Game Theoretic Solutions for Precoding Strategies over the Interference Channel

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Abstract—In this paper, precoding strategies over interference channels are analyzed from a game-theoretic perspective. The Nash equilibrium and Nash bargaining solutions of precoding matrices, as the optimal precoding strategies in non-cooperative and cooperative cases, respectively, are derived for a two-player game over both flat fading and frequency selective channels. It is shown that the non-cooperative and cooperative solutions of precoding matrices are the same over multiple-input single-output (MISO) flat fading interference channels under a total power constraint. The solution in flat fading channel case is also extended to an $M$-player case.

Keywords – precoding, interference channel, Nash equilibrium, Nash bargaining.

I. INTRODUCTION

Spectrum sharing among users is critical in wireless communications with scarce radio resources [1], [2]. Operating in the same frequency band, the users interfere with each other if they communicate simultaneously. The communication channel in this case is called an interference channel. While information-theoretic studies of interference channels have a relatively long history [3]-[5], the investigation of such channels from a game theoretic perspective is a recent research topic.

From a game theoretical perspective, spectrum sharing over an interference channel can be modeled as a game, and then the equilibrium concepts or bargaining axioms can be applied. A Nash bargaining solution for frequency division multiplexing (FDM) rate region of two players with a flat fading interference channel is computed in [6] and the condition for existence of Nash bargaining solution is also derived. In [7], the aforementioned result is extended to $N$ players with a frequency selective channel and joint time division multiplexing (TDM)/FDM scheme. It is shown that at most a single frequency bin is shared by any two users in the optimal solution of TDM/FDM cooperative game over a frequency selective channel. The competitive and cooperative solution of beamforming vectors and the corresponding rates for the two-user multiple-input single-output (MISO) interference channel are investigated in [8]. The advantage of Nash bargaining solution over Nash equilibrium one is demonstrated and the necessity of cooperation in spectrum sharing is observed. The games in preceding references, such as the power allocation game on a frequency selective channel in [7] and the beamforming game in [8] are both vector-valued games. A matrix-valued game of finding optimal linear precoding strategies based on Nash equilibrium for multiuser non-cooperative wireless systems is analyzed in [9], and the algorithms for computing the Nash equilibrium solution are developed in [10]. The existence and performance of Nash equilibrium are also discussed.

It is well recognized that generally Nash equilibria are not optimal solutions and sometimes can be quite inefficient, referred to as in [6]-[8]. Therefore the objective of this research is to study the precoding strategy from a game perspective and find Nash bargaining solutions of precoding matrices that lead to Pareto-optimal solutions for both flat fading channel and frequency selective channel cases.

II. SYSTEM MODEL

The signal model of a two-user multiple-input multiple-output (MIMO) interference channel can be written as:

\[
\begin{align*}
\mathbf{y}_1 &= \mathbf{H}_{11}\mathbf{F}_1\mathbf{s}_1 + \mathbf{H}_{21}\mathbf{F}_2\mathbf{s}_2 + \mathbf{n}_1 \\
\mathbf{y}_2 &= \mathbf{H}_{22}\mathbf{F}_2\mathbf{s}_2 + \mathbf{H}_{12}\mathbf{F}_1\mathbf{s}_1 + \mathbf{n}_2
\end{align*}
\]

where $\mathbf{H}_{ij}$ is the $N \times N$ channel matrix between transmitter $i$ and receiver $j$, $\mathbf{F}_i$ is the $N \times N$ precoding matrix of user $i$, $\mathbf{s}_i$ is the $N \times 1$ information symbol block of user $i$, $\mathbf{n}_i$ is the $N \times 1$ additive Gaussian noise vector with $E\{\mathbf{n}_i\mathbf{n}_i^H\} = \sigma_n^2 \mathbf{I}$, and $\mathbf{I}$ denotes identity matrix. The information symbols are assumed to be uncorrelated and with unit-energy, i.e., $E\{\mathbf{s}_i\mathbf{s}_i^H\} = \mathbf{I}$.

In our game model, the wireless users are considered as players while the choices of precoding matrices are their strategies and the corresponding transmission rates for them are the payoffs. There may or may not be cooperation between players while the channel can be flat fading or frequency selective fading. For flat fading case, we consider a simplified MISO model in which the received symbol $\mathbf{y}$ and noise $\mathbf{n}$ are scalars and channel $\mathbf{h}_{ij}$ is an $N \times 1$ vector. The maximum total transmit power at each transmitter is denoted as $P_{\text{max}}$. Without loss of generality, $P_{\text{max}}$ is set to unity throughout the paper. For the frequency selective channel, a spectral mask constraint is adopted to limit the power which each user can allocate on each frequency bin. It is assumed that the receivers know the channel information perfectly and feed it back to transmitters without any errors.

III. PRECODING STRATEGY OVER THE FLAT FADING CHANNEL

In this section, the precoding strategy over the flat fading interference channel is studied. The non-cooperative (Nash...
equilibrium) solution over MISO interference channel for two users is shown and the result is then extended to the M-user case. Based on these results, the solutions for the cooperative game are derived using Nash bargaining theory.

Due to the total power constraint $P_{\text{max}} = 1$, the precoding matrices for the MISO flat fading case should satisfy the following condition:

$$
E(\{F_i, s_i\}) = \text{Trace}(F_i F_i^H) \leq 1, \quad \forall i \in \{1, 2\} \quad (2)
$$

where $\| \cdot \|$ denotes the Euclidian norm of a vector, $\text{Trace} \{ \cdot \}$ stands for the trace of a matrix and $(\cdot)^H$ stands for Hermitian transpose. The corresponding rate of user $i$ is given by [9]:

$$
R_i = \log(\|I + F_i^H h_i^T R_i^{-1} h_i^T F_i\|), \quad \forall i \in \{1, 2\} \quad (3)
$$

where $R_i = \sigma_i^2 + h_i^T F_i^H h_i^*$ (in which $j \in \{1, 2\}\setminus\{i\}$) is the noise plus interference for user $i$, $\| \cdot \|$ denotes the determinant of a matrix, and $(\cdot)^T$ and $(\cdot)^*$ stand for transpose and complex conjugation, respectively.

A. Non-cooperative game and Nash equilibrium solution

In the non-cooperative case where the two users are selfish and competing with each other, the result of the game will be a Nash equilibrium that satisfies

$$
R_1(F_1^{\text{NE}})|_{F_2 = F_2^{\text{NE}}} \geq R_1(F_1)|_{F_2 = F_2^{\text{NE}}}
$$

$$
R_2(F_2^{\text{NE}})|_{F_1 = F_1^{\text{NE}}} \geq R_2(F_2)|_{F_1 = F_1^{\text{NE}}}
$$

(4)

for any precoding matrices $F_1$ and $F_2$ under constraint (2).

**Proposition 1:** There is a unique Nash equilibrium for the non-cooperative game over the flat fading MISO interference channel under the total power constraint, and the corresponding precoding matrices are

$$
F_1^{\text{NE}} = \frac{i \otimes h_{11}^*}{\sqrt{\|h_{11}\|}}
$$

$$
F_2^{\text{NE}} = \frac{i \otimes h_{22}^*}{\sqrt{\|h_{22}\|}}
$$

(5)

with $i = [1, 1, \ldots, 1]_{1 \times N}$, and $\otimes$ representing Kronecker product.

**Proof:** We can rewrite the rate $R_1$ in (3) as:

$$
R_1 = \log(1 + \|h_{11}^T F_1\|^2/(\sigma_i^2 + \|h_{21}^T F_2\|^2)).
$$

(6)

From the inequality $\|h_{11}^T F_1\| \leq \|h_{11}^T\| F_1^H F_1$, and the power constraint $\|F_1\| \leq 1$, we have $h_{11}^T F_1^H F_1 \leq \|h_{11}\|$, with equality only when $F_1 = i \otimes h_{11}^*/(\sqrt{\|h_{11}\|})$. Therefore given that user 2 uses the Nash equilibrium solution $F_2^{\text{NE}}$, the best precoding matrix that user 1 can use to maximize $R_1$ is $F_1^{\text{NE}}$. The case for user 2 is the same. The uniqueness of the Nash equilibrium follows from the fact that, given that user 2 uses any $F_2$ under the power constraint, user 1’s best strategy keeps to be the same, i.e., $F_1^{\text{NE}}$ in (5), which maximizes the numerator in (6). Actually $\{F_1^{\text{NE}}, F_2^{\text{NE}}\}$ here is a special case of Nash equilibrium called dominant strategy equilibrium [11], corresponds to the case when each user has a fixed best strategy regardless of the choice of other users.

This result can be extended to an $M$-player game. The precoding matrix for user $k$ ($1 \leq k \leq M$) corresponding to the unique Nash equilibrium in an $M$-player non-cooperative game over the flat fading MISO interference channel is

$$
F_k^{\text{NE}} = \frac{i \otimes h_{kk}^*}{\sqrt{M\|h_{kk}\|}}, \quad \forall k \leq M.
$$

(7)

B. Nash bargaining solution

In a cooperative game, it is still assumed that the players are rational and selfish, but they agree to cooperate to increase their benefits. Four axioms are developed in [12], which should be satisfied in the bargaining solutions. Define the following Nash function for a two-player game [7]:

$$
F = (R_1 - R_1^{\text{NE}})(R_2 - R_2^{\text{NE}}).
$$

(8)

The Nash bargaining solution $\{R_1^{\text{NB}}, R_2^{\text{NB}}\}$ is the set of rates that maximizes $F$. However, Nash bargaining solution exists only when the rate region $(R_1, R_2)$ of the two users is convex. Although in general the rate region is not convex, the convex hull of it can be obtained using the TDM and/or FDM mode [8].

Here FDM is used to obtain the Nash bargaining solution of the cooperative game in the flat fading scenario. The portion of the total bandwidth which each user occupies is determined by the following convex optimization problem

$$
\max_{\alpha, F_1, F_2} \quad \alpha R_1(\alpha) - R_1^{\text{NE}}(\alpha) - (1 - \alpha) R_2^{\text{NE}}
$$

subject to: $0 \leq \alpha \leq 1$

$$
\text{Trace}(F_1 F_1^H) \leq 1
$$

$$
\text{Trace}(F_2 F_2^H) \leq 1
$$

(9)

where

$$
R_1(\alpha) = \alpha \log\left(1 + \frac{\|h_{11}^T F_{1\text{NB}}\|^2}{\sigma_i^2}\right)
$$

$$
R_2(1 - \alpha) = (1 - \alpha) \log\left(1 + \frac{\|h_{22}^T F_{2\text{NB}}\|^2}{(1 - \alpha)\sigma_i^2}\right)
$$

(10)

and $\alpha$ and $(1 - \alpha)$ are the portions of total bandwidth that users 1 and 2 occupy, respectively.

**Proposition 2:** The cooperative solution of precoding matrices are the same as the Nash equilibrium solution, i.e., $F_1^{\text{NB}} = F_1^{\text{NE}}$ and $F_2^{\text{NB}} = F_2^{\text{NE}}$.

The proof is omitted here due to space limitations. Similarly it can be proved that, for an $M$-player FDM cooperative game over the flat fading channel, the Nash bargaining solution of precoding matrix for each user is the same as the Nash equilibrium solution.

IV. PRECODING STRATEGY OVER THE FREQUENCY SELECTIVE CHANNEL UNDER A SPECTRAL MASK CONSTRAINT

For frequency selective channels, existing research focuses on the precoding strategy in non-cooperative cases [9]. So far no research exists in the open literature on investigating precoding strategies in cooperative cases. Since cooperative
solution is expected to achieve better transmission rates, it is our research target in this section. Particularly, we first find the non-cooperative solution for our special case. Then the Nash bargaining solution for a cooperative game over frequency selective channels is derived. Two-player game is considered next.

With proper cyclic prefix incorporated in transmitted symbols, the channel matrix $H_{ij}$ in (1) can be diagonalized as $H_{ij} = \omega_i \Omega_{ij} W^H$, with $W$ being an $N \times N$ IFFT matrix and $\Omega_{ij}$ being a diagonal matrix given by [13]:

$$\Omega_{ij} = \begin{pmatrix} H_{ij}(0) \\ \vdots \\ H_{ij}(N) \end{pmatrix}$$

(11)

where $H_{ij}(k)$ is the channel frequency-response of the $k$th frequency bin from transmitter $i$ to receiver $j$, and $N$ is the total number of frequency bins. The rates that the two players obtain can be written as

$$R_1 = \frac{1}{N} \log \left( |I + F^H_1 H_{ij}^H R^{-1}_j H_{ii} F_1| \right)$$

$$R_2 = \frac{1}{N} \log \left( |I + F^H_2 H_{ij}^H R^{-1}_j H_{ii} F_2| \right)$$

(12)

where $R_j = \sigma_j^2 I + H_{ij}^H F_j H_{ij}$ (in which $j \in \{1, 2\} \setminus \{i\}$) is the noise plus interference for user $i$.

A. Non-cooperative solution: Nash equilibrium

In [9], the non-cooperative solution of precoding matrices for a general case has been derived, and the Nash equilibrium solution of precoding strategies for a Q-player non-cooperative game over the frequency selective channel has been found. The game with both total power and spectral mask constraints is considered, and the matrix-valued game is simplified to an equivalent vector-valued power allocation game given as

$$F^\text{NE}_q = W \sqrt{\text{diag}(p_q)}, \quad \forall q \in \{1, 2, ..., Q\}$$

(13)

where $p_q \triangleq (p_q(k))_{k=1}^N$ is the optimal power allocation plan for user $q$, which can be solved from a vector-valued game.

The interpretation of this Nash equilibrium solution is as follows. All users first allocate power to their best frequency bins, where “best” means the frequency bin with highest signal to interference plus noise ratio (SINR). In our specific model, however, only spectral mask constraint is considered. Let us assume that the maximum power which any user can allocate to $k$th frequency bin is $p_{\text{max}}(k)$. It can be shown that the NE solution is

$$F^\text{NE}_i = W \sqrt{\text{diag}(p_{\text{max}})}, \quad i = 1, 2$$

(14)

where $p_{\text{max}} \triangleq (p_{\text{max}}(k))_{k=1}^N$.

It can be seen from (14) that each user uses maximum allowable power on all frequency bins to maximize its rate. The pair $\{F^\text{NE}_1, F^\text{NE}_2\}$ constitutes a dominant strategy equilibrium. However, the dominant strategy equilibrium solution may not exist if the total power which each user can use is limited.

B. Cooperative solution using Nash bargaining

In [7], the Nash bargaining solution for a multiple-player game over frequency selective channels is analyzed, where a mask constraint is adopted and the cooperative solution is achieved through TDM/FDM among users. It is known that TDM/FDM rate region is not the capacity region of the interference channel. However, the TDM/FDM rate region is convex and can be implemented easily. Therefore, we also adopt the TDM/FDM scheme in our model. The difference lies in that the optimal precoding matrices for the users need to be derived here, which makes the problem more complex.

**Proposition 3:** Precoding matrices corresponding to the Nash bargaining solution for a two-player cooperative game with TDM/FDM over the frequency selective channel should have the form of:

$$F_1 = W A_1$$
$$F_2 = W A_2$$

(15)

where $A_1$ and $A_2$ are diagonal matrices satisfying

$$A_1 + A_2 = \sqrt{\text{diag}(\Lambda)}$$
$$A_1 A_2 = 0$$

(16)

The proof is omitted here due to space limitations.

**Proposition 4:** The Nash bargaining solution for the two-player cooperative game with TDM/FDM over frequency selective channel can be achieved using the optimal precoding strategy containing at most two sets of precoding matrices that satisfy (15) and (16) denoted as $\{F^1_1, F^1_2\}$ and $\{F^2_1, F^2_2\}$. And if two sets exist, $F^1_1$ and $F^2_2$ should satisfy

$$\text{Trace}(A^1_1 - A^1_2) = \text{Trace}(A^2_1 - A^2_2) \in \{\sqrt{\text{diag}(1)}, \sqrt{\text{diag}(2)}, ..., \sqrt{\text{diag}(N)}\}$$

(17)

The proof follows from the fact that the Nash bargaining solution can be achieved by sharing at most a single frequency bin between the two users [7]. The complete proof is omitted here due to space limitations.

Let us assume that all frequency bins are numbered so that $R_2(m)/R_1(m) \geq R_2(n)/R_1(n)$ if $m \geq n (m,n \in \{1,2,\ldots,N\})$. Also let $k^*$ denote the frequency bin shared between two users in the Nash bargaining solution. Then, according to Propositions 3 and 4, the optimal sets of precoding matrices can be expressed as

**Set 1:**

$$F^1_1 = W A^1_1 = W \sqrt{\text{diag}(p^{1*}_1)}$$
$$F^2_1 = W A^2_1 = W \sqrt{\text{diag}(p^{2*}_1)}$$

(18)

where $p^{1*}_1 = [p_{\text{max}}(1), 0, \ldots, 0]$ and $p^{2*}_1 = [0, \ldots, 0, 0, \ldots, 0, p_{\text{max}}(k^* + 1), \ldots, p_{\text{max}}(N)]$.

**Set 2:**

$$F^1_2 = W A^1_2 = W \sqrt{\text{diag}(p^{1*}_2)}$$
$$F^2_2 = W A^2_2 = W \sqrt{\text{diag}(p^{2*}_2)}$$

(19)

where $p^{1*}_2 = [p_{\text{max}}(1), p_{\text{max}}(2), \ldots, p_{\text{max}}(k^* - 1), 0, \ldots, 0]$ and $p^{2*}_2 = [0, \ldots, 0, 0, p_{\text{max}}(k^*), \ldots, p_{\text{max}}(N)]$. 

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In the case when the frequency bin \( k \) is shared, the rates in (12) can be written as

\[
R_i(\alpha_i, k) = \alpha_i R_i(k) + \sum_{m=1, m\neq k}^{N} R_i(m) \tag{20}
\]

where

\[
R_i(m) = \log(1 + |H_{ii}(m)|^2/\sigma^2) \tag{21}
\]

is the rate on the frequency bin \( m \) for user \( i \), \( \alpha_i \) is the fraction of time that user \( i \) uses frequency bin \( k \), \( H_{ii}(m) \) is the \( m \)th element of \( \Omega_{ii} \) defined in (11), and \( \lambda_{ii}^l(m) \) \((i \in \{1,2\}, l \in \{1,2\})\) is the larger one among the \( m \)th diagonal elements on the diagonal of \( \Lambda_{ii}^k \) defined in (18) and (19). Note that \( \alpha_i \) should be non-negative and \( \alpha_1 + \alpha_2 \) should be equal to unity in order to ensure the optimality of the solution.

It is also possible that in the Nash bargain solution the two players do not share any frequency bin and thus they only have one set of optimal precoding matrices. This can be considered as a special case of sharing a frequency bin with \( \alpha_i = 0 \) for one user and \( 1 - \alpha_i = 1 \) for the other. Hence, only the general case is considered in the sequel.

To find the Nash bargaining solution, the shared frequency bin \( \alpha^* \) and time fraction \( \alpha_i^{NB} \) can be determined by solving the following optimization problem

\[
\begin{align*}
\max_{k \in \{1, \ldots, N\}, \alpha_1, \alpha_2} & \quad \left( R_1(\alpha_1, k) - R_1^{NE}\right)\left( R_2(\alpha_2, k) - R_2^{NE}\right) \\
\text{subject to:} & \quad 0 \leq \alpha_1 \leq 1 \\
& \quad 0 \leq \alpha_2 \leq 1 \\
& \quad \alpha_1 + \alpha_2 = 1.
\end{align*} \tag{22}
\]

Then, the bargaining solution \( R_i^{NB} \) can be obtained from (20). In the Nash bargaining solution of precoding strategy, set 1 of precoding matrices is used for portion \( \alpha_i^{NB} \) of time and set 2 is used for portion \( \alpha_2^{NB} \).

The Nash bargaining solution exists if and only if there exist \( k, \alpha_1 \) and \( \alpha_2 \) such that i) conditions in (22) are satisfied; and ii) the corresponding \( R_i \)’s satisfy \( R_1 \geq R_1^{NE} \) and \( R_2 \geq R_2^{NE} \).

In other worlds, the Nash bargaining solution exists only when both players can achieve higher rates through cooperation.

V. SIMULATION RESULTS

The simulation results of Nash equilibrium solution and Nash bargaining solution in both flat fading and frequency selective channels are provided.

In flat fading channel scenario, we first calculate the Nash equilibrium and Nash bargaining solutions according to Propositions 1 and 2, with parameters \( h_{11} = h_{22} = [0.9, 0.4]^T \), \( h_{12} = [0.4, 0.1]^T \), \( h_{21} = [0.7, 0.1]^T \), and \( \sigma_1^2 = \sigma_2^2 = 0.02 \). The result is shown in Fig. 1. Here the rate for user 1 is lower than that for the user 2 in Nash equilibrium solution because it suffers more interference. However, in Nash bargaining solution, both users obtain the same rate that is higher than the rates in Nash equilibrium solution, since there is no interference and \( h_{11} = h_{22} \). In Fig. 2, the same solutions are computed for user 1 under different SNR. The channels for both users are assumed to be symmetric (\( h_{11} = h_{22} \) and \( h_{12} = h_{21} \)) so that \( R_1 = R_2 \), and thus only \( R_1 \) is shown here versus SNR. From the figure, it can be verified that the advantage of Nash bargaining solution over Nash equilibrium solution becomes more notable when SNR increases. Fig. 3 plots the value of Nash function at Nash bargaining solution versus variable channel parameters. Note that Nash function reflects the advantage of cooperation in the system. In this example we assume \( h_{11} = h_{22} = [\alpha, \alpha]^T \) and \( h_{12} = h_{21} = [\beta, \beta]^T \), and calculate \( F = (R_1^{NB} - R_1^{NE})(R_2^{NB} - R_2^{NE}) \) under different \( \alpha \) and \( \beta \). From Fig. 3, we can see how the channel parameters affect the benefit of cooperation.

![Fig. 1. The FDM rate region, Nash equilibrium, and Nash bargaining solutions over the flat fading channel.](image1)

![Fig. 2. Nash equilibrium and Nash bargaining solutions versus SNR over the flat fading channel.](image2)
In frequency selective channel scenario, we assume that users have four available frequency bins to share. The channels are assumed to be Rayleigh fading and the noise power $\sigma^2$ equals 0.01 for both users. The Nash equilibrium and Nash bargaining solutions are found according to Propositions 3 and 4, and are shown in Fig. 4. Fig. 5 shows the value of Nash function under different FDM/TDM frequency bin allocation schemes, where $k$ is the frequency bin being shared and $\alpha$ is the fraction of time user 1 uses this frequency bin. The largest value of $F$ in the figure corresponds to the optimal scheme that provides Nash bargaining solution.

VI. CONCLUSIONS

In this paper, we derive the precoding strategies for flat fading and frequency selective interference channels. It is found that the non-cooperative solution over MISO flat fading channel under total power constraint constitutes a special form of Nash equilibrium, namely dominant strategy equilibrium, and thus is also the solution of cooperative game with FDM approach. We also derive the solution of optimal precoding matrices for the cooperative game over frequency selective channel. Simulation results demonstrate the advantage of Nash bargaining solution over Nash Equilibrium one. The impact of channel parameters, SNR, and spectrum allocation scheme on system performance is also shown.

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