{c}{N-N_{su}} < g$. When $\frac{c}{N-N_{su}} \geq g$, if $r \geq \frac{c}{N-N_s} \geq g$, following the discussion in Case 1, $x^* = 0$ and no one buys; while if $r < \frac{c}{N-N_s}$, following the discussion in Case 4, there are not sufficient buyers to afford the relay service. Therefore, with $\frac{c}{N-N_{su}} \geq g$, the game will end with the relay portion throughput being zero.

Fig. 5 shows an example of ESS x^* at different price r and with different number $(N - N_{su})$ of unsuccessful users. The total number of users in the network is $N = 12$. The cost to forward one segment is $c = 2$ and the gain of correctly receiving one segment is $g = 1$. We first study x^* at different price with a fixed number of unsuccessful users and use $(N - N_{su}) = 5$ as an example. We observe that when $r < \frac{c}{N-N_{su}} = 0.4$ (i.e., in price ranges Γ_6, Γ_7...), the number of buyers is not sufficient and the game ends with no relay service. If $r \in [0.4, 1)$, we observe that at a lower price, the game requires more buyers to pay the relay service, and thus, unsuccessful users have a smaller probability to free ride with a larger x^*. For $r \geq g = 1$, no user buys and the ESS is $x^* = 0$. Fig. 5 also shows the ESS with a different number of unsuccessful users, $(N - N_{su}) = 8$. It can be seen that for a given price r in the price range Γ_5 to Γ_3, more unsuccessful users give a smaller ESS x^*. This is because when the number of unsuccessful users is large, each unsuccessful user expects other unsuccessful users to purchase the relay service and he/she has a higher tendency to free ride.

D. Price Setting Game and Throughput Optimization

From the previous discussion, at different price r selected by the local agent, we may have different ESS, and thus different relay portion throughput. Therefore, x^* is a function of r. For presentation simplicity, we use x^* to represent $x^*(r)$ in later discussion. In the following, we will analyze the optimal price that maximizes the system throughput.

Given $(N - N_{su})$ unsuccessful users and price r, each unsuccessful user follows the ESS x^* to play the buyers' game, and the relay portion throughput is

$$T_R(x^*|N_{su}) = \sum_{k=\lceil c/r \rceil}^{N-N_{su}} \binom{N - N_{su}}{k} (x^*)^k (1 - x^*)^{N-N_{su}-k}, \tag{7}$$

where the summation term denotes the probability that there are sufficient buyers to afford the relay service. The local agent aims to find the optimal r^* that can maximize $T_R(x^*|N_{su})$,

$$r^* = \arg\max_r T_R(x^*|N_{su}). \tag{8}$$

From Theorem 1, when $r \geq g$, $x^* = 0$ and $T_R(x^*|N_{su}) = 0$. When $r < \frac{c}{N-N_{su}}$, the number of buyers is insufficient and the game ends also with zero relay portion throughput. Therefore, the optimal price r^* is in the range $\left[\frac{c}{N-N_{su}}, g\right)$.

From Theorem 1, when $\frac{c}{N-N_{su}} \leq r < g$, there are two possible ESS, $x^* = 1$ when $\frac{c}{N-N_{su}} \leq r < \frac{c}{N-N_{su}-1}$ and $x^* \in (0, 1)$ when $\frac{c}{N-N_{su}-1} \leq r < g$. Comparing the relay portion throughput when $x^* = 1$ and $x^* \in (0, 1)$, we have

$$\begin{aligned} & T_R(x^*|N_{su})|_{x^*=1} \\ = & \sum_{k=\lceil c/r \rceil}^{N-N_{su}} \binom{N - N_{su}}{k} 1^k (1 - 1)^{N-N_{su}-k} = 1, \end{aligned} \tag{9}$$

and

$$\begin{aligned} & T_R(x^*|N_{su})|_{x^* \in (0,1)} \\ = & \sum_{k=\lceil c/r \rceil}^{N-N_{su}} \binom{N - N_{su}}{k} (x^*)^k (1 - x^*)^{N-N_{su}-k} \\ < & \sum_{k=0}^{N-N_{su}} \binom{N - N_{su}}{k} (x^*)^k (1 - x^*)^{N-N_{su}-k} = 1. \end{aligned} \tag{10}$$

Therefore, $x^* = 1$ gives a higher relay portion throughput, and the optimal price r^* should be in the price range $\Gamma_{N-N_{su}}$ that gives $x^* = 1$.

Fig. 6 shows an example of x^* and $T_R(x^*|N_{su})$ at different price r with $N - N_{su} = 10$, $c = 3$, and $g = 1$. Following the previous discussion, the optimal price should lie in $\Gamma_{N-N_{su}}$, which corresponds to $[0.30, 0.333)$ in Fig. 6. In this price

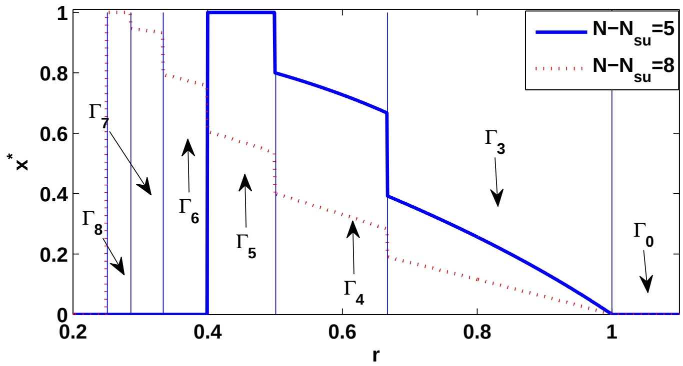


Fig. 5: An example of x^*.

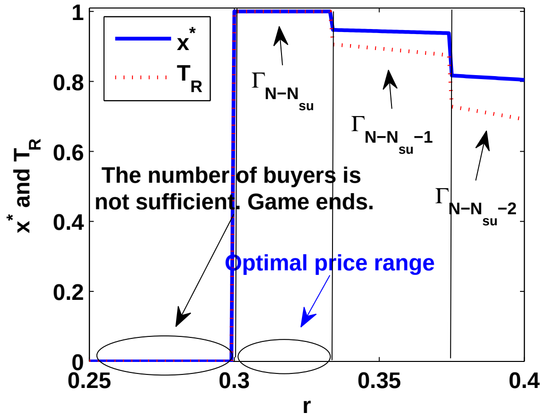


Fig. 6: An example of x^* and T.

range, we observe that $x^* = 1$, and $T_R(x^*|N_{su}) = 1$, which is the maximum relay portion throughout. In this scenario, all unsuccessful users have to buy together to afford the relay service. Any free riding behavior will result in the failure of purchasing a relay, which lowers the utilities of all unsuccessful users. Thus, the optimal price drives the buyers' game to the equilibrium where all unsuccessful users buy and share the cost to afford the relay service.

E. Equilibrium of the Stackelberg Game

To summarize, the subgame perfect Nash Equilibrium (SPNE) of the Stackelberg game is

- Stage 1, the sellers' game. **S** is a weakly dominant strategy, and all successful users decide to be sellers.
- Stage 2, the price setting game. Given $(N - N_{su})$ unsuccessful users, if $\frac{c}{N-N_{su}} < g$, the optimal price can be any value in the range $\Gamma_{N-N_{su}}$. Otherwise, the game ends.
- Stage 3, the buyers' game. At the optimal price r^* selected in Stage 2, all unsuccessful users decide to be buyers with probability $x^* = 1$.
- Stage 4, the transaction game. Since under the optimal price r^*, all unsuccessful users are buyers and $r^*(N - N_{su}) \geq c$, being a relay is profitable, and the selected seller forwards the segment, and gets payment $r^*(N - N_{su})$.

At this SPNE, the relay portion throughput is 1 if $\frac{c}{N-N_{su}} < g$, and 0 otherwise.

IV. Wireless Multicast With Heterogeneous Users

In wireless multicast, users may use different mobile devices with different remaining energy, whose costs to forward a segment are different. For example, the cost to forward a segment using a smart phone with little remaining energy is much higher than that when using a laptop with much more energy left. In this section, we will study cooperative wireless multicast when users have different costs of forwarding a segment, referred to as *heterogeneous users*.

A. Game Model for Heterogeneous Users

In this work, we assume that each user's cost is his/her private information, which is independent and identically distributed following the same distribution $\phi(\cdot)$ in the range $[L, H]$, where L and H are the lower and upper bounds of a user's cost. Thus, successful users may request different payments to forward a segment. Since the costs are their private information, they may lie to others if cheating can help improve their own utilities. For example, successful users may claim high costs so that to ask high payments for providing the relay service. However, if the asked payments are too high, unsuccessful users may not be able to afford, and thus, the system efficiency may be reduced. To encourage successful users to tell their true costs, we use the second-price sealed-bid auction, which is a truth-telling auction [27]. To help readers have the whole picture of the system, the detailed game model is illustrated below.

Stage 1: The Sellers' Auction Game. After the broadcast portion, assume that there are N_{su} successful users. Each of them decides whether to sell relay service. Let $\{\mathbf{S}, \mathbf{NS}\}$ denote their strategy set, including being a seller ($\mathbf{S}$) and not being a seller ($\mathbf{NS}$). Assume that $N_s(\leq N_{su})$ successful users decide to be sellers. They will enter the auction game, where a seller, say user i, submits to the local agent his/her bid including his/her ID and the payment b_i he/she asks for. As discussed in Section II-B, messages from the sellers are encrypted and then sent simultaneously using CDMA technology in the selling part in Fig. 2. So only the local agent can decrypt them, which can prevent others from overhearing the transmission and avoid potential leak of their bidding information.

Stage 2: The Price Setting Game. The local agent, as the host of the auction, decides the winner of the auction, and the winning bid, denoted by b^w, that the winner will get paid after relaying the segment. Following the second-price sealed-bid protocol, the winner is the seller with the lowest bid, and b^w is the second lowest bid or the reserve bid $b^r(N_s)$, whichever is less. Here $b^r(N_s)$ is a function of N_s and denotes the highest payment that buyers will accept. From the discussion in Section III-C, no user will buy if $r \geq g$. Here r is still the relay price that will be charged to each buyer, and g is still the utility gain of correctly receiving a segment. Given that there are N_s sellers, the number of buyers is no more than $(N - N_s)$, and the highest total payment from all buyers is less than $(N - N_s)g$. Thus, the reserve bid should satisfy $b^r(N_s) < (N - N_s)g$. In this work, we set $b^r(N_s) = (N - N_s)g - \epsilon$, where ϵ is an arbitrarily small positive number. Note that if the bids of all the sellers are larger than $b^r(N_s)$, there is no winner of this auction game, and the game ends with no relay service.

The local agent also selects a relay price r that will be charged to each buyer for the relay service, and then announces (in the price announcement part in Fig. 2) to all users the number N_s of sellers, the winning bidder's ID, the winning bid b^w, and the relay price r. The local agent selects the optimal relay price r to maximize the system throughput. As will be shown in our analysis in Section IV-B.3, the optimal price r depends on the number of users who do not participate in the auction game (i.e., $(N - N_s)$) and users' cost distribution $\phi(\cdot)$.

Stage 3: The Buyers' Game. Based on information announced by the local agent, unsuccessful users decide whether to be buyers. All buyers broadcast their IDs simultaneously in the buying part in Fig. 2. The winning bidder listens to buyers' messages, and knows the number N_b of buyers and the total payment $N_b r$ the buyers offer.

Stage 4: The Transaction Game. For the winning bidder, say user i, if the winning bid b^w is not less than his/her cost c_i, user i is willing to forward the segment. However, the local agent pays user i only when the total payment $N_b r$ from the buyers is not less than the winning bid b^w and user i forwards the segment. Thus, user i will forward the segment if $N_b r \geq b^w \geq c_i$, and the local agent charges r from each buyer and pays b^w to user i. Otherwise, user i will not relay the segment and the game ends.

Note that after the transaction, there might be extra unused payment of $(N_b r - b^w)$. The local agent will keep this unused payment and accumulate it from each segment transmission. Once the accumulated amount is larger than the winning bid b^w in one round, the local agent uses it to pay the relay service and all unsuccessful users enjoy a free segment forwarding in that round. In this paper, we consider the scenario where users frequently join and leave the multicast service. They may leave before the next free relay service. Thus, we ignore the impact of free relay service on buyers' utility in our analysis.

B. Subgame Perfect Nash Equilibrium Analysis

Note that the transaction game ensures that the relay will always receive a non-negative net utility gain in the game. This result can simplify the sellers' auction game. Thus, similar to the analysis in Section III, next we first study the sellers' auction game, followed by the buyers' game and the price setting game.

1) The Sellers' Auction Game: In this stage, each successful user decides whether to bid and how to bid in the auction game. Note that in this stage, the reserve bid $b^r(N_s)$ is unknown (as the number N_s of sellers is unknown), and a successful user's decision will also affect $b^r(N_s)$, which should be taken into consideration when choosing his/her strategy. For a successful user i, we have the following proposition.

Proposition 1: For a successful user i with cost c_i to forward a segment, if $c_i > b^r(1)$, he/she should not enter the auction game. Otherwise, he/she should participate in the auction game and bid $b_i = c_i$. This is a weakly dominant strategy.

Proof: We first show that a successful user, say user i, should not enter the auction game if $c_i > b^r(1)$, and he/she should participate in the game otherwise.

Note that $b^r(1)$ is the highest possible reserve bid (i.e., the reserve bid when there is only one successful user), and thus, it is the highest payment a seller can receive. If user i's cost c_i is larger than $b^r(1)$, he/she cannot benefit from serving as a relay, and thus should not enter the auction game. When $c_i \leq b^r(1)$, if user i takes strategy $\mathbf{NS}$ and does not bid, his/her utility in the relay portion is 0. However, if he/she participates in the auction, he/she has a positive probability to win the auction and make a non-negative net utility gain by forwarding the segment. Thus, using strategy $\mathbf{S}$, his/her expected utility in the relay portion is non-negative, and $\mathbf{S}$ is a weakly dominant strategy over $\mathbf{NS}$. Therefore, user i should enter the auction game as long as $c_i \leq b^r(1)$.

In the following, we prove that when the successful user i decides to participate in the auction game, bidding his/her real cost is a weakly dominant strategy. To illustrate this, we define $\hat{b}_i = \min_{j \neq i} b_j$ as the smallest bid excluding b_i, and consider two different scenarios.

- $c_i \leq \min(b^r(N_s), \hat{b}_i)$**:** In this case, user i can win the auction by bidding any value in the range $\Big(0, \min(b^r(N_s), \hat{b})\Big]$, and this range includes his/her true cost c_i. Then, his/her winning bid is $b^w = \min(b^r(N_s), \hat{b}_i)$. Thus, he/she has the chance to make a net utility gain of $b^w - c_i \geq 0$. However, if he/she bids a price higher than $\min(b^r(N_s), \hat{b})$, he/she cannot win the auction, and his/her utility in the relay portion is zero. Thus, $b_i = c_i$ is a weakly dominant strategy.
- $c_i > \min(b^r(N_s), \hat{b}_i)$**:** In this case, user i cannot win the auction by bidding any value in the range $\Big(\min(b^r(N_s), \hat{b}_i), +\infty\Big)$, and this range includes his/her true cost c_i. Then, his/her payoff in the relay portion is zero. On the other hand, if he/she bids $b_i \leq \min(b^r(N_s), \hat{b}_i)$, he/she can win the auction, but will not relay, as the payment for relaying is $b^w = \min(b^r(N_s), \hat{b}_i)$ which is less than his/her

cost c_i. Thus, his/her payoff is also zero. Therefore, in this scenario, for any b_i, user i's payoff is zero.

From the above discussions, the successful user i should bid $b_i = c_i$ if $c_i \leq b^r(1)$, and it is a weakly dominant strategy. ■

From Proposition 1, each seller bids his/her real cost in the auction game. Thus, if there is a winner of the auction (i.e., the lowest bid is no larger than the reserver bid, $b^r(N_s)$), the winning bidder is the bidder who has the lowest cost among all bidders, and the winning bid b^w is no less than the winner's cost. Thus, in the transaction game, as long as $N_b r \geq b^w$, the winning bidder will forward the segment.

2) The Buyers' Game: In the buyers' game, based on the number N_s of bidders, the winning bid b^w, and the price r determined by the local agent, each unsuccessful user decides whether to be a buyer. Similar to the game with homogeneous users, we also model unsuccessful users' interaction as an evolutionary game, while the analysis is more complicated. This is because given N_s bidders, the rest $(N - N_s)$ users include unsuccessful users who may or may not purchase the relay service as well as successful users who do not bid in the sellers' auction game. Unsuccessful users should take this into consideration when choosing their strategies. Specifically, Given N_s bidders, for unsuccessful user i, let $\mathcal{Y}_{-i}$ denote the set of all other unsuccessful users and successful users who do not bid in the sellers' auction game. So $|\mathcal{Y}_{-i}| = l \triangleq N - N_s - 1$. Recall that a successful user will bid if his/her cost is below $b^r(1)$. Since each user's cost follows the probability distribution $\phi(\cdot)$, the probability that a successful user does not bid is $1 - \Phi(b^r(1))$, where $\Phi(\cdot)$ is the cumulative distribution function of $\phi(\cdot)$. Then for each user in $\mathcal{Y}_{-i}$, the probability that he/she is a successful user is

$$p_s = \frac{p_1\Big[1 - \Phi(b^r(1))\Big]}{p_1\Big[1 - \Phi(b^r(1))\Big] + (1 - p_1)}. \tag{11}$$

Let n be the number of unsuccessful users in $\mathcal{Y}_{-i}$, and it follows Binomial distribution $B(l, 1 - p_s)$.

Based on the above discussion on the number of unsuccessful users, following a similar analysis in Section III-C, for each unsuccessful user i, we first derive his/her average utility $\bar{V}_B(x)$ and $\bar{V}_{NB}(x)$ by using the strategy $\mathbf{B}$ and $\mathbf{NB}$, respectively, and then find the ESS x^*.[7]

Recall that each unsuccessful user purchases the relay service with probability x. Thus, given n unsuccessful users in $\mathcal{Y}_{-i}$, the conditional number k of buyers in $\mathcal{Y}_{-i}$ follows Binomial distribution $B(n, x)$. In this context, if user i decides to be a buyer, the total number of buyers is $k+1$, and thus, the total payment from all buyers is $(k+1)r$. If $(k+1)r \geq b^w$, the winning bidder forwards the segment, and user i receives the segment correctly and pays the price r; otherwise, there is no relay service and user i's utility in the relay portion is 0. Therefore, in the relay portion, user i's average utility of strategy $\mathbf{B}$ is

$$\bar{V}_B(x) = (g - r)\sum_{n=0}^{l}\binom{l}{n}(1-p_s)^n p_s^{l-n}\Bigg\{ \sum_{k=0}^{n}\binom{n}{k}x^k(1-x)^{n-k} I[(k+1)r \geq b^w]\Bigg\}, \tag{12}$$

and his/her average utility of strategy $\mathbf{NB}$ is

$$\bar{V}_{NB}(x) = g\sum_{n=0}^{l}\binom{l}{n}(1-p_s)^n p_s^{l-n}\Bigg\{ \sum_{k=0}^{n}\binom{n}{k}x^k(1-x)^{n-k} I[kr \geq b^w]\Bigg\}. \tag{13}$$

Let $h(x) = \bar{V}_B(x) - \bar{V}_{NB}(x)$, and Appendix II shows that $h(x)$ can be rewritten as

$$\begin{aligned} h(x) =& g\binom{l}{k^*}[x(1-p_s)]^{k^*}[1 - x(1-p_s)]^{(l-k^*)} \\ &- r\sum_{k=k^*}^{l}\binom{l}{k}[x(1-p_s)]^k[1-x(1-p_s)]^{(l-k)} \\ =& f\Big((1-p_s)x\Big). \end{aligned} \tag{14}$$

In (14), $k^* = \lceil b^w/r \rceil - 1$ where $\lceil b^w/r \rceil$ is the minimum number of buyers required to afford the relay service, and $f(x)$ is defined in (4).

Similar to the analysis in Section III-C, at the stable state x, we have $\dot{x} = \eta x(1-x)h(x) = 0$. Thus, we have three possible solutions: $x = 0$, $x = 1$, and x satisfies $h(x) = 0$, all of which can be ESS x^*. To study the ESS x^* at different prices, similar to the analysis in Section III-C.2, we let $\underline{j} = \lfloor \frac{b^w}{g} \rfloor + 1$ and partition the whole price range $[0, +\infty)$ into subintervals

$$\Gamma_0 = [g, +\infty), \Gamma_{\underline{j}} = \left[\frac{b^w}{\underline{j}}, g\right), \text{ and}$$
$$\Gamma_j = \left[\frac{b^w}{j}, \frac{b^w}{j-1}\right) \text{ for } j > \underline{j}. \tag{15}$$

In price range Γ_j with $j \geq \underline{j}$, at least j buyers are required to afford the relay service.

• Case 1, $r \in \Gamma_0$, i.e., $r \geq g$: Same as the analysis in the game with homogeneous users, no user buys and $x^* = 0$ is the ESS.

• Case 2, $r \in \Gamma_j$ with $\underline{j} \leq j \leq N - N_s - 1$: Same as in Section III-C.2, we study the ESS when $j = \underline{j} = 1$ (which happens only when $b^w < g$) and when $1 < \underline{j} < N - N_s$, separately.

When $j = \underline{j} = 1$, i.e, $r \in [b^w, g)$, we have $k^* = \lceil b^w/r \rceil - 1 = 0$. From the analysis in Section III-C.2, when $k^* = 0$, $f(x)$ in (4) is a decreasing function of x for $0 \leq x \leq 1$, and $f(x) = 0$ has a single root $\breve{x}_f \in (0, 1)$. From $h(x) = f\Big((1-p_s)x\Big)$, $h(x)$ is a decreasing function of x in the interval $\Big(0, \frac{1}{1-p_s}\Big)$, and $h(x) = 0$ has a single root in the interval $\Big(0, \frac{1}{1-p_s}\Big)$, given as $\breve{x}_h = \frac{\breve{x}_f}{1-p_s}$. When $\breve{x}_f \geq 1 - p_s$, or equivalently, $h(1) = f(1-p_s) \geq 0$, as shown in Fig. 7a. Thus, $h(x) \geq 0$ for all $0 \leq x \leq 1$, and strategy $\mathbf{B}$

[7] Similar to the homogeneous case, the evolutionary game in the heterogenous case is also a one-shot game with complete information, where each user's gain g, the probability distribution function of the number of unsuccessful users, and the winning bid b^w are all public information. Thus, we can directly derive the ESS by solving (1).

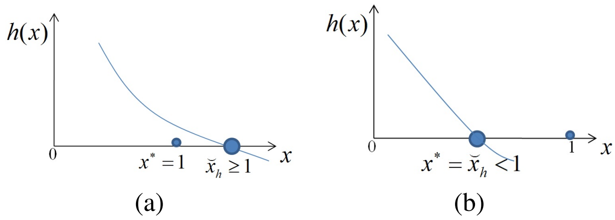


Fig. 7: $r \in \Gamma_1$. (a): $h(1) > 0$, (b): $h(1) < 0$.

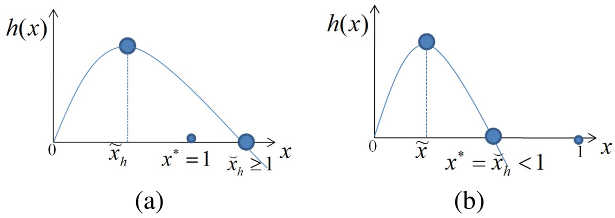


Fig. 8: $r \in \Gamma_j$, where $1 < j < N - N_s$. (a): $h(1) > 0$, (b): $h(1) < 0$.

gives a higher utility than strategy **NB**. So all unsuccessful users buy with $x^* = 1$ being the ESS. When $\breve{x}_f < 1 - p_s$, that is, $h(1) = f(1 - p_s) < 0$, $\breve{x}_h$ is in the interval $(0, 1)$. Similar to the homogeneous user case, $x^* = \breve{x}_h$ is the ESS, as shown in Fig. 7b.

When $r \in \Gamma_j$ with $1 < j < N - N_s$, we have $k^* = \lceil b^w/r \rceil - 1 > 0$. From the analysis in Section III-C.2, when $k^* > 0$, $f'(x) = 0$ has a single root $\tilde{x}_f \in (0, 1)$, where $f'(x) > 0$ when $0 < x < \tilde{x}_f$, and $f'(x) < 0$ when $x > \tilde{x}_f$. Also, $f(x) = 0$ has a single root $\breve{x}_f$ in the interval $(\tilde{x}_f, 1)$. Note that $h'(x) = (1-p_s) f'\Big((1-p_s)x\Big)$. Therefore, $h'(x) = 0$ has a single root $\tilde{x}_h = \frac{\tilde{x}_f}{1-p_s}$ in the interval $(0, \frac{1}{1-p_s})$, $h(x)$ is an increasing function of x when $0 < x < \tilde{x}_h$, $h(x)$ is a decreasing function when $\tilde{x}_h < x < \frac{1}{1-p_s}$, and $h(x) = 0$ has a single root $\breve{x}_h = \frac{\breve{x}_f}{1-p_s}$ in the interval $(\tilde{x}_h, \frac{1}{1-p_s})$. If $\breve{x}_f \geq 1 - p_s$, or equivalently, $h(1) = f(1 - p_s) \geq 0$, as shown in Fig. 8a, $h(x) \geq 0$ for all $0 \leq x \leq 1$ and $x^* = 1$ is the ESS since strategy **B** always outperforms strategy **NB**. If $\breve{x}_f < 1 - p_s$, or equivalently, $h(1) = f(1 - p_s) < 0$, as shown in Fig. 8b, $\breve{x}_h$ is in the interval $(0, 1)$. So similar to the homogeneous user case, $x^* = \breve{x}_h$ is the ESS.

• Case 3, $r \in \Gamma_{N-N_s} = [\frac{b^w}{N-N_s}, \frac{b^w}{N-N_s-1})$: In this price range, at least $(N - N_s)$ buyers are required to afford the relay service, while there are at most $(N-N_s)$ possible buyers. Therefore, there is no chance to free ride, and all unsuccessful users will buy with $x^* = 1$. Mathematically, when $r \in \Gamma_{N-N_s}$, $h(x) = f\Big(x(1-p_s)\Big) = (g - r)[x(1 - p_s)]^l \geq 0$ for all $x \in [0, 1]$. Therefore, the strategy **B** always outperforms strategy **NB**, and $x^* = 1$.

• Case 4, $r \in \Gamma_j$ with $j \geq N - N_s + 1$: At least $(N - N_s + 1)$ buyers are required to afford the relay service. However, the number of total potential buyers is no more than $(N - N_s)$. Thus, there are not sufficient buyers to afford the relay service, and the game ends.

In summary, we have the following theorem.

Theorem 2: Given N_s bidders in Stage 1, the winning bid b^w, and the relay price r,

- Case 1, when $r \geq g$ (i.e., $r \in \Gamma_0$), $x^* = 0$ is the ESS, and no successful user buys;
- Case 2, when $\frac{b^w}{N-N_s-1} \leq r < g$ (i.e., $r \in \Gamma_{\underline{j}} \cup \Gamma_{\underline{j}+1} \cup ... \cup \Gamma_{N-N_s-1}$), if $h(1) \geq 0$, $x^* = 1$ is the ESS and all unsuccessful users buy. If $h(1) < 0$, for $x \in (0, 1)$, $h(x) = 0$ has a single root $0 < \breve{x}_h < 1$, which is the ESS, $x^* = \breve{x}_h$;
- Case 3, when $\frac{b^w}{N-N_s} \leq r < \frac{b^w}{N-N_s-1}$ (i.e., $r \in \Gamma_{N-N_s}$), $x^* = 1$ is the ESS and all unsuccessful users buy;
- Case 4, when $r < \frac{b^w}{N-N_s}$ (i.e., $r \in \Gamma_{N-N_s+1} \cup \Gamma_{N-N_s+2} \cup ...$), there are not sufficient buyers to afford the relay service, and the game ends.

3) The Price Setting Game: In the price setting game, the local agent finds the optimal price r^* to maximize the relay portion throughput. Given the number N_s of bidders and the winning bid b^w, with the ESS x^* (which is a function of relay price r) from Theorem 2, we have the relay portion throughput

$$T_R(x^*|N_s, b^w) = \sum_{n=0}^{N-N_s} \binom{N-N_s}{n} (1-p_s)^n p_s^{(N-N_s-n)} \Bigg\{ \sum_{k=\lceil b^w/r \rceil}^{n} \binom{n}{k} (x^*)^k (1-x^*)^{n-k} \Bigg\}. \quad (16)$$

To maximize $T_R(x^*|N_s, b^w)$, based on Theorem 2, the optimal price r^* should be in the range $\Gamma_{\underline{j}} \cup \Gamma_{\underline{j}+1} \cup \cdots \cup \Gamma_{N-N_s}$ where $x^* > 0$. Fig. 9 shows an example of x^* and the corresponding throughput T_R where there are a total of $N = 12$ users in the system, and $N_s = 2$ of them bid in Stage 1 with the winning bid $b^w = 2$. For each user, the probability to correctly receive the segment in the broadcast portion is $p_1 = 0.4$, and the gain of correctly receiving the segment is $g = 1$. Users' costs $\{c_i\}$ are uniformly distributed in the range $[L = 1, H = 17]$. From (11), the probability that a user who does not bid is a successful user is $p_s = 0.2$. From Fig. 9, when $r < b^w/(N - N_s) = 0.2$ (i.e., in price ranges $\Gamma_{11}, \Gamma_{12}, ...$), there are not sufficient buyers and the relay portion throughput is zero. When $r \in [0.2, 1.0)$ (i.e., in price ranges $\Gamma_{10}, ..., \Gamma_3$), we observe that at a low price (e.g., in Γ_{10}), the game requires a large number of buyers to afford the relay service. However, since there are some successful users who do not bid in the sellers' auction game, even if all unsuccessful users buy with $x^* = 1$, it is still possible that the total payment $N_b r$ is smaller than the winning bid b^w, and therefore, the relay portion throughput is small. At a high price, e.g., when $r \in \Gamma_3$, the minimum number of required buyers is small. Thus, unsuccessful users have a high tendency to free ride, and the probability that there are not sufficient buyers is high, which also results in a low relay portion throughput. Therefore, the optimal price should be appropriately selected to address this tradeoff, and T_R is maximized when $r = b^w/7 = 0.286$ in this example.

To efficiently find the optimal price, we have the following proposition, whose proof is in Appendix III.

Proposition 2: In each price range Γ_j with $j \in \{\underline{j}, ..., N - N_s\}$, $T_R(x^*|N_s, b^w)$ is a non-increasing function of r.

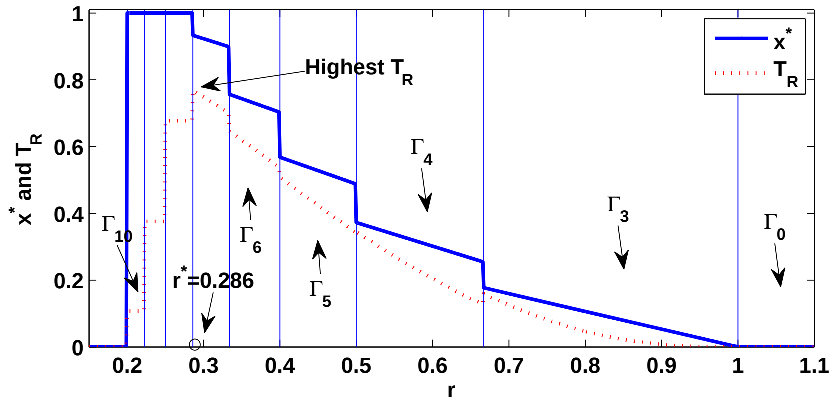


Fig. 9: An example of x^* and T_R.

This can also be observed from the example in Fig. 9. In each price range Γ_j with $3 \leq j \leq 10$, T_R is a non-increasing function of r and is maximized at the left boundary $r = b^w/j$. Based on this observation, we propose Algorithm 1 to efficiently find the global optimal price that maximizes the relay portion throughput. Specifically, Algorithm 1 compares $T_R(x^*|N_s, b^w)$ at $r = b^w/j$ when $j = \underline{j}, \cdots, N - N_s$, and chooses the optimal price r^* that maximizes $T_R(x^*|N_s, b^w)$.

Algorithm 1: Optimal Price Selection

1: $T_R^* = 0$, $r^* = 0$
2: **for** $j = \underline{j}$ to $(N - N_s)$ **do**
3: Set $r = b^w/j$ and use Theorem 2 to find x^*, and use (16) to calculate $T_R(x^*|N_s, b^w)$
4: **if** $T_R(x^*|N_s, b^w) > T_R^*$ **then**
5: $T_R^* = T_R(x^*|N_s, b^w)$ and $r^* = b^w/j$
6: **end if**
7: **end for**

We now discuss the properties of the optimal price r^*. We first study the ESS x^* at the optimal price r^* when $(N - N_s)$ takes different values. For the system in Fig. 9, we vary the value of $(N - N_s)$ from 5 to 11, and other parameters are the same as in Fig. 9. In Table II, for different $(N - N_s)$, we list $(N - N_s)(1 - p_s)$, the average number of unsuccessful users among the $(N - N_s)$ users who do not bid, $\lceil b^w/r^* \rceil$, the minimal number of users required to afford the relay service, and the ESS x^* at the optimal price r^*. From Table II, we observe that at the optimal price, $\lceil b^w/r^* \rceil$ has a similar value to $(N - N_s)(1 - p_s)$. It means that the optimal price is chosen carefully to let each unsuccessful user has a small or zero probability to free ride (i.e., x^* is close to 1 as shown in Table II), while ensuring that the total payment from unsuccessful users is sufficient to pay the relay service.

We then study the optimal price r^* at different values of $(N - N_s)$ (the number of users who do not bid) and p_s (probability that a user who does not bid is a successful user). For the system in Fig. 9, we vary $(N - N_s)$ from 4 to 10, and vary p_1 so that p_s is 0.2, 0.4, or 0.6. Other parameters are the same. The optimal price r^* is shown in Fig. 10. We can see

TABLE II: x^* at the optimal price with different number N_s of bidders.

$(N - N_s)$	5	6	7	8	9	10	11
$(N - N_s)(1 - p_s)$	4	4.8	5.6	6.4	7.2	8	8.8
$\lceil b^w/r^* \rceil$	4	5	5	6	7	7	8
x^*	1.0	1.0	0.90	0.99	1.0	0.93	0.99

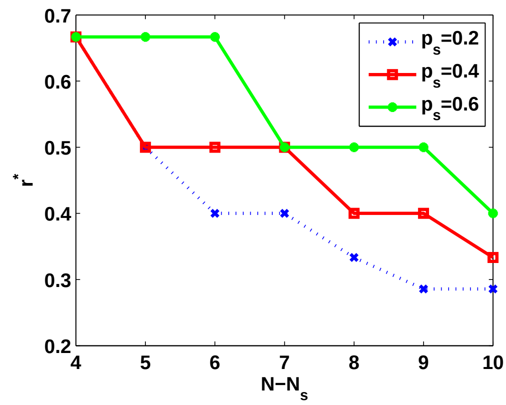


Fig. 10: An example of the optimal price r^*.

that given a fixed winning bid (i.e., $b^w = 2$ in the example), the optimal price increases when $(N - N_s)$ decreases. This is because, with a smaller $(N - N_s)$ and thus potentially fewer unsuccessful users, each buyer needs to pay more to purchase the relay service. Similarly, with a larger p_s, there are fewer unsuccessful users, and each buyer also has to pay a higher price to purchase the relay service. Note that, in Fig. 10, when $(N - N_s)$ and p_s vary, r^* takes values from a common finite set. This is because from Proposition 2, the optimal price can only take values in the finite set $\left\{\frac{b^w}{\underline{j}}, \frac{b^w}{\underline{j}+1}, \cdots, \frac{b^w}{N-N_s}\right\}$.

4) SPNE of the Stackelberg Game with Heterogeneous Users: To summarize, the SPNE of the multi-buyer multi-seller game with heterogeneous users is:

- Stage 1, the sellers' auction game. Successful users whose costs are no larger than $b^r(1)$ will enter the auction game, and bid their true costs.
- Stage 2, the price setting game. The local agent follows the second-price sealed-bid auction protocol to select the

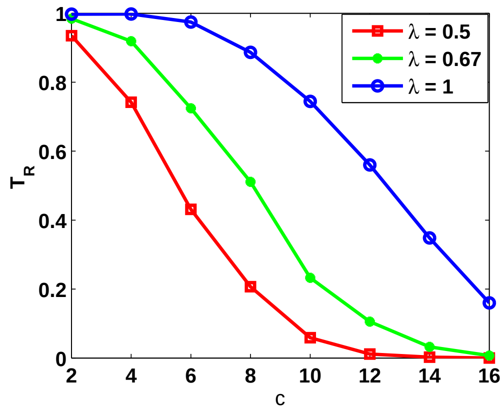


Fig. 11: Relay portion throughput with homogeneous users under different c and arrival rate λ.

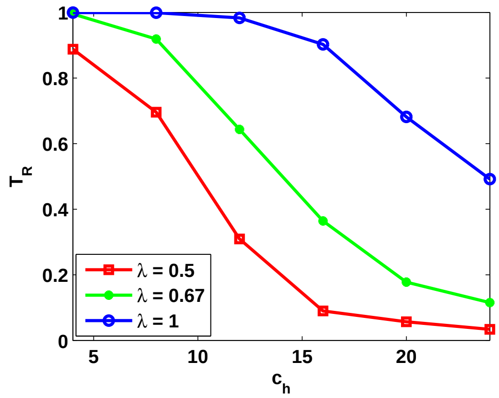


Fig. 12: Relay portion throughput with heterogeneous users under different H and arrival rate λ.

winner of the auction and the winning bid b^w. Then the local agent uses Algorithm 1 to select the optimal price r^*.

- Stage 3, the buyers' game. Based on the number N_s of sellers, the winning bid b^w, and the price r^* selected by the local agent, each unsuccessful user follows Theorem 2 to find the ESS x^*, and decides to be a buyer with probability x^*.
- Stage 4, the transaction game. Given the number N_b of buyers, if $r^* N_b \geq b^w$, the auction winner relays the segment, and receives a payment of b^w. Otherwise, there is no relay and the game ends.

V. Simulation Results

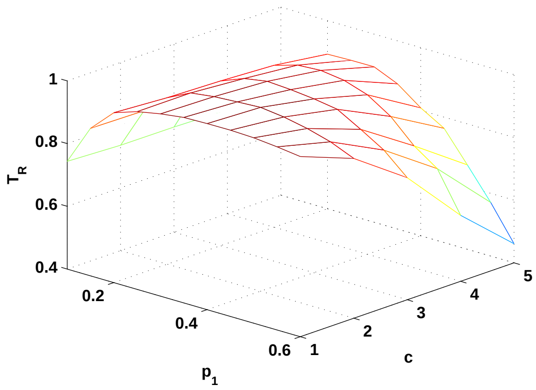
In our simulation, we consider a multicast network with a BS and a group of users who dynamically join and leave the multicast service. For each user, the probability p_1 of receiving a segment[8] correctly from the BS is 0.37, and the utility gain of receiving a segment correctly is $g = 1$. The initial number of users is 10. Users join the multicast service according to a Poisson process with an average arrival rate of λ users per segment duration (i.e., the length of the broadcast portion in Fig. 2). The sojourning period of each user in the system follows an exponential distribution with an average of μ segments. In our simulation, we fix $\mu = 20$ and test the system when $\lambda = 0.5$, 0.67 and 1, which correspond to the average network size of $N = 9.8$, 13.6 and 20.2, respectively.

Fig. 11 shows the system performance with homogeneous users when we have different c and λ. For each segment transmission, users follow the SPNE as discussed in Section III-E. Fig. 11 shows the average results for 5000 segment transmissions. From this figure, we observe that when c increases, the relay portion throughput decreases. This is because, when c increases, the probability that the condition $\frac{c}{N-N_{su}} < g$ is not satisfied increases. Recall that, if the condition $\frac{c}{N-N_{su}} < g$ is not satisfied, there is no relay service, as shown in Section III-E. So a higher c leads to a lower relay portion throughput. In Fig. 11, we also observe that when λ increases, the relay portion throughput increases. This is because a larger λ will on average give a larger network size, and thus more unsuccessful users. Then, the probability that the number of buyers is sufficient increases, which gives a higher relay portion throughput.

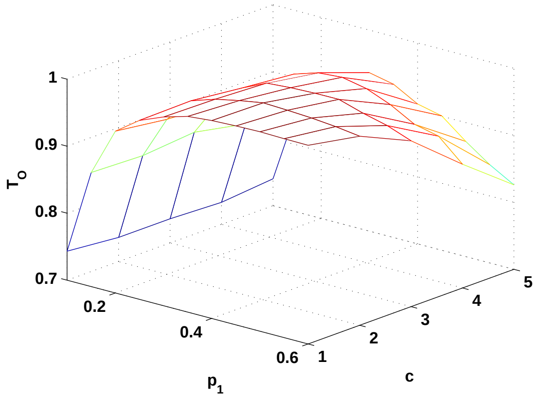
Fig. 12 shows the system performance with heterogeneous users with different H and λ. In this simulation, users' costs are uniformly distributed and randomly generated in $[L, H]$ with a fixed $L = 4$, and we test the system performance with different H. For each segment transmission, users follow the SPNE as discussed in Section IV-B.4. We also test the system for 5000 segment transmissions. From this figure, we observe that when H increases, the relay portion throughput decreases. This is because when H increases, on average, each user has a higher cost to relay a segment. Therefore, the winning bid b^w increases, and it requires more buyers to afford the relay service. Thus, with a higher H, the probability that the number of buyers is sufficient decreases, causing a decrease in the relay portion throughput. Furthermore, we observe from Fig. 12 that when λ increases, the relay portion throughput increases. This is because a larger λ gives, on average, a larger network size N (thus a larger $b^r(1) = (N-1)g - \epsilon$) and more successful users (a larger N_{su}). From Proposition 1, a successful user will bid if its cost is lower than $b^r(1)$. So with a larger λ, more successful users will bid, resulting in a lower winning bid b^w. In addition, a larger λ gives on average more unsuccessful users, which, together with the fact that the winning bid b^w decreases, increases the probability that there are sufficient buyers to afford the relay service, and thus, increases the relay portion throughput.

In this payment-based game, the numbers of buyers and sellers in each round are affected by the probability p_1. A larger p_1 will on average give a larger number of sellers but a smaller number of buyers, which may affect the system performance. We define T_O as the overall throughput, which is the average percentage of users who receive the segment correctly after the broadcast and the relay portions. Fig. 13 compares the relay portion and the overall throughput in homogeneous case when c and p_1 vary. The simulation setup

[8] In this simulation, we do not specify the segment length. This is because it only affects the probability p_1, and in our simulation results, e.g., Fig. 13, we study the system performance under different p_1, which reflects how the segment length impacts the system performance.



(a)



(b)

Fig. 13: System throughput with homogeneous users, when we have different p_1 and c. (a) The relay portion throughput. (b) The overall throughput.

is similar to that in Fig. 11. We fix $\lambda = 0.67$, and other parameters are the same. From this figure, when c is low, the relay portion and the overall throughput increases as p_1 increases. However, for a large c, such as when c is set to 5, as p_1 increases from 0.2 to 0.6, Fig. 13a illustrates that the relay portion throughput first increases and then decreases. This is because when p_1 is small, for example $p_1 = 0.2$, the probability that there are successful users who can provide relay service is small. As p_1 increases, the probability that successful users exist increases. Thus, the relay portion throughput increases, which also helps increase the overall throughput. However, as p_1 keeps increasing, the relay portion throughput starts to decrease. This is because when the cost is high, the required number of buyers is also large. When p_1 becomes large, there are fewer unsuccessful users. Thus, the probability that there are sufficient buyers decreases, and the relay portion throughput decreases. The overall throughput also decreases, which means that, for a larger p_1, the decrease in the relay portion throughput dominates the increase in the broadcast portion throughput. Therefore, using a large p_1 may not always be the optimal decision, especially when c is high.

For a given range of p_1,[9] the BS can search the optimal p_1^* in this range, which gives the maximal overall throughput while keeping the consumed transmission power of the BS low.

VI. Further Discussion: Impacts of Honest and Altruistic Users

In this work, to simplify the analysis, we assume that all users are selfish users, who aim to maximize their own utilities, and may even cheat if cheating can improve their payoffs. However, in practice, honest and altruistic users may exist, where honest users are always willing to reveal their private information, i.e., the costs to forward a segment in the heterogeneous user case, while altruistic users are always willing to contribute to the system. In this section, we discuss how our proposed framework may be affected by these two kinds of users.

To integrate honest users in our framework is straightforward. This is because our system is cheat-proof by using the second-price sealed-bid auction game. For each user, telling the true information is the dominant strategy. Thus, no matter whether a user is honest or not, he/she has incentive to reveal his/her private information.

To integrate altruistic users in our system, we consider two scenarios:

Scenario 1: some successful users are altruistic, and they are willing to relay the segment without receiving any payment. This scenario can be easily handled in our 4-stage Stackelberg game. In Stage 1, the sellers' game (the selling part in Fig. 2), altruistic users will inform the local agent that they are altruistic. Then, in Stage 2, the price setting game (the price announcement part in Fig. 2), the local agent will randomly select one altruistic user as relay, and announce that no payment is required for the relay service. Thus, unsuccessful users do not need to buy, and the selected altruistic user will forward the segment. Then, the game ends for this segment transmission.

Scenario 2: successful users are all selfish users, while some unsuccessful users might be altruistic, who are altruistic buyers and always willing to buy for any given price r. First, altruistic buyers do not affect our game with homogeneous users. This is because in the homogenous user case, the required number of buyers is the same as the number of unsuccessful users at the optimal price. Since no one can free ride, each unsuccessful user has to buy no matter whether he/she is altruistic. However, altruistic buyers may affect our game with heterogeneous users. Specifically, in heterogeneous user case, Stage 1 of our game stays the same as described in Section IV-A, since we consider that all successful users are selfish. However, altruistic buyers affect the Stage 2 and Stage 3 a lot. For example, in Stage 3, the buyers' game (the buying part in Fig. 2), each selfish unsuccessful user needs to take potential altruistic buyers into consideration to decide his/her own strategy. For instance, if there are enough altruistic buyers to afford the relay service, he/she does not have to buy. However, since he/she does not know the exact number of altruistic

[9]For any given broadcast channel condition, p_1 is jointly affected by the segment length, the transmission power, channel coding and modulation.

buyers, he/she needs to estimate this number, and then decides his/her optimal strategy to maximize his/her expected payoff. Furthermore, due to the backward induction that we use to derive the SPNE, the change of the buyers' game then affects the optimal price decided in Stage 2, the price setting game. The effect of altruistic buyers deserves further investigation in our future work.

VII. Conclusions

In a wireless multicast system, the cooperation among users can significantly improve the system performance. However, successful users may not be willing to help unsuccessful users, as forwarding costs their transmission power. In addition, due to the broadcast nature of wireless communications, unsuccessful users may prefer to free ride rather than buying the relay service. In this work, to stimulate user cooperation, we formulate the interaction among users as a multi-seller multi-buyer payment-based game, where users pay to receive relay service and get paid if they forward their successfully received segments to others. In either homogeneous user case or heterogeneous user case, we derive the ESS of the buyers' game, and further derive the optimal price to maximize the relay portion throughput. It is shown that, under the optimal price, there is no chance for an unsuccessful user to free ride in the homogeneous user case, since the required number of buyers at the optimal price is equal to the number of unsuccessful users. There is also a very small probability for an unsuccessful user to free ride in the heterogeneous user case, since the required number of buyers at the optimal price is very close to the number of unsuccessful users. Therefore, our mechanisms have the merits of improving system efficiency and stimulating users to cooperate (i.e., successful users sell, and unsuccessful users buy).

Appendix I

For $r \in \Gamma_j$ with $1 < j < N - N_{su}$, $f'(x) = 0$ has a single root $\tilde{x}_f \in \left(0, \frac{j-1}{N-N_{su}-1}\right)$, and $f'(x) > 0$ when $x \in (0, \tilde{x}_f)$ and $f'(x) < 0$ when $x \in (\tilde{x}_f, 1)$.

Proof: From (4), $f(x)$ can be rewritten as

$$f(x) = \underbrace{g\binom{l}{k^*}x^{k^*}(1-x)^{(l-k^*)}}_{\triangleq X(x)} - \underbrace{r\sum_{k=k^*}^{l}\binom{l}{k}x^k(1-x)^{(l-k)}}_{\triangleq Y(x)} = X(x) - Y(x), \quad (17)$$

where $l = N - N_{su} - 1$ and $k^* = \lceil c/r \rceil - 1 = j - 1$ with $1 \leq k^* \leq N - N_{su} - 2$. We then study the monotonicity of $X(x)$ and $Y(x)$, respectively. First, we have $X'(x) = g\binom{l}{k^*}x^{(k^*-1)}(1-x)^{(l-k^*-1)}(k^* - lx)$, and thus, $X'(x) > 0$ when $x \in (0, k^*/l)$, and $X'(x) \leq 0$ when $x \in [k^*/l, 1)$. Similarly, we have $Y'(x) = r\sum_{k=k^*}^{l}\binom{l}{k}x^{(k-1)}(1-x)^{(l-k-1)}(k - lx)$. When $x \in (0, k^*/l)$, we have $(k - lx) > 0$, and thus $Y'(x) > 0$. When $x \in [k^*/l, 1)$, since $lx > 0, 1, ..., k^* - 1$, we have

$$\begin{aligned} Y'(x) &= r\sum_{k=k^*}^{l}\binom{l}{k}x^{(k-1)}(1-x)^{(l-k-1)}(k - lx) \\ &> r\sum_{k=0}^{l}\binom{l}{k}x^{(k-1)}(1-x)^{(l-k-1)}(k - lx) \\ &= r\frac{d}{dx}\left[\sum_{k=0}^{l}\binom{l}{k}x^k(1-x)^{(l-k)}\right] \\ &= r\frac{d(1)}{dx} = 0. \end{aligned} \quad (18)$$

Therefore, we have $Y'(x) > 0$ for $x \in (0, 1)$.

Based on the above analysis, when $x \in [k^*/l, 1)$, we have $X'(x) \leq 0$ and $Y'(x) > 0$, and therefore, $f'(x) = X'(x) - Y'(x) < 0$. When $x \in (0, k^*/l)$, we have $X'(x) > 0$ and $Y'(x) > 0$. Thus, we need to further investigate $f'(x)$, and have (19). In the last line of (19), when $x \in (0, k^*/l)$, except for the term $W(x)$, all the other terms are larger than zero. To study $W(x)$, we derive $W'(x)$ in (20). In (20), since $k \geq k^*$ and $x \in (0, k^*/l)$, we have $W'(x) < 0$. We then study the function value of $W(x)$ when x approaches 0 and k^*/l, respectively. Since $r \in \Gamma_j$ with $2 \leq j \leq N - N_{su} - 1$, we have $r < g$. Then we have

$$\begin{aligned} \lim_{x\to 0} W(x) &= \binom{l}{k^*} - \frac{r}{g}\binom{l}{k^*}\frac{k^* - 0}{k^* - 0} \\ &= \binom{l}{k^*}\left(1 - \frac{r}{g}\right) > 0, \end{aligned} \quad (21)$$

$$\lim_{x\to k^*/l} W(x) = \binom{l}{k^*} - \infty = -\infty. \quad (22)$$

Therefore, $W(x) = 0$ has a single root, $\tilde{x}_f$, in the range $(0, k^*/l)$. From (19), $\tilde{x}_f$ is also the single root of $f'(x) = 0$. Thus, when $x \in (0, \tilde{x}_f)$, $W(x) > 0$, and $f'(x) > 0$. When $x \in (\tilde{x}_f, k^*/l)$, $W(x) < 0$, and $f'(x) < 0$.

Based on the above analysis, $f'(x) = 0$ has a single root, $\tilde{x}_f$, in the range $(0, k^*/l)$ ($k^* = j-1$ and $l = N - N_{su} - 1$), and $f(x) > 0$ when $x \in (0, \tilde{x}_f)$ and $f(x) < 0$ when $x \in (\tilde{x}_f, 1)$. ■

Appendix II
Proof of (14)

Proof: Let $k^* = \lceil b^w/r \rceil - 1$, and we first derive $h(x) = \bar{V}_B(x) - \bar{V}_{NB}(x)$ in (23). We then rewrite $G(x)$ in (24). Similarly, $F(x) = r\sum_{k=k^*}^{l}\left\{\binom{l}{k}[(1-p_s)x]^k[(1-p_s)(1-x)+p_s]^{(l-k)}\right\}$. Thus,

$$\begin{aligned} h(x) = &g\binom{l}{k^*}[x(1-p_s)]^{k^*}[1 - x(1-p_s)]^{(l-k^*)} \\ &- r\sum_{k=k^*}^{l}\binom{l}{k}[(1-p_s)x]^k[1 - x(1-p_s)]^{(l-k)}. \end{aligned} \quad (25)$$

■

$$
\begin{aligned}
f'(x) &= g\binom{l}{k^*}x^{(k^*-1)}(1-x)^{(l-k^*-1)}(k^*-lx) - r\sum_{k=k^*}^{l}\binom{l}{k}x^{(k-1)}(1-x)^{(l-k-1)}(k-lx) \\
&= gx^{(k^*-1)}(1-x)^{(l-k^*-1)}(k^*-lx)\underbrace{\left\{\binom{l}{k^*} - \frac{r}{g}\sum_{k=k^*}^{l}\binom{l}{k}\left(\frac{x}{1-x}\right)^{(k-k^*)}\frac{k-lx}{k^*-lx}\right\}}_{\triangleq W(x)} \\
&= gx^{(k^*-1)}(1-x)^{(l-k^*-1)}(k^*-lx)W(x). \qquad (19)
\end{aligned}
$$

$$
W'(x) = -\frac{r}{g}\sum_{k=k^*}^{l}\binom{l}{k}\left\{\left(\frac{x}{1-x}\right)^{(k-k^*-1)}\frac{k-k^*}{(1-x)^2}\frac{k-lx}{k^*-lx} + \left(\frac{x}{1-x}\right)^{(k-k^*)}\frac{l(k-k^*)}{(k^*-lx)^2}\right\}. \qquad (20)
$$

$$
\begin{aligned}
h(x) &= \underbrace{\sum_{n=k^*}^{l}\binom{l}{n}(1-p_s)^n p_s^{l-n}\binom{n}{k^*}x^{k^*}(1-x)^{n-k^*}g}_{\triangleq G(x)} - \underbrace{\sum_{n=k^*}^{l}\binom{l}{n}(1-p_s)^n p_s^{l-n}\left\{\sum_{k=k^*}^{n}\binom{n}{k}x^k(1-x)^{(n-k)}r\right\}}_{\triangleq F(x)} \\
&= G(x) - F(x). \qquad (23)
\end{aligned}
$$

$$
\begin{aligned}
G(x) &= g\sum_{n=k^*}^{l}\frac{l!}{n!(l-n)!}(1-p_s)^n p_s^{l-n}\frac{n!}{k^*!(n-k^*)!}x^{k^*}(1-x)^{n-k^*} \\
&= g\frac{l!}{k^*!(l-k^*)!}(1-p_s)^{k^*}x^{k^*}\sum_{n=k^*}^{l}\frac{(l-k^*)!}{(n-k^*)!(l-n)!}[(1-p_s)(1-x)]^{(n-k^*)}p_s^{l-n} \\
&\overset{m\triangleq n-k^*}{=\!=\!=} g\binom{l}{k^*}[x(1-p_s)]^{k^*}\sum_{m=0}^{l-k^*}\frac{(l-k^*)!}{m!(l-k^*-m)!}[(1-p_s)(1-x)]^m p_s^{l-k^*-m} \\
&= g\binom{l}{k^*}[x(1-p_s)]^{k^*}[(1-p_s)(1-x)]+p_s]^{(l-k^*)}. \qquad (24)
\end{aligned}
$$

Appendix III
Proof of Proposition 2

Proof: For $r \in \Gamma_j$ with $j \in \{\underline{j}, ..., N-N_s\}$, we first prove that, $\partial T_R/\partial x^* \geq 0$, and then prove that $\partial x^*/\partial r \leq 0$. Therefore, we have $\frac{\partial T_R}{\partial r} = \frac{\partial T_R}{\partial x^*}\frac{\partial x^*}{\partial r} \leq 0$ and T_R is a non-increasing function of r.

To prove that $\partial T_R/\partial x^* \geq 0$, we first define $q(n,x^*) \overset{\triangle}{=} \sum_{k=\lceil b^w/r\rceil}^{n}\binom{n}{k}(x^*)^k(1-x^*)^{n-k}$ and rewrite (16) as $T_R(x^*|N_s,b^w) = \sum_{n=0}^{N-N_s}\binom{N-N_s}{n}(1-p_s)^n p_s^{(N-N_s-n)}q(n,x^*)$. The first derivative of $q(n,x^*)$ over x^* is

$$
\frac{\partial q(n,x^*)}{\partial x^*} = \sum_{k=\lceil b^w/r\rceil}^{n}\binom{n}{k}(x^*)^{(k-1)}(1-x^*)^{(n-k-1)}(k-nx^*). \qquad (26)
$$

Similar to (18), we can prove that $\partial q(n,x^*)/\partial x^* \geq 0$ for $x^* \in [0,1]$. Therefore, we have

$$
\frac{\partial T_R}{\partial x^*} = \sum_{n=0}^{N-N_s}\binom{N-N_s}{n}(1-p_s)^n p_s^{(N-N_s-n)}\frac{\partial q(n,x^*)}{\partial x^*} \geq 0. \qquad (27)
$$

We then prove for $r \in \Gamma_j$ with $j \in \{\underline{j}, ..., N-N_s\}$, $\frac{\partial x^*}{\partial r} \leq 0$. First, if $r \in \Gamma_{N-N_s}$ (Case 3 in Theorem 2), $x^* = 1$ for any $r \in \Gamma_{N-N_s}$. Therefore, when $r \in \Gamma_{N-N_s}$, we have $\partial x^*/\partial r = 0$.

If $r \in \Gamma_j$ with $j \in \{\underline{j}, ..., N-N_s-1\}$ (Case 2 in Theorem 2), from Theorem 2, it can be seen that x^* is the non-zero root, denoted $\breve{x}_h$, of $h(x) = 0$ if $\breve{x}_h < 1$, and $x^* = 1$ otherwise. When the price r increases within Γ_j, k^* keeps the same, and thus, $h(x)$ decreases based on (14). From Fig. 7 and Fig. 8, it can be seen that, if $h(x)$ decreases, the non-zero root of $h(x) = 0$ decreases. Therefore, when r increases, the ESS x^* either keeps at $x^* = 1$, or decreases. This means $\partial x^*/\partial r \leq 0$. ■

Acknowledgments

The authors would like to thank the anonymous reviewers for their constructive comments and suggestions which helped to improve the quality of the manuscript.

References

[1] I. Güvenc, U. C. Kozat, M. R. Jeong, F. Watanabe, and C. C. Chong, "Reliable multicast and broadcast services in relay-based emergency

communications," *IEEE Wireless Commun.*, vol. 15, no. 3, pp. 40–47, June 2008.

[2] H. V. Zhao and W. Su, "Cooperative wireless multicast: performance analysis and power/location optimization," *IEEE Trans. Wireless Commun.*, vol. 9, no. 6, pp. 2088–2100, June 2010.

[3] J. Si, Z. Li, Z. Liu, and X. Chen, "Energy efficient cooperative broadcasting in wireless networks," in *Proc. 2009 IEEE Int. Conf. Commun. (ICC)*, June 2009, pp. 5235–5240.

[4] O. Alay, T. Korakis, Y. Wang, E. Erkip, and S. Panwar, "Layered wireless video multicast using omni-directional relays," in *Proc. IEEE Int. Conf. on Acoustics, Speech and Signal Process. (ICASSP)*, pp. 2149–2152, Mar.-Ap. 2008.

[5] X. Liu, G. Cheung, and C. Chuah, "Structured network coding and cooperative wireless ad-hoc peer-to-peer repair for WWAN video broadcast," *IEEE Trans. Multimedia*, vol. 11, no. 4, pp. 730–741, June 2009.

[6] B. Niu, H. Jiang, and H. V. Zhao, "A cooperative multicast strategy in wireless networks," *IEEE Trans. Veh. Technol.*, vol. 59, no. 6, pp. 3136–3143, July 2010.

[7] B. Wang, Y. Wu, and K. J. R. Liu, "Game theory for cognitive radio networks: An overview," *Computer Networks*, vol. 54, no. 14, pp. 2537–2561, 2010.

[8] Y. Wu, B. Wang, K. J. R. Liu, and T. C. Clancy, "Repeated open spectrum sharing game with cheat-proof strategies," *IEEE Trans. Wireless Commun.*, vol. 8, no. 4, pp. 1922–1933, Apr. 2009.

[9] D. Niyato and E. Hossain, "Competitive pricing for spectrum sharing in cognitive radio networks: Dynamic game, inefficiency of Nash equilibrium, and collusion," *IEEE J. Sel. Areas Commun.,*, vol. 26, no. 1, pp. 192–202, Jan. 2008.

[10] R. Etkin, A. Parekh, and D. Tse, "Spectrum sharing for unlicensed bands," *IEEE J. Sel. Areas Commun.*, vol. 25, no. 3, pp. 517–528, April 2007.

[11] W. Yu and K. J. R. Liu, "Game theoretic analysis of cooperation stimulation and security in autonomous mobile ad hoc networks," *IEEE Trans. Mobile Computing*, vol. 6, no. 5, pp. 507–521, May 2007.

[12] W. S. Lin, H. V. Zhao, and K. J. R. Liu, "Cooperation stimulation strategies for peer-to-peer wireless live video-sharing social networks," *IEEE Trans. Image Process.*, vol. 19, no. 7, pp. 1768–1784, July 2010.

[13] B. Niu, H. V. Zhao, and H. Jiang, "A cooperation stimulation strategy in wireless multicast networks," *IEEE Trans. Signal Process.*, vol. 59, no. 5, pp. 2355–2369, May 2011.

[14] S. Zhong, J. Chen, and Y. R. Yang, "Sprite: a simple, cheat-proof, credit-based system for mobile ad-hoc networks," in *Proc. IEEE INFOCOM*, pp. 1987–1997, Mar.-Apr. 2003.

[15] L. Buttyán and J.-P. Hubaux, "Stimulating cooperation in self-organizing mobile ad hoc networks," *ACM/Kluwer Mobile Network Applications*, vol. 8, no. 5, pp. 579–592, Oct. 2003.

[16] Z. Ji, W. Yu, and K. J. R. Liu, "A game theoretical framework for dynamic pricing-based routing in self-organized manets," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 7, pp. 1204–1217, Sept. 2008.

[17] S. Eidenbenz, G. Resta, and P. Santi, "Commit: a sender-centric truthful and energy-efficient routing protocol for Ad-hoc networks with selfish nodes," in *Proc. 19th IEEE Int. Parallel and Distributed Processing Symposium*, Apr. 2005.

[18] M. J. Neely, "Optimal pricing in a free market wireless network," in *Proc. IEEE INFOCOM*, pp. 213–221, May 2007.

[19] D. Niyato, E. Hossain, and Z. Han, "Dynamics of multiple-seller and multiple-buyer spectrum trading in cognitive radio networks: A game-theoretic modeling approach," *IEEE Trans. Mobile Computing*, vol. 8, no. 8, pp. 1009–1022, Aug. 2009.

[20] F. Milan, J. J. Jaramillo, and R. Srikant, "Performance analysis of reputation-based mechanisms for multi-hop wireless networks," in *Proc. 40th Annual Conf. Information Sciences and Systems*, pp. 12–17, Mar. 2006.

[21] P. Michiardi and R. Molva, "Core: A collaborative reputation mechanism to enforce node cooperation in mobile ad hoc networks," in *Proc. the IFIP TC6/TC11 6th Joint Working Conference on Communications and Multimedia Security: Advanced Communications and Multimedia Security*, pp. 107–121, 2002.

[22] J. J. Jaramillo and R. Srikant, "A game theory based reputation mechanism to incentivize cooperation in wireless ad hoc networks," *Ad Hoc Networks*, vol. 8, no. 4, pp. 416–429, 2010.

[23] Y. Chen and K. J. R. Liu, "Indirect reciprocity game modelling for cooperation stimulation in cognitive networks," *IEEE Trans. Commun.*, vol. 59, no. 1, pp. 159–168, Jan. 2011.

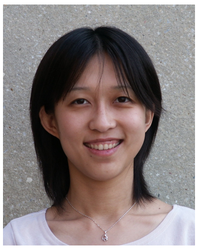
[24] J. W. Weibull, *Evolutionary Game Theory*. MIT Press, 1995.

[25] B. Wang, K. J. R. Liu, and T. C. Clancy, "Evolutionary cooperative spectrum sensing game: how to collaborate?" *IEEE Trans. Commun.*, vol. 58, no. 3, pp. 890–900, Mar. 2010.

[26] Y. Chen, B. Wang, W. S. Lin, Y. Wu, and K. J. R. Liu, "Cooperative peer-to-peer streaming: An evolutionary game-theoretic approach," *IEEE Trans. Circuits and Systems for Video Technol.*, vol. 20, no. 10, pp. 1346–1357, Oct. 2010.

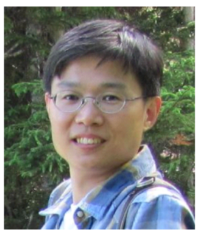
[27] V. Krishna, *Auction Theory*. Academic Press, 2nd Edition, 2009.

Bo Hu (S'12) received the B.E. degree in Computer Science and Technology from University of Science and Technology of China, Hefei, China in 2007. He is currently working toward the Ph.D. degree with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, Alberta, Canada. His research interests include media-sharing social networks, and multimedia signal processing.



H. Vicky Zhao (M'05) received the B.S. and M.S. degree from Tsinghua University, China, in 1997 and 1999, respectively, and the Ph. D degree from University of Maryland, College Park, in 2004, all in electrical engineering. She was a Research Associate with the Department of Electrical and Computer Engineering and the Institute for Systems Research, University of Maryland, College Park from Jan. 2005 to July 2006. She is currently an Associate Professor with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, Canada.

Dr. Zhao's research interests include media-sharing social networks, information security and forensics, digital communications and signal processing. Dr. Zhao received the IEEE Signal Processing Society (SPS) 2008 Young Author Best Paper Award. She is a co-author of "Multimedia Fingerprinting Forensics for Traitor Tracing" (Hindawi, 2005) and "Behavior Dynamics in Media-Sharing Social Networks" (Cambridge University Press, 2011). She was the Associate Editor for IEEE Signal Processing Letters from Nov. 2008 to Oct. 2012, a guest editor of the March 2012 special issue on Signal and Information Processing for Social Learning and Networking of IEEE Signal Processing Magazine, and is the Associate Editor for Elsevier Journal of Visual Communication and Image Representation.



Hai Jiang (M'07) received the B.Sc. and M.Sc. degrees in electronics engineering from Peking University, Beijing, China, in 1995 and 1998, respectively, and the Ph.D. degree (with an Outstanding Achievement in Graduate Studies Award) in electrical engineering from the University of Waterloo, Waterloo, Ontario, Canada, in 2006.

Since July 2007, he has been a faculty member at the University of Alberta, Edmonton, Alberta, Canada, where he is currently an Associate Professor in the Department of Electrical and Computer Engineering. His research interests include radio resource management, cognitive radio networking, and cross-layer design for wireless multimedia communications.

Dr. Jiang is an Associate Editor for the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY and the IEEE WIRELESS COMMUNICATIONS LETTERS. He served as a Co-Chair for the Wireless and Mobile Networking Symposium at the IEEE International Conference on Communications (ICC) in 2010. He received an Alberta Ingenuity New Faculty Award in 2008 and a Best Paper Award from the IEEE Global Communications Conference (GLOBECOM) in 2008.