Voltage Sag State Estimation for Power Distribution Systems

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Abstract—The increased awareness on power quality has resulted in the need to quantify the voltage sag performance of a distribution feeder; similar to what has been done on characterizing the reliability performance of a feeder. Since it is impossible to measure the sag level at every node of a distribution feeder, estimation of sag characteristics at unmetered nodes becomes necessary. This paper proposes the concept of “voltage sag state estimation” and associated algorithms to achieve this goal. The proposed method has the following characteristics: 1) It makes use of the radial connection characteristic of a distribution feeder, 2) it is based on a limited number of metering points, and 3) it employs a least-square method to predict the sag profile along a distribution line. The results of the proposed sag state estimator can be used to calculate the feeder power quality performance indices such as the System Average RMS Frequency Index (SARFIx).

Index Terms—Distribution system, power quality, state estimation, voltage sags.

I. INTRODUCTION

Voltage sag and interruption are the most important power quality disturbances for customers due to their significant adverse impact on electronic equipment [1], [2]. In the early 90s, the impact or severity of sag disturbances was measured using magnitude and duration indices and compared with the power quality envelope [3], [4]. This per-disturbance based approach was found insufficient by late 90s due to the increasing need to quantify the power quality characteristics for specific customer sites. As a result, a new index, frequency of occurrence, was introduced [5]. This index has since received wide acceptance by industry and the per-site based power quality performance characterization has become an important aspect of PQ management.

In recent years, the need to characterize the feeder-level power quality performance has emerged. This is driven by the need to compare the PQ performance among utility companies or among various feeders. This concept is borrowed from the utility reliability management practice. For example, it has become common practice for regulators to use indices such as SAIFI, CAIFI and CAIDI to measure the customer service performance of utility companies [6]. Inspired by the feeder-level reliability concept, the IEEE PES Task Force on Voltage Sag Indices is recommending a set of indices to aid the assessment of service quality for a particular feeder or circuit area [7].

One of the representative indices is the System Average RMS (Variation) Frequency Index, SARFIx [8]. This index represents the average number of specified RMS variation events that occurred over the assessment period per customer served. The specified disturbances are those with a magnitude less than x for sags or a magnitude greater than x for swells. The definition of SARFIx is as follows:

$$SARFI_x = \frac{\sum N_i}{N_T}$$  \hspace{1cm} (1)

where

$x$: RMS voltage threshold with possible values of 140, 120, 110, 90, 80, 70, 50, and 10%. These eight values are not arbitrary and are chosen to coincide with the equipment response characteristics;

$N_i$: number of customers experiencing short-duration voltage deviations with magnitudes above $x\%$ for $x > 100\%$ or below $x\%$ for $x < 100\%$ due to event $i$;

$N_T$: number of customers served from the section of the system to be assessed.

It can be seen that the index is similar to the System Average Interruption Frequency Index (SAIFIx) used for reliability assessment, but it includes not only interruptions but also sags and swells. However, to determine the SARFIx is a much more difficult task since one must estimate the number of customers experiencing a specified level of voltage disturbances, unless all customers are equipped with power quality monitors.

In response to the need for feeder-level PQ performance characterization, this paper introduces the concept of sag state estimation and proposes an algorithm to estimate the sag levels for all customers in a feeder. Its primary application is to support the calculation of feeder-level PQ performance indices based on a limited number of metering points.

This paper is organized as follows. The metering technology available for sag state estimation is discussed in Section II, which establishes the scope of and constraints on the estimation technique to be developed. Section III presents the basic idea of the proposed sag state estimation method. The practical implementation issues of the proposed technique are analyzed in Section IV. A case study example is shown in Section V to demonstrate the characteristics of the proposed method. In Section VI conclusions are drawn.

II. CONCEPT OF VOLTAGE SAG STATE ESTIMATION

Any sag state estimation (SSE) technique must rely on existing or anticipated instrumentation technology. Unfortunately, the
technology choices available for SSE are limited due to cost constraints. It is not possible to install sag monitors at every node of a feeder. As a result, the problems faced by SSE are not the same as those of the traditional power system estimation where redundant metering data are available. In addition, the sag monitors are unlikely to be synchronized beyond their date/time stamps, which further complicates the problem.

The concept of SSE proposed in this paper is inspired and is made possible by the following trend in power distribution systems: advanced automatic meter reading (AMR) systems and outage mapping systems are now able to log sag disturbances. For example, the Turtle AMR system is capable to log the occurrence of voltage sags and their duration. The Sentry outage mapping system has a similar capability. The next generation of these devices could well possess the capability of recording the sag magnitudes as well.

Both types of systems have been widely used in the power distribution systems of USA and Canada. They share many common characteristics as far as SSE is concerned. These characteristics are as follows.

1) They record the instants of voltage sag occurrences to the accuracy of 1 s to 0.01 s.
2) They also record the duration of the voltage sag disturbances to the accuracy of 1 s to 0.01 s.
3) We can only expect that these devices record the sag magnitude. It would be too demanding for them to record current, sag waveforms, voltage phase information etc.
4) There is no redundancy in the device placement. In the case of outage mapping, not all customers are monitored using the devices.

The above conditions define the instrumentation platform available for sag estimation. It can be seen that the problems encountered here are quite different from those faced by the traditional state estimation techniques [9]. In fact, the traditional techniques are unable to solve such a sag estimation problem. There is a need to formulate the problem differently.

The sag state estimation problem formulation given in this paper estimates the voltage sag magnitudes for the unmetered customer points using the magnitudes obtained at a limited number of metering points. This problem is generally unsolvable if no additional information is available. Fortunately, there are two pieces of information that can make the problem solvable. One piece is that the system has a radial connection and other is that sags and interruptions are normally associated with faults in the system.

III. BASIC METHOD FOR SAG STATE ESTIMATION

The SSE problem can be best understood with the sample system shown in Fig. 1. \( U_0 \ldots U_6 \) are the metered voltages of the system. If there is a fault at point \( F \), the voltage profile of the feeder will have the form shown in Fig. 2. In this figure, the circles indicate the metered points and the corresponding voltage values. It can be seen that if we can determine the voltage profile curve using the metered values, it is possible to estimate the sag values at any point of the feeder. The problem of SSE can therefore be reformulated as the estimation of voltage profile along a feeder using a limited number of metered results.

It can be seen that this is essentially a curve-fitting problem. In order to improve the accuracy of fitting, we need to explore the characteristics of the voltage profile curve further. The fault behavior of a radial feeder tells us that the voltage profile follows approximately a two segment piecewise straight line as shown in Fig. 3, where the feeder length is normalized on the basis of feeder impedance values. As a result, the SSE problem can be stated as follows.

Given measured voltage values \( U_1, U_2, \ldots, U_N \), find parameters \( a, b, c \) and \( Z_f \), so that the voltage profile curve defined as

\[
U(Z) = \begin{cases} 
  aZ + b & Z < Z_f \\
  c & Z \geq Z_f 
\end{cases}
\]

(2)

Yields least estimation error for the following index:

\[
\varepsilon = \sum_{i=1,N} [(U(Z_i) - U_i)^2] = \min.
\]

(3)

A number of algorithms can be developed to solve the above problem. In this paper, we propose a step-by-step practical solution method for the problem. The method is described as follows.

Step 1) Compute the following slope parameters:

\[
S_{01} = \frac{U_0 - U_1}{Z_{01}}, \quad S_{12} = \frac{U_1 - U_2}{Z_{12}}, \quad \ldots, \quad S_{56} = \frac{U_5 - U_6}{Z_{56}}.
\]

According to (2), the slopes will take two possible values, one is \( a \) (for the upstream segment) and the other is 0 for the segment downstream of the fault point.

Step 2) A criterion is used to classify the slope values into two groups, one belongs to slope \( a \) and the other belongs to slope 0. This situation is shown in Fig. 4. Note that there is always one slope value that will reside between 0 and \( a \). This point is ignored.

Step 3) Parameters \( a \) and \( b \) are solved by using the least square estimation method on the data points belonging to the first group. Parameter \( c \) is also solved using the least square method. The result is that \( c \) is the average value of the voltages belonging to the s group.

Step 4) The intersection point of the two curve segments is the parameter \( Z_f \). In other words, \( Z_f \) is solved as

\[
Z_f = \frac{c - b}{a}.
\]

(4)

The simple radial feeder shown in Fig. 5 is used to demonstrate the above method. It is a part of the IEEE 123-bus (280 node) distribution test system [10]. The voltage profile of three-phase feeder is shown in Fig. 6 with a three-phase fault at point 52.
Fig. 1. Sample radial distribution system.

It can be seen that the voltage profile follows two segment straight lines. Parameters $a$, $b$, $c$ and $Z_f$ can be solved from the available nodal voltage data. The results are

$$U(Z) = \begin{cases} \frac{-100.2Z + 37.89}{Z < 0.228} \\ 0 & \text{if } Z \geq 0.228 \end{cases} \quad (5)$$

IV. PRACTICAL IMPLEMENTATION ISSUES

The basic method of SSE has been introduced and demonstrated using a three-phase fault case in the last section. In practice, however, most faults are unbalanced faults and distribution network topology is more complicated [11]. There is a need to expand the method to deal with more realistic situations. Such issues are addressed in this section.

A. Fault Path Search

For general radial systems, the basic SSE method should be applied to the fault path. A fault path is defined as the path linking the fault point and the feeder-sending end (see Fig. 7). Accordingly, it is necessary to search for the fault path first. The following method is proposed for fault path search.

1) The search starts at the feeder-sending end. This node is called the root node.

2) Examine the voltages at the nodes that have a direct connection to the root node. The node having the lowest voltage is the one considered to be on the fault path.

3) This node is renamed as the new root node and the search goes back to step 2).

4) If no more voltage reduction is found in the search process, the fault path search is completed.

The search process for the system of Fig. 7 is shown in Fig. 8. The above procedure will eventually trace to the fault location. The fault path forms the one line distribution system required by the SSE method. Parameters $a$, $b$, $c$ and $Z_f$ are then determined
for the fault path, which in turn will enable one to calculate any (unmetered) nodal voltages along the fault path. The voltages of the nodes that are not in the fault path are essentially equal to the voltages at the nodes where they are branched off from the fault path.

B. Effect of Fault Impedance

The fault impedance has a significant impact on the straight-line voltage profile assumption indicated in (2). In fact, the voltage profile as a function of the line impedance is no longer linear in this case. This can be understood by considering the voltage at node \( x \) that is \( k \) kilometers upstream of the fault location

\[
U_x = U_F + kZ_{\text{per km}}I_F
\]

(6)

where \( U_x \) is the voltage phasor at bus \( x \), \( U_F \) is the fault bus voltage phasor, \( Z_{\text{per km}} \) is the line impedance per kilometer, \( I_F \) is the fault current phasor. The magnitude of \( U_x \) has the following form:

\[
|U_x| = \sqrt{(U_{FR} + k \Delta U_R)^2 + (U_{FI} + k \Delta U_I)^2}
\]

(7)

where \( U_F = U_{FR} + jU_{FI} \) and \( Z_{\text{per km}}I_F = \Delta U_R + j\Delta U_I \). It can be seen that \( |U_x| \) is a nonlinear equation of \( k \). As a result, the voltage profile is not a straight line. Fig. 9 shows the voltage profile for a fault at bus 52 with different fault impedance.

A method to solve the above problem is to introduce a new variable \( |U_x|^2 \). This variable is the 2nd-order function of the fault distance \( k \). The profile of \( |U_x|^2 \) for the faults described in Fig. 9 is shown in Fig. 10. Also plotted in this figure is the 2nd-order polynomial fitting of the squared voltage. It can be seen that there is a good match between the two curves. As a result, the following improved voltage profile fitting method is proposed.

Given measured voltage values \( U_1, U_2, \ldots, U_N \), find parameters \( a, b, c, d \) and \( Z_f \), so that the voltage profile curve defined as

\[
V(Z) = U(Z)^2 = \begin{cases} 
    aZ^2 + bZ + c & Z < Z_f \\
    d & Z \geq Z_f
\end{cases}
\]

(8)

Yields least estimation error for the following index:

\[
\varepsilon = \sum_{i=1}^{n} [V(Z_i) - U_i^2]^2 = \min.
\]

(9)

C. Unbalanced Fault

The SSE method is applied to each phase of the system regardless the fault type. For example, all metered voltages of phase A will be used to estimate the voltage profile of phase A. In order to understand this approach more clearly, the fault path voltage profiles for the system in Fig. 5 are shown in Figs. 11–13.

It can be seen that voltage profile of each phase all follows a linear profile if the fault point voltage is zero or a 2nd-order profile (for \( U^2 \)) if there is a fault resistance or it is a phase-to-phase fault. As a result, the proposed method can be applied to each phase of the feeder independently. Furthermore, there is no need to determine the type of faults before applying the sag state estimation algorithm. The phases experiencing voltage swell can also be estimated similarly.

D. Transformer Connection

The metered voltages are sary voltages. We need to estimate the primary voltage for voltage profile estimation. If a
single-phase transformer is involved, the primary voltage at the transformer location can be considered to be equal to the meter reading. If a three-phase transformer is involved, it is generally not possible to determine the primary voltage magnitudes based on the sary voltage magnitudes if the transformer is not connected [12]. There are two methods to deal with this problem. One is to omit this metering point. Another is to record both the magnitudes and phases of the sary voltages and then derive the primary voltages based on the results. This method is doable since most three phase loads are large and their revenue meters have the capability to record voltage phasors.

V. CASE STUDIES

A simplified one-line diagram of the system used as a case example is shown in Fig. 14. It is a part of the IEEE 123-bus (280 node) distribution test system [10]. Table I presents the voltage magnitudes of the metered nodes based on 120 V.

Voltage profile of phase A fluctuates lightly, but voltages of phase B and C all drop down from the root node to node 8, so either phase of B and C can be chosen as a search objective phase, and fault path is shown as

\[ 150 \rightarrow 1 \rightarrow 7 \rightarrow 8 \rightarrow 13 \rightarrow 18 \rightarrow 135 \rightarrow 35 \rightarrow 40 \rightarrow 42 \rightarrow 44 \rightarrow 47 \rightarrow 48 \text{ (or 49)} \]

where the metered node is marked with underline.
The approximation yields more estimation error. As a result, the voltage fitting curve of phase B and C is estimated as

\[ V_B(Z) = \begin{cases} 
9094.9Z^2 - 12103Z + 7387.2 & Z < 0.549 \\
3064.92 & Z \geq 0.549
\end{cases} \quad (10) \]

\[ V_C(Z) = \begin{cases} 
10148Z^2 - 10133Z + 6160.2 & Z < 0.557 \\
3064.92 & Z \geq 0.557
\end{cases} \quad (11) \]

It can be seen that the slopes cannot be divided into two groups with equal values. Similarly the situation occurs for phase C, so sag state estimation with a 2nd-order polynomial fitting for \( U^2 \) of phases B and C is applied. The profile of \( |U_x|^2 \) and the 2nd-order polynomial fitting for phase B and C is shown in Figs. 15 and 16. Also plotted in the figure is the linear approximation curve.

It can be seen that there is a better match between the 2nd order polynomial fitting and the voltage profile. Linear approximation yields more estimation error. As a result, the voltage profile curve of phase B and C is estimated as

Voltages of the unmetered nodes for phase A can be estimated with the linear least square method. They equal to the average value of the metered voltages. Then unmetered voltages of system can all be estimated and results are shown and are compared with actual values in Table II. It was found that the sag state estimation error is less than 0.35%.

VI. CONCLUSION

The concept of “voltage sag state estimation” and associated solution algorithm has been presented in this paper. The ob-
jective is to estimate the voltage profiles of a feeder based on limited number of metering points. This is achieved by making use of the radial connection characteristic of a distribution feeder and by estimating the voltage sag profile along the fault path. This paper has discussed various implementation issues and their solutions. Case study results have shown that the proposed method has good performance for the simulated cases. The method can be used for calculating feeder power quality performance indices such as SARFmix. It is also helpful for distribution engineers to determine the power quality impact of a particular disturbance on a power distribution system.

The ideas and results presented in this paper can only be considered as an early attempt to solve a complex technical problem. They are far from implementation. We hope that this paper will raise the awareness on the need for sag state estimation research and will serve as a step stone for other researchers. The following are some of the topics requiring further investigations.

- **Improvement of the method for more realistic situations.** The induction motor loads are expected to contribute to the fault current, which will make the feeder voltage profile much more complicated. Load currents are likely affect the profile of nonfault caused sags. The method assumes that the sags have reached steady state. This may not be true for sags lasting less than two to three cycles. All these are challenging problems that must be addressed.

- **Refinement of the algorithm for robustness and efficiency.** What has been presented is only one of many possible algorithms to solve the SSE problem. The idea such as fault path search is based on intuitive understanding. One can also cast the SSE problem in a more complete mathematical framework and develop rigorous solution algorithms.

- **Verification of the proposed SSE formulation and solution method using field data.** The impact of metering errors needs to be analyzed as well.

- **Other research subjects.** A further evolvement of SSE research could include 1) the study on optimal meter placement and bad data detection, 2) the improvement of existing metering technology to cater for SSE, and 3) new applications of the sag state estimation results.

### REFERENCES


