Power Allocation in Multi-User Wireless Relay Networks through Bargaining

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***Abstract*—In this paper, we consider a multi-user single-relay wireless network, where the relay facilitates transmissions of the users' signals to the destination. We study the relay power allocation among the users, and use bargaining theory to model the negotiation among the users on relay power allocation. By assigning a bargaining power to each user to indicate its transmission priority, we propose an asymmetric Nash bargaining solution (NBS)-based relay power allocation scheme. We also propose a distributed implementation for this solution, where each user only requires its local channel state information (CSI). We analytically investigate the impact of the bargaining powers on the relay power allocation and show that via proper selection of the bargaining powers, the proposed power allocation can achieve a balance between the network sum-rate and the user fairness. Then we generalize the NBS-based power allocation and its distributed implementation to multi-user multi-relay networks. Simulation results are shown to compare the proposed power allocation with sum-rate-optimal power allocation and even power allocation. The impact of the bargaining powers on the power allocation is also demonstrated via simulations.**

Index Terms: Wireless relay network, power allocation, Nash bargaining solution (NBS), dual problem, gradient projection method.

I. Introduction

Cooperative relay network is a promising concept to improve the performance of communication in a wireless network. The basic idea is to have multiple nodes in the network help each other's transmission to achieve diversity [1]. It does not require multiple antennas to be equipped on communication devices, which is especially important for devices with strict size and complexity limitations. There are numerous works on cooperative strategies that optimize the global network performance and the analysis of fundamental limits in cooperative network. A widely used cooperative strategy with low computational load at the relays is amplify-and-forward (AF) where the relays simply forward amplified versions of the signals they receive.

While pioneering efforts in cooperative network focus on single-user network (e.g., [3]–[8]), research on multi-user relay network, in which multiple transmissions of different users can be supported by assisting relays, are important to meet the demands of future communication systems [9]–[18]. Compared with single-user relay network, multi-user relay network is more challenging since the users have competitive demands for limited relay resources. In the literature, the major objectives of resource allocation in multi-user relay networks fall into two categories: achieving optimal network throughput and achieving user fairness. In [12], the optimal relay power allocation problem is considered to maximize the network throughput in an AF multi-user two-way single-relay network. The problem is solved based on Lagrange dual decomposition approach. In [13], the joint subcarrier allocation and power allocation problems are investigated to maximize the capacity in a relay aided uplink multi-user network. A suboptimal solution is presented to solve the joint problem. Considering a total power constraint at the relay and users, the sum-rate maximization problem is addressed in [14] for a DF multi-user single-relay network.

If sum-rate maximization is the main objective in relay resource allocation as in [12]–[14], users with bad channel conditions may starve since more resources are assigned to users with good channel conditions. Thus, for some applications, it is also important to fairly distribute relay resources to guarantee the quality of service of all users. The relay resource allocation problem with fairness concerns is investigated in [15]–[18]. In [15], the joint subcarrier pairing and power allocation in the downlink multi-destination single-relay network is investigated with proportional fairness constraint. [16] considers the subcarrier allocation problem in a multi-user multi-relay network where a minimum rate requirement must be satisfied to all users. The work in [17], [18] are on a two-user network where each user can work as a relay for the other. They consider the scenario where the sources are selfish and try to optimize their own quality-of-service, and are willing to cooperate with each other only when cooperation is beneficial. Cooperative game theory is used to study how the sources negotiate to address their conflicting objectives. By employing a two-source bargaining game, fair bandwidth allocation [17] and power allocation [18] are found using Nash bargaining solution (NBS).

To our best knowledge, all prior papers in the literature focused exclusively on either global performance optimality, e.g., [12]–[14] or user fairness, e.g., [15]–[18]. However, in practical networks, different applications may require different balances between fairness and global performance, e.g., some applications prefer fairness among the users while others desire better global network performance. Even for the same network application, the desired balance between global performance and fairness may change from time to time. Motivated by this, in this paper we use bargaining game and propose an asymmetric NBS-based power allocation solution, which can jointly address these two issues. In addition, most previous works assume that there exists a trusted central controller who collects all the required channel state information and who has sufficient computation capability to implement the proposed solutions. This is impractical in systems such as ad hoc networks and sensor networks, where centralized controllers do not exist. Such systems therefore require a distributed cooperative protocol. To improve the scalability of our scheme for such scenarios, we provide a distributed implementation of the NBS-based power allocation scheme in which users with local information only are able to independently decide how to cooperate with other users and relays.

In this paper, we consider a multi-user single-relay AF network, and use game theory to analyze the relay power allocation among the users. We model the interaction among the users as a bargaining problem, where they negotiate with each other on relay power allocation. The distinctive novelty and contributions of this paper are briefly summarized as follows.

1) We propose a new *asymmetric* NBS-based relay power allocation scheme, which can achieve a balance between global network performance and user fairness. Existing literature only consider one of the two conflicting issues. The proposed scheme has potential in satisfying different and changing requirements of network applications.
2) The effect of bargaining power selection on network performance are investigated. We show analytically that via appropriate bargaining power selection, the proposed scheme can achieve the sum-rate-optimal solution for best global performance and even power allocation for best fairness.
3) A centralized implementation of the proposed power allocation is provided. More importantly, to improve the scalability of the proposed scheme, we propose a distributed implementation of our solution, which only requires local CSI at the users. Convergence conditions are provided for this distributed algorithm.
4) We generalize the proposed NBS-based power allocation scheme and its distributed implementation to multi-user multi-relay networks.

The rest of the paper is organized as follows. Section II elaborates the network model and the relay power allocation problem. The NBS-based relay power allocation scheme is proposed and studied in Section III. In Section IV, we propose a centralized and a distributed schemes to implement the proposed relay power allocation. Discussions on bargaining power selection and how it can balance different network requirements are given in Section V. In Section VI, we show the simulation results. Conclusion is given in Section VIII.

II. SYSTEM MODEL

Consider a wireless network with N users communicating with their destinations with the help of one relay as shown in Figure 1. Denote the channel from User i to the relay as f_i, the channel from User i to Destination i (the direct link) as h_i, and the channel from the relay to Destination i as g_i. We consider two channel models in this paper, Rayleigh flat-fading channel and path-loss channel. We denote the transmit power of User i as Q_i and the maximum transmit power of the relay as P. We also denote the power the relay uses in helping User i as P_i.

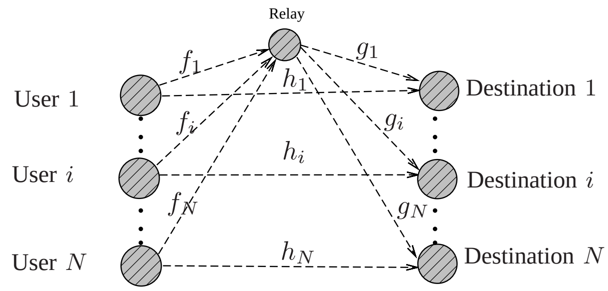

Fig. 1. Multi-user single-relay network.

Frequency division multiple access (FDMA) is used, so transmissions of different users are orthogonal and interference free. Without loss of generality, we consider the transmission of User i's message on Channel i. To send one symbol from User i to Destination i, we use the popular half-duplex two-step AF protocol. In the first step, User i transmits $\sqrt{Q_i}s_i$, where s_i is the information symbol normalized as $\mathbf{E}(|s_i|^2)=1$. The signals received by the relay and Destination i are $y_{iR}=\sqrt{Q_i}s_if_i+n_{iR}$ and $y_{iD}=\sqrt{Q_i}s_ih_i+n_{iD}$, respectively, where n_{iR} and n_{iD} are additive noises at the relay and Destination i in the first step, respectively. They are assumed to be independent Gaussian following the distribution $\mathcal{CN}(0,1)$. In the second step, the relay amplifies y_{iR} and forwards it with power P_i on Channel i. The signal received at Destination i in the second step can be shown to be

$$y_{Ri}=\sqrt{\frac{Q_iP_i}{Q_i|f_i|^2+1}}s_if_ig_i+\sqrt{\frac{P_i}{Q_i|f_i|^2+1}}g_in_{iR}+n_{iRD}, \tag{1}$$

where n_{iRD} is the additive noise at Destination i in Step 2, which is assumed to be independent to other noises with the same distribution, $\mathcal{CN}(0,1)$.

To simplify the presentation, we introduce two variables, namely the *noise forwarding rate* and the *signal forwarding rate*. We define

$$\xi_i \triangleq \frac{|g_i|^2}{Q_i|f_i|^2+1}, \tag{2}$$

which is the power of the second term at the right hand side of (1) when $P_i=1$. In this paper, we call ξ_i the noise forwarding rate corresponding to User i since its physical meaning is the noise power that the relay forwards to Destination i if unit relay power is used. Intuitively, a large noise forwarding rate means low quality in the user's relay-path. Similarly, we define the signal forwarding rate of User i as

$$\rho_i=\frac{Q_i|f_ig_i|^2}{Q_i|f_i|^2+1}. \tag{3}$$

It is the power of the first term at the right hand side of (1) when $P_i=1$. Its physical meaning is the signal power that the relay forwards to Destination i if unit relay power is used. A large signal forwarding rate intuitively means high quality in the user's relay-path.

After maximum ratio-combining of both the direct and relay paths, the effective received signal-to-noise-ratio (SNR) of

User i's transmission can be shown straightforwardly to be

$$\mathrm{SNR}_{iRD} = \frac{\rho_i P_i}{\xi_i P_i + 1} + Q_i |h_i|^2. \quad (4)$$

If User i's transmission is not helped by the relay and only the direct transmission is active, its received SNR becomes

$$\mathrm{SNR}_{iD} = Q_i |h_i|^2. \quad (5)$$

III. NBS-Based Power Allocation

We can see from (4) that all users desire the relay to allocate as much power as possible to help their own transmissions so they can achieve the highest SNRs. But the relay power is limited, so allocating more relay power to one user means less power available for the rest. To address this conflict among users, we model the interaction among the users as a bargaining game, and derive a fair relay power allocation scheme based on the NBS of the game.

A. Bargaining Game Model

In this section, we use bargaining game model to analyze the conflict and interaction among independent users.[1] The first step to formulate the power allocation problem as a bargaining game is to design the utility function. As in [11], we define User i's utility function to be the effective received SNR of User i given in (4), that is,

$$u_i(P_i) \triangleq \mathrm{SNR}_{iRD} = \frac{\rho_i P_i}{\xi_i P_i + 1} + Q_i |h_i|^2. \quad (6)$$

It represents the received quality-of-service, and is directly related to the performance of the communication. It can be seen that $u_i(P_i)$ is an increasing function of P_i. Given the N users in our relay network, we define the utility vector as $\mathbf{u} = (u_1\ u_2\ \cdots\ u_N)$. We denote the disagreement point as $\mathbf{u}_0 = (u_{1,0}\ u_{2,0}\ \ldots\ u_{N,0})$, which is the vector of the minimal utility that each user expects if they do not reach an agreement and play non-cooperatively. Thus,

$$u_{i,0} \triangleq \mathrm{SNR}_{iD} = Q_i |h_i|^2, \quad (7)$$

which is the utility of User i when it does not get any power from the relay and uses the direct transmission only, i.e. $P_i = 0$[2].

Given the above definitions of the utility function and the disagreement point, a utility vector $\mathbf{u} = (u_1\ u_2\ \cdots\ u_N)$ is called feasible if there exists a power allocation strategy $(P_1\ P_2 \cdots\ P_N)$ where $P_i \geq 0$ and $\sum_{i=1}^{N} P_i \leq P$ that gives User i utility u_i for all $i = 1, \cdots, N$. Let $\mathcal{S}$ be the set of all feasible utility vectors. Thus

$$\mathcal{S} \triangleq \left\{ (u_1 \cdots u_N) \middle| \sum_{i=1}^{N} P_i \leq P, P_i \geq 0 \right\}. \quad (8)$$

The first inequality in (8), $\sum_{i=1}^{N} P_i \leq P$, is from the relay power constraint. Power allocations that do not satisfy this constraint are infeasible. The second inequality, $P_i \geq 0$, says that each user has to be allocated non-negative relay power, a natural condition from practical point of view. This inequality also guarantees that when cooperates, each user gets no less utility compared to the case that it does not cooperate and only the direct link is used for communication. This is a necessary condition for the game theory formulation of feasible set.

In our relay power bargaining game among the users, we consider the scenario where different users may have different priorities in obtaining the relay power. To model this, users are assigned bargaining powers, denoted as $\beta_1, \cdots, \beta_N$, that they agree upon before transmission [20]. The bargaining powers are normalized as $\sum_{i=1}^{N} \beta_i = 1$. In Section V, we will investigate the effect of bargaining power selection on the proposed NBS-based power allocation and provide bargaining power allocation schemes that can bridge between global network performance and user fairness.

B. Nash Bargaining Solution (NBS)

In our bargaining game model for the relay power allocation, given the feasible set $\mathcal{S}$ and the disagreement point $\mathbf{u}_0$, the users negotiate and select one feasible utility vector in $\mathcal{S}$ and the corresponding power allocation strategy. Depending on how they define "fairness", the users may choose different solutions in $\mathcal{S}$. In this paper, we choose the asymmetric NBS [20] as the bargaining game solution for the following reasons. First, it has been proved in [21] that NBS is Pareto optimal, where no user can further improve its utility without decreasing others'. Thus NBS ensures that all relay power is efficiently utilized by the users, which is preferred on system design perspective. Second, NBS achieves proportional fairness by dividing the additional utility among users in a ratio that is equal to the rate at which this utility can be transferred [20]. Third, as will be discussed in Section V, NBS has flexibility in bargaining power selection, which provides us a way to balance between global network performance and user fairness. In this paper, we look for the NBS-based relay power allocation. For this purpose, we first prove the following two lemmas.

Lemma 1: Given the utility function $u_i(P_i)$ in (6), the feasible set $\mathcal{S}$ defined in (8) is convex.

Proof: From (6) and (7),

$$u_i(P_i) = \frac{\rho_i P_i}{\xi_i P_i + 1} + u_{i,0}. \quad (9)$$

It is a strictly increasing function of P_i and $\lim_{P_i \to \infty} u_i = \rho_i/\xi_i + u_{i,0} = Q_i |f_i|^2 + u_{i,0}$. Also, we can show that $P_i = \frac{u_i - u_{i,0}}{\rho_i - (u_i - u_{i,0})\xi_i}$. So $\mathcal{S}$ can be rewritten as

$$\mathcal{S} = \left\{ \mathbf{u} \middle| \phi(\mathbf{u}) \triangleq \sum_{i=1}^{N} \frac{u_i - u_{i,0}}{\rho_i - (u_i - u_{i,0})\xi_i} \leq P, \right. \\ \left. u_{i,0} \leq u_i < Q_i |f_i|^2 + u_{i,0}, i = 1, \cdots, N \right\}, \quad (10)$$

where the last constraint ensures that $0 \leq P_i < \infty$ for all i's.

[1] As defined in [21], bargaining theory studies the situation "in which two (or more) players can mutually benefit from reaching a certain agreement but have conflicting interests on the terms of the agreement". This fits our problem where the users have conflicting demands for relay power, and they have an interest in agreeing on the share, so they can all benefit and improve their SNR (achievable rate).

[2] This is a natural choice since if the users do not agree on the relay power allocation, the relay will not allocate any power to any user. Similar disagreement point setting is adopted in [18], [22]–[26].

Define $\mathcal{S}_1 \triangleq \{\mathbf{u}|u_i \geq u_{i,0}, i = 1, \cdots, N\}$ and $\mathcal{S}_2 \triangleq \{\mathbf{u}|\phi(\mathbf{u}) \leq P, u_i < Q_i|f_i|^2 + u_{i,0}, i = 1, \cdots, N\}$. We thus have $\mathcal{S} = \mathcal{S}_1 \cap \mathcal{S}_2$. $\mathcal{S}_1$ is a convex set by definition. To prove that $\mathcal{S}$ is convex, we only need to show that $\mathcal{S}_2$ is also convex.

We first prove that $\phi(\mathbf{u})$ is a convex function. From the definition of ϕ in (10), the Hessian or the second-order derivative of $\phi(\mathbf{u})$ is

$$\nabla^2 f(\mathbf{u}) = \begin{bmatrix} \frac{\partial^2 f(\mathbf{u})}{\partial u_1^2} & 0 & \cdots & 0 \\ 0 & \frac{\partial^2 f(\mathbf{u})}{\partial u_2^2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\partial^2 f(\mathbf{u})}{\partial u_N^2} \end{bmatrix}, \quad (11)$$

which is a diagonal matrix whose ith diagonal element is

$$\frac{\partial^2 \phi}{\partial u_i^2} = \frac{2Q_i|f_i|^2}{\xi_i \left[Q_i|f_i|^2 - (u_i - u_{i,0})\right]^3}. \quad (12)$$

For any finite P_i , we have $Q_i|f_i|^2 - (u_i - u_{i,0}) > 0$, so $\frac{\partial^2 \phi}{\partial u_i^2} > 0$ for all $i = 1, \cdots, N$. Thus, $\nabla^2 \phi(\mathbf{u})$ is positive definite, which shows that $\phi(\mathbf{u})$ is a convex function. Consequently, from the definition of $\phi(\mathbf{u})$, $\mathcal{S}_2$ is convex [29], and this completes the proof. ■

Lemma 2: There is at least one point in $\mathcal{S}$ with $u_i > u_{i,0}$ for all $i = 1, \cdots, N$.

Proof: We show this lemma by construction. Consider the even power allocation where $P_i = P/N$ for all $i = 1, \cdots, N$. Since u_i is an increasing function of P_i, we have $u_i > u_{i,0}$ for all $i = 1, \cdots, N$. ■

With the results in Lemma 1 and Lemma 2, the asymmetric NBS is the solution to the following optimization problem [20]

$$\arg \max_{P_1, \cdots, P_N} \prod_{i=1}^{N} (u_i - u_{i,0})^{\beta_i}, \text{ s.t. } P_i \geq 0, \sum_{i=1}^{N} P_i \leq P, \quad (13)$$

where β_i is User i's bargaining power. This problem can be simplified by the following lemma.

Lemma 3: The optimization problem in (13) is equivalent to the following problem:

$$\begin{aligned} &\arg \max_{P_1, \cdots, P_N} \sum_{i=1}^{N} \beta_i \log \left(\frac{\rho_i P_i}{\xi_i P_i + 1} \right) \\ &\text{s.t.} \quad P_i > 0, \quad \sum_{i=1}^{N} P_i = P. \end{aligned} \quad (14)$$

Proof: As the logarithm function is monotonically increasing, we can take the logarithm of the objective function in (13) without changing its solution. Thus the objective function in (14) is obtained using the definitions in (6) and (7).

Furthermore, notice that when $P_i = 0$ for some i, the objective function of (14) becomes $-\infty$. This is obviously non-optimal since any feasible power allocation with non-zero P_i for all i's (e.g., $P_i = P/N$) will result in a higher objective function. Thus, we can replace $P_i \geq 0$ by $P_i > 0$. This ensures that all users will enter the bargaining game.

Next, we show by contradiction that the optimal solution, denoted as $\mathbf{P}^* = (P_1^* \ \cdots \ P_N^*)$ satisfies $\sum_{i=1}^{N} P_i^* = P$. Assume that the optimal solution $\mathbf{P}^*$ gives the utility vector $\mathbf{u}^* = (u_1^*, u_2^*, \cdots, u_N^*)$ and satisfies $\sum_{i=1}^{N} P_i^* < P$. Let $\Delta_P = P - \sum_{i=1}^{N} P_i^*$. We consider another power allocation strategy $\mathbf{P}' \triangleq (P_1^* + \Delta_P, P_2^*, \cdots, P_N^*)$, which gives the utility vector $\mathbf{u}' = (u_1' \ u_2' \ \cdots \ u_N')$. It is straightforward to show that $\mathbf{u}'$ is in the feasible set $\mathcal{S}$, $u_1' > u_1^*$, and $u_i' = u_i^*$ for $i = 2, \cdots, N$. Thus this new solution results in a higher objective function than $\mathbf{P}^*$, which contradicts the assumption that $\mathbf{P}^*$ is optimal. This completes the proof. ■

Thus, to find the NBS-based relay power allocation, we should solve (14). Define $\mathbf{P} \triangleq \left[\ P_1 \ \ \cdots \ \ P_N \ \right]$. We write the Lagrangian function for problem (14) as

$$\begin{aligned} \mathcal{L}(\mathbf{P}, \alpha) \triangleq &\sum_{i=1}^{N} \beta_i \log \frac{\rho_i P_i}{\xi_i P_i + 1} \\ &- \sum_{i=1}^{N} \lambda_i P_i + \alpha \left(P - \sum_{i=1}^{N} P_i \right). \end{aligned} \quad (15)$$

Here λ_i and α are Lagrangian multipliers associated with the inequality and equality constraints. In (14), the objective function can be shown straightforwardly to be concave, the inequality constraint functions are convex, and the equality constraint function is affine, so it is a convex optimization problem. Its first-order Karush-Kuhn-Tucker (KKT) conditions, which are necessary and sufficient for the solution of (14) (see (5.49) on Page 243 in [29]) are

$$\frac{\partial \mathcal{L}(\mathbf{P}, \alpha)}{\partial P_i} = \frac{\beta_i}{(\xi_i P_i + 1) P_i} - \lambda_i - \alpha = 0, \quad (16)$$

$$-P_i < 0, \quad \sum_{i=1}^{N} P_i = P, \quad \lambda_i \geq 0, \quad \lambda_i P_i = 0, \quad (17)$$

for $i = 1, \cdots, N$. As $P_i > 0$, we have $\lambda_i = 0$ and thus

$$P_i = \frac{2\beta_i}{\alpha} \left(\sqrt{1 + \frac{4\xi_i \beta_i}{\alpha}} + 1 \right)^{-1} \quad \text{and} \quad \sum_{i=1}^{N} P_i = P. \quad (18)$$

Using (18), we have

$$\frac{2}{\alpha} \sum_{i=1}^{N} \beta_i \left(\sqrt{1 + \frac{4\xi_i \beta_i}{\alpha}} + 1 \right)^{-1} = P. \quad (19)$$

It can be shown that when α changes from 0 to ∞, the left-hand-side of (19) monotonically decreases from ∞ to 0. Thus, (19) has a unique positive solution and the solution can be found using bisection method[3]. Once the optimal α satisfying (19) is found, the NBS-based relay power allocation solution can be found using (18).

[3]The range of α can be set as $(0, \frac{1}{P})$. The upper bound of α can be derived as follows. Since $\xi_i = \frac{|g_i|^2}{Q_i|f_i|^2+1} > 0$ and $\beta_i > 0$, from (19), we have $P = \frac{2}{\alpha} \sum_{i=1}^{N} \beta_i \left(\sqrt{1 + \frac{4\xi_i \beta_i}{\alpha}} + 1 \right)^{-1} < \frac{2}{\alpha} \sum_{i=1}^{N} \frac{\beta_i}{2} = \frac{1}{\alpha}$, which gives $\alpha < \frac{1}{P}$. For the lower bound of α, we can set it to 0 since α is nonnegative.

IV. Implementation of the NBS-based Relay Power Allocation

In this section, we give possible implementations of the proposed NBS-based relay power allocation. First, we propose a centralized implementation, which requires no iterations and no computation at the users. But it requires global and perfect CSI at the relay. Also, the centralized implementation is based on the assumption that the relay is trustworthy. We then propose a distributed implementation, which requires only local CSI at each user and no computation is required at the relay.

A. Centralized Implementation

For the centralized implementation of the proposed relay power allocation, the relay, assumed to have global and perfect CSI, computes the NBS-based power allocation solution proposed in Section III and uses the corresponding power values to help the users. To get the NBS-based relay power allocation solution, the relay first finds the α that satisfies (19) using bisection method, then finds the NBS-based relay power allocation solution using (18). For the relay to know the channel gains from the users to itself, $f_1, \cdots, f_N$, training and channel estimations can be performed. For the relay to know the channel gain g_i from itself to Destination i, Destination i first estimates g_i, then feeds the coefficient back to the relay.

With this implementation, we actually assume that the relay is trustworthy. All users believe that 1) the relay will not change the parameter values (e.g., the bargaining powers and the CSI) to favor any user, and 2) the relay follows the NBS-based power allocation results to help all users in their transmissions.

B. Distributed Implementation

In practical wireless networks, especially for networks with a large number of users, it may be impractical to implement the aforementioned NBS-based power allocation in a centralized way at the relay. The reasons are threefold. First, the centralized scheme assumes accurate and complete CSI at the relay, which brings overhead for training, channel estimation, and CSI feedback from the destinations to the relay. Second, in the centralized scheme, all computational load is at the relay, which may not have high computational capability for many real network applications or may not be willing to conduct such computations. Third, in some applications, the users may distrust the relay and are unwilling to have the relay being the controller in power allocation.

To overcome these problems, we propose a distributed algorithm[4] to solve (19) at the users, each having local CSI only, i.e., User i knows f_i and g_i. Similarly, the CSI can be obtained via training and feedback channel. Similar to [31], [32], we implement the distributed algorithm based on the gradient projection of the dual problem associated with the original problem (14).

[4]It should be noted that, compared with the centralized implementation, the distributed methods have drawbacks such as estimation error accumulation, delay, quantization error, and extra bandwidth cost. In this paper, since the focus is on providing a possible distributed implementation scheme, we use the ideal assumption that the estimation/quantization error accumulation and delay are negligible.

The dual problem of (14) is:

$$\min_{\alpha \geq 0} D(\alpha), \tag{20}$$

where $D(\alpha)$ is the dual function defined as follows:

$$\begin{aligned} D(\alpha) &\triangleq \max_{\mathbf{P}} \mathcal{L}(\mathbf{P}, \alpha) \\ &= \max_{\mathbf{P}} \left\{ \sum_{i=1}^{N} \left(\beta_i \log \frac{\rho_i P_i}{\xi_i P_i + 1} - \alpha P_i \right) + \alpha P \right\}. \end{aligned} \tag{21}$$

$\mathcal{L}(\mathbf{P}, \alpha)$ is the Lagrangian function defined in (15). We have shown that $\lambda_i = 0$, so the term with λ_i is omitted.

Note that the summation term in $\mathcal{L}(\mathbf{P}, \alpha)$ is separable in P_i. Hence, we have from (21)

$$D(\alpha) = \sum_{i=1}^{N} \left\{ \underbrace{\max_{P_i} \left(\beta_i \log \frac{\rho_i P_i}{\xi_i P_i + 1} - \alpha P_i \right)}_{\triangleq F_i(P_i)} + \alpha P \right\}. \tag{22}$$

Since Problem (14) is a convex optimization problem, by duality theory, if α^* is the optimal solution of the dual problem in (20), $(P_1(\alpha^*), \cdots, P_N(\alpha^*))$ calculated from (18) is the optimal solution of (14). Therefore, we can focus on the dual problem (20).

The gradient of $D(\alpha)$ can be calculated to be:

$$\frac{\partial D(\alpha)}{\partial \alpha} = P - \sum_{i=1}^{N} P_i(\alpha). \tag{23}$$

We can now solve the dual problem with the gradient projection method [33] where α is adjusted in the opposite direction to $\frac{\partial D(\alpha)}{\partial \alpha}$ as:

$$\begin{aligned} \alpha(t+1) &= \max \left\{ 0, \alpha(t) - \gamma \frac{\partial D}{\partial \alpha}(\alpha(t)) \right\} \\ &= \max \left\{ 0, \alpha(t) - \gamma \left[P - \sum_{i=1}^{N} P_i(\alpha(t)) \right] \right\}, \end{aligned} \tag{24}$$

where $\gamma > 0$ is the step-size.

The gradient projection method generates a sequence of α values: $\alpha(0), \cdots, \alpha(t), \alpha(t+1), \cdots$ that approaches the optimal solution α^*. With a constraint on the step size γ, the convergence of the gradient projection method can be guaranteed, which is stated in the following theorem.

Theorem 1: Let $\beta_{\min} \triangleq \min\{\beta_1, \cdots \beta_N\}$ and $|g_{\max}| \triangleq \max\{|g_1|, \cdots, |g_N|\}$. If the step-size satisfies $0 < \gamma < \frac{2\beta_{\min}}{NP^2(|g_{\max}|^2 P+1)}$, for any initial $\alpha(0) \geq 0$, the gradient projection method will converge to the primal and dual optimal point, i.e.,

$$\lim_{t \to \infty} \alpha(t) = \alpha^*, \quad \lim_{t \to \infty} P_i(\alpha(t)) = P_i^*. \tag{25}$$

Proof: See the Appendix. ■

We now comment on the convergence speed of the distributed scheme. Using Tylor's theorem to $\frac{\partial D(\alpha(t))}{\partial \alpha(t)}$ at the optimal α^*, it can be readily shown that $\alpha(t) - \alpha^* \cong$

$\frac{\partial D(\alpha(t))}{\partial \alpha(t)}\left(\frac{\partial^2 D(\alpha(t))}{\partial^2 \alpha(t)}\right)^{-1} + o(\alpha^* - \alpha(t))$. Combining the tth and $(t+1)$th iterations, we get that around α^*, $S = \frac{\alpha(t+1)-\alpha^*}{\alpha(t)-\alpha^*} \cong 1 + \gamma \sum_{i=1}^{N} \frac{1}{\Theta_i''(P_i(\alpha))}$, where $\sum_{i=1}^{N} \frac{1}{\Theta_i''(P_i(\alpha))}$ is non-positive (as can be seen from the proof of Lemma 5 in the appendix). Note that S determines the convergence speed [28] and a larger S means a faster convergence speed. So when $1 + \gamma \sum_{i=1}^{N} \frac{1}{\Theta_i''(P_i(\alpha))}$ is positive, a larger step size gives a higher convergence speed. When $1+\gamma \sum_{i=1}^{N} \frac{1}{\Theta_i''(P_i(\alpha))}$ is negative, however, oscillation of the gradient projection method might occur, which impedes the convergence speed of our distributed algorithm.

We have shown how to get the NBS-based power allocation based on gradient projection method of the dual problem. Now, we discuss the distributed implementation of the proposed NBS-based power allocation scheme based on the above results.

Assume that each user has local CSI only. In each iteration of the distributed scheme, User i individually calculates $P_i(\alpha)$ according to (18) and broadcasts this information to all other users. Then each user updates α according to (24). We assume that user updates are synchronized. This cycle repeats until convergence. The distributed implementation is written as Algorithm 1.

Algorithm 1 Distributed Relay Power Allocation

1: Initialize α and γ, e.g., $\alpha = \frac{1}{P}$ and $\gamma = \frac{\beta_{\min}}{NP^2(|g_{\max}|^2 P+1)}$.
2: Each user calculates $P_i(\alpha)$ according to (18) and broadcasts it to all other users.
3: Each user updates α according to (24). Go to Step 2 until convergence.

To guarantee convergence, as specified in Theorem 1, the step size in updating α needs to satisfy the condition $0 < \gamma < \frac{2\beta_{\min}}{NP^2(|g_{\max}|^2 P+1)}$. Thus the users need to know $\beta_{\min}$ and $|g_{\max}|$ to agree on a step size. $\beta_{\min}$ is the smallest bargaining power, which is pre-determined and known to all users. For the users to know $|g_{\max}|$, a distributed scheme based on timer [34] can be used: each user starts a timer whose value is an increasing function of $1/|g_i|$. The timer of the user with the smallest $1/|g_i|$ stops first, then it broadcasts its $|g_i|$, which is also $|g_{\max}|$. Other users will hear this signalling and get $|g_{\max}|$. Then the users decide on a step size inside the interval for convergence, e.g., $\gamma = \frac{\beta_{\min}}{NP^2(|g_{\max}|^2 P+1)}$.

This distributed scheme based on updating $\alpha(t)$ can be seen as "price-based" power allocation. The parameter α can be interpreted as the price per unit power charged by the relay depending on the requested power from the users, and $F_i(P_i)$ defined in (22) represents the maximum benefit that User i can receive at price α. Equation (24) says that at time t, if the total demand $\sum_{i=1}^{N} P_i(\alpha(t))$ is larger than the available relay power P, the price should be raised; otherwise it should be reduced.

For the broadcasting of $P_i(\alpha)$, we can adopt a scheme similar to that in [24]: For each channel assigned to the users, a portion of the frequency band is used as the guard channel. Since the guard channels are orthogonal, users can broadcast their power demands simultaneously on these channels. To manage the error accumulation problem, we can use error-correcting codes [35] when broadcasting $P_i(\alpha)$.

V. Investigation on Bargaining Power Selection

In this section, we discuss the impact of the bargaining powers on the relay power allocation and show that by proper selection of the bargaining powers, the proposed NBS-based power allocation can bridge the even power allocation, which has the best fairness, and the sum-rate-optimal power allocation, which has the best global performance.

A. Impact of Bargaining Power Selection on Power Allocation

First, we investigate the effect of bargaining power selection on the proposed NBS-based power allocation. In the following proposition, we show that a user's bargaining power determines its priority and thus its allocated relay power.

Proposition 1: If User k's bargaining power β_k is increased while other users' bargaining powers are either decreased or remain unchanged, more power will be allocated to User k.

Proof: We use contradiction to prove this lemma. For a given set of bargaining powers $\beta_1, \cdots, \beta_N$, let $(P_1 \ \cdots \ P_N)$ be the solution to (14), which satisfies (16)-(18). From (16) and the fact that $\lambda_i = 0$, we have

$$\psi(P_i) \triangleq (\xi_i P_i + 1) P_i = \beta_i \alpha^{-1}, \text{ for all } i. \tag{26}$$

Therefore, $\frac{\psi(P_k)}{\psi(P_j)} = \frac{\beta_k}{\beta_j}$. Now consider another set of bargaining powers $\beta_1', \cdots, \beta_N'$ with $(P_1' \ \cdots \ P_N')$ being the solution to (14). For the same reason, we have $\psi(P_k')/\psi(P_j') = \beta_k'/\beta_j'$.

Assume that $\beta_k' > \beta_k$ and $\beta_j' \leq \beta_j$ for all $j \neq k$ but $P_k' \leq P_k$. We have $\frac{\beta_k'}{\beta_j'} > \frac{\beta_k}{\beta_j}$ and thus

$$\frac{\psi(P_k')}{\psi(P_j')} > \frac{\psi(P_k)}{\psi(P_j)} \text{ for all } j \neq k. \tag{27}$$

Note that $\psi(P_i)$ is a strictly increasing function of P_i. So $\psi(P_k') \leq \psi(P_k)$ due to the assumption that $P_k' \leq P_k$. Consequently, from (27), we have $\psi(P_j') < \psi(P_j)$, and thus $P_j' < P_j$ for all $j \neq k$, since $\psi(\cdot)$ is monotonically increasing. Thus, $\sum_{i=1}^{N} P_i' < \sum_{i=1}^{N} P_i = P$ and $(P_1' \ \cdots \ P_N')$ cannot be a solution to (14). This completes the proof. ■

In this paper, we assume that bargaining powers of users are determined by service providers and they are initiated before the bargaining process. Proposition 1 implies that we can adjust the NBS-based relay power allocation solution via adjusting the user bargaining powers. Priorities of users can be materialized with this adjustment. For example, in scenarios where service providers aim to receive the most monetary revenue, larger bargaining powers can be assigned to users who pay higher price for higher priority. In this way, according to Proposition 1, these users will receive more relay power.

B. Bridging between Global Sum-Rate Optimum and Fairness

In this subsection, we connect the proposed NBS-based relay power allocation with even power allocation, which has the best fairness, and the global sum-rate-optimal power allocation, which has the best global performance. We show that via appropriate bargaining power selection, the proposed NBS-based solution provides a balance between fairness and global performance.

In the even power allocation, the amount of power the relay allocates to each user is P/N. The following proposition is proved.

Proposition 2: If

$$\beta_i = \frac{N + P\xi_i}{N^2 + P\sum_{j=1}^{N}\xi_j}, \tag{28}$$

the proposed NBS-based power allocation is the same as even power allocation.

Proof: It is shown in the proof of Proposition 1 that with given bargaining powers $\beta_1, \cdots, \beta_N$, the NBS-based power allocation satisfies (26). With the value of β_i in (28), we have

$$\frac{(\xi_i P_i + 1)P_i}{(\xi_j P_j + 1)P_j} = \frac{\beta_i}{\beta_j} = \frac{N + P\xi_i}{N + P\xi_j}. \tag{29}$$

By observation, we can see that this is true if and only if $P_i = P_j = P/N$ for any i, j, which shows that the NSB-based power allocation coincides with the even power allocation when β_i is selected as in (28). ■

Recall that ξ_i defined in (2) is the noise forwarding rate of User i. From (28) we can see that to achieve even power allocation, a user with a larger noise forwarding rate (whose relay-path has a lower quality) should be assigned a larger bargaining power.

The sum-rate-optimal power allocation is the power allocation that maximizes the sum-rate of all users in the network. The sum-rate optimization problem of the network is as follows

$$\begin{aligned} &\arg\max_{\mathbf{P}}(C_{1RD} + \cdots + C_{NRD}) \\ &= \arg\max_{\mathbf{P}} \sum_{i=1}^{N} \log_2\left(\frac{\rho_i P_i}{\xi_i P_i + 1} + Q_i|h_i|^2 + 1\right), \\ &\text{s.t.} \quad \sum_{i=1}^{N} P_i \le P. \end{aligned} \tag{30}$$

Using the same techniques as in (16)-(19), we can show that the solution of (30) satisfies (31) on the next page, where α_1 is the Lagrangian multiplier associated with the equality constraint. The solution to (31), denoted as $\mathbf{P}^o$ (the superscript ‘o’ stands for sum-rate-optimal), can be found by first using bisection method to solve the optimal α_1 using the second equation in (31), then using the value of α_1 in the first equation in (31) to obtain the P_i’s.

Once $\mathbf{P}^o$ is found, we can find the bargaining powers that equate the NBS-based power allocation with the sum-rate-optimal solution as

$$\beta_i = \frac{\psi(P_i^o)}{\sum_{i=1}^{N}\psi(P_i^o)}, \tag{32}$$

where ψ is defined in (26). The proof of this result is similar to the proof of Proposition 2, thus is omitted.

We would like to note that the representation of the bargaining power in (32) is not in a closed-form but in an implicit form. To find the values, a numerical bisection method as explain above is required. The purpose of the discussion is to show that through proper selection of the bargaining powers, the proposed NBS-based power allocation can achieve the global sum-rate-optimal.

In order to better understand how to select the bargaining powers for global performance, in the following, we use a high SNR approximation for further investigations. One of the widely-used high SNR approximations is to neglect the noise term that is forwarded by the relay, i.e., $\sqrt{\frac{P_i}{Q_i|f_i|^2+1}}gn_{iR}$. This approximation has shown to be sufficiently tight [27], especially in medium to high SNR regions, e.g., when the users are transmitting with a high power, or the relay is close to users. In the following proposition, we give the bargaining powers that equate the NBS-based power allocation with the sum-rate-optimal power allocation.

Proposition 3: Let

$$\beta_i = \frac{1}{N} + \sum_{j=1}^{N}\frac{Q_j|h_j|^2+1}{\rho_j NP} - \frac{Q_i|h_i|^2+1}{\rho_i P}. \tag{33}$$

For high SNR, if the relay noise is neglected, the proposed NBS-based power allocation maximizes the network sum-rate.

Proof: When the noise at the relay is neglected, the utility of User i is approximated as

$$\text{SNR}'_{iRD} = \rho_i P_i + Q_i|h_i|^2. \tag{34}$$

The disagreement point of User i is the same as in (7). So NBS is the solution to the following optimization problem:

$$\begin{aligned} &\arg\max_{P_1,\cdots,P_N}\sum_{i=1}^{N}\beta_i\log(\rho_i P_i) \\ &\text{s.t.} \quad P_i > 0, \quad \sum_{i=1}^{N}P_i = P. \end{aligned} \tag{35}$$

Using the same optimization techniques in (16)-(19), we can show straightforwardly that the solution to (35) is

$$P_i^{NBS} = \beta_i P. \tag{36}$$

For sum-rate-optimal solution, with the high-SNR approximation, (30) is equivalent to the following problem:

$$\begin{aligned} &\arg\max_{P_1,\cdots,P_N}\sum_{i=1}^{N}\log\left(\rho_i P_i + Q_i|h_i|^2 + 1\right) \\ &\text{s.t.} \quad P_i > 0, \quad \sum_{i=1}^{N}P_i = P. \end{aligned} \tag{37}$$

Again by using the KKT conditions, the solution is

$$P_i^o = \frac{P}{N} + \sum_{j=1}^{N}\frac{Q_j|h_j|^2+1}{\rho_j N} - \frac{Q_i|h_i|^2+1}{\rho_i} \tag{38}$$

When β_i is defined as in (33), we have $P_i^{NBS} = P_i^o$. ■

We can see that the first two terms in (38) are the same for all users. So the last term is the dominant factor in the bargaining power selection in achieving global sum-rate-optimal. Recall that ρ_i defined in (3) is the signal forwarding rate of User i. (33) shows that for global optimum, a user with

$$P_i = \frac{-\left(\frac{Q_i|f_i|^2}{Q_i|h_i|^2+1}+2\right)+\sqrt{\left(\frac{Q_i|f_i|^2}{Q_i|h_i|^2+1}+2\right)^2+4\left(\frac{Q_i|f_i|^2}{Q_i|h_i|^2+1}+1\right)\left(\frac{\rho_i}{\alpha_1(Q_i|h_i|^2+1)}-1\right)}}{2\xi_i\left(\frac{Q_i|f_i|^2}{Q_i|h_i|^2+1}+1\right)} \quad \text{and} \quad \sum_{i=1}^{N} P_i = P, \tag{31}$$

a larger signal forwarding rate (whose relay-path has a higher quality) should be assigned a larger bargaining power. This has the opposite trend as the even power allocation case. The other coefficient $(Q_i|h_i|^2+1)$ in the last terms relates to the direct link and is independent of the relay link.

Based on the above discussions, for networks with different requirements, we can adjust the NBS-based relay power allocation toward the requirements by adjusting the bargaining powers. For example, in a network design, if the global sum-rate-optimal power allocation is desired, users whose relay-paths have higher quality should be allocated more relay power. With the proposed NBS-based power allocation, we can obtain good network sum-rate by assigning larger bargaining powers to such users. On the other hand, if fairness is the major concern, we can assign larger bargaining powers to users whose relay-paths have lower quality. Those users can thus obtain more relay powers to ensure a certain level of quality, which helps the fairness consideration of the network. But this improved fairness is at the cost of lower network sum-rate.

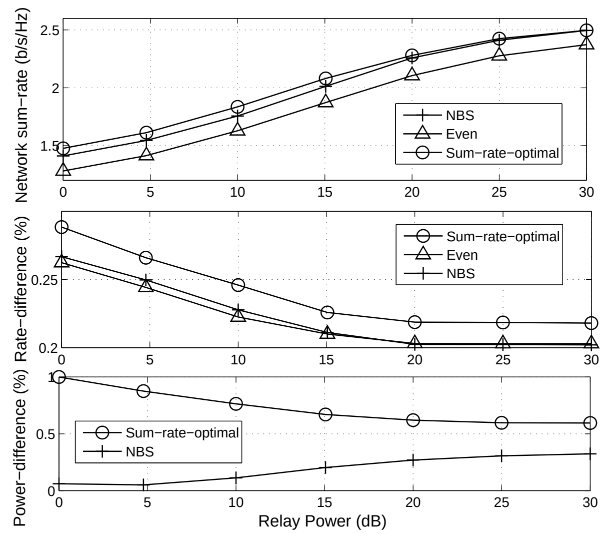

Fig. 2. Sum-rate, normalized-rate-difference, and normalized-power-difference of a 3-user network with Rayleigh channels.

VI. Simulation Results

In this section, we show the performance of our NBS-based power allocation solution and compare it with the sum-rate-optimal solution, the even power solution, and the rate-fair solution. The sum-rate-optimal solution is the relay power allocation that maximizes the network sum-rate while fairness is not considered. With the even power solution, the relay power assigned to each user is P/N. It has the best fairness in the sense of power. The rate-fair solution is the relay power allocation that makes all users in the network have the same achievable rate. It has the best fairness in the sense of achievable rate. It is not always possible, depending on the values of the channel coefficients. We compare four parameters: network sum-rate, individual achievable rate γ_i, the normalized-rate-difference, which is defined as $\mathbf{E}\{[max_i(\gamma_i) - min_i(\gamma_i)]/max_i(\gamma_i)\}$, and the normalized-power-difference $\mathbf{E}\{[max_i(P_i) - min_i(P_i)]/max_i(P_i)\}$. A smaller normalized-rate-difference (or normalized-power-difference) indicates a fairer solution. Other fairness metrics, e.g. Jain's fairness index [36], show the same performance trend. Two channel models are considered: Rayleigh flat-fading channels and static channels with path-loss only.

A. Rayleigh Flat-Fading Channels

For the Rayleigh flat-fading model, the channel gains, f_i, h_i, and g, are modeled as independent and identically distributed (i.i.d.) random variables following the distribution $\mathcal{CN}(0,1)$. We consider a three-user network and all users have the same bargaining power: $\beta_1 = \beta_2 = \beta_3 = 1/3$. The transmit power of each user is set to be 10 dB. The relay power constraint P is in the range of 0 to 30 dB. Since for this channel mode, rate-fair solution is not always possible, the proposed solution is only compared with the sum-rate-optimal and the even power solutions.

Figure 2 compares the average sum-rate, normalized-rate-difference, and normalized-power-difference of the sum-rate-optimal solution, even power allocation, and the NBS-based power allocation. For even power allocation, as the relay allocates the same power to all three users, the normalized-power-difference is 0, thus is not shown in Figure 2. It can be seen that in the simulated power range, the sum-rate difference between the proposed NBS-based and the sum-rate-optimal solutions is within 4%, while it is within 14% between the sum-rate-optimal and the even power solutions. The proposed solution is about 4 dB superior to the even power solution in global performance. From the normalized-rate-difference, we find that our NBS-based solution has similar rate-fairness to the even power solution and is fairer than the sum-rate-optimal solution. From the normalized-power-difference, we find that our NBS-based solution is fairer in the sense of power than the sum-rate-optimal solution.

B. Static Channels With Path-Loss Only

In this section, we consider a static network whose channels are only related to the path-loss, which is inverse proportional to the distance squared. The network has two users, one relay, and two destinations. The relative positions of the nodes are shown in Figure 3, where the coordinates of User 1, User 2, the relay, Destination 1, and Destination 2 are (-9, 0), (-3, 0),

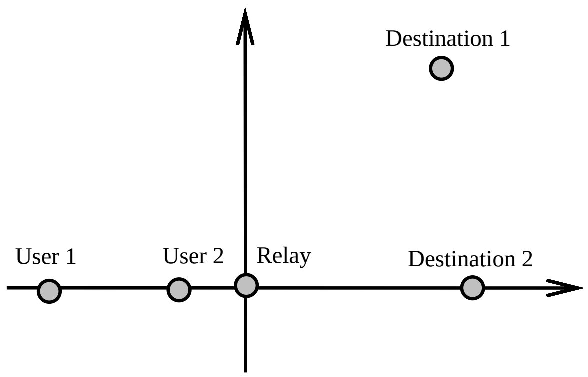

Fig. 3. Two-user network with static channels.

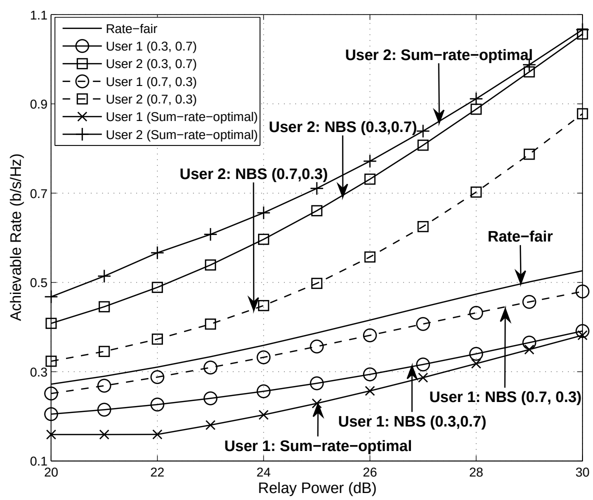

Fig. 4. Achievable rates of a two-user network with static channels.

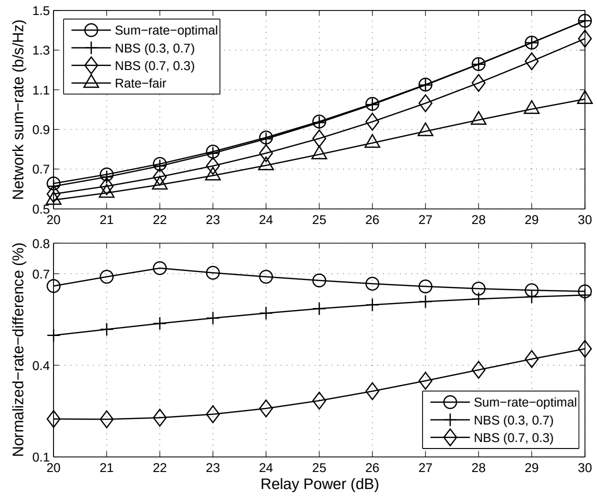

Fig. 5. Sum-rate and normalized-rate-difference of a two-user network with static channels.

(0, 0), (7, 12), and (13, 0), respectively. Thus, User 2 has a better relay channel. The transmission power of both users are 20 dB, and the relay power constraint P ranges from 20 dB to 30 dB.

To investigate the global network sum-rate, the fairness, and the effect of the bargaining powers on network performance, we show the individual achievable rates of the users (in Figure 4), network sum-rate, and the normalized-rate-difference (in Figure 5) under the proposed solutions with two different sets of bargaining powers: $\beta_1 = 0.3, \beta_2 = 0.7$ and $\beta_1 = 0.7, \beta_2 = 0.3$. For comparison, the individual achievable rates under the sum-rate-optimal solution and the rate-fair solution are also shown. As the achievable rates of the two user are the same for the rate-fair solution, the normalized-rate-difference is 0 for this scheme and is not shown in Figure 5.

Comparing the two NBS-based power allocation schemes with different bargaining powers, we can see from the two figures that a user achieves a higher rate with a larger bargaining power, and the bargaining power can be tuned to gain the desired balance between the global network sum-rate and individual rate-fairness. When User 2, who has a better channel, is assigned a higher bargaining power, the NBS-based solution emphasizes more on the network sum-rate and allocates more relay power to User 2. In Figure 4 and 5, the sum-rate performance of the NBS-based solution with $\beta_1 = 0.3, \beta_2 = 0.7$ is very close to that of the sum-rate-optimal solution. In this case, User 1, with a worse channel, experiences low achievable rate, which is 37% to 50% of the achievable rate of User 2. On the contrary, when a larger bargaining power 0.7 is assigned to User 1, the NBS-base solution allocates more power to User 1, and the performance is closer to the rate-fair solution. In this case, the network sum-rate is reduced to 90% of that of the sum-rate-optimal solution when P is small and 93% when P is large. The normalized-rate-difference justifies the above-mentioned analysis, which shows that NBS-based solution with $\beta_1 = 0.7, \beta_2 = 0.3$ is fairer in the sense of rate than the other two schemes.

To further illustrate the effect of the bargaining powers on network performance, we show the network sum-rate, normalized-rate-difference, and normalized-power-difference (in Figure 6) under the proposed solution with the bargaining power of User 1 changing from 0 to 1. We consider three relay powers: 25 dB, 30 dB, and 35 dB. Other network conditions are the same as the static network shown in Figure 3. When $\beta_1 = 0$ or $\beta_1 = 1$, all relay power is allocated to User 2 or User 1, so the normalized-rate-difference is 1. For the three different relay powers, network sum-rate is maximized at approximately $\beta_1 = 0.25$. After that, we can see a reduction in the network sum-rate as β_1 increases, which verifies the conclusion in Section V-B: by assigning a larger bargaining power to User 2 which has a higher signal forwarding power, the solution approaches the sum-rate-optimal solution. For fairness in the sense of both rate and power, the normalized-rate-difference and the normalized-power-difference decrease as β_1 increases until rate-fair or power-fair is achieved. For $P = 25, 30$, and 35 dB, when $\beta_1 = 0.6, 0.64$, and 0.675, the proposed NBS-based power allocation becomes even power

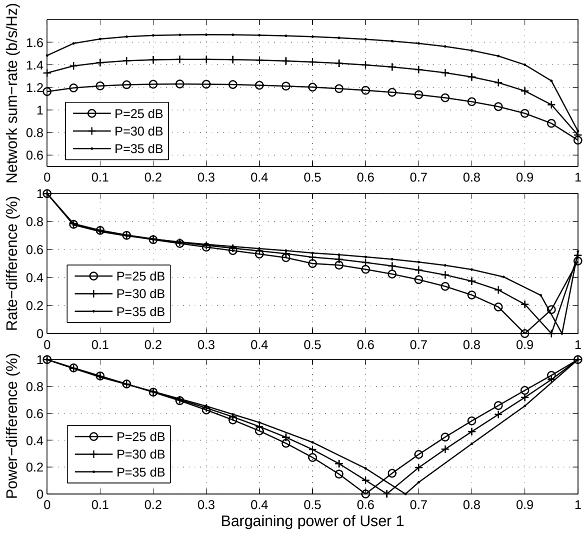

Fig. 6. Sum-rate, normalized-rate-difference, and normalized-power-difference of a two-user network with different relay powers and varying bargaining powers.

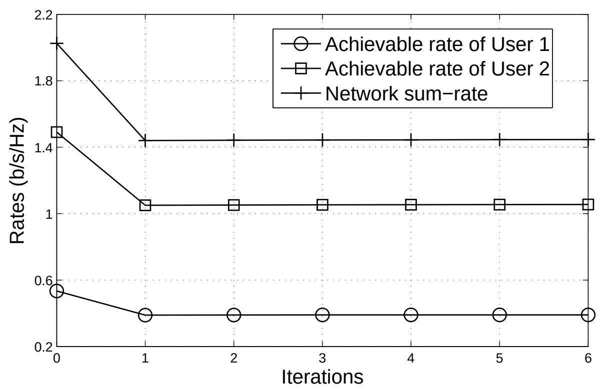

Fig. 7. Convergence of a 2-user network.

allocation. These values of β_1 are the same as been calculated with Proposition 2. This verifies our claim in Section V-B that on the contrary to sum-rate optimum, power fairness can be approached by assigning higher bargaining power to User 1 which has a larger noise forwarding rate, and thus lower quality in the relay path. Similar to power-fairness, for rate-fairness, the user with a higher noise forwarding rate should be assigned a higher bargaining power. For $P = 25, 30$, and 35 dB, rate-fair power allocation can be achieved using the proposed NBS-based power allocation when $\beta_1 = 0.9, 0.95$, and 0.97, respectively.

Figure 7 illustrates the convergence of the distributed relay power allocation. In this simulation, the relay power is set to be 30 dB, the bargaining powers of the two users are $\beta_1 = 0.7, \beta_2 = 0.3$, and all other network settings are the same as the network in Figure 3. α is initialized as 0.1. We can see from Figure 7 that the proposed distributed scheme converges after 2 iterations and similar performance is verified with different initial values of α.

VII. Extension to Multi-user Multi-relay Networks

In this section, we discuss the extension of our work to multi-user multi-relay networks where users can receive help from multiple relays. Assume that there are N users and R relays. Assume that the relays also use orthogonal channels. Denote the channel gain from User i to Relay r as f_{ir}, and the channel gain from Relay r to Destination i as g_{ir}. Denote the power constraint of Relay r as $P^{(r)}$ and Relay r uses power P_{ir} to help User i. So the power allocation for all users from all relays can be denoted as a matrix $\{P_{ir}\}$, where the row index is the user index and the column index is the relay index. Denote $\mathbf{P}_i = [P_{i1}, P_{i2}, \cdots, P_{iR}]^T$ as the power allocation for User i from all relays, and $\mathbf{P}^{(r)} = [P_{1r}, P_{2r}, \cdots, P_{Nr}]^T$ as the power allocation vector of Relay r for all users. Define $\xi_{ir} \triangleq \frac{|g_{ir}|^2}{Q_i|f_{ir}|^2+1}$ and $\rho_{ir} \triangleq \frac{Q_i|f_{ir}g_{ir}|^2}{Q_i|f_{ir}|^2+1}$ as the noise forwarding rate and signal forwarding rate of User i at Relay r. Other assumptions and notation are the same as the single-relay case.

With maximum ratio-combining, the received SNR of User i's transmission is $\mathrm{SNR}_{iRD} = \sum_{r=1}^{R} \frac{\rho_{ir}P_{ir}}{\xi_{ir}P_{ir}+1} + Q_i|h_i|^2$. Similarly, define the utility of User i as:

$$u_i(\mathbf{P}_i) \triangleq \mathrm{SNR}_{iRD}. \tag{39}$$

$u_{i,0} = Q_i|h_i|^2$ is the minimum utility that User i expects.

Similar to the single-relay case, to use the NBS-based power allocation, we first need to prove that the feasible set

$$\mathcal{S}^M \triangleq \left\{ (u_1 \cdots u_N) \middle| P_{ir} \geq 0, \quad \sum_{i=1}^{N} P_{ir} \leq P^{(r)}, \quad r = 1 \cdots R \right\} \tag{40}$$

is convex.

Lemma 4: Given the utility function $u_i(\mathbf{P}_i)$ in (39), the feasible set $\mathcal{S}^M$ in (40) is convex.

Proof: Given $\{x_{ir}\}$ as a feasible power allocation matrix where x_{ir} is the power allocation from Relay r to User i, denote $\mathbf{x}^{(r)} = [x_{1r} \cdots x_{Nr}]^T$ as the power allocation vector at Relay r for all users and $\mathbf{x}_i = [x_{i1} \cdots x_{iR}]^T$ as the power allocation for User i from all relays. To prove that $\mathcal{S}^M$ is convex, we need to show that given two arbitrary power allocation matrices $\{x_{ir}\}$ and $\{y_{ir}\}$ and the corresponding utility vectors $\mathbf{u} = [u_1(\mathbf{x}_1), u_2(\mathbf{x}_2), \cdots, u_N(\mathbf{x}_N)]^T$ and $\mathbf{v} = [u_1(\mathbf{y}_1), u_2(\mathbf{y}_2), \cdots, u_N(\mathbf{y}_N)]^T$ in the feasible set $\mathcal{S}^M$, we have $\theta\mathbf{u} + (1-\theta)\mathbf{v} \in \mathcal{S}^M$ for any $0 \leq \theta \leq 1$.

Note that

$$\begin{aligned}
\mathbf{u} =& \begin{bmatrix} u_1(\mathbf{x}_1) = \frac{\rho_{11}x_{11}}{\xi_{11}x_{11}+1} + \cdots + \frac{\rho_{1R}x_{1R}}{\xi_{1R}x_{1R}+1} + u_{1,0} \\ u_2(\mathbf{x}_2) = \frac{\rho_{21}x_{21}}{\xi_{21}x_{21}+1} + \cdots + \frac{\rho_{2R}x_{2R}}{\xi_{2R}x_{2R}+1} + u_{2,0} \\ \cdots \\ u_N(\mathbf{x}_N) = \frac{\rho_{N1}x_{N1}}{\xi_{N1}x_{N1}+1} + \cdots + \frac{\rho_{NR}x_{NR}}{\xi_{NR}x_{NR}+1} + u_{N,0} \end{bmatrix} \\
=& \begin{bmatrix} \frac{\rho_{11}x_{11}}{\xi_{11}x_{11}+1} \\ \frac{\rho_{21}x_{21}}{\xi_{21}x_{21}+1} \\ \vdots \\ \frac{\rho_{N1}x_{N1}}{\xi_{N1}x_{N1}+1} \end{bmatrix} + \cdots + \begin{bmatrix} \frac{\rho_{1R}P_{1R}}{\xi_{1R}P_{1R}+1} \\ \frac{\rho_{2R}P_{2R}}{\xi_{21}P_{2R}+1} \\ \vdots \\ \frac{\rho_{NR}P_{NR}}{\xi_{N1}P_{NR}+1} \end{bmatrix} + \begin{bmatrix} u_{1,0} \\ u_{2,0} \\ \vdots \\ u_{N,0} \end{bmatrix} \\
=& \mathbf{f}^1\left(\mathbf{x}^{(1)}\right) + \mathbf{f}^2\left(\mathbf{x}^{(2)}\right) + \cdots + \mathbf{f}^R\left(\mathbf{x}^{(R)}\right) + \mathbf{u}_0,
\end{aligned} \tag{41}$$

where $\mathbf{f}^r\left(\mathbf{P}^{(r)}\right) \triangleq \left[\frac{\rho_{1r}x_{1r}}{\xi_{1r}x_{1r}+1} \cdots \frac{\rho_{Nr}x_{Nr}}{\xi_{Nr}x_{Nr}+1}\right]^T$ for $r = 1, \cdots, R$. Similarly, given the power allocation matrix $\{y_{ir}\}$, the corresponding utility vector is $\mathbf{v}=\mathbf{f}^1\left(\mathbf{y}^{(1)}\right)+\mathbf{f}^2\left(\mathbf{y}^{(2)}\right)+\cdots+\mathbf{f}^R\left(\mathbf{y}^{(R)}\right)+\mathbf{u}_0$. Therefore, we have

$$\begin{aligned}\theta\mathbf{u}+(1-\theta)\mathbf{v}=&\left[\theta\mathbf{f}^1\left(\mathbf{x}^1\right)+(1-\theta)\mathbf{f}^1\left(\mathbf{y}^1\right)\right]\\+\cdots+&\left[\theta\mathbf{f}^R\left(\mathbf{x}^R\right)+(1-\theta)\mathbf{f}^R\left(\mathbf{y}^R\right)\right]+\mathbf{u}_0.\end{aligned}\tag{42}$$

Note that $\left\{\mathbf{f}^r\left(\mathbf{P}^{(r)}\right)+\mathbf{u}_0\middle|P_{ir}\geq 0,\sum_{i=1}^N P_{ir}\leq P^{(r)}\right\}$ is the feasible set of a network with a single relay r, and is convex from Lemma 1. Therefore, for any Relay r and any $0\leq\theta\leq 1$, we can find another power allocation vector $\mathbf{z}^{(r)}$ with $z_{ir}\geq 0$ and $\sum_{i=1}^N z_{ir}\leq P^{(r)}$ such that $\mathbf{f}^r\left(\mathbf{z}^{(r)}\right)=\theta\mathbf{f}^r\left(\mathbf{x}^{(r)}\right)+(1-\theta)\mathbf{f}^r\left(\mathbf{y}^{(r)}\right)$. Combining the power allocation vectors $\{\mathbf{z}^{(r)}\}$ for all relays, we can find the feasible power allocation matrix $\{z_{ir}\}$ such that $\theta\mathbf{u}+(1-\theta)\mathbf{v}{=}\mathbf{f}^1\left(\mathbf{z}^{(1)}\right)+\mathbf{f}^2\left(\mathbf{z}^{(2)}\right)+\cdots+\mathbf{f}^R\left(\mathbf{z}^{(R)}\right)+\mathbf{u}_0\in\mathcal{S}^M$. This completes the proof. ■

In addition, Lemma 2 is also valid for the multi-relay case, that is, there is at least one point in $\mathcal{S}^M$ with $u_i>u_{i,0}$ for all $i=1,\cdots,N$. Therefore, the asymmetric NBS for the multi-relay network is the solution of the following optimization problem:

$$\begin{aligned}&\arg\max_{\mathbf{P}_1,\cdots,\mathbf{P}_N}\sum_{i=1}^N\beta_i\log\left(\sum_{r=1}^R\frac{\rho_{ir}P_{ir}}{\xi_{ir}P_{ir}+1}\right)\\&\text{s.t.}\quad P_{ir}>0,\quad\sum_{i=1}^N P_{ir}=P^{(r)}.\end{aligned}\tag{43}$$

This is a convex optimization problem and can be solved efficiently using standard convex optimization techniques [29] for centralized implementation.

To implement the distributed NBS-based power allocation, we can follow the same technique in Section IV.B. First, we write the Lagrangian function for (43) as

$$\begin{aligned}\mathcal{L}(\{P_{ir}\},\vec{\alpha})=&\sum_{i=1}^N\beta_i\log\left(\sum_{r=1}^R\frac{\rho_{ir}P_{ir}}{\xi_{ir}P_{ir}+1}\right)\\-\sum_{i=1}^{NR}\lambda_{ir}P_{ir}-&\sum_{r=1}^R\alpha_r\left(\sum_{i=1}^N P_{ir}-P^{(r)}\right).\end{aligned}\tag{44}$$

Here λ_{ir} and $\vec{\alpha}=[\alpha_1\cdots\alpha_R]$ are Lagrangian multipliers associated with the inequality and equality constraints. Same as the analysis of the single-relay networks in Section III.B, we have $\lambda_{ir}=0$ for all $i=1,\cdots,N$ and $r=1,\cdots R$ as $P_{ir}>0$.

Then, similar to the analysis of the single relay network in Section IV.B, the dual problem of (43) is: $\min_{\vec{\alpha}\geq 0}D^M(\vec{\alpha})$, where $D^M(\vec{\alpha})$ is the dual function defined as in (45) on the next page. As explained in the single-relay case, the equality in (45) holds since the summation term in $\mathcal{L}(\{P_{ir}\},\vec{\alpha})$ is separable in $\mathbf{P}_i$.

The gradient of $D^M(\vec{\alpha})$ is

$$\frac{\partial D^M(\vec{\alpha})}{\partial\alpha_r}=P^{(r)}-\sum_{i=1}^N P_{ir}(\vec{\alpha}),\quad r=1,\cdots,R,\tag{46}$$

where $\{P_{ir}(\vec{\alpha})\}_{i=1}^N$ is the maximizer of $F_i(\mathbf{P}_i)$ in (45) for a given $\vec{\alpha}$. Since $F_i(\mathbf{P}_i)$ is a convex function, $\{P_{ir}(\vec{\alpha})\}_{i=1}^N$ can be calculated with standard convex optimization techniques.

The dual problem can be solved with the gradient project method where α_r can be adjusted in the opposite direction to $\frac{\partial D^M(\vec{\alpha})}{\partial\alpha_r}$ as:

$$\begin{aligned}\alpha_r(t+1)=&\max\left\{0,\alpha_r(t)-\gamma_r\frac{\partial D}{\partial\alpha_r}(\vec{\alpha}(t))\right\}\\=&\max\left\{0,\alpha_r(t)-\gamma_r\left[P^{(r)}-\sum_{i=1}^N P_{ir}(\overrightarrow{\alpha})\right]\right\}\end{aligned}\tag{47}$$

Similar to Theorem 1, we can show that if the step-size satisfies $0<\gamma_r<\frac{2\beta_{\min}}{NP^{(r)2}(|g_{\max}^{(r)}|^2P^{(r)}+1)}$ for all relays, the gradient projection method converges to the primal and dual optimal point. Here, $|g_{\max}^{(r)}|\triangleq\max\{|g_{1r}|,\cdots,|g_{Nr}|\}$.

Assume that each user has local CSI only. In each iteration of the distributed scheme, User i individually calculates $P_{ir}(\overrightarrow{\alpha})$ (for $r=1,\cdots,R$) and broadcasts this information to all other users. Then each user updates $\overrightarrow{\alpha}$ according to (47). This cycle repeats until convergence. The distributed implementation of the NBS-based power power allocation for multi-relay networks can be summarized as in Algorithm 2.

Algorithm 2 Distributed Relay Power Allocation for Multi-relay Networks

1: Initialize α_r and γ_r, e.g., $\alpha_r=\frac{1}{P^{(r)}}$ and $\gamma_r=\frac{\beta_{\min}}{NP^{(r)2}(|g_{\max}^{(r)}|^2P^{(r)}+1)}$, for $r=1,\cdots,R$.
2: Each user calculates $P_{ir}(\overrightarrow{\alpha})$ (for $r=1,\cdots,R$) that maximizes $F_i(\mathbf{P}_i)$ in (45) and broadcasts this information to all other users.
3: Each user updates $\overrightarrow{\alpha}$ according to (47). Go to Step 2 until convergence.

VIII. Conclusion

In this paper, we consider a multi-user single-relay wireless network, and conduct the game-theoretic analysis of relay power allocation among the users. We propose an asymmetric NBS-based power allocation solution, where each user is assigned a bargaining power indicating its transmission priority. We first proposed a centralized algorithm to implement the NBS-based power allocation at the relay. Then, considering the scalability of the network, we propose a distributed algorithm for the NBS-based power allocation and its convergence conditions are provided. We show that bargaining powers can be adjusted to accommodate different requirements in different applications. After that, we generalize our NBS-based power allocation solution and its distributed implementation to multi-user multi-relay networks. Simulations are conducted to compare the proposed NBS-based power allocation with the sum-rate-optimal power allocation, the even power allocation, and the rate-fair power allocation. We find that the proposed NBS-based scheme has better sum-rate than even and rate-fair power allocation and is fairer than the sum-rate-optimal solution. Via simulation, we also demonstrate the impact of the bargaining powers on the proposed relay power allocation solution. We show that the proposed scheme can bridge the sum-rate-optimal power allocation, which has the best global

$$
\begin{aligned}
D^M(\vec{\alpha}) &\triangleq \max_{\{P_{ir}\}} \mathcal{L}(\{P_{ir}\}, \vec{\alpha}) \\
&= \max_{\{P_{ir}\}} \left\{ \sum_{i=1}^{N} \beta_i \log\left(\sum_{r=1}^{R} \frac{\rho_{ir} P_{ir}}{\xi_{ir} P_{ir} + 1} \right) - \sum_{r=1}^{R} \alpha_r \left(\sum_{i=1}^{N} P_{ir} - P^{(r)} \right) \right\} \\
&= \sum_{i=1}^{N} \left\{ \max_{\mathbf{P}_i} \underbrace{\left[\beta_i \log\left(\sum_{r=1}^{R} \frac{\rho_{ir} P_{ir}}{\xi_{ir} P_{ir} + 1} \right) - \sum_{r=1}^{R} \alpha_r P_{ir} \right]}_{\triangleq F_i(\mathbf{P}_i)} + \sum_{r=1}^{R} \alpha_r P^{(r)} \right\}. \qquad (45)
\end{aligned}
$$

performance and the even power allocation, which has the best fairness, by proper selection of bargaining powers.

APPENDIX

To prove Theorem 1, we first prove the following lemma:

Lemma 5: Functions $\Theta_i(P_i) = \beta_i \log \frac{\rho_i P_i}{\xi_i P_i + 1}, i = 1, \cdots, N$ are increasing, strictly concave and twice continuously differentiable. The curvatures of $\Theta_i(P_i)$ are bounded away from zero on feasible set $\mathcal{S}$.

Proof: Lemma 5 is straightforward due to the following facts.

$$
\begin{aligned}
\Theta_i'(P_i) &= \frac{\beta_i}{P_i(\xi_i P_i + 1)} > 0, \\
\Theta_i''(P_i) &= \frac{-\beta_i(2\xi_i P_i + 1)}{P_i^2(\xi_i P_i + 1)^2} < 0, \text{and continuous.}
\end{aligned}
$$

Since $\frac{\xi_i P_i}{\xi_i P_i + 1} > 0$, we have

$$
\begin{aligned}
-\Theta_i''(P_i) &= \frac{\beta_i(2\xi_i P_i + 1)}{P_i^2(\xi_i P_i + 1)^2} = \frac{\beta_i(1 + \frac{\xi_i P_i}{\xi_i P_i + 1})}{P_i^2(\xi_i P_i + 1)} \\
&\geq \frac{\beta_i}{P_i^2(\xi_i P_i + 1)}.
\end{aligned}
$$

■

From Lemma 5, we get that the dual objective function is convex, lower bounded, and continuously differentiable. To optimize $\Theta_i(P_i)$, the equation $\Theta_i'(P_i) = \alpha$ must be satisfied. Thus $P_i = \max\{0, \Theta_i'^{-1}(\alpha)\}$, where $\Theta_i'^{-1}$ is the inverse function of Θ_i'. Then we get $\frac{\partial P_i(\alpha)}{\partial \alpha} = \max\left\{0, \frac{1}{\Theta_i''(P_i(\alpha))}\right\}$. From (23), we get $\frac{\partial D(\alpha)}{\partial \alpha} = P - \sum_{i=1}^{N} P_i(\alpha)$, and hence

$$
\frac{\partial^2 D(\alpha)}{\partial^2 \alpha} = -\sum_{i=1}^{N} \frac{1}{\Theta_i''(P_i(\alpha))}.
$$

By using Taylor theorem, there exists a $t \in [0, 1]$, such that

$$
\frac{\partial D(\alpha)}{\partial \alpha} - \frac{\partial D(\beta)}{\partial \beta} = \frac{\partial^2 D(\mu)}{\partial^2 \mu}(\alpha - \beta),
$$

where $\mu = t\alpha + (1 - t)\beta$. Thus,

$$
\left| \frac{\partial D(\alpha)}{\partial \alpha} - \frac{\partial D(\beta)}{\partial \beta} \right| \leq \left| \frac{\partial^2 D(\mu)}{\partial^2 \mu} \right| |(\alpha - \beta)|.
$$

Now, from Lemma 5,

$$
\left| \frac{\partial^2 D(\mu)}{\partial^2 \mu} \right| = \sum_{i=1}^{N} \frac{1}{|\Theta_i''(P_i(\alpha))|} \leq \sum_{i=1}^{N} \frac{P_i^2(\xi_i P_i + 1)}{\beta_i}.
$$

As $Q_i|f_i|^2 + 1 > 1$, $\xi_i < |g_i|^2$. We have

$$
\sum_{i=1}^{N} \frac{P_i^2(\xi_i P_i + 1)}{\beta_i} \leq \frac{NP^2(|g_{max}|^2 P + 1)}{\beta_{min}}.
$$

From the analysis above, we conclude that $\frac{\partial D(\alpha)}{\partial \alpha}$ is Lipschitz [28] and the Lipschitz constant is $\kappa = NP^2(|g_{max}|^2 P + 1)/\beta_{min}$. Let γ be the step-size. If $\gamma \in (0, \frac{2}{\kappa})$, then any accumulation point α^* generated by sequence $\alpha(t)$ is dual optimal. We can then follow the same proof statements in [32] to show that $P_i(\alpha(t))$ will converge to the unique primal optimal point P_i^*.

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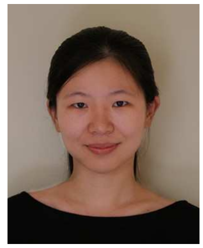

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