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Brief paper

Event based agreement protocols for multi-agent networks[☆]

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ABSTRACT

This paper considers an average consensus problem for multiple integrators over fixed, or switching, undirected and connected network topologies. Event based control is used on each agent to drive the state to their initial average eventually. An event triggering scheme is designed based on a quadratic Lyapunov function. The derivative of the Lyapunov function is made negative by an appropriate choice of the event condition for each agent. The event condition is sampled-data and distributed in the sense that the event detector uses only neighbor information and local computation at discrete sampling instants. The event based protocol design is illustrated with simulations.

1. Introduction

Numerous contributions have been given in the literature for multi-agent systems by research papers (Arcak, 2007; Cortés, 2008; Jadbabaie, Lin, & Morse, 2003; Lin, Broucke, & Francis, 2004; Moreau, 2005; Olfati-Saber & Murray, 2004; Tanner, Jadbabaie, & Pappas, 2007; Xiao & Wang, 2008) and monographs (Mesbahi & Egerstedt, 2010; Ren & Beard, 2008). Continuous communication between neighboring agents is often used for distributed consensus protocol design. While continuous communication is an ideal assumption, it is more realistic to interact intermittently at discrete sampling instants (Chen & Francis, 1995). One choice is to use periodic synchronous sampling (Xie, Liu, Wang, & Jia, 2009a,b); however, it is undesirable and unnecessary to update the control actions for all agents at the same time.

Event based control is an alternative to time triggered control (Henningsson, Johannesson, & Cervin, 2008; Lunze & Lehmann, 2010). The distinct feature of event based control is that control action is updated only when some specific event occurs. For example, a logic condition is violated or the network topology is changed. By comparison with time triggered control, event based control has the often cited advantage on communication reduction. Since the pioneering paper (Åström & Bernhardsson, 2002), event based control has been studied extensively in networked control systems (Wang & Hovakimyan, 2012), decentralized systems (Mazo & Tabuada, 2011; Wang & Lemmon, 2011), and in many cases it outperforms the traditional time triggered control (Meng & Chen, 2012). It has also been proved especially useful in multi-agent systems, such as consensus algorithm (Dimarogonas, 2011; Dimarogonas & Frazzoli, 2009; Dimarogonas, Frazzoli, & Johansson, 2012; Dimarogonas & Johansson, 2009; Liu & Chen, 2010, 2011; Seyboth, Dimarogonas, & Johansson, 2013; Shi & Johansson, 2011), formation control (Tang, Liu, & Chen, 2011), tracking control (Hu, Chen, & Li, 2011a,b), and path planning (Teixeira, Dimarogonas, Johansson, & Sousa, 2010a,b).

The focus here is the event based consensus problem, which arises in a variety of domains including cooperative control of multiple autonomous vehicles, cooperative robotics, and wireless sensor networks. Interested readers are referred to the above cited references on theoretic research on event based consensus protocols. A common feature of these references is continuous communication and event based control updating. Such continuous detection and updating do not meet the original purpose of introducing event based control as a means for reducing communication requirements between interconnected subsystems, since to implement the continuous event detector requires delicate hardware to monitor and check the event condition constantly, which may also become a major source of energy consumption.

Based on the above observation, the concept of sampled-data event detection is defined as periodic evaluation of the event condition. This paper is devoted to the development and analysis of distributed event based algorithms with sampled-data event detection for solving average consensus problems that are defined

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over undirected, connected network topologies. The analysis is begun with consensus problems over a fixed topology. A relatively straightforward extension to the analysis of switching topologies is also presented. To the best knowledge of the authors, this paper is the first to address consensus problems of multi-agent systems using a sampled-data event detector, which is an improvement over continuous event detectors. Besides the sampled-data event detector here admits a minimum inter-event time which is lower bounded by the synchronous sampling period. This is beneficial for the event detector design of each agent to reduce communication between neighboring agents and save sensor energy for event detection. A Lyapunov-based approach is used which is instrumental in recent studies on the consensus of multi-agent systems using event driven communication. In contrast to commonly used Lyapunov functions in existing work, a new Lyapunov function is introduced as abstraction of the detailed dynamical models. It is shown that the parameters of the event detector can be selected so that the time derivative of the Lyapunov function calculated along the trajectories of the closed-loop system is negative semi-definite. With the aid of LaSalle's invariance principle, each agent can be shown to converge to the initial average of all agents.

There are two main contributions in this paper. The first one is to provide a new event based consensus algorithm with sampled-data event detection for multi-agent systems. This approach is fundamentally different from previously developed methods, and the differences facilitate our implementation of event detectors in a sampled-data fashion. The second main contribution is the proposal of new event based consensus algorithms for switching network topologies with distributed and sampled-data event detection that demonstrates a close link between the fixed topology and switching topology.

The remainder of this paper is organized as follows. Section 2 is devoted to an introduction of some concepts in algebraic graph theory and a formal statement of the problem; whereas Section 3 states the main results, which will be extended to switching topologies in Section 4. In Section 5, the simulation results are presented to validate our analysis results. Finally, Section 6 discusses conclusions and possible extensions.

2. Preliminaries and problem formulation

2.1. Algebraic graph theory

Some concepts and facts about algebraic graph theory will be examined since the interaction topology of multi-agent networks can be modeled by an undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, which consists of a finite vertex set $\mathcal{V} = \{v_1, v_2, \ldots, v_n\}$, representing n agents, and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, corresponding to the communication links between agents (Godsil & Royle, 2001). If $v_iv_j \in \mathcal{E}$ is an edge, then v_i and v_j are adjacent or v_j is a neighbor of v_i, and for an undirected graph, $v_iv_j \in \mathcal{E}$ iff $v_jv_i \in \mathcal{E}$. Analogously, the neighborhood $\mathcal{N}_i(\mathcal{G})$ of agent v_i can be mathematically defined as

$$\mathcal{N}_i(\mathcal{G}) = \left\{ j \,\middle|\, v_iv_j \in \mathcal{E}, j \neq i \right\},$$

which contains all indexes of agents that agent v_i can communicate with. A path of length r from v_{i_0} to v_{i_r} in a graph is a sequence of $r+1$ distinct vertices starting with v_{i_0} and ending with v_{i_r}

$$v_{i_0}, v_{i_1}, \ldots, v_{i_r},$$

such that for $k = 0, 1, \ldots, r-1$, the consecutive vertices v_{i_k} and $v_{i_{k+1}}$ are adjacent. Graph $\mathcal{G}$ is connected if there is a path between any two vertices of a graph $\mathcal{G}$.

A graph also admits matrix representations. Some of these matrices, such as the adjacency matrix, the degree matrix, and the Laplacian matrix, will be reviewed subsequently.

The adjacency matrix $\mathcal{A}(\mathcal{G})$ encoding of the adjacency relationship in the graph $\mathcal{G}$ is defined such that

$$a_{ij} = \begin{cases} 1 & \text{if } v_iv_j \in \mathcal{E}, \\ 0 & \text{otherwise}, \end{cases}$$

where a_{ij} is the (i, j) entry of the adjacency matrix $\mathcal{A}(\mathcal{G}) \in \mathcal{R}^{n\times n}$. The adjacency matrix of an undirected graph is symmetric because $a_{ij} = a_{ji}$ for all $i \neq j$.

The degree matrix $\mathcal{D}(\mathcal{G})$ for an undirected graph $\mathcal{G}$ is a diagonal matrix

$$\text{diag}\,\{d_1, d_2, \ldots, d_n\}$$

with d_i being the cardinality of agent v_i's neighbor set $\mathcal{N}_i(\mathcal{G})$.

The Laplacian matrix $\mathcal{L}(\mathcal{G})$ associated with an undirected graph $\mathcal{G}$ is defined as

$$\mathcal{L}(\mathcal{G}) = \mathcal{D}(\mathcal{G}) - \mathcal{A}(\mathcal{G}),$$

where $\mathcal{D}(\mathcal{G})$ is the degree matrix of $\mathcal{G}$ and $\mathcal{A}(\mathcal{G})$ is its adjacency matrix. For undirected graphs, the Laplacian matrix $\mathcal{L}(\mathcal{G})$ is symmetric and positive semi-definite, that is, $\mathcal{L}(\mathcal{G}) = \mathcal{L}(\mathcal{G})^T \geq 0$; hence its eigenvalues are real and can be ordered as

$$\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$$

with $\lambda_1 = 0$ and λ_2 is the smallest nonzero eigenvalue for connected graphs. The vector $\mathbf{1}$, with all entries equal to 1, is an eigenvector of $\mathcal{L}(\mathcal{G})$ associated with eigenvalue 0.

2.2. Consensus problem

The dynamics associated with each agent $v_i \in \mathcal{V}$ is described by the following equation:

$$\dot{x}_i(t) = u_i(t), \tag{1}$$

where $x_i \in \mathbb{R}$ is the state, $u_i \in \mathbb{R}$ is the control input of the ith agent. The following remarks are in order.

Remark 1. In order not to overshadow the main idea and complicate the notation, the case that scalar agents over unweighted graphs is considered. However, the framework proposed in this paper can be extended to design event based consensus protocols for multi-agent systems over weighted topology and with higher dimensional agents, that is, $x_i \in \mathbb{R}^p$.

The overall goal is to propose an event based control mechanism to reduce communication between neighboring agents along with the energy consumption of event detection for each agent while preserving asymptotic property of consensus. Therefore, an event detector is configured at each agent which is used to determine when the sampled local information should be used to update the control actions of itself and its neighbors. The event condition for agent v_i has the following form

$$\left\| e_i\left(t_k^i + lh\right)\right\|_2^2 \leq \sigma_i \left\| z_i\left(t_k^i + lh\right)\right\|_2^2, \quad l = 1, 2, \ldots \tag{2}$$

where σ_i is a positive scalar to be determined later, t_k^i is the kth event instant for agent v_i and is an integer multiple of h, h is the sampling period for all agents synchronized physically by a clock, $e_i\left(t_k^i + lh\right)$ is defined as the difference between the state at the last event time and the currently sampled state

$$e_i\left(t_k^i + lh\right) = x_i\left(t_k^i\right) - x_i\left(t_k^i + lh\right),$$

and

$$z_i\left(t_k^i + lh\right) = \sum_{j\in\mathcal{N}_i(\mathcal{G})} \left(x_i(t_k^i + lh) - x_j\left(t_k^i + lh\right)\right).$$

Remark 2. At each sampling instant, each agent broadcasts its state information to the neighbors and also receives state information from its neighbors for event detection. If the condition in (2) is satisfied, no further action is required for agent v_i; otherwise, agent v_i will update its own control action and notify its neighbors to update their control actions by using its current state information. The violation of the inequality in (2) has the effect of resetting the error $e_i\left(t_k^i + lh\right)$ to zero; at the same time, the event condition is satisfied again. The event instants for agent v_i are thus defined iteratively by

$$t_{k+1}^i = t_k^i + h \inf\left\{l : \|e_i(t_k^i + lh)\|_2^2 > \sigma_i \|z_i(t_k^i + lh)\|_2^2\right\},$$

where $t_0^i = 0$ is the initial time. Obviously, all the measurements $x_i\left(t_k^i\right)$ are subsequence of the sampled state $x_i(kh)$, that is to say, the event instants $\left\{t_0^i, t_1^i, \ldots\right\} \subseteq \{0, h, 2h, \ldots\}$. This means that the inter-event times $\left\{t_{k+1}^i - t_k^i, k = 0, 1, \ldots\right\}$ are at least lower bounded by the sampling period h for all agents.

Remark 3. While the proposed event based consensus scheme and the sampled-data consensus in Xie et al. (2009a,b) share a common sampling interval in information exchange, they are fundamentally different. For the sampled-data consensus, all the data sampled are used for actuation; for the event based consensus, all the data sampled are used for event detection; if the event condition of agent v_i is satisfied at the sampling instant kh, then the state information $x_i(kh)$ will not be used for updating its own and neighbors' control laws. However, agent v_j with $j \notin \mathcal{N}_i(\mathcal{G})$ may update its actuation at the sampling instant kh. Therefore, the average actuator updating period is larger than the sampling period h since only a part of the data sampled are used for actuation. Moreover, the proposed event based actuator updates are asynchronous in general. This is in contrast to the sampled-data consensus in which the actuator updates are synchronous. Specially, when $\sigma < 0$, the event condition in (2) is not satisfied at each sampling instant, and the event based consensus thus reduces to the sampled-data consensus.

Remark 4. The advantages of the event condition in (2) over existing ones are obvious. Firstly, different from centralized event detectors in Dimarogonas and Johansson (2009), Dimarogonas et al. (2012), and Liu and Chen (2010), that is, every agent has to be aware of the global information, the event detector in (2) is distributed in the sense that each agent needs only the information from its neighbors to decide the updating instants. Secondly, different from the distributed event detector in Dimarogonas and Johansson (2009), the event detector in (2) does not need to know the rendezvous location in advance and access to its global position. Each agent needs only the relative displacements with respect to its neighbors and the relative displacement itself at different times. Thirdly, different from the continuous event detector in Seyboth et al. (2013), which requires continuous local event detection and the continuous event detectors in Dimarogonas and Frazzoli (2009); Dimarogonas et al. (2012); Dimarogonas and Johansson (2009); Liu and Chen (2010), which require both continuous local event detection and continuous communication between neighboring agents, the event detector in (2) can greatly reduce the sensor energy consumption and network bandwidth usage by checking the event condition at discrete sampling instants only. Finally, it is worth noting that existing results on distributed methods can only guarantee the nonexistence of accumulation points, but fail to provide the minimum inter-event time. However, the event detector in (2) inherently admits a minimum inter-event time h as mentioned previously.

To reduce clutter in the notation, define

$$\hat{x}_i(t) \triangleq x_i\left(t_k^i\right), \quad \text{for } t_k^i \le t < t_{k+1}^i,$$

which converts the discrete-time signal $x_i\left(t_k^i\right)$ into the continuous-time signal $\hat{x}_i(t)$ simply by holding it constant until the next event occurs. With the notation defined above, an event based consensus algorithm is given by

$$u_i(t) = -\sum_{j \in \mathcal{N}_i(\mathcal{G})} \left(\hat{x}_i(t) - \hat{x}_j(t)\right). \tag{3}$$

Remark 5. Note that the control law is not piecewise constant between the event times $\left\{t_0^i, t_1^i, \ldots\right\}$ but piecewise constant between the sampling instants $\{0, h, 2h, \ldots\}$ since the control law will be updated both at its own event times $\left\{t_0^i, t_1^i, \ldots\right\}$ as well as the event times of its neighbors $\bigcup_{j \in \mathcal{N}_i(\mathcal{G})} \left\{t_0^j, t_1^j, \ldots\right\}$, but at discrete sampling instants only.

The asymptotic consensus problem is said to be solved if one can find an event based protocol such that for all $x_i(0)$, and all $i, j = 1, \ldots, n$, $\left\|x_i(t) - x_j(t)\right\|_2 \longrightarrow 0$ as $t \longrightarrow \infty$.

3. Multi-agent networks with fixed topology

Tentatively, the topology is assumed to be fixed, then the dependence on the graph $\mathcal{G}$ can be dropped in the corresponding notation.

Under the control law given in the previous section, the closed-loop system for agent v_i can be obtained that

$$\dot{x}_i(t) = -\sum_{j \in \mathcal{N}_i} \left(\hat{x}_i(t) - \hat{x}_j(t)\right).$$

Combining the definition of $e\left(t_k^i + lh\right)$, the dynamics of agent v_i for $t \in \left[t_k^i + lh, t_k^i + lh + h\right)$ is then given by

$$\begin{aligned}
\dot{x}_i(t) &= -\sum_{j \in \mathcal{N}_i} \left(x_i\left(t_k^i\right) - x_j\left(t_{k'}^j\right)\right) \\
&= -\sum_{j \in \mathcal{N}_i} \left(x_i\left(t_k^i + lh\right) - x_j\left(t_k^i + lh\right)\right) \\
&\quad - \sum_{j \in \mathcal{N}_i} \left(x_i\left(t_k^i\right) - x_i\left(t_k^i + lh\right)\right) \\
&\quad + \sum_{j \in \mathcal{N}_i} \left(x_j\left(t_{k'}^j\right) - x_j\left(t_k^i + lh\right)\right) \\
&= -\sum_{j \in N_i} \left(x_i\left(t_k^i + lh\right) - x_j\left(t_k^i + lh\right)\right) \\
&\quad - \sum_{j \in \mathcal{N}_i} \left(e_i\left(t_k^i + lh\right) - e_j\left(t_k^i + lh\right)\right),
\end{aligned}$$

where $t_{k'}^j$ is defined as

$$t_{k'}^j = \max\left\{t \,\middle|\, t \in \{t_k^j, k = 0, 1, \ldots\},\ t \le t_k^i + lh\right\}.$$

The equations above for $t \in [kh, (k+1)h)$ can also be written in compact form as

$$\dot{x}(t) = -\mathcal{L}x(kh) - \mathcal{L}e(kh), \tag{4}$$

where

$$x = [x_1, \ldots, x_n]^T, \qquad e = [e_1, \ldots, e_n]^T,$$

and $\mathcal{L}$ is the Laplacian matrix.

Denote the state average of agents as

$$\bar{x}(t) = \frac{1}{n} \sum_{i=1}^{n} x_i(t),$$

then under the event based protocol in (3)

$$\dot{\bar{x}}(t) = \frac{1}{n} \sum_{i=1}^{n} \dot{x}_i(t) = \frac{1}{n} \mathbf{1}^T \dot{x}(t) = -\frac{1}{n} \mathbf{1}^T \mathcal{L} \hat{x}(t) \equiv 0$$

since $\mathbf{1}^T \mathcal{L} = \mathbf{0}^T$. Therefore, it is time-invariant, and define the disagreement vector as

$$\delta(t) = x(t) - \bar{x}(t)\, \mathbf{1} = x(t) - \bar{x}\mathbf{1}.$$

Given a connected graph $\mathcal{G}$, consider the following Lyapunov functional candidate:

$$V(x(t)) = \frac{1}{2} x^T(t)\, x(t), \tag{5}$$

that is, half of the sum of squares of the states.

Remark 6. It is worth mentioning that existing results on event based consensus algorithm resort mostly to a Lyapunov-type argument, that is, define the following Lyapunov function

$$V(t) = \frac{1}{2}\delta^T(t)\,\delta(t) \quad \text{or} \quad V(t) = \frac{1}{2}x(t)^T\mathcal{L}x(t)$$

and assess the convergence to the origin. Different from existing results, LaSalle's invariance principle will be introduced to analyze the convergence of an event based agreement protocol to the agreement subspace instead of the origin, where the meeting location for multi-agent systems over an undirected, connected graph is exactly $x_1 = x_2 = \cdots = x_n = \bar{x}$.

Remark 7. A claim is made that the function in (5) must decrease to reach the agreement subspace, and one can never increase the function to achieve consensus at their initial average. To see this, apply the Jensen's inequality to the convex function $f(y) = y^2$,

$$V(x) = \frac{n}{2}\sum_{i=1}^{n}\frac{1}{n}x_i^2 \geq \frac{n}{2}\left(\sum_{i=1}^{n}\frac{1}{n}x_i\right)^2 = \frac{n}{2}\bar{x}^2 = V(\bar{x}\mathbf{1}).$$

Therefore, a valid event based protocol candidate would be the one which can make the function in (5) decrease with respect to t.

Now consider the time evolution of the function $V(x(t))$ in (5) along the trajectory generated by (4) for any $t \in [kh, (k+1)h)$, which is given by

$$\begin{aligned}\dot{V}(t) &= -x^T(t)\,\mathcal{L}\,(x(kh)+e(kh))\\ &= (t-kh)\,(x(kh)+e(kh))^T\,\mathcal{L}^2\,(x(kh)+e(kh))\\ &\quad -x^T(kh)\,\mathcal{L}\,(x(kh)+e(kh))\\ &\leq -x^T(kh)\,\mathcal{L}\,(x(kh)+e(kh))\\ &\quad +h\lambda_n\,(x(kh)+e(kh))^T\,\mathcal{L}\,(x(kh)+e(kh))\\ &= -(1-h\lambda_n)x^T(kh)\,\mathcal{L}x(kh) - x^T(kh)\,\mathcal{L}e(kh)\\ &\quad + h\lambda_n e^T(kh)\,\mathcal{L}e(kh) + 2h\lambda_n x^T(kh)\,\mathcal{L}e(kh).\end{aligned}$$

Using the inequality

$$x^T(kh)\,\mathcal{L}e(kh) \leq \frac{1}{2}x^T(kh)\,\mathcal{L}x(kh) + \frac{e^T(kh)\,\mathcal{L}e(kh)}{2}$$

$\dot{V}(t)$ can be bounded as

$$\dot{V}(t) \leq -\frac{1}{2}x^T(kh)\,\mathcal{L}x(kh) + \frac{1}{2}e^T(kh)\,\mathcal{L}e(kh)$$

with $2h\lambda_n \leq 1$. Combining the event condition in (2), we get

$$\dot{V}(t) \leq -\frac{1}{2}(1-\lambda_n^2\sigma_{\max})x^T(kh)\,\mathcal{L}x(kh)$$

where $\sigma_{\max} = \max\{\sigma_i, i = 1, \ldots, n\}$. Thereby

$$\dot{V}(t) \leq 0$$

for any $k \in \{0, 1, 2, \ldots\}$ and $t \in [kh, (k+1)h)$ if

$$0 < h \leq \frac{1}{2\lambda_n} \quad \text{and} \quad 0 < \sigma_{\max} < \frac{1}{\lambda_n^2}.$$

Moreover, based on the fact that the underlying communication topology $\mathcal{G}$ is connected, the largest invariant set contained in the set is

$$\left\{x \in \mathbb{R}^n \,\middle|\, \dot{V}(t) = 0\right\} = \operatorname{span}\{\mathbf{1}\}.$$

Thus, from LaSalle's invariance principle, $\dot{V}(t) \leq 0$ for $\forall t \geq 0$ implies consensus for all agents.

Hence, the following theorem can be concluded.

Theorem 8. *Consider the system in (1) over a connected communication graph with the protocol in (3) driven by event condition in (2). Then all agents are asymptotically converging to their initial average if*

$$0 < h \leq \frac{1}{2\lambda_n} \quad \textit{and} \quad 0 < \sigma_{\max} < \frac{1}{\lambda_n^2}.$$

Remark 9. The choices of the sampling period h and the parameters σ_i, $i = 1, 2, \ldots, n$ require some global information about the topology. An upper bound on the largest eigenvalue λ_n can be found by

$$\lambda_n \leq 2d_{\max} \leq 2(n-1),$$

based on the result in Grone and Merris (1994) and the fact that $d_{\max} \leq n-1$. Therefore, the sampling period h and the parameters σ_i can be chosen with the constraints

$$0 < \sigma_{\max} < \frac{1}{4(n-1)^2},$$

and

$$0 < h \leq \frac{1}{4(n-1)}.$$

There is a way to choose the sampling period h locally and realize sampling synchronization for all agents if each agent knows n, the total number of agents. This can be done by scaling the maximum sampling period by a common scalar α with $0 < \alpha < 1$ known by all agents, that is, each agent chooses

$$h = \frac{\alpha}{4(n-1)}$$

as its local sampling period. Also notice that h and σ_i, $i = 1, 2, \ldots, n$, have only upper bound constraints; therefore, small enough α and σ_i are always appropriate. It is more realistic to approximate the continuous event detection by a high fast rate sampled-data event detection. Intuitively speaking, the smaller σ_i will lead to higher frequency of control update and faster convergence rate for the system, so there is a trade-off between the performance and control updating cost in this sense.

Remark 10. According to Xie et al. (2009a), the maximum stabilizing sampling period to solve the average consensus problem for undirected and connected graphs is $2/\lambda_n$. Although this maximum sampling period is four times higher than the one presented in Theorem 8, the average actuator updating period in this paper is determined by both the sampling period and event detectors, and it is larger than the sampling period in general. In addition, our design is performed in continuous time, whereas the sampled-data consensus approach is a purely discrete-time design, which completely ignores what is happening between sampling instants. Therefore, there might be large inter-sample amplitudes.

4. Multi-agent networks with switching topology

In this section, the event based protocol will be extended to the case when the underlying undirected communication topology $\mathcal{G}$ switches among possible connected graphs with the same finite vertex set:

$$\{\mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_m\}$$

with the index set $J = \{1, \ldots, m\}$. The switching networks can be modeled using a piecewise constant switching signal

$$s(t) : [0, +\infty) \rightarrow J.$$

The switching times are defined by

$$0 = T_0 < T_1 < T_2 < \cdots.$$

Denote the active topology at the sampling instant kh as $\mathcal{G}_{s(kh)}$ and the corresponding Laplacian matrix by $\mathcal{L}(\mathcal{G}_{s(kh)})$.

Remark 11. The topology can switch not only at sampling instants but also between sampling instants. There may be several switches taking place between two consecutive sampling instants, but only the recent one to the current sampling instant has influence on controllers and event detectors. Agents whose neighborhood relation remain the same at two consecutive sampling instants will not be affected by switching; agents with communication link changes from two consecutive sampling instants have to evaluate their event conditions and control laws using the current set of neighbors.

In the case of switching topology, the event condition and event based consensus protocol can be defined similarly as the one in (2) and (3), respectively.

The common Lyapunov function

$$V(t) = \frac{1}{2} x^T(t)\, x(t)$$

can be used to investigate the convergence of the event based consensus protocol for switching topologies over undirected and connected graphs. Then, with respect to (1), the derivative of $V(t)$ in the time interval $[kh, (k+1)h)$ is given by

$$\dot{V}(t) = -x^T(t)\, \mathcal{L}(\mathcal{G}_{s(kh)})\, (x(kh) + e(kh)).$$

If

$$0 < \sigma_{\max} < \frac{1}{\lambda_n^2\left(\mathcal{G}_{s(kh)}\right)},$$

and

$$0 < h \le \frac{1}{2\lambda_n\left(\mathcal{G}_{s(kh)}\right)},$$

then similar to the fixed topology case, it can be proved

$$\dot{V}(t) \le -\frac{1}{2}\left(1 - \lambda_n^2\left(\mathcal{G}_{s(kh)}\right)\sigma_{\max}\right) x^T(kh)\, \mathcal{L}(\mathcal{G}_{s(kh)}) x(kh),$$

with $\lambda_n\left(\mathcal{G}_{s(kh)}\right)$ being the largest eigenvalue of the Laplacian matrix $\mathcal{L}\left(\mathcal{G}_{s(kh)}\right)$. Since the set

$$\left\{x \in \mathbb{R}^n \,\middle|\, \dot{V}(t) = 0\right\} = \text{span}\{\mathbf{1}\}$$

is independent of any individual topology as it switches among a number of connected graphs, the following theorem can thus be obtained.

Theorem 12. *Consider the system in (1) switches over a number of connected graphs with the protocol in (3) driven by the event condition in (2). Then all agents are asymptotically converging to their initial average if*

$$0 < \sigma_{\max} < \frac{1}{\lambda_{\max}^2}, \quad \text{and} \quad 0 < h \le \frac{1}{2\lambda_{\max}},$$

where $\lambda_{\max} = \max\{\lambda_n(\mathcal{G}), \mathcal{G} \in \{\mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_m\}\}$ *with* $\lambda_n(\mathcal{G})$ *being the largest eigenvalue of the Laplacian matrix* $\mathcal{L}(\mathcal{G})$.

5. Numerical simulations

The event based consensus protocols proposed are now illustrated by computer simulations.

Example 13. Consider a scenario where four agents are to meet at a single location. Fig. 1 shows the corresponding communication topology among these agents, which is used in Dimarogonas et al. (2012) as well. Note that the graph is connected. Based on the communication topology, the adjacency matrix $\mathcal{A}$ and the degree matrix $\mathcal{D}$ are

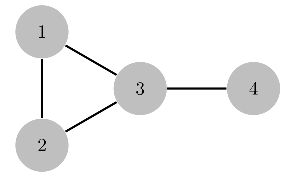


Fig. 1. Communication topology.

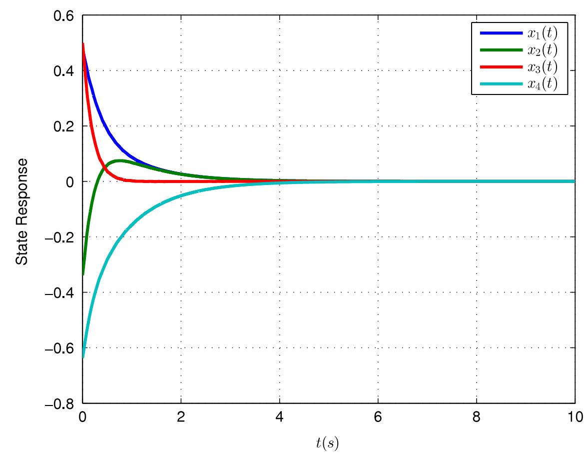


Fig. 2. Evolution of each agent.

$$\mathcal{A} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \qquad \mathcal{D} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

and the Laplacian matrix is thus given by

$$\mathcal{L} = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}.$$

The largest eigenvalue of the Laplacian matrix is $\lambda_n = 4$. The parameters of the event detector for each agent and the sampling period for all agents are chosen as

$$\sigma_1 = \sigma_2 = 0.033, \qquad \sigma_3 = 0.02, \qquad \sigma_4 = 0.06,$$
$$h = 0.002,$$

which satisfy the conditions that

$$\sigma_{\max} < 0.0625, \qquad h \le 0.125.$$

The initial values of agents are chosen as $x(0) = [0.4773 \ -0.3392 \ 0.5 \ -0.6381]^T$. Using the event condition in (2), a simulation is conducted for $t \in [0, 10)$. The evolution of the state and the norm of the disagreement vector $\|x(t) - \bar{x}\mathbf{1}\|$ using event based consensus protocol are shown in Figs. 2 and 3, respectively. It can be seen in both figures that the agents reach consensus at their initial average. The control signal and the time instants when the events occur for each agent are shown in Figs. 4 and 5, respectively. It can be seen that the number of actuator control updates is greatly reduced to reach average consensus compared with continuous communication scheme. Fig. 6 shows the evolution of $\|e_i(kh)\|$ for $i = 1, 2, 3, 4$. In these figures, an event is generated when the error signal norm reaches the threshold $\sqrt{\sigma_i}\|z_i(kh)\|$, and therefore the error signal $\|e_i(kh)\|$ is reset to zero immediately. In addition, the

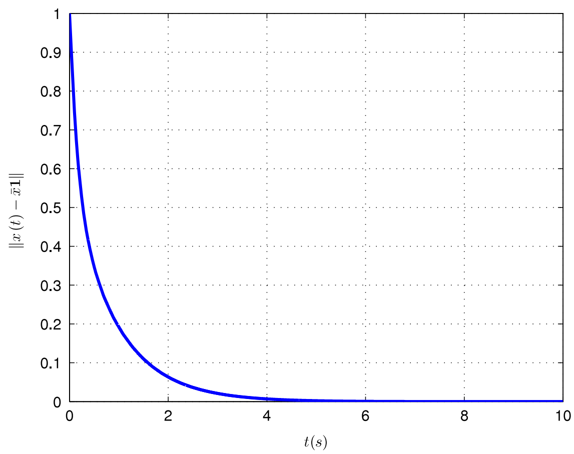


Fig. 3. Evolution of $\|x(t) - \bar{x}\mathbf{1}\|$.

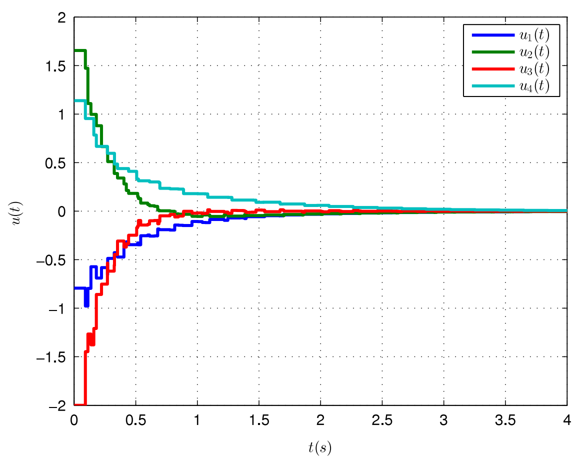


Fig. 4. Control inputs for the agents.

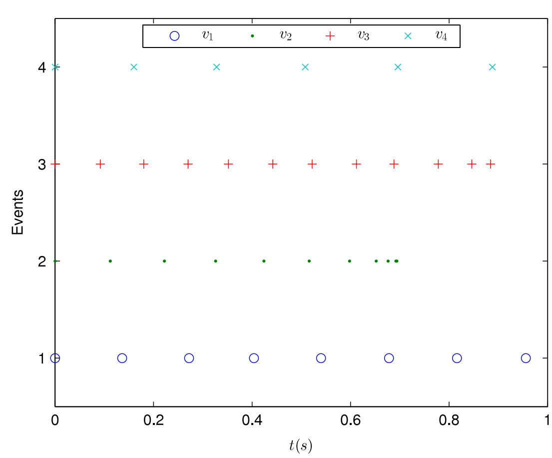


Fig. 5. Event times for each agent.

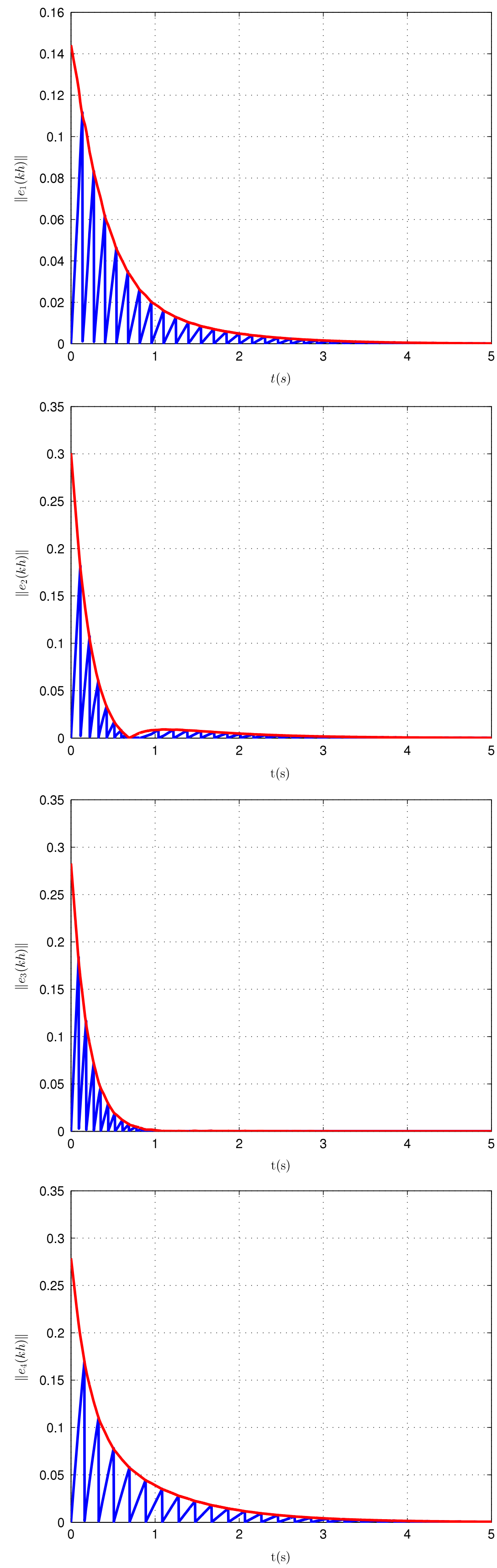


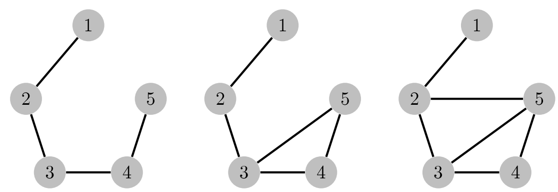
Fig. 6. Evolution of error signals for each agent.

simulation result for each agent is also reported in Table 1. It can be seen from the table that the actual minimum inter-event times for agents v_1 and v_4 are greater than the sampling period 0.002 except agents v_2 and v_3 whose minimum inter-event time is equal to 0.002. Note that the actuation updates are not invoked when the system is in steady state.

Example 14. Five agents switching over three possible interaction topologies are illustrated in Fig. 7. Note that all the graphs are connected. The initial value of each agent is generated randomly from the uniform distribution on the interval $[-10, 10]$, and the

Table 1
Event intervals for the agents.

Agent	v_1	v_2	v_3	v_4
Event times	66	69	75	52
Min interval	0.1320	0.002	0.002	0.1600
Mean interval	0.1516	0.1453	0.1309	0.1955
Max interval	0.1560	0.3460	0.6140	0.1980



(a) $\mathcal{G}_1$. (b) $\mathcal{G}_2$. (c) $\mathcal{G}_3$.

Fig. 7. Switching communication topology.

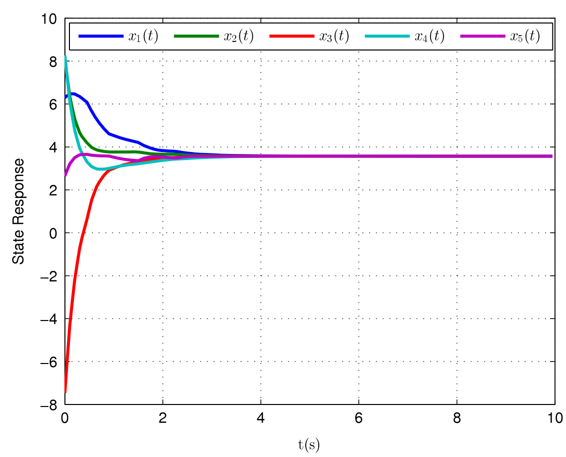


Fig. 8. Evolution of each agent.

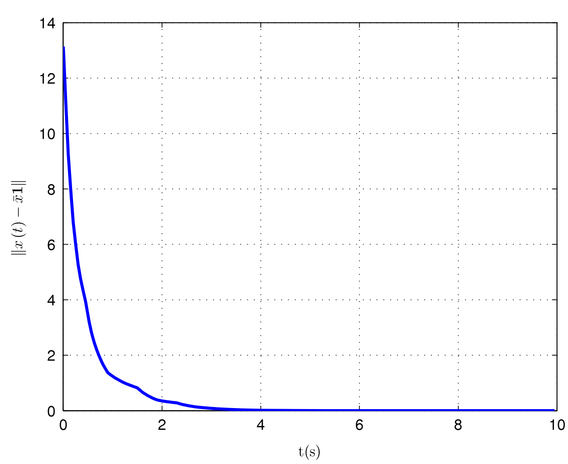


Fig. 9. Evolution of $\|x(t) - \bar{x}\mathbf{1}\|$.

initial network topology is $\mathcal{G}_1$. After the dwell time which is randomly chosen from the uniform distribution on the interval $[0.1, 0.5]$, the network topology switches to another graph which is chosen randomly from the uniform distribution on the index set $J = \{1, 2, 3\}$. Such randomly switching process continuous until the end of simulation. The parameters used of the event detector for each agent and the sampling period for all agents are $\sigma_i = 0.02$, $i = 1, 2, 3, 4, 5$, and $h = 0.05$, respectively. The evolution of the state and the norm of the disagreement vector $\|x(t) - \bar{x}\mathbf{1}\|$ are shown in Figs. 8 and 9, respectively. It can be seen that the

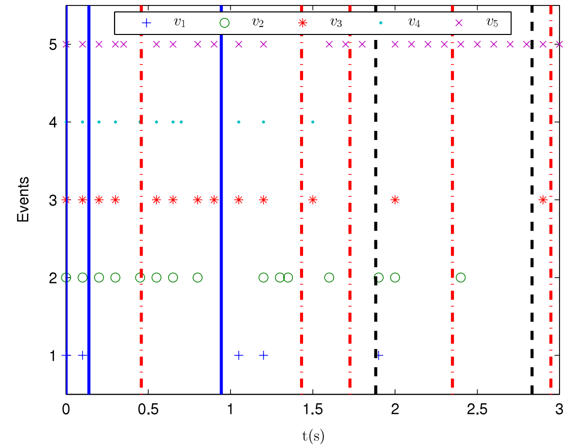


Fig. 10. Event times for each agent.

event conditions work well in the case of switching topology. The simulation result of event times for each agent is shown in Fig. 10, where the solid vertical line denotes switching to topology $\mathcal{G}_1$, the dashed vertical line denotes switching to topology $\mathcal{G}_2$, and the dash-dotted vertical line denotes switching to topology $\mathcal{G}_3$.

6. Conclusions

In this paper, event based control algorithms have been proposed to make multi-agent systems with fixed topology contractive in the sense of consensus. A new Lyapunov function was introduced, and the time derivative of the Lyapunov function was made negative semi-definite by an appropriate choice of event conditions. Based on this Lyapunov function, sampled-data event detectors were designed to drive the states to their initial average. Based on the results for fixed topologies, an event based consensus algorithm for switching topology was also given. These designs were illustrated with simulations. Future work will address the generalization to directed topology networks with communication delays as well as disturbances. Moreover, the utilization of a common sampling period for all agents might be restrictive in distributed networks. Employing different sampling periods for different agents would be an interesting extension but may require new tools for the analysis.

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