Two-Stage Based Design for Phased-MIMO Radar With Improved Coherent Transmit Processing Gain

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Abstract—We consider the problem of two-dimensional (2D) transmit beamforming design for phased-MIMO Radar with a limited number of transmit power amplifiers. Subarray partitioning is used in MIMO radar where individual subarrays operate in a phased-array mode leading to a reduction in the number of power amplifiers required. However, the use of subarray partitioning results in poor transmit beampattern characteristics due to the reduced physical aperture of the subarrays as compared to the aperture of the full transmit array. To address this problem, we introduce a new method for achieving a desired transmit beampattern while applying the concept of phased-MIMO radar. Our design consists of two cascaded stages where the first stage involves mapping a set of finite number of orthogonal waveforms into another set of cross-correlated waveforms using a linear mixing operator. The second stage involves partitioning the transmit array into a finite number of transmit subarrays where each subarray is used to radiate one of the cross-correlated waveforms in phased-array mode. The mixing matrix used in the first stage is appropriately designed to ensure that the overall transmit beampattern, i.e., the summation of all beampatterns of the individual subarrays, is as close as possible to a desired transmit beampattern. The number of power amplifiers required is finite and equals to the number of subarrays. One of the advantages of the new method is that it can achieve coherent transmit gain that is comparable to the coherent transmit gain of a phased-array radar while implementing the concept of MIMO radar. Simulation examples are used to validate the proposed method capabilities.

Index Terms—Phased-MIMO radar, transmit coherent processing gain, transmit beamforming, transmit power amplifier.

I. INTRODUCTION

The essence of the emerging concept of multiple-input-multiple-output (MIMO) radar [1]– [5] is to transmit multiple orthogonal waveforms using multiple transmit colocated (or widely separated) antennas and to jointly process the received echoes due to all transmitted waveforms. As compared to the phased-array radar, the use of MIMO radar with colocated antennas enables improving angular resolution, increasing the upper limit on the number of detectable targets, improving parameter identifiability, extending the array aperture by virtual sensors, and enhancing the flexibility for transmit/receive beampattern design [3]–[7]. However, MIMO radar with colocated transmit antennas suffers from the loss of coherent transmit processing gain as a result of omnidirectional transmission of orthogonal waveforms [3]. To address the latter problem, various solutions have been reported in the literature such as the phased-MIMO radar technique [6], the transmit energy focusing method [7], and other transmit beamforming methods [8]–[10]. However, the aforementioned methods are primarily developed for the case of MIMO radar with one-dimensional transmit arrays.

Motivated by the great practical interest in two-dimensional (2D) transmit arrays, the idea of 2D transmit beamforming design has been reported in [11]–[13]. The number of transmit elements used in 2D arrays is typically large and can range from several dozens to few thousands. Applying the concept of MIMO radar in its conventional form to large arrays requires feeding every transmit antenna with an independent waveform. In such a case, each transmit antenna is required to have its own transmit power amplifier. This dramatically increases the cost required to build a practical MIMO radar system with large transmit arrays. One way to reduce the cost is to use sparse 2D transmit arrays. The use of carefully designed sparse 2D arrays also enables realizing dual-band radar systems where antennas that operate within one frequency band are not allowed to co-exist with antennas that operate within the other frequency band. Another way to reduce the number of required transmit power amplifiers is to use subarray partitioning, e.g., the phased-MIMO radar concept [6]. However, the use of subarray partitioning results in poor transmit beampattern characteristics due to the reduced physical aperture of the subarrays as compared to the aperture of the full array.

In this paper, we address the latter problem and develop a new method for achieving a desired transmit beampattern while applying the concept of phased-MIMO radar. We introduce a new method for designing phased-MIMO radar which consists of two cascaded stages. At the first stage a set of finite number of orthogonal waveforms is mapped into another set of cross-correlated waveforms using a linear mixing operator. The second stage involves partitioning the transmit array into a finite number of transmit subarrays where each subarray is used to radiate one of the cross-correlated waveforms in phased-array mode. It is also possible to design a transmit beamforming vector for each subarray if the user requires...
to illuminate a certain desired 2D spatial sector, i.e., if a wider transmit beam is required. Our new method can be applied to full uniform rectangular transmit arrays as well as sparse 2D transmit arrays. We appropriately design a mixing matrix that maps the set of orthogonal waveforms into a set of cross-correlated ones while ensuring that the overall transmit beampattern, i.e., the summation of all beampatterns of the individual subarrays, is as close as possible to a desired transmit beampattern. The number of power amplifiers required is finite and equals to the number of transmit subarrays. In addition to achieving waveform diversity, our new method achieves an overall transmit beampattern that is as narrow as the transmit beampattern of phased-array radar, i.e., it can achieve coherent transmit gain that is comparable to the coherent transmit gain of phased-array radar. Simulation examples are used to validate the effectiveness and capabilities of the proposed method.

II. MIMO Radar Signal Model

Consider a mono-static radar system with a sparse planar transmit array. The transmit antenna elements are assumed to be located on a uniform rectangular grid of size $M \times N$, where $M$ is the number of points on a given column and $N$ is the number of points on a given row. The grid points on any given column are assumed to be equally spaced with displacement $d_x$. Similarly, the grid points on any given row are assumed to be equally spaced with displacement $d_y$. Let $Z$ be an $M \times N$ matrix of ones and zeros where the $mn$-th entry equals one if a transmit antenna is located at the $mn$-th point on the grid. A zero entry in $Z$ denoted the absence of a transmit antenna at the corresponding point on the grid.

Let the $M \times N \times 1$ steering vector of the transmit array be represented as

$$ a(\theta, \phi) = \text{vec} \left( Z \odot [u(\theta, \phi)v^T(\theta, \phi)] \right) $$

where $\text{vec} \cdot$ stands for the operator that stacks the columns of a matrix in one column vector, $(\cdot)^T$ denotes the transpose, $\odot$ stands for the Hadamard product, $\theta$ and $\phi$ denote the elevation and azimuth angles, respectively, and $u(\theta, \phi)$ and $v(\theta, \phi)$ are vectors of dimensions $M \times 1$ and $N \times 1$, respectively, that are defined as follows

$$ u(\theta, \phi) = \left[ 1, e^{j2\pi d_x \sin \theta \cos \phi}, \ldots, e^{j2\pi(M-1)d_x \sin \theta \cos \phi} \right]^T $$

$$ v(\theta, \phi) = \left[ 1, e^{j2\pi d_y \sin \theta \sin \phi}, \ldots, e^{j2\pi(N-1)d_y \sin \theta \sin \phi} \right]^T. $$

It is worth noting that the steering vector $a(\theta, \phi)$ is sparse. In practical applications sparsity is used to reduce the implementation cost and allow for implementing multiple band systems on the same platform [11]–[14]. Therefore, the sparsity in $a(\theta, \phi)$ should be taken into account to reduce the associated computational cost. However, in this paper we use $a(\theta, \phi)$ directly in our formulations.

A. Transmit Array Subaperturing

Due to cost and/or implementation considerations, we assume that the number of orthogonal waveforms to be used by the MIMO radar system is limited to $K \ll MN$ waveforms. Let $\psi(t) = [\psi_1(t), \ldots, \psi_K(t)]$ be the $K \times 1$ vector of predesigned independent waveforms which satisfy the orthogonality condition $\int_T \psi(t)\psi^H(t) = I_K$, where $T$ is the radar pulse duration, $I_K$ is the identity matrix of size $K \times K$, and $(\cdot)^H$ stands for the Hermitian transpose.

Using transmit array partitioning, we divide the transmit array into $K$ non-overlapped subarrays, i.e., each transmit antenna is allowed to be included in one subarray only. Each orthogonal waveform is transmitted via one of the subarrays in a phased-array mode, i.e., one power amplifier is needed for each subarray in addition to a number of phase-shifters equal to the number of elements in the concerned subarray. Let $Z_k$, $k = 1, \ldots, K$, be the $M \times N$ matrix of ones and zeros that defines the $k$-th subarray, i.e., the $mn$-th entry in $Z_k$ is one when the antenna element located at the $mn$-th point of the grid belongs to the $k$-th subarray and zero otherwise. The sparse $MN \times 1$ steering vector associated with the $k$-th subarray can be defined as

$$ a_k(\theta, \phi) = \text{vec} \left( Z_k \odot [u(\theta, \phi)v^T(\theta, \phi)] \right), \quad k = 1, \ldots, K. $$

Note that the subarray steering vector in (4) is very sparse, and, therefore, the $Q_k \times 1$ squeezed steering vector can be defined as

$$ \bar{a}_k(\theta, \phi) = \text{SQ} (a_k(\theta, \phi)), \quad k = 1, \ldots, K $$

where $\text{SQ} \cdot$ is the operator that retains the non-zero entries and discards the zero ones from a sparse vector, and $Q_k$ is the number of non-zero entries in $a_k(\theta, \phi)$. Let $w_k$, $k = 1, \ldots, K$, be the $Q_k \times 1$ transmit beamforming weight vector associated with the $k$-th subarray. The transmit beamforming weight vectors can be appropriately designed to form beams towards a certain direction in space or to focus the transmit energy within a 2D spatial sector defined by $\Theta = [\theta_1, \theta_2]$ in the elevation domain and $\Phi = [\phi_1, \phi_2]$ in the azimuth domain.

Assuming that $w_k$, $k = 1, \ldots, K$, are already designed, let us define the $K \times 1$ transmit gain vector as

$$ g(\theta, \phi) = [w_1^H \bar{a}_1(\theta, \phi), \ldots, w_K^H \bar{a}_K(\theta, \phi)]^T. $$

The complex envelope (i.e., the baseband representation) of the signal radiated towards a hypothetical target located at direction $(\theta, \phi)$ in the far-field can be modeled as

$$ e(t; \theta, \phi) = g^T(\theta, \phi)\psi(t) $$

$$ = \sum_{k=1}^K w_k^H \bar{a}_k(\theta, \phi)\psi_k(t). $$

The transmit beampattern summed over all transmitted signals
be written as

\[
P_c(\theta, \phi) = \int_T g^T(\theta, \phi) s(t) s^H(t) g^*(\theta, \phi) dt
= \int_T g^T(\theta, \phi) M \psi(t) \psi^H(t) M^H g^*(\theta, \phi) dt
= g^T(\theta, \phi) M \left( \int_T \psi(t) \psi^H(t) dt \right) M^H g^*(\theta, \phi)
= \|M^H g^*(\theta, \phi)\|^2 = \sum_{k=1}^K |m_k^H g^*(\theta, \phi)|^2.
\]

It is worth noting that \(g(\theta, \phi)\) in (10) is assumed to be known, i.e., it can be pre-calculated as in (6). The form (10) enables us to optimize the overall transmit beampattern by appropriately designing the waveform mixing matrix \(M\).

One meaningful way to optimize the overall beampattern is to minimize the difference between a certain desired transmit beampattern and the actual one while satisfying transmit power distribution constraints, e.g., transmit power distribution among different transmit subarrays. The associated optimization problem can be formulated as

\[
\min_{M} \max_{\theta, \phi} \left| \|M^H g^*(\theta, \phi)\|^2 - P_d(\theta, \phi) \right|,
\]

s.t. \(\theta \in \left[ -\pi, \frac{\pi}{2} \right], \forall \phi \in [0, \pi]\)

where \((\cdot)_{[k,j]}\) denotes the \((k, j)\)-th entry of a matrix and \(E_k\) is the maximum transmit energy that can be transmitted via the \(k\)-th subarray. It is worth noting that the optimization problem (11)–(12) can be used to achieve an arbitrary desired beampattern, i.e., it can be used to focus the transmit energy within one or multiple 2D spatial sectors. Although the optimization problem (11)–(12) is non-convex, it can be solved using positive semidefinite relaxation techniques.

When the radar operation involves target tracking or spatial scanning, it is desirable to focus the transmit energy towards a certain spatial direction defined by \((\theta_d, \phi_d)\). In this case, one meaningful approach that enables implementing the phased-MIMO concept is to minimize the sidelobe levels while maintaining a distortionless response towards the desired direction. Therefore, the mixing matrix design problem can be formulated as the following optimization problem

\[
\min_{M} \max_{\theta, \phi} \left| \|M^H g^*(\theta, \phi)\| \right|, \quad \forall \theta \in \widehat{\Theta}, \forall \phi \in \widehat{\Phi}
\]

s.t. \(m_k^H g^*(\theta, \phi) = \kappa e^{-j\varphi_k}, \quad k = 1, \ldots, K\)

\[
\sum_{j=1}^K |M_{[k,j]}|^2 \leq \frac{E_k}{\|w_k\|^2}, \quad k = 1, \ldots, K
\]

where \(\varphi_k, k = 1, \ldots, K\) are pre-known phases of user choice, \(\widehat{\Theta}\) and \(\widehat{\Phi}\) are the out-of-sector regions in the Elevation and Azimuth domains, respectively, and \(\kappa\) is a scaling constant that can be used to ensure that the optimization problem has a feasible solution. The constraints in (15) are used to ensure...
that the transmit power radiated through the \( k \)th subarray does not exceed the power budget \( E_k \) that is pre-assigned for the \( k \)th subarray. The optimization problem (13)–(15) is convex and can be efficiently solved using the interior-point methods.

IV. Simulation Results

In our simulations, we assume a transmit array whose elements are located on an \( 13 \times 17 \) uniform rectangular grid, i.e., the total number of points on the grid that can host a transmit antenna is 221. The vertical displacement between any two adjacent points on the grid is 0.59 wavelength while the horizontal displacement between any two adjacent points is 0.55 wavelength. A thinned array of 56 transmit antennas distributed on the grid points as shown in Fig. 2 (see [14] for thinned array design). We wish to form a 2D transmit beam to illuminate a target located at \( \theta_d = 40^\circ \), \( \phi_d = 100^\circ \). We compare the transmit beampattern shape obtained using three methods. (i) The conventional 2D phased-array radar. (ii) The phased MIMO radar using subaperturing. (iii) The proposed two-stage phased-MIMO radar. For the phased-MIMO radar given in Subsection II-A and the two-stage phased-MIMO radar developed in Sec. III, the transmit array is partitioned into six disjoint subarrays as shown in Fig. 2. For the two-stage phased-MIMO radar, the optimization problem (13)–(15) is used to design the matrix \( M \). The latter optimization problem is solved using the CVX toolbox [15]. The transmit power assigned to the individual subarrays is normalized to the total number of transmit antennas included in each subarray, i.e., \( E_k = 8 \) is used for subarrays 1, 2, 5, and 6 while \( E_k = 12 \) is used for subarrays 3 and 4. The value of \( \kappa = 1 \) is used.

Fig. 3 shows the normalized transmit beampattern versus the Elevation angle calculated at \( \phi = 100^\circ \) for all methods tested. Fig. 3 shows the normalized transmit beampattern versus the Azimuth angle calculated at \( \theta = 40^\circ \) for all methods tested. It is worth noting that phased-array radar achieves the best possible transmit coherent processing gain. However, it does not offer waveform diversity. It can be seen from Figs. 3 and 4 that the main beam of the phased MIMO radar is much wider than the main beam of the conventional phased-array radar due to the reduced aperture of the subarrays as compared to the whole array. As we can see from these two figures, the transmit beampattern of the two-stage phased-MIMO radar is very close in shape to the transmit beampattern of the conventional phased-array radar, i.e., the effect of reduced subarray aperture is effectively cancelled out by the new design.

V. Conclusions

The problem of 2D phased-MIMO radar design under the practically important constraint that the number of power amplifiers required at the transmit array is small has been investigated. The concept of subarray partitioning where the transmit array is partitioned into several subarrays is adopted. Each subarray operates in a phased-array mode leading to a reduction in the number of power amplifiers required. However, the use of subarray partitioning results in poor transmit beampattern characteristics due to the reduced physical aperture of the individual subarrays as compared to the physical aperture of the full transmit array. A two-stage design bases phased-MIMO radar that achieves a desired transmit beampattern while applying the concept of transmit subarray partitioning.
has been developed. The new method consists of two cascaded stages. At the first stage a set of finite number of orthogonal waveforms is mapped into another set of cross-correlated waveforms using a linear mixing operator. The second stage involves partitioning the transmit array into a finite number of transmit subarrays where each subarray is used to radiate one of the cross-correlated waveforms in phased-array mode. The mixing matrix used in the first stage is appropriately designed to ensure that the overall transmit beampattern is as close as possible to a desired beampattern. The latter design problem is formulated as a convex optimization problem and solved efficiently using the interior point methods. The number of transmit power amplifiers required is finite and equals to the number of subarrays. Simulation examples are used to validate the proposed method capabilities and effectiveness.

REFERENCES


