ROBUST MULTI-ANTENNA BROADCASTING WITH IMPERFECT CHANNEL STATE INFORMATION

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ABSTRACT
The problem of robust multi-antenna broadcasting is considered in the case of erroneous channel state information (CSI) at the transmitter. The criterion of minimum transmission power is used subject to the constraints guaranteeing that the signal-to-noise ratio (SNR) of each intended receiver satisfies a prescribed quality of service (QoS) requirement for the worst-case norm-bounded CSI mismatch. The resulting robust broadcasting problem is non-convex and, therefore, is difficult to solve. However, a suitable semi-definite relaxation (SDR) of this problem is proposed that enables to transform it to a convex form and to compute the optimal transmitter weight vector.

1. INTRODUCTION
Consider a scenario in which a multi-antenna transmitter has to broadcast information to multiple receivers within some service area [1]. The traditional broadcasting strategy is to transmit the available power isotropically or using a fixed transmit beampattern. However, in such a case the transmit CSI is ignored and, as a result, the users with weak channels can experience a severe QoS degradation. In the case of exact transmit CSI, it has been suggested in [1] to minimize the transmitted power subject to constraints guaranteeing that the receive SNRs of all intended receivers satisfy certain QoS requirements. The authors of [1] have used the SDR approach to relax the originally NP-hard broadcasting problem to a suitable convex form and, subsequently, to solve the resulting relaxed problem using convex optimization techniques.

In practical situations, there are always some errors in the transmit CSI. Therefore, robust broadcast designs are of great interest. Motivated by this fact, a promising robust extension of the broadcasting strategy of [1] has been proposed in [2] where the authors have adopted the concept of worst-case beamformer design [3] to develop a robust version of the algorithm of [1]. The robust approach of [2] amounts to a simple re-scaling of the solution obtained in [1] to make the QoS constraints resistant to CSI errors. However, as it will be demonstrated below, because of several approximations involved in the design, the approach of [2] may lead to unnecessary high transmitted power values in the case of high-SNR QoS requirements.

In this paper, we propose a new approach to robust multi-antenna broadcasting in the case of erroneous CSI. Our approach also uses the concept of worst-case beamformer design, but, in contrast to [2], we apply the worst-case design strategy directly to the original problem of [1]. More specifically, we minimize the transmission power subject to the constraints guaranteeing that the SNR of each intended receiver satisfies some prescribed QoS requirement for the worst-case norm-bounded CSI mismatch. Although the resulting robust broadcasting problem appears to be non-convex and NP-hard, a suitable SDR of this problem is proposed that enables us to transform it to a convex form and solve it using convex optimization techniques.

2. PROBLEM FORMULATION
Let us consider a multi-antenna transmitter with \( N \) sensors and \( M \) single-antenna receivers. The transmitter broadcasts common information to all the receivers. The \( N \times 1 \) vector \( \mathbf{h}_i \) denotes the frequency-flat quasi-static channel between the transmitter and the \( i \)th receiver, and the \( N \times 1 \) vector \( \mathbf{w} \) denotes the transmitter weight vector, respectively.

The receive SNR of the \( i \)th user can be defined as

\[
\text{SNR}_i = \frac{|\mathbf{w}^H \mathbf{h}_i|^2}{\sigma_i^2}
\]  (1)

where \( \sigma_i^2 \) is the noise variance of the \( i \)th receiver and \( (\cdot)^H \) is the Hermitian transpose.

Let us now assume that all channel vectors are known with a certain errors \( \delta_i \) and that these errors are all norm-bounded, that is \( ||\delta_i|| \leq \varepsilon \), where \( ||\cdot|| \) is the 2-norm of a vector, and the parameter \( \varepsilon \) is assumed to be known.

The robust broadcasting design problem can be then formulated as

\[
\min_{\mathbf{w}} ||\mathbf{w}||^2
\]

subject to

\[
\min_{||\delta_i|| \leq \varepsilon} |\mathbf{w}^H (\mathbf{h}_i + \delta_i)|^2 \geq C_i \quad \forall \ i = 1, \ldots, M
\]  (2)
where $C_i = \text{SNR}_{\text{min}_i} \sigma_i^2$ and $\text{SNR}_{\text{min}_i}$ is the minimum SNR required for the $i$th receiver.

### 3. Problem Approximation

Let us first modify inequality constraints in (2) to simplify the problem. Using the triangle inequality, we have

$$|w^H (h_i + \delta_i)| \geq |w^H h_i| - |w^H \delta_i|. \quad (3)$$

Using the Cauchy-Schwarz inequality along with the fact that $\|\delta_i\| \leq \varepsilon$, we have

$$|w^H \delta_i| \leq \|w\| \|\delta_i\| \leq \varepsilon \|w\|. \quad (4)$$

And

$$\max_{\|\delta_i\| \leq \varepsilon} |w^H \delta_i| = \varepsilon \|w\|. \quad (5)$$

Hence, we obtain that

$$\min_{\|\delta_i\| \leq \varepsilon} |w^H (h_i + \delta_i)|^2 \geq (|w^H h_i| - \varepsilon \|w\|)^2. \quad (6)$$

Now, expanding the right hand side of (6), we have

$$(|w^H h_i| - \varepsilon \|w\|)^2 = |w^H h_i|^2 + \varepsilon^2 \|w\|^2 - 2\varepsilon |w^H h_i| \geq |w^H h_i|^2 + \varepsilon^2 \|w\|^2 - 2\varepsilon \|w\| \|h_i\| \geq |w^H h_i|^2 + \varepsilon (\|h_i\| - \varepsilon \|h_i\|) \|w\|^2 \quad (7)$$

where the Cauchy-Schwarz inequality has been used again in the second line.

Using (6) and (7), we can modify the original problem (2) by approximating (strengthening) the constraints in (2). Then, the modified problem becomes

$$\min_{w} \|w\|^2$$

s.t. $|w^H h_i|^2 + \varepsilon (\|h_i\| - \varepsilon \|h_i\|) \|w\|^2 \geq C_i \quad (8)$

$\forall i = 1, \ldots, M.$

To simplify the problem (8), let us convert it to a real-valued form. Using the following definitions

$$\tilde{w} \triangleq \text{Re}\{w\}^T \text{Im}\{w\}^T$$

$$\tilde{h}_i \triangleq \text{Re}\{h_i\}^T \text{Im}\{h_i\}^T$$

$$\tilde{h}_i \triangleq \text{Im}\{h_i\}^T - \text{Re}\{h_i\}^T$$

$$Q_i \triangleq h_i \tilde{h}_i^T + \tilde{h}_i h_i^T$$

we obtain that

$$\|w\|^2 = \tilde{w}^T \tilde{w}$$

$$|w^H h_i|^2 = (\tilde{w}^T h_i)^2 + (\tilde{w}^T \tilde{h}_i)^2 = \tilde{w}^T Q_i \tilde{w}$$

$$\|h_i\| = \sqrt{\tilde{h}_i^T h_i}$$

where $(\cdot)^T$ denotes the transpose and $\text{Re}\{\cdot\}$ and $\text{Im}\{\cdot\}$ denote the real and imaginary parts, respectively. Therefore, the inequality constraint in (8) can be rewritten as

$$\tilde{w}^T Q_i \tilde{w} + \varepsilon \left(2 \sqrt{\tilde{h}_i^T h_i} \right) \tilde{w} + \varepsilon^2 \|w\|^2 = C_i \quad \forall i = 1, \ldots, M \quad (9)$$

or, equivalently

$$\tilde{w}^T A_i \tilde{w} + C_i \leq 0 \quad \forall i = 1, \ldots, M \quad (10)$$

where

$$A_i \triangleq \varepsilon \left(2 \sqrt{\tilde{h}_i^T h_i} - \varepsilon \right) I - Q_i$$

and $I$ is the identity matrix.

Hence, we can rewrite (8) as

$$\min_{w} \|w\|^2$$

s.t. $\tilde{w}^T A_i \tilde{w} + C_i \leq 0 \quad (11)$

$\forall i = 1, \ldots, M.$

This problem is a quadratically constrained quadratic programming (QCQP) problem. As the matrix $A_i$ is in general indefinite, the constraints are non-convex.

Let us now use the SDR approach to relax (10) to a suitable convex form. First, we rewrite this problem as

$$\min_{\tilde{w}} \text{trace}(\tilde{w} \tilde{w}^T)$$

s.t. $\text{trace}(\tilde{w} \tilde{w}^T A_i) + C_i \leq 0 \quad (12)$

$\forall i = 1, \ldots, M.$

where we use the fact that $\tilde{w}^T A_i \tilde{w} = \text{trace}(\tilde{w} \tilde{w}^T A_i)$. Introducing $W = \tilde{w} \tilde{w}^T$, we can further reformulate our problem as

$$\min_{W} \text{trace}(W)$$

s.t. $\text{trace}(WA_i) + C_i \leq 0 \quad (13)$

$\forall i = 1, \ldots, M,$

$W \geq 0, \ \text{rank}(W) = 1$

where the inequality $W \geq 0$ means that the matrix $W$ is symmetric positive semi-definite. Both the objective function and the trace constraints in (12) are linear and, therefore, convex. The constraint $W \geq 0$ in (12) is convex as well. However, the rank constraint $\text{rank}(W) = 1$ in (12) is non-convex. Therefore, following the SDR approach, we drop the latter non-convex constraint to obtain the following relaxed problem:

$$\min_{W} \text{trace}(W)$$

s.t. $\text{trace}(WA_i) + C_i \leq 0 \quad (14)$

$\forall i = 1, \ldots, M,$

$W \geq 0$

which belongs to the class of convex SDP problems that can be efficiently solved using interior point methods [1], [4].
4. THE PROPOSED ALGORITHM

Let \( \mathbf{W}_{\text{opt}} \) denote the optimal solution to (13). If this matrix is rank-one, then the optimal weight vector can be recovered from \( \mathbf{W}_{\text{opt}} \) straightforwardly, using the principal eigenvector corresponding to the only non-zero eigenvalue. However, because of the SDR step, \( \mathbf{W}_{\text{opt}} \) will not be rank-one in general.

A powerful approach that has recently emerged in the optimization community is to obtain an approximate solution to the original problem from the solution to its relaxed version using the so-called randomization. Various randomization techniques have been developed so far, see [5]-[7] and references therein. A common idea of these techniques in application to our problem is to generate a set of candidate vectors \( \{\tilde{\mathbf{w}}_{\text{cand},l}\}_{l=1}^{L} \) using \( \mathbf{W}_{\text{opt}} \) and choose the best solution from these candidate vectors. Here, \( L \) is the number of randomizations used.

To obtain the candidate vectors, we calculate the eigendecomposition of \( \mathbf{W}_{\text{opt}} \) in the form

\[
\mathbf{W}_{\text{opt}} = \mathbf{U}\mathbf{\Sigma}\mathbf{U}^T
\]

and select

\[
\tilde{\mathbf{w}}_{\text{cand},l} = \mathbf{U}\mathbf{\Sigma}^{1/2}\mathbf{v}_l
\]

(14)

where \( \mathbf{v}_l \) is a zero-mean real Gaussian vector whose covariance matrix is \( \mathbf{I} \). This ensures that \( E\{\tilde{\mathbf{w}}_{\text{cand},l}\tilde{\mathbf{w}}_{\text{cand},l}^H\} = \mathbf{W}_{\text{opt}} \) where \( E\{\cdot\} \) denotes the statistical expectation. From \( \tilde{\mathbf{w}}_{\text{cand},l} \), the candidate weight vector \( \mathbf{w}_{\text{cand},l} \) can be directly recovered as

\[
\mathbf{w}_{\text{cand},l} = \tilde{\mathbf{w}}_{\text{cand},l}(1:N) + j\tilde{\mathbf{w}}_{\text{cand},l}(N+1:2N).
\]

(15)

Depending on a particular realization \( \mathbf{v}_l \), the so-obtained candidate weight vectors may violate the worst-case QoS constraints. Therefore, for each candidate weight vector, it is useful to check whether the constraints are violated. If some constraints are indeed violated, then this vector can be properly re-scaled to make sure that all these constraints are finally satisfied after such a re-scaling.

The proposed algorithm is summarized in Table 1.

Table 1. The proposed robust broadcasting algorithm.

- Solve (13) using some SDP solver.
- Use randomization to generate the set of candidate vectors \( \{\mathbf{w}_{\text{cand},l}\}_{l=1}^{L} \), and then use (15) to compute \( \{\mathbf{w}_{\text{cand},l}\}_{l=1}^{L} \).
- For each \( \mathbf{w}_{\text{cand},l} \), identify the constraint that is violated at most by checking \(|\mathbf{w}_{\text{cand},l}^H\mathbf{h}_i| - \varepsilon \mathbf{w}_{\text{cand},l}||^2 \geq C_i, \forall i = 1, \ldots, M.\)
- Re-scale each vector \( \mathbf{w}_{\text{cand},l} \) so that the most violated constraint is satisfied with equality.
- Pick the smallest-norm weight vector among the re-scaled candidate weight vectors.

\( L = 3000 \) randomizations are used in each simulation run in our algorithm and the algorithms of [1] and [2], respectively. Our reason for choosing different values of \( L \) in different algorithms is that for the techniques of [1] and [2], we follow the authors recommendations given in [1] to use three different types of randomization, while for our technique, only the randomization of the type of (14) is exploited.

Figs. 1 (a) and (b) display the transmitted power versus the minimal required SNR for \( \varepsilon = 0.1 \) and \( \varepsilon = 0.15 \), respectively. Figs. 2 (a) and (b) show histograms of the number of constraints versus the normalized constraint value

\[ g = \frac{|\mathbf{w}^H(\mathbf{h}_i + \delta_i)|^2}{C_i} \]

for SNR = 7 dB and SNR = 9 dB, respectively. In both plots of this figure, \( \varepsilon = 0.15 \). Clearly, all parts of the histograms in Fig. 2 that correspond to the values \( g < 1 \) indicate the percentage of users for which the QoS requirements are violated.

From Fig. 1, it can be seen that the approach of [2] may lead to unnecessary high transmitted power values. This is especially true in the case of high-SNR QoS requirements. At the same time, the proposed technique provides substantially lower transmitted power values which are comparable to that of the non-robust approach of [1].

From Fig. 2, we observe that, as expected, the non-robust technique of [1] leads to violated QoS constraints for a significant part of users, while both the proposed technique and the approach of [2] satisfy QoS constraints for all users. However, it can be clearly seen that the approach of [2] is overly conservative. More specifically, it oversatisfies the QoS constraints much more than our technique. The latter observation is in agreement with the results of Fig. 1 showing that the method of [2] leads to substantially higher values of the transmitted powers as compared to our method.

5. SIMULATION RESULTS

Throughout our simulations, we assume that \( N = 4, M = 8, \) and \( C_i = C \) for all \( i = 1, \ldots, M \). Each user noise is additive white Gaussian noise (AGWN) with variance \( \sigma_i^2 = 1 \) \( (i = 1, \ldots, M) \). For each user, a Rayleigh channel with unit variance has been assumed. For each channel vector \( \mathbf{h}_i \), the corresponding error vector \( \delta_i \) has been uniformly randomly generated in a sphere centered at zero with the radius \( \varepsilon \). The value of \( \varepsilon \) used in the proposed algorithm is always the same as that utilized for generating the channel error vectors.

In all examples, the proposed algorithm is compared to the non-robust technique of [1] and robust technique of [2]. All results are averaged over 300 simulation runs. \( L = 2000 \) and \( L = 3000 \) randomizations are used in each simulation run in our algorithm and the algorithms of [1] and [2], respectively. Our reason for choosing different values of \( L \) in different algorithms is that for the techniques of [1] and [2], we follow the authors recommendations given in [1] to use three different types of randomization, while for our technique, only the randomization of the type of (14) is exploited.

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Fig. 1. Transmitted power versus minimal required receiver SNR.

6. REFERENCES


