Status and Future Directions of Power System Harmonic Analysis

Wilsun Xu, Senior Member, IEEE

Abstract--This paper provides a concise review on the main developments and conclusions in the area of power system harmonic analysis. Commonly accepted methods for conducting harmonic studies have been summarized. The area of harmonic analysis is still a very fertile ground for exploration. With the help of four example research topics, the paper also demonstrates possible future directions of power system harmonic analysis research.

Index Terms--Harmonics, Harmonic Analysis

I. INTRODUCTION

Power system harmonic analysis is to determine the impact of harmonic producing loads on a power system. Harmonic analysis has been widely used for system planning, operation criteria development, equipment design, troubleshooting, verification of standard compliance, and so on. Over the past two decades, significant efforts and progresses have been made in the area of power system harmonic analysis. Well-accepted component models, simulation methods and analysis procedures for conducting harmonic studies have been established. Harmonic studies are becoming an important component of power system analysis and design.

With the widespread use of digital computers, computer simulation has become the preferred method to conduct harmonic analysis. This has transformed the subject of harmonic analysis into two main areas: computer modeling of power systems for harmonic analysis and computer simulation of harmonic propagation in power systems. Research work in the above two areas has resulted in a number of software packages for harmonic analysis and simulation. It is important to note, however, that harmonic analysis still requires a good knowledge on the subject area and the skill of analytical thinking.

One of the purposes of this paper is to review the status of computer-aided harmonic analysis methods. The most commonly accepted techniques for harmonics modeling and simulation are discussed. The readers will find from this review that the research and application of harmonic analysis have entered a relatively mature stage. This naturally leads to the following question: what are the future directions of harmonic analysis? In the second part of this paper, some thoughts to this question are presented using examples of research work conducted at the University of Alberta and other places. It is hoped that these examples or thoughts will serve as a step stone for continued research efforts in this area. For people who are interested in applying harmonic analysis techniques, this paper could serve as a channel to obtain feedback from industry.

II. MODELING OF POWER SYSTEM COMPONENTS FOR HARMONIC ANALYSIS

Harmonics are a high frequency phenomenon involving frequencies from 50/60Hz to about 3000Hz. For harmonic studies, it is important to take into account the component response characteristics at the harmonic frequency range. Since most harmonic analysis tools are based on frequency domain algorithms, component models for harmonic analysis normally takes the frequency domain form as well. A detailed summary on this topic can be found from reference [1] and other papers presented in this panel session. In this section, we will focus on the development status of component modeling research.

Overhead Lines and Underground Cables: There is a consensus that lines and cables can be modeled with a multiphase coupled equivalent pi-circuit. For balanced harmonic analysis, the model can be further simplified into a single-phase pi-circuit determined from the positive sequence impedance data of the component. It is important to include the shunt element and its associated long-line effect in the model. The shunt admittance of the component, though small at the fundamental frequency, can become quite significant at higher frequencies. This effect can be easily represented using the exact or equivalent pi-circuit model [1].

Transformers: A transformer affects harmonic flows through its series impedance, winding connection, and magnetizing branch. Many research results in this topic have concluded that a transformer can be modeled using its short-circuit impedance for most harmonic studies. It is also important to include the transformer phase-shift effect. This phase-shifting effect could lead to significant harmonic cancellations in a system [2]. The magnetizing branch of a transformer is a harmonic source. Inclusion of the saturation characteristic of the branch is important only when the harmonics generated by a transformer are of primary concern.

Rotating Machines: This component includes synchronous and induction machines. The consensus is that a machine can be represented using its short-circuit impedance for most harmonic analysis tasks, although more complex models are available for advanced applications.

W. Xu is with the Department of Electrical and Computer Engineering, University of Alberta, Canada. He is also an adjunct professor of Shandong University, China. (e-mail: wxu@ee.ualberta.ca).
**Aggregate Loads**: Aggregate loads refer to a group of load buses that are treated as one component in harmonic analysis. Typical aggregate loads are distribution feeders seen from a substation bus or a customer plant seen at the point of common coupling. Although such loads typically contain harmonic sources, the main concern is the frequency response characteristics of its equivalent impedance. If the harmonic sources are of concern, the load should not be treated as an aggregate one.

The model for aggregate loads, therefore, has the form of a frequency-dependent impedance. Research has shown that the impedance is not only a function of the individual loads contained in the component but also dependent on the lines or cables connecting the loads. For example, the distribution feeder conductors and shunt capacitors can have a larger impact on the frequency-dependent impedance seen at the feeder terminal than those of the loads connected to the feeder. As a result, it is almost impossible to use a set of general formulas to construct an adequate impedance model for aggregate loads. Although a few models for aggregate loads have been proposed in the past, the validity of these models has not been fully verified. Modeling of aggregate loads is one of the weakest areas in the harmonics modeling field. A lot of research work is still needed. In author’s opinion, solutions to the following topics would be valuable:

- Verified techniques to construct aggregate load models.
- The impact of model inaccuracy on harmonic analysis results.
- Measured harmonic impedance data for aggregate loads and analysis on the correlation of the measured data with the load composition.

**External System**: External system for power quality analysis typically refers to either the utility supply system seen at the point of common coupling from a customer's perspective or the neighboring networks of a utility system under study. Reference [1] has more information on this subject.

**Power Electronic Devices**: Power electronic devices can be a load or a compensator. In terms of harmonic analysis, a common characteristic of them is that they are harmonic sources. As a result, a number of different models have been proposed to represent such devices. Among these models, the most commonly accepted one is the current source model. The model simply treats a power electronic device as a harmonic current source. The magnitude and phase of the source can be calculated, for example, from the typical harmonic current spectrum of the device. A more accurate procedure to establish the current source model is as follows:

1) The power electronic load device is treated as a PQ load at the fundamental frequency, and the fundamental frequency power flow of the system is determined;
2) The current injected from the load to the system is then calculated and is denoted as \( I_{h} \);
3) The magnitude of the harmonic current source representing the load can be determined as follows:

\[
I_h = \frac{I_{h\text{-spectrum}}}{I_{1\text{-spectrum}}}
\] (1)

4) The phase angle of the harmonic current source can be calculated using the following formula:

\[
\theta_h = \theta_{h\text{-spectrum}} + \frac{\theta_f}{\theta_{1\text{-spectrum}}}
\] (2)

where subscript “spectrum” standards for the typical harmonic current spectrum of the load. This data can be measured, obtained from manufacturers, or calculated according to formulas. The current source model is the most common one used in commercial power system harmonic analysis programs. Its main disadvantage is the use of typical harmonic spectrum and, as such, assessment of cases involving non-typical operating conditions becomes difficult. This has prompted the development of other modeling techniques such as detailed time-simulation based models and a hybrid of current source model and the detailed model.

It is reasonable to say that power electronic device modeling techniques for harmonic calculations have become a mature subject. What is needed is to improve our understanding on the harmonic characteristics of the devices for other applications. For example, harmonic sources are commonly treated as current sources for harmonic source detection applications. There is an urgent need to determine the degree of validity of this assumption. Another case involves the generation of interharmonics from variable frequency drives. As will be demonstrated later, the traditional models are insufficient to analyze some aspects of the interharmonics problems.

**III. Network Solution Techniques for Harmonic Analysis**

Over the past two decades, considerable progress has been made in the area of computing harmonic power flows for a power system. Mature techniques are now available for assessing harmonic distortions in a network containing significant harmonic sources. In this section, two most common and useful harmonic analysis techniques are presented and discussed.

**A. Frequency Scan Analysis**

Frequency scan is the simplest and most commonly used technique for harmonic analysis [3]. The input data requirement is minimal. It calculates the frequency response of a network seen at a particular bus or node. Typically, an one per unit sinusoidal current is injected into the bus of interest and the voltage response is calculated. This calculation is repeated using discrete frequency steps throughout the range of interest. Mathematically, the process is to solve the following network equation at frequency \( f \):

\[
[Y_f][V_f] = [I_f]
\] (3)

where \([I_f]\) is the known current vector and \([V_f]\) is the nodal voltage vector to be solved. In a typical frequency scan analysis, only one entry of \([I_f]\) is nonzero. In other analysis, a set of positive or zero sequence currents may be injected into three phases of a bus respectively. The results are the positive
or zero sequence driving-point impedance of the network. Frequency scan analysis is the most effective tool to detect harmonic resonance conditions in a system. It has also been widely used for filter design.

B. Harmonic Power Flow Analysis

If one needs to find the harmonic distortion levels for certain operating conditions, a network harmonic power flow analysis should be conducted. Several harmonic power flow techniques have been proposed in the past [4], [5]. Many years of application experiences have shown that a non-iterative technique that represents harmonic sources as current sources is sufficient for many common harmonic analysis tasks. This technique is summarized as follows:

Step 1: Compute the fundamental frequency power flow of the network. The results define the operating scenario for harmonic analysis. In this step, the harmonic-producing loads are modeled as constant power loads.

Step 2: Determine the harmonic current source models for the harmonic-producing loads. The model consists of the magnitudes and phase angles of current source at various harmonic frequencies. Equations needed to establish the model are described in Section II, Equations (1) and (2).

Step 3: Calculate the network harmonic voltages and currents by solving the following network nodal equation for harmonic number $h$ of interest:

\[
[Y_h][V_h] = [I_h]
\]

(4)

where $[I_h]$ is a known vector that has all harmonic current sources included, and $[V_h]$ is the nodal voltage vector to be solved. Equation (4) is solved for all harmonics interested.

Step 4: The results of Steps 1 and 3 jointly define the harmonic power flow conditions of the study system. Harmonic indices such as total harmonic distortions and transformer k factors can be calculated from the results.

The main disadvantage of the above method is the use of typical harmonic spectra to represent harmonic-producing devices. It prevents an adequate assessment of cases involving non-typical operating conditions. Such conditions include, for example, partial loading of harmonic-producing devices, excessive harmonic voltage distortions and unbalanced network conditions. The applications of harmonic power flow analysis include 1) network harmonic distortion assessment, 2) harmonic limit compliance verification, 3) filter performance evaluation, and 4) equipment de-rating calculation.

C. Comments on Network Harmonic Analysis Techniques

The development of harmonic analysis techniques followed mainly two directions over the past many years. One direction is to improve solution algorithms for harmonic power flow calculation. Examples of this direction are the iterative harmonic method [4] and the Newton-Raphson based solution method [5]. The second direction is to include more complete network models in the solution. A good example of this direction is the three-phase or multiphase harmonic analysis [6]. This direction is needed since certain harmonic problems such as harmonic-telephone interference, must be analyzed in three phase.

For both directions, the focus has been on the analysis of networks with dominant harmonic sources. This is a reflection of the industry situation in the past, where harmonic-producing loads are large and are concentrated in a few limited locations. With the proliferation of power electronic loads, a power system may contain many harmonic sources with comparable sizes. How to analyze such systems has become one of the challenges in harmonic analysis research.

IV. Future Directions of Harmonic Analysis

It is always dangerous to predict research directions for the future. Taking into account the fact that there is only a single author for this paper and the author’s knowledge on the subject is limited, possibilities of error are high. Instead of predicting the future, what we attempt to do in this section is to illustrate some interesting research topics in the area of harmonic analysis. It is hoped that the examples or thoughts presented will serve as a step stone for continued research efforts in the area.

A. Distributed Harmonic Sources

A noticeable trend in power systems nowadays is the emergence of distributed harmonic-producing loads. These loads typically have comparable sizes and are distributed all over an electric network. Traditional techniques for harmonic power flow analysis generally have difficulties in determining the collective impact of these sources. Determination of harmonic distortions for systems with distributed sources is a new and challenging research topic in the area of harmonic analysis. In order to make the problem manageable, some researchers have sub-divided the problems into two. One assumes that the harmonic-producing loads are deterministic and the other assumes the loads vary randomly.

The first sub-problem can be described as the following: a distribution system has a number of harmonic-producing loads of comparable sizes. The loads operate at a constant load level. An office building with a lot of PCs is a typical example of such a case. It is required to determine the harmonic distortion level for this type of systems. Traditional methods to solve this problem cannot take into account the diversity and attenuation effects of the harmonic sources [7]. Consequently, the results can be quite conservative, since both effects help to reduce the over all harmonic distortion level in the system. Although several papers have been published on this subject [7], [8], almost no method has been proposed to estimate the harmonic distortion levels for such systems.

At the University of Alberta, we attempt to solve this problem by enabling the traditional methods to include the diversity and attenuation effects. One attractive idea is based on the observation that each type of harmonic-producing loads has unique characteristic curves representing its harmonic diversity and attenuation effects. Fig. 1 shows an example curve for PC power supplies. As a result, one could use the following generic iterative method to analyze systems with distributed harmonic sources:
1) Determine attenuation and diversity characteristic curves by using, for example, measurements for the loads to be studied. The relationship can be represented using the following equations:

\[ I_h = \alpha_h(VTHD), \quad \theta_h = \beta_h(VTHD) \] (5)

The above equations state that the voltage THD at the terminal of the harmonic-producing load will affect the spectrum of the harmonic current produced by the load.

2) The harmonic power flow results are first obtained using the traditional methods. The bus voltage THDs are determined from the results.

3) Using the THD results, the harmonic current spectrum of the sources are adjusted according to the characteristics curves defined in (5): This adjustment takes into account the attenuation and diversity effects.

4) Using the adjusted harmonic current spectrum data, the system harmonic power flow is re-solved. This provides improved harmonic distortion results. The bus voltage THDs are determined.

5) With the updated THD values, the calculation is redirected to Step 2). This iterative procedure stops when there is no significant change on the bus voltage THD level.

Sample results for a system multiple personal computers are shown in Fig. 2. It compares the results obtained by the proposed method, the traditional method, and the measured results. The x-axis represents different sizes of the supply system impedance. The results show that the traditional methods are very conservative and the proposed method is significant more accurate than the traditional methods.

Verification of the proposed methods has been conducted using test systems. The results are compared with those obtained by Monte Carlo simulation. Three sets of results were obtained. The results labeled as “Monte Carlo” are obtained from Monte Carlo simulation without any approximations. They can therefore be considered as the “correct” results. The results labeled as “Normal Distribution” are obtained by assuming they follow a Normal distribution. Parameters of the distribution (mean value and standard deviation) are extracted from the Monte Carlo simulation results. A good agreement of these results with the "Monte Carlo" results confirms the validity of the normal distribution. Parameters of the distribution (mean value and standard deviation) are extracted from the Monte Carlo simulation results. A good agreement of these results with the "Monte Carlo" results confirms the validity of the normal distribution. Parameters of the distribution (mean value and standard deviation) are extracted from the Monte Carlo simulation results. A good agreement of these results with the "Monte Carlo" results confirms the validity of the normal distribution.

Fig. 1. Characterization of the attenuation effects for single-phase power supply.

B. Distributed Random Harmonic Sources

A more difficult subject is to consider the random variation of the loads in addition to its distributed nature. A typical case can be described as the following: a distribution system has a number of harmonic-producing loads of comparable sizes. The loads vary randomly from \( P_i \text{min} \) to \( P_i \text{max} \), where subscript “i” is the bus number of a particular load. How can one determine the mean value and the 95% probability value of the harmonic distortion levels in the system? The 95% probability level is defined as the THD or IHD values that could be exceeded only for 5% probability. A representative case for this problem is a distribution system with DC drive powered ski lifts. For the sake of simplicity, let’s omit the diversity and attenuation effects at present stage of analysis.

A lot of work has been done to establish methods that can characterize the total impact of randomly operating harmonic sources [9]. A consensus on the subject has been reached recently [10]. It is generally agreed that if there are many harmonic sources in a system the net impact follows the normal distribution. Since a normal distribution can be characterized by two parameters, mean value and standard deviation, the problem becomes how to find the two parameters for a given set of load variation patterns. Several methods, such as Monte Carlo simulation, have been proposed to find the parameters [9-14]. The University of Alberta has also investigated this problem. Our approach is not to develop a brand new method, as its practical application could be limited due to the need for the users to learn another computer program. We try to use existing deterministic power flow methods to estimate answers with reasonable accuracy.

Verification of the proposed methods has been conducted using test systems. The results are compared with those obtained by Monte Carlo simulation. Three sets of results were obtained. The results labeled as “Monte Carlo” are obtained from Monte Carlo simulation without any approximations. They can therefore be considered as the “correct” results. The results labeled as “Normal Distribution” are obtained by assuming they follow a Normal distribution. Parameters of the distribution (mean value and standard deviation) are extracted from the Monte Carlo simulation results. A good agreement of these results with the "Monte Carlo” results confirms the validity of the normal distribution. Parameters of the distribution (mean value and standard deviation) are extracted from the Monte Carlo simulation results. A good agreement of these results with the "Monte Carlo” results confirms the validity of the normal distribution. Parameters of the distribution (mean value and standard deviation) are extracted from the Monte Carlo simulation results. A good agreement of these results with the "Monte Carlo” results confirms the validity of the normal distribution. Parameters of the distribution (mean value and standard deviation) are extracted from the Monte Carlo simulation results. A good agreement of these results with the "Monte Carlo” results confirms the validity of the normal distribution. Parameters of the distribution (mean value and standard deviation) are extracted from the Monte Carlo simulation results. A good agreement of these results with the "Monte Carlo” results confirms the validity of the normal distribution.
C. Assessment of Interharmonics

Inter-harmonics are those spectra components whose frequencies are not integer multiples of the fundamental frequency. In recent years, interharmonics have gained some attention because they can cause voltage flicker. A voltage flicker can be viewed as the modulation of the peak (or RMS) value of a voltage waveform. The inter-harmonics can beat with the fundamental frequency component to cause modulated peak voltage. Fig. 4 shows the waveform composed by a 175 Hz component and a 60Hz component. It can be seen that the waveform magnitude beats at about 5 Hz.

Fig. 4. Voltage flicker waveform caused by a 175Hz inter-harmonic.

Variable frequency drives, particularly the current source inverter based and the voltage source inverter based drives, can be a source of interharmonics. The drive can be considered as a frequency converter. It converts a 60Hz input into an output with a different frequency. Technically speaking, the system can be considered as bi-directional, meaning the motor side voltages and currents could also be converted to the supply system side (Fig. 5). For example, if the motor is running at 50Hz, this frequency will be seen at the DC link as 50Hz ripples. As a result, the current of the DC link actually contains both 60Hz ripples (due to supply voltage) and 50Hz ripples (due to motor voltage). Since the supply side current is related to the DC link current through the converter, the 50Hz ripples will penetrate into the supply side and present themselves as inter-harmonics (since 50Hz is not an integer multiplier of 60Hz). Reference [15] provides an excellent description on this phenomenon. The inter-harmonic components are normally very small and they vary with the drive output frequency. If their frequencies coincide with the system resonance frequency, the inter-harmonics can be amplified. Problems could also arise if the drive produces excessive inter-harmonics due to poor design or aggressive cost cutting.

Although several papers have been published on the subject of predicting interharmonics for such drive systems, a lot more work is still needed. One topic of good interest to industry and research communities is to develop an adequate inter-harmonic model for such a drive system. This is a difficult task due to the following reasons:

1) The inter-harmonics cannot be modeled as a pure current source. This is because the inverter produced DC-link ripple is voltage ripple (assuming the inverter side has a strong AC source to simplify this analysis). The current ripple is therefore a function of the impedances of the DC link and the supply system. If the inverter side is a weak system, the impedance of the inverter side could also play a role.

2) The frequency and magnitude of the inter-harmonics vary with the drive operating condition. As a result, the analysis of the impact of interharmonics have to cover a wide range of operating conditions.

These research results can be very valuable to the analysis of drive-caused voltage flicker problems and the design of mitigation measures to such problems.

D. Advanced analysis of harmonic resonance

Although the phenomenon of harmonic resonance is well known, tools available to analyze the phenomenon are very limited. Frequency scan analysis can normally identify the existence of resonance. It is more important to know which bus can excite the resonance more easily and which power system components play more significant role for the resonance. One approach that has the potential to answer these questions is to analyze the eigen-structure of the system admittance matrix $Y_f$ as shown in (3).

Imagine a system experiences a sharp parallel resonance at frequency $f$ according to the frequency scan analysis. It implies that some elements of the voltage vector of (3) have large values at $f$. This in turn implies that the inverse of the $[Y_f]$ matrix has large elements. This phenomenon, in turn, is primarily caused by the fact that one of the eigen-values of the $Y$ matrix is close to zero. In fact, if the system had no damping, the $Y$ matrix would become singular due to one of its eigenvalues becomes zero. The above reasoning leads us to believe that the characteristics of the smallest eigenvalue of the $[Y_f]$ matrix could contain useful information about the cause of the harmonic resonance. The above analysis can be formally stated as follows:

Let

$$Y = ??T = T^{-1}??T$$
be the eigen-decomposition of the $Y$ matrix at frequency $f$. $\Psi$ and $T$ are the left and right eigenvector matrices of $Y$ (the subscript $f$ is omitted here to simplify notations) and $\Lambda$ is the corresponding eigenvalue matrix. $\Psi = T^{-1}$ is due to the fact that $Y$ is 'block' symmetric. Define

$$I_m = TI$$
$$V_m = TV$$

(6)

as the modal current and voltage vectors respectively. It can be seen that the original frequency scan equation has been transformed into the following form:

$$
\begin{bmatrix}
V_{m1} \\
V_{m2} \\
\vdots \\
V_{m3}
\end{bmatrix} =
\begin{bmatrix}
\lambda_1^{-1} & 0 & 0 & 0 \\
0 & \lambda_2^{-1} & 0 & 0 \\
0 & 0 & \ldots & 0 \\
0 & 0 & 0 & \lambda_n^{-1}
\end{bmatrix}
\begin{bmatrix}
I_{m1} \\
I_{m2} \\
\vdots \\
I_{m3}
\end{bmatrix}
$$

(7)

With the above equation, one can easily see that if $\lambda_i = 0$ or is very small, a small injection of mode 1 current will lead to a large mode 1 voltage. On the other hand, the other modal voltages will not be affected since they have no 'coupling' with mode 1 current. In other words, one can easily identify the 'locations' of resonance in the modal domain. After identifying the critical mode of resonance, it is easy to find the 'participation' of each bus in the resonance. This can be done using the well-known participation factor theory [16]. Fig. 6 shows a sample analysis on an IEEE 14 bus harmonic test system. The size of the circle shows the degree of participation of each bus to the resonance occurred at $f=10.3$. It can be seen from the figure that this particular resonance condition mainly involves buses 1, 2 and 5. At the University of Alberta, we have studied harmonic resonance using modal analysis for sometime and a lot of very interesting results have been obtained. In this paper, we can only present very limited information due to space limitation.

V. CONCLUSIONS

In this paper, we have reviewed some significant progresses in the area of power system harmonic analysis. Commonly accepted methods for conducting harmonic studies have been summarized. The area of harmonic analysis is still a very fertile ground for exploration. With the help of four example research topics, we attempt to demonstrate possible future directions of harmonic analysis research. The conclusion is that we need to approach the subject from a wider perspective. It is important to remember that there are still many problems remaining to be solved. The most difficult part facing us is how to extract the problems into a form that can be researched with clear directions.

VI. ACKNOWLEDGMENT

The sample research works presented in this paper are supported by the Natural Science and Engineering Research Council of Canada and other granting agencies. Some figures are provided by my graduate students.

VII. REFERENCES


VIII. BIOGRAPHIES

Wilsun Xu (M'90, SM'95) received Ph.D. from the University of British Columbia, Vancouver, Canada in 1989. He worked in BC Hydro from 1990 to 1996 as an electrical engineer. Dr. Xu is presently a professor at the University of Alberta and an adjunct professor of the Shandong University of China. His main research interests are harmonics and power quality. He can be reached at wxu@ee.ualberta.ca.