Direction Finding for MIMO Radar with Colocated Antennas Using Transmit Beamspace Preprocessing

(Invited Paper)

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Abstract—The problem of direction finding for multiple targets in mono-static multiple-input multiple-output (MIMO) radar systems is considered. Assuming that the targets are located within a certain spatial sector, we focus the energy of multiple (two or more) transmitted orthogonal waveforms within that spatial sector using appropriately designed transmit beamforming. The transmit beamformers are designed so that matching the received data to the waveforms yields multiple (two or more) data sets with rotational invariance property that allows applying search-free direction finding techniques such as ESPRIT. Unlike previously reported MIMO radar ESPRIT-based direction finding techniques, our method is applicable to arbitrary arrays and achieves better estimation performance at lower computational cost.

Index Terms—MIMO radar, direction-of-arrival estimation, transmit beamspace, rotational invariance, search-free methods.

I. INTRODUCTION AND MOTIVATION

The development of multiple-input multiple-output (MIMO) radar is recently the focus of intensive research [1]- [3]. A MIMO radar is generally defined as a radar system with multiple transmit linearly independent waveforms and it enables joint processing of data received by multiple receive antennas. MIMO radar can be either equipped with widely separated antennas [2] or colocated antennas [3].

Estimating direction-of-arrivals (DOAs) of multiple targets from measurements corrupted by noise at the receiving array of antennas is one of the most important radar applications frequently encountered in practice. Many DOA estimation methods have been developed for traditional single-input multiple-output (SIMO) radar [4]- [11]. Among these methods the estimation of signal parameters via rotational invariance techniques (ESPRIT) is one of the most popular due to its simplicity and high-resolution capabilities [8], [11]. ESPRIT algorithm is a special and computationally efficient case of a more general decomposition technique of high-dimension (higher than 2) data arrays known as parallel factor analysis (PARAFAC) [12], [13].

More recently, some algorithms have been developed for DOA estimation of multiple targets in MIMO radar systems equipped with \( M \) colocated transmit antennas and \( N \) receive antennas [14]- [18]. The algorithms proposed in [14] and [15] require an exhaustive search over the unknown parameters and, therefore, mandate prohibitive computational cost if the search is performed over a fine grid. On the other hand, the search-free ESPRIT-based algorithms of [16], [17] and PARAFAC-based algorithm of [18] utilize the rotational invariance property of the so-called extended virtual array to estimate the DOAs at a moderate computational cost. It is worth noting that in the case of MIMO radar, the advantages of the aforementioned DOA estimation methods over similar ESPRIT-based and PARAFAC-based DOA estimation methods for SIMO radar appear due to the fact that the extended virtual array of \( MN \) virtual antennas can be obtained in MIMO radar case by match filtering the data received by the \( N \)-antenna receiving array to the transmitted waveforms. Therefore, the effective aperture of the virtual array can be significantly extended that leads to improved resolution.

In the methods of [16]- [18], the rotational invariance property is achieved by partitioning the receiving array to two (in ESPRIT case) or multiple (in PARAFAC case) overlapped subarrays. Then, the rotational invariance is also presumed for the virtual enlarged array of \( MN \) virtual antennas. However, the methods of [16]- [18] employ full waveform diversity, i.e., the number of transmitted waveforms equals the number of transmit antennas, at the price of reduced transmit energy per waveform. In other words, for fixed total transmit energy \( E \), each waveform has energy \( E/M \) transmitted omnidirectionally. It results in a reduced signal-to-noise ratio (SNR) per virtual antenna. On the other hand, it is well-known that the estimation accuracy of subspace-based techniques suffers from high SNR threshold, that is, the root-mean-squared error (RMSE) of DOA estimates approaches the corresponding Cramer-Rao bound only for relatively high SNRs [19], [20]. Therefore, the higher the SNR, the better the DOA estimation performance can be achieved.

The SNR per virtual antenna can be increased by (i) transmitting less waveforms of higher energy and/or (ii) focusing transmitted energy within spatial sectors where the targets are likely to be located. Therefore, in this work, we consider the possibility of obtaining the rotational invariance property while transmitting \( 1 < K \leq M \) orthogonal waveforms using different beamforming weight vectors and focusing the transmitted energy per waveform. In other words, for fixed total transmit energy \( E \), each waveform has energy \( E/M \) transmitted omnidirectionally. It results in a reduced signal-to-noise ratio (SNR) per virtual antenna. On the other hand, it is well-known that the estimation accuracy of subspace-based techniques suffers from high SNR threshold, that is, the root-mean-squared error (RMSE) of DOA estimates approaches the corresponding Cramer-Rao bound only for relatively high SNRs [19], [20]. Therefore, the higher the SNR, the better the DOA estimation performance can be achieved.
energy on a certain spatial sector where the targets are located. It enables us to design search-free ESPRIT/PARAFAC-based DOA estimation methods. Note that by using less waveforms the energy available for each transmitted waveform can be increased, that is, the SNR per each virtual antenna can be improved, while the aperture of the virtual array decreases to $KN$. Moreover, the SNR per virtual antenna can be further increased by focussing the transmitted energy in a certain sector where the targets are located.

The paper is organized as follows. In Section II, MIMO radar signal model is briefly introduced. A new approach to DOA estimation based on transmit beamspace preprocessing of the signal transmitted by the $m$-modeled as array and the pulse width. This approach using ESPRIT method is given in Section IV. Simulation results which show the advantages of the proposed method are reported in Section V followed by conclusions drawn in Section VI.

II. MIMO RADAR SIGNAL MODEL

Consider a mono-static radar system with $M$-antenna transmit array and $N$-antenna receive array. The complex envelope of the signal transmitted by the $m$th transmit antenna is modeled as

$$s_m(t) = \sqrt{\frac{E}{M}} \phi_m(t), \quad m = 1, \ldots, M$$

where $t$ is the fast time index, i.e., the time index within one radar pulse, $E$ is the total transmit energy within one radar pulse, and $\phi_m(t)$ is a baseband waveform. Assume that the waveforms emitted by different transmit antennas are orthogonal. Also, the waveforms are normalized to have unit-energy, i.e., $\int_T |\phi_m(t)|^2 dt = 1, \quad m = 1, \ldots, M$, where $T$ is the pulse width.

Assuming that $L$ targets are present, the $N \times 1$ received complex vector of array observations can be written as

$$x(t, \tau) = \sum_{l=1}^{L} r_l(t, \tau) b(\theta_l) + z(t, \tau)$$

where $\tau$ is the slow time index, i.e., the pulse number, $b(\theta)$ is the steering vector of the receive array, $z(t, \tau)$ is the $N \times 1$ zero-mean white Gaussian noise term, and

$$r_l(t, \tau) \triangleq \sqrt{\frac{E}{M}} \beta_l(\tau) a^T(\theta_l) \phi(t)$$

is the radar return due to the $l$th target. In (3), $\beta_l(\tau)$, $\theta_l$, and $a(\theta_l)$ are the reflection coefficient with variance $\sigma^2_\beta$, spatial angle, and steering vector associated with the $l$th target, respectively, $\phi(t) \triangleq [\phi_1(t), \ldots, \phi_M(t)]^T$ is the waveform vector, and $(\cdot)^T$ stands for the transpose. Note that the reflection coefficient $\beta_l(\tau)$ for each target is assumed to be constant during the whole pulse, but varies from pulse to pulse, i.e., it obeys the Swirling II target model [18].

By mach-filtering $x(t, \tau)$ to each of the waveforms $\{\phi_m\}_{m=1}^M$ and stacking the results in one column vector, we obtain the $MN \times 1$ virtual data vector [3]

$$y(\tau) \triangleq [x_1^T(\tau) \ldots x_M^T(\tau)]^T = \sqrt{\frac{E}{M}} \sum_{l=1}^{L} \beta_l(\tau) a(\theta_l) \otimes b(\theta_l) + \tilde{z}(\tau)$$

where $\otimes$ denotes the Kronker product, $\tilde{z}(\tau)$ is the $MN \times 1$ noise term whose covariance is given by $\sigma^2_\tilde{z} I_{MN}$, and

$$x_m(\tau) \triangleq \int_T x(t, \tau) \phi_m^*(t) dt.$$ 

The data model (4) is used in [16] for DOA estimation using ESPRIT. In particular, $y(\tau)$ in (4) is partitioned into $y_1(\tau) \triangleq [x_1^T(\tau), \ldots, x_{M-1}^T(\tau)]^T$ and $y_2(\tau) \triangleq [x_2^T(\tau), \ldots, x_M^T(\tau)]^T$. It has been shown in [16] that $y_1$ and $y_2$ obey rotational invariance property which enables the use of ESPRIT for DOA estimation. However, the rotational invariance property is valid only in the case when the transmit array is a uniform linear array (ULA). Therefore, the method of [16] is limited by the transmit array structure and may suffer from performance degradation in the presence of array perturbation errors. Moreover, it suffers from low SNR per virtual antenna as a result of dividing the total transmit energy over $M$ different waveforms.

III. DOA ESTIMATION USING TRANSMIT BEAMSPACE PREPROCESSING

Instead of transmitting omn-directionally, we propose to focus the transmitted energy within a sector $\Theta$ by forming $K$ directional beams where an independent waveform is transmitted over each beam. Note that the spatial sector $\Theta$ can be estimated using any low-resolution DOA estimation technique of low complexity.

Let $W \triangleq [w_1, \ldots, w_K]^T$ be the transmit beamspace matrix of dimension $M \times K$, where $w_k$ is the $M \times 1$ unit-norm weight vector used to form the $k$th beam. The beamspace matrix can be properly designed to maintain constant beampattern within the sector of interest $\Theta$ and to minimize the energy transmitted in the out-of-sector areas. The $K \times 1$ basebeam beamspace signal vector transmitted by the array can be modeled as

$$s(t) = \sqrt{\frac{E}{K}} W^* \phi_K(t)$$

where $\phi_K \triangleq [\phi_1(t), \ldots, \phi_K(t)]^T$ and $(\cdot)^*$ is the conjugation operator. It is worth noting that the beamspace energy, i.e., the energy of (6) transmitted within one radar pulse is given by

$$E_{\text{beam}} \triangleq \int_T ||s(t)||^2 dt = \frac{E}{K} \sum_{k=1}^{K} ||w_k||^2 \int_T ||\phi_k(t)||^2 dt = E. \quad (7)$$

At the receive array, the $N \times 1$ complex vector of array observations can be expressed as

$$x_{\text{beam}}(t, \tau) = \sqrt{\frac{E}{K}} \sum_{l=1}^{L} \beta_l(\tau) b(\theta_l) u^T(\theta_l) \phi_K(t) + z(t, \tau)$$

where $u(\theta) \triangleq W^H a(\theta)$ is the transmit beamspace vector and $(\cdot)^H$ stands for the Hermitian transpose.
By match-filtering $x(t, \tau)$ to each of the waveforms \( \{\phi_k\}_{k=1}^K \), we obtain the $KN \times 1$ virtual data vector

\[
y_{\text{beam}}(\tau) \triangleq [y_1^T(\tau) \cdots y_K^T(\tau)]^T
\]

\[
= \sqrt{\frac{E}{K}} \sum_{l=1}^L \beta_l(\tau) u(\theta_l) \otimes b(\theta_l) + \tilde{z}_K(\tau) \quad (9)
\]

where \( \tilde{z}_K(\tau) \) is the $KN \times 1$ noise term whose covariance is given by \( \sigma_z^2 I_{KN} \), and \( y_k(\tau) \triangleq \int_{\tau} x_{\text{beam}}(t, \tau) \phi_k^*(t) dt \).

In the noise free case, the data vectors \( \{y_k(\tau)\}_{k=1}^K \) can be expressed as

\[
y_k(\tau) = B_k \beta(\tau) \quad (10)
\]

where

\[
\beta(\tau) \triangleq [\beta_1(\tau), \ldots, \beta_L(\tau)]^T
\]

\[
B_k \triangleq [b(\theta_1), \ldots, b(\theta_L)] \Psi_k
\]

\[
\Psi_k \triangleq \text{diag} \left\{ \tilde{w}_1^H a_1(\theta_1), \ldots, \tilde{w}_L^H a_L(\theta_L) \right\} \tag{13}
\]

It is worth noting that the matrices \( \{B_k\}_{k=1}^K \) are related to each other as

\[
B_k = B_j \Psi_j^{-1} \Psi_k, \quad k, j = 1, \ldots, K. \tag{14}
\]

By carefully designing the beamspace weight matrix \( W \), (14) enjoys the rotational invariance property. Therefore, the data model (9) lends itself straightforwardly to the rotational invariance based direction finding techniques such as ESPRIT [8], [16], [17] and PARAFAC [12], [18].

IV. TRANSMIT BEAMSPACE BASED ESPRIT

Consider the case when only two transmit beams are formed. Then, the transmit beamspace matrix is \( W = [w_1, w_2] \). Assume also that the transmit weight vectors have the forms \( w_1 = [0 \tilde{w}_1^T]^T \) and \( w_2 = [\tilde{w}_2 0]^T \), where \( \tilde{w}_1 \) and \( \tilde{w}_2 \) are of dimension \((M - 1) \times 1\). In this case, (14) simplifies to

\[
B_2 = B_1 \Psi \quad (15)
\]

where

\[
\Psi \triangleq \text{diag} \left\{ \tilde{w}_1^H a_1(\theta_1), \ldots, \tilde{w}_L^H a_L(\theta_1) \right\} \tag{16}
\]

and \( a_1(\theta) \) and \( a_2(\theta) \) are the \((M - 1) \times 1\) vectors which contain the first \( M - 1 \) and the last \( M - 1 \) elements of \( a(\theta) \), respectively. It is worth noting that (16) can be rewritten as

\[
\Psi = \text{diag} \left\{ A(\theta_1)e^{j\Omega(\theta_1)}, \ldots, A(\theta_L)e^{j\Omega(\theta_L)} \right\} \tag{17}
\]

where \( A(\theta) \) and \( \Omega(\theta) \) are the magnitude and angle of \( \tilde{w}_2^H a_2(\theta)/\tilde{w}_1^H a_1(\theta) \), respectively. It can be seen from (15) and (17) that the vectors \( y_1 \) and \( y_2 \) (see also (10)) enjoy the rotational invariance property. Therefore, ESPRIT-based DOA estimation techniques can be used to estimate \( \Psi \) and \( \{\theta_l\}_{l=1}^L \) can be obtained from \( \Psi \) by looking up a table that converts \( \Omega(\theta) \) to \( \theta \).

In the special case when the transmit array is a ULA, choosing \( w_1 = \tilde{w}_2 \) yields \( w_2^H a_2(\theta) = \tilde{w}_1^H a_1(\theta)e^{-j\frac{2\pi}{\lambda}d \cdot \sin \theta} \), where \( d \) is the displacement between any of two adjacent antennas of the transmit array and \( \lambda \) is the propagation wavelength. In this case, (16) boils down to

\[
\Psi = \text{diag} \left\{ e^{-j\frac{2\pi}{\lambda}d \cdot \sin \theta_1}, \ldots, e^{-j\frac{2\pi}{\lambda}d \cdot \sin \theta_K} \right\}. \tag{18}
\]

It is worth noting that the computational complexity of applying ESPRIT to the data models (9) for \( K = 2 \) is of \( O(2^3N^3) \) as compared to \( O(M^3N^3) \) for the ESPRIT applied to the data model (4), i.e., the method of [16]. Moreover, as compared to the method of [16], our method enjoys SNR improvement of \( M|\tilde{w}_2^H a_1(\theta)|^2/2 \) due to energy focusing.

V. SIMULATION RESULTS

In our simulations, we assume a ULA of \( M = 10 \) omni-directional antennas at the transmitter. \( N = 10 \) omni-directional antennas are also assumed at the receiver. The additive noise is assumed Gaussian zero-mean unit-variance spatially and temporally white. Two targets are located at directions \( 1^\circ \) and \( 3^\circ \), respectively. The sector of interest is \([-5^\circ, 5^\circ]\). ESPRIT-based DOA estimation is performed using our method as well as the method of [16]. For our method, the transmit array is partitioned into two overlapped subarrays of 9 antennas each, and the weight vector \( \tilde{w}_1 = \tilde{w}_2 \) is obtained by averaging the two eigenvectors associated with the maximum two eigevalues of the matrix \( \int_{\theta} a_1(\theta)a_1^H(\theta)d\theta \) yielding \( \tilde{w}_1 = [-0.5623 -0.5076 -0.4358 -0.3501 -0.2542 -0.1524 -0.0490 0.0512 0.1441]^T \). For the method of [16], the receiving array is partitioned into two overlapped subarrays of 9 antennas each. For both methods tested, the total transmit energy is fixed to \( E = M \).

The transmit beamspace beampattern is shown in Fig. 1. It can be seen from this figure that the transmitted energy is focussed within the sector of interest yielding the SNR gain of approximately 7 dB. The probability of target resolution (see for the definition [20, Ch. 9]) and the DOA estimation RMSE versus SNR \( \triangleq E\sigma_{\beta}/M\sigma_{\lambda}^2 = \sigma_{\beta}/\sigma_{\lambda}^2 \) are shown in Figs. 2 and 3, respectively. It can be seen from Fig. 2 that our proposed method has better probability of target resolution than that of the method of [16]. Fig. 3 shows that the proposed method has better estimation performance at low and moderate

![Fig. 1. Transmit beamspace beampattern](image-url)
SNR values than the method of [16]. We also note from Fig. 3 that at high SNRs, the algorithm of [16] has lower RMSE than the proposed algorithm. This is consistent with what was concluded in [2] that radar systems with waveform diversity outperform phased-array radars at high SNR and vice versa.

VI. CONCLUSION

The new ESPRIT/PARAFAC-based DOA estimation method for mono-static MIMO radar is developed. Different from the previous ESPRIT/PARAFAC-based methods in which the rotational invariance property is ensured by partitioning the receiving array into a number (two or more) of overlapped subarrays, the same property is achieved in our method by focusing the energy of multiple (two or more) transmitted orthogonal waveforms within a certain spatial sector using appropriately designed transmit beamforming. Particularly, the transmit beamformers are designed so that match-filtering the received data to the waveforms yields multiple (two or more) data sets with rotational invariance property that allows applying search-free direction finding techniques such as ESPRIT and PARAFAC. Our method is applicable to arbitrary arrays and achieves better estimation performance at lower computational cost as compared to the previous search-free methods. Better estimation performance is attributed to the fact that better SNR per virtual antenna is obtained by transmitting less waveforms and focussing the transmitted energy within a certain sector.

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