Network-Based $\mathscr{H}_\infty$ Output Tracking Control

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***Abstract*—This paper is concerned with the problem of $\mathscr{H}_\infty$ output tracking for network-based control systems. The physical plant and the controller are, respectively, in continuous time and discrete time. By using a sampled-data approach, a new model based on the updating instants of the holder is formulated, and a linear matrix inequality (LMI)-based procedure is proposed for designing state-feedback controllers, which guarantee that the output of the closed-loop networked control system tracks the output of a given reference model well in the $\mathscr{H}_\infty$ sense. Both network-induced delays and data packet dropouts have been taken into consideration in the controller design. The network-induced delays are assumed to have both an upper bound and a lower bound, which is more general than those used in the literature. The introduction of the lower bound is shown to be advantageous for reducing conservatism. Moreover, the controller design method is further extended to more general cases, where the system matrices of the physical plant contain parameter uncertainties, represented in either polytopic or norm-bounded frameworks. Finally, an illustrative example is presented to show the usefulness and effectiveness of the proposed $\mathscr{H}_\infty$ output tracking design.**

***Index Terms*—Model reference control, networked control systems, output tracking, sampled-data systems.**

I. Introduction

OUTPUT tracking control (also called model reference control) has wide applications in dynamic processes in industry, economics, and biology. The main objective of tracking control is to make the output of the plant, via a controller, track the output of a given reference model as close as possible. Output tracking is widely used in robot control [11], [30], flight control [3], [23], motor control [19], [34], etc. It has been well recognized that tracking control design is more general and more difficult than stabilization, and in the last few decades, many important results on output tracking have been reported (see, [2], [8]–[10], [24], [31], [33], [38] and the references therein).

The existing results on output tracking control mainly focus on designing continuous-time controllers for continuous-time physical plants. However, in many practical systems, such as computer-based control systems, the plant is controlled by a discrete-time controller with sample and hold devices. In such cases, the system can be expressed as a sampled-data model [6], [7]. Unfortunately, little progress has been made toward solving the problem of output tracking control for sampled-data systems. In addition, it is worth pointing out that the aforementioned results are based on the implicit assumption that the measurements and control commands transmitted between the physical plant and the controller do NOT exhibit aftereffect phenomena, that is, no signal transmission delays have been taken into consideration in the design of output tracking controllers.

It is well known that in many practical systems, the original plant, controller, sensor, and actuator are difficult to be located at the same place, and thus, signals are required to be transmitted from one place to another. In modern industrial systems, these components are often connected over network media, giving rise to the so-called networked control systems (NCSs). Due to its great advantages (such as low cost, reduced weight and power requirements, simple installation and maintenance, and high reliability), NCS receives more and more attention in recent years. Therefore, modeling, analysis, and control of network-based systems with limited communication capability has emerged as a topic of significant interest to the control community, which is highlighted by the recent special issue edited by Antsaklis and Baillieul [1]. Among the reported results on NCS, to mention a few, the stability issue is investigated in [28], [35], and [47], stabilizing controllers are designed in [42], [43], [45], [46], and [48], performance preserved control is studied in [22], [25], [26], [32], and [44], and moving horizon control is proposed in [17]. To the best of the authors' knowledge, however, the problem of output tracking for networked control systems has not been investigated and still remains challenging, which motivates the present study.

In this paper, we investigate the problem of $\mathscr{H}_\infty$ output tracking for network-based control systems. In our study, the controlled plant is in continuous time while the controller is in discrete time, which represents a typical computer-based control scheme. The network-induced delays (from sensor to controller and from controller to actuator) are assumed to have both an upper bound and a lower bound, which is more general than those used in the literature (where the lower bound is assumed to be zero). By using a sampled-data approach, a new model based on the updating instants of the zero-order hold (ZOH) is formulated, and a linear matrix inequality (LMI)-based procedure is proposed for designing state-feedback controllers, which guarantee the output of the closed-loop networked control system tracks the output of a given reference model well in the $\mathscr{H}_\infty$ sense. Both network-induced delays and data packet dropouts have been taken into consideration in the controller design. It is shown, via a numerical example, that the introduction of the lower delay bound is advantageous for reducing conservatism. Moreover, the controller design method is further extended to

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more general cases, where the system matrices of the physical plant contain parameter uncertainties, represented in either polytopic or norm-bounded frameworks. Finally, an illustrative example is presented to show the usefulness and effectiveness of the proposed $\mathscr{H}_\infty$ output tracking design.

The remainder of this paper is organized as follows. The problem of network-based $\mathscr{H}_\infty$ output tracking control is formulated in Section II. Section III presents the main results on $\mathscr{H}_\infty$ output tracking performance analysis and controller design, based on which the problem of robust $\mathscr{H}_\infty$ output tracking control is solved in Section IV. Section V gives an illustrative example and we conclude the paper in Section VI.

Notation: The notation used throughout the paper is fairly standard. The superscript "T" stands for matrix transposition; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and the notation $P > 0\,(\geq 0)$ means that P is real symmetric and positive definite (semidefinite). In symmetric block matrices or complex matrix expressions, we use an asterisk $(*)$ to represent a term that is induced by symmetry and $\mathrm{diag}\{\ldots\}$ stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. The space of square-integrable vector functions over $[0, \infty)$ is denoted by $L_2[0, \infty)$, and for $w = \{w(t)\} \in L_2[0, \infty)$, its norm is given by

$$\|w\|_2 = \sqrt{\int_{t=0}^{\infty} |w(t)|^2\, dt}.$$

II. Problem Formulation

Consider a typical networked control system shown in Fig. 1. Suppose the physical plant is given by the following linear system:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Ew(t) \\ y(t) &= Cx(t) + Du(t). \end{aligned} \tag{1}$$

Here, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^p$ is the control input, $y(t) \in \mathbb{R}^q$ is the output, and $w(t) \in \mathbb{R}^l$ is the disturbance input that satisfies $w = \{w(t)\} \in L_2[0, \infty)$. $A, B, C, D,$ and E are system matrices with appropriate dimensions.

Our purpose is to design a controller, such that the output $y(t)$ of the closed-loop networked control system tracks a reference signal to meet the required tracking performance. Suppose the reference signal $y_r(t)$ is generated by

$$\begin{aligned} y_r(t) &= Hx_r(t) \\ \dot{x}_r(t) &= Gx_r(t) + r(t) \end{aligned} \tag{2}$$

where $y_r(t)$ has the same dimension as $y(t)$; $x_r(t), r(t) \in \mathbb{R}^r$ are, respectively, the reference state and the energy bounded reference input; and G and H are appropriately dimensioned constant matrices with G Hurwitz. It is assumed that both $x(t)$ and $x_r(t)$ are online measurable, and the measurements of $x(t)$ and $x_r(t)$ are transmitted with a single packet. In addition, it is assumed that the sensor is clock-driven, while the controller and ZOH are event-driven. The sampling period is assumed to be h, where h is a positive real constant. Now denote the *updating instant* of the ZOH as t_k, and suppose that the updating signal (successfully transmitted signal from the sampler to the controller and to the ZOH) at the instant t_k has experienced signal transmission delays η_k ($\eta_k = \tau_k + d_k$, where τ_k is the delay from the sampler to the controller and d_k is the delay from the controller to the ZOH). Therefore, the state-feedback controller takes the following form

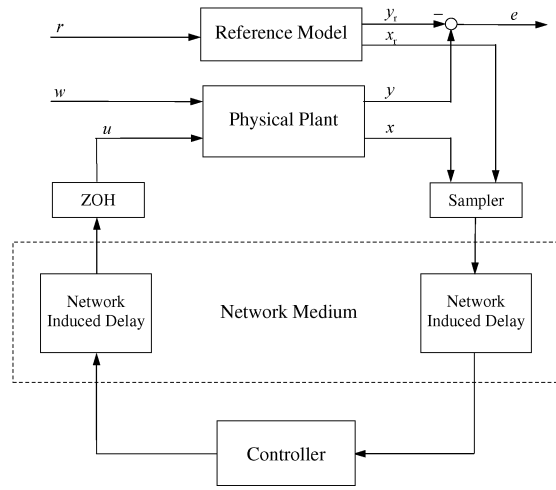

Fig. 1. Networked tracking control system.

$$u(t_k) = K_1 x(t_k - \eta_k) + K_2 x_r(t_k - \eta_k) \tag{3}$$

where K_1 and K_2 are the state-feedback control gains. Thus, considering the behavior of the ZHO, we have

$$u(t) = K_1 x(t_k - \eta_k) + K_2 x_r(t_k - \eta_k), \quad t_k \leq t < t_{k+1} \tag{4}$$

with t_{k+1} being the next updating instant of the ZOH after t_k.

A natural assumption on the network-induced delays η_k can be made as follows:

$$\eta_m \leq \eta_k \leq \eta_M \tag{5}$$

where η_m and η_M denote the lower and upper delay bounds, respectively.

In addition, at the updating instant t_k, the number of accumulated data packet dropouts since the last updating instant t_{k-1} is denoted as δ_k. We assume that the maximum number of data packet dropouts is $\bar{\delta}$, that is

$$\delta_k \leq \bar{\delta}. \tag{6}$$

Then, it can be seen from (5) and (6) that

$$t_{k+1} - t_k = (\delta_{k+1} + 1)\, h + \eta_{k+1} - \eta_k. \tag{7}$$

Therefore, from (1)–(4), we can obtain the following augmented closed-loop system:

$$\begin{aligned} \dot{\zeta}(t) &= \bar{A}\zeta(t) + \bar{B}\zeta(t_k - \eta_k) + \bar{E}v(t) \\ e(t) &= \bar{C}\zeta(t) + \bar{D}\zeta(t_k - \eta_k) \\ & t_k \leq t < t_{k+1} \end{aligned} \tag{8}$$

where

$$\zeta(t) = \begin{bmatrix} x(t) \\ x_r(t) \end{bmatrix}, e(t) = y(t) - y_r(t), v(t) = \begin{bmatrix} w(t) \\ r(t) \end{bmatrix},$$
$$\bar{A} = \begin{bmatrix} A & 0 \\ 0 & G \end{bmatrix}, \bar{B} = \begin{bmatrix} BK_1 & BK_2 \\ 0 & 0 \end{bmatrix}, \bar{E} = \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix},$$
$$\bar{C} = [\,C \quad -H\,], \bar{D} = [\,DK_1 \quad DK_2\,]. \tag{9}$$

Then, the tracking requirements are expressed as follows

1) The augmented closed-loop system in (8) with $v(t) \equiv 0$ is asymptotically stable;
2) The effect of $w(t)$ and $r(t)$ on the tracking error $e(t)$ is attenuated below a desired level in the $\mathscr{H}_\infty$ sense. More specifically, it is required that
$$\|e\|_2 < \gamma \|v\|_2 \tag{10}$$
for all nonzero $v \in L_2[0, \infty)$ under zero initial condition, where $\gamma > 0$.

We say the $\mathscr{H}_\infty$ output tracking performance γ is achieved if the aforementioned two requirements are met.

The model of the networked control system formulated earlier deserves some remarks.

Remark 1: It is important to note that in (4), t_k refers to the *updating instant* of the ZOH. While in [42], the controller is expressed as

$$u(t) = F\bar{x}(t_k), \quad t_k \le t < t_{k+1} \tag{11}$$

with t_k standing for the *sampling instant*. It should be noted that when the controller and actuator are event-driven, we cannot use the sampling instant to express the behavior of the ZOH. The reason is that the signal transmission delays may not necessarily be integer multiples of the sampling period, and thus, the ZOH may be updated between sampling instants. By using the updating instant in this paper, we do not need to synchronize the ZOH and the sampler, and thus, the networked control model formulated here is essentially different from that in [42], and is more general, though they appear to be similar.

Remark 2: The assumption on the network-induced delays η_k made in (5) is more general than those in [42]–[45]. The main difference lies in the lower bound that we introduce. By assuming $\eta_m = 0$, (5) is the same as those in [42]–[45]. In the next section, we will present a new approach that deals with the general assumption on η_k in (5). The introduction of the lower-bound η_m will be shown, via a numerical example, to be advantageous for reducing conservatism.

Remark 3: In the literature, there are some approaches that consider stochastic models for the dropout and delay on the packets transmitted over networks [41], [46]. These models usually consider a discrete-time system controlled by a discrete-time controller, while our approach treats the sampled-data problem directly, which constitutes the main difference between our model and the existing stochastic models. In addition, these stochastic models usually assume a probabilistic structure on the delay or dropout, while our approach places bounds on the delay or number of packet dropouts.

Remark 4: In this paper, we adopt the standard $\mathscr{H}_\infty$ norm to measure the tracking performance, which assumes zero initial conditions. Generally speaking, the system response is composed of two parts: zero-state response (due to input only) and zero-input response (due to initial conditions only). By using the standard $\mathscr{H}_\infty$-norm characterization, the problem we formulated earlier only considers the tracking performance due to zero-state response, and zero-input response is not considered. However, it is not difficult to further adapt the results developed in this paper to the case where both the zero-state response and zero-input response are considered. In addition, from another point of view, the response due to nonzero initial conditions could also be seen as a past-time input response. As can be seen in Example 2 in Section V, the tracking controller designed in this paper can also guarantee a good tracking performance for nonzero initial conditions.

Remark 5: In a recent Ph.D. thesis [21], Lian presented a comprehensive study on the analysis, design, modeling, and control of networked control systems. This thesis covers a broad range of problems related to NCSs. However, it is worth commenting that the problem of network-based output tracking control considered in this paper is not covered in [21] as well as any other existing reference. In addition, the method used in this paper is quite different from those in [21], in that the sampled-data behavior is dealt with via a delay system approach that allows an effective treatment of model uncertainties, and all the conditions are characterized using LMIs that can be efficiently solved via standard numerical software.

III. $\mathscr{H}_\infty$ Output Tracking Control Design

A. Main Idea

It is noted that the augmented closed-loop system in (8) is in the form of a sampled-data system. As the time sequence $\{t_k\}$ depends on both the network-induced delays and data packet dropouts, the period $t_{k+1} - t_k$ for the sampled-data system in (8) is variable and uncertain.

Now, let us represent $t_k - \eta_k$ in (8) as

$$t_k - \eta_k = t - t + t_k - \eta_m + \eta_m - \eta_k = t - \eta_m - \eta(t) \tag{12}$$

where

$$\eta(t) = t - t_k + (\eta_k - \eta_m). \tag{13}$$

Then, from (7), we have

$$0 \le \eta(t) \le \kappa \tag{14}$$

where

$$\kappa = \eta_M - \eta_m + \left(\bar{\delta} + 1\right) h. \tag{15}$$

By substituting (12) into (8), we obtain

$$\begin{aligned} \dot{\zeta}(t) &= \bar{A}\zeta(t) + \bar{B}\zeta(t - \eta_m - \eta(t)) + \bar{E}v(t) \\ e(t) &= \bar{C}\zeta(t) + \bar{D}\zeta(t - \eta_m - \eta(t)). \end{aligned} \tag{16}$$

Remark 6: It is worth noting that in the aforesaid transformed system, η_m is a constant delay, and $\eta(t)$ is a nondifferentiable time-varying delay with bound κ. Our main idea in the earlier transformation is to represent the sampled-data system in (8) as a continuous-time system with two successive delay components in the state. The $\mathscr{H}_\infty$ output tracking problem will be solved based on this new model.

Remark 7: In the literature, most of the assumptions on the network-induced delays are given as: $0 \le \eta_k \le \eta_M$. Based on this assumption, the closed-loop system takes the following form:

$$\begin{aligned}\dot{\zeta}(t) &= \bar{A}\zeta(t) + \bar{B}\zeta(t-\bar{\eta}(t)) + \bar{E}v(t)\\ e(t) &= \bar{C}\zeta(t) + \bar{D}\zeta(t-\bar{\eta}(t))\end{aligned} \tag{17}$$

with $0 \le \bar{\eta}(t) \le \bar{\kappa}$, where

$$\bar{\kappa} = \eta_M + \left(\bar{\delta}+1\right)h. \tag{18}$$

In some situations, however, we may know the lower bound of the network-induced delays, that is, $\eta_m \le \eta_k \le \eta_M$. If we still use (17) for this case, the lower bound η_m cannot be taken into consideration. In addition, by comparing (17) and (16), we can see that the upper bound of the nondifferentiable time-varying delay $\eta(t)$ in (16) is smaller by η_m than that of $\bar{\eta}(t)$ in (17), that is, $\bar{\kappa} - \kappa = \eta_m$. In other words, when using (17) for the case $\eta_m \le \eta_k \le \eta_M$, $\eta_m + \eta(t)$ will be treated as an integrated nondifferentiable time-varying delay $\bar{\eta}(t)$, and the value of η_m cannot be taken into consideration in the controller design. In this paper, we will derive results based on (16) by considering η_m and $\eta(t)$ as two successive delay components, which will be shown to be less conservative via a numerical example in the next subsection. However, the existing results on NCS, such as [42]–[45], do not offer including the lower bound η_m in design.

B. $\mathscr{H}_\infty$ Output Tracking Performance Analysis

Section III-B is concerned with the problem of $\mathscr{H}_\infty$ output tracking performance analysis. More specifically, assuming that the matrices A, B, C, D, E, G, H and the controller gains K_1 and K_2 are known, we shall study the conditions under which the augmented closed-loop system in (8) achieves the $\mathscr{H}_\infty$ output tracking performance γ. The following theorem shows that the $\mathscr{H}_\infty$ tracking performance can be guaranteed if there exist some matrices satisfying certain LMIs. This theorem will play an instrumental role in the controller design problem.

Theorem 1: Consider the networked control system in Fig. 1. Given the matrices A, B, C, D, E, G, H and the controller gains K_1 and K_2, the augmented closed-loop system in (8) achieves the $\mathscr{H}_\infty$ output tracking performance γ if there exist matrices $P > 0, Q > 0, M_i > 0, U_i, V_i, i = 1, 2$, satisfying (19), shown at the bottom of the page, where

$$\begin{aligned}\Psi_{11} &= P\bar{A} + \bar{A}^T P + Q + U_1^T + U_1 + \bar{A}^T\Psi\bar{A} + \bar{C}^T\bar{C}\\ \Psi_{22} &= -Q - V_1^T - V_1 + U_2^T + U_2\\ \Psi_{33} &= \bar{B}^T\Psi\bar{B} - V_2^T - V_2 + \bar{D}^T\bar{D}\\ \Psi &= \eta_m M_1 + \kappa M_2.\end{aligned} \tag{20}$$

Proof: We first establish the asymptotic stability of the augmented closed-loop system in (8) with $v(t) \equiv 0$. Choose the following Lyapunov–Krasovskii functional:

$$\begin{aligned}V(t) &= V_1(t) + V_2(t) + V_3(t)\\ V_1(t) &= \zeta^T(t)P\zeta(t)\\ V_2(t) &= \int_{t-\eta_m}^{t} \zeta^T(\alpha)Q\zeta(\alpha)\,d\alpha\\ V_3(t) &= \int_{-\eta_m}^{0}\int_{t+\beta}^{t} \dot{\zeta}^T(\alpha)M_1\dot{\zeta}(\alpha)d\alpha d\beta\\ &\quad + \int_{-\eta_m-\kappa}^{-\eta_m}\int_{t+\beta}^{t} \dot{\zeta}^T(\alpha)M_2\dot{\zeta}(\alpha)d\alpha d\beta\end{aligned} \tag{21}$$

where $P > 0$, $Q > 0$, $M_i > 0$ are matrices to be determined. Then, along the solution of system (16) with $v(t) \equiv 0$, the time derivative of $V(t)$ is given by

$$\dot{V}_1(t) = 2\zeta^T(t)P\left[\bar{A}\zeta(t) + \bar{B}\zeta(t-\eta_m-\eta(t))\right] \tag{22}$$

$$\dot{V}_2(t) = \zeta^T(t)Q\zeta(t) - \zeta^T(t-\eta_m)Q\zeta(t-\eta_m) \tag{23}$$

$$\begin{aligned}\dot{V}_3(t) &= \eta_m\dot{\zeta}^T(t)M_1\dot{\zeta}(t) - \int_{t-\eta_m}^{t}\dot{\zeta}^T(\alpha)M_1\dot{\zeta}(\alpha)d\alpha\\ &\quad + \kappa\dot{\zeta}^T(t)M_2\dot{\zeta}(t) - \int_{t-\eta_m-\kappa}^{t-\eta_m}\dot{\zeta}^T(\alpha)M_2\dot{\zeta}(\alpha)d\alpha\\ &\le \dot{\zeta}^T(t)\Psi\dot{\zeta}(t) - \int_{t-\eta_m}^{t}\dot{\zeta}^T(\alpha)M_1\dot{\zeta}(\alpha)d\alpha\\ &\quad - \int_{t-\eta_m-\eta(t)}^{t-\eta_m}\dot{\zeta}^T(\alpha)M_2\dot{\zeta}(\alpha)d\alpha\end{aligned} \tag{24}$$

where Ψ is given in (20).

By the Newton–Leibniz formula, we have

$$\int_{t-\eta_m}^{t}\dot{\zeta}(\alpha)d\alpha = \zeta(t) - \zeta(t-\eta_m) \tag{25}$$

$$\int_{t-\eta_m-\eta(t)}^{t-\eta_m}\dot{\zeta}(\alpha)d\alpha = \zeta(t-\eta_m) - \zeta(t-\eta_m-\eta(t)). \tag{26}$$

$$\begin{bmatrix}\Psi_{11} & -U_1+V_1^T & P\bar{B}+\bar{A}^T\Psi\bar{B}+\bar{C}^T\bar{D} & U_1 & 0 & P\bar{E}+\bar{A}^T\Psi\bar{E}\\ * & \Psi_{22} & -U_2+V_2^T & V_1 & U_2 & 0\\ * & * & \Psi_{33} & 0 & V_2 & \bar{B}^T\Psi\bar{E}\\ * & * & * & -\eta_m^{-1}M_1 & 0 & 0\\ * & * & * & * & -\kappa^{-1}M_2 & 0\\ * & * & * & * & * & -\gamma^2 I+\bar{E}^T\Psi\bar{E}\end{bmatrix} < 0 \tag{19}$$

Then, for any matrices $U_i, V_i, i = 1, 2$, we have

$$\Lambda_1 = 2\begin{bmatrix}\zeta(t)\\ \zeta(t-\eta_m)\end{bmatrix}^T \begin{bmatrix}U_1\\ V_1\end{bmatrix} \times \left[\zeta(t) - \zeta(t-\eta_m) - \int_{t-\eta_m}^{t} \dot{\zeta}(\alpha)\,d\alpha\right] = 0 \quad (27)$$

$$\Lambda_2 = 2\begin{bmatrix}\zeta(t-\eta_m)\\ \zeta(t-\eta_m-\eta(t))\end{bmatrix}^T \begin{bmatrix}U_2\\ V_2\end{bmatrix} \times \left\{\zeta(t-\eta_m) - \zeta(t-\eta_m-\eta(t)) - \int_{t-\eta_m-\eta(t)}^{t-\eta_m} \dot{\zeta}(\alpha)d\alpha\right\} = 0. \quad (28)$$

In addition, for any matrices $\begin{bmatrix}X_i & Y_i\\ Y_i^T & Z_i\end{bmatrix} \geq 0, i = 1, 2$, we have

$$\Psi_1 = \eta_m \Psi_3 - \int_{t-\eta_m}^{t} \Psi_3 d\alpha = 0 \quad (29)$$

$$\Psi_2 = \kappa\Psi_4 - \int_{t-\eta_m-\eta(t)}^{t-\eta_m} \Psi_4 d\alpha \geq 0 \quad (30)$$

where

$$\Psi_3 = \begin{bmatrix}\zeta(t)\\ \zeta(t-\eta_m)\end{bmatrix}^T \begin{bmatrix}X_1 & Y_1\\ Y_1^T & Z_1\end{bmatrix} \begin{bmatrix}\zeta(t)\\ \zeta(t-\eta_m)\end{bmatrix}$$

$$\Psi_4 = \begin{bmatrix}\zeta(t-\eta_m)\\ \zeta(t-\eta_m-\eta(t))\end{bmatrix}^T \begin{bmatrix}X_2 & Y_2\\ Y_2^T & Z_2\end{bmatrix} \times \begin{bmatrix}\zeta(t-\eta_m)\\ \zeta(t-\eta_m-\eta(t))\end{bmatrix}.$$

Then, from (21)–(30), we obtain

$$\dot{V}(t) \leq \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \sum_{i=1}^{2}(\Lambda_i + \Psi_i) = \phi^T(t)\bar{\Pi}\phi(t) + \int_{t-\eta_m}^{t} \phi_1^T(t,\alpha)\Pi_1\phi_1(t,\alpha)d\alpha + \int_{t-\eta_m-\eta(t)}^{t-\eta_m} \phi_2^T(t,\alpha)\Pi_2\phi_2(t,\alpha)d\alpha \quad (31)$$

where

$$\phi(t) = \begin{bmatrix}\zeta(t)\\ \zeta(t-\eta_m)\\ \zeta(t-\eta_m-\eta(t))\end{bmatrix}$$

$$\phi_1(t,\alpha) = \begin{bmatrix}\zeta(t)\\ \zeta(t-\eta_m)\\ \dot{\zeta}(\alpha)\end{bmatrix}$$

$$\phi_2(t,\alpha) = \begin{bmatrix}\zeta(t-\eta_m)\\ \zeta(t-\eta_m-\eta(t))\\ \dot{\zeta}(\alpha)\end{bmatrix}$$

$$\Pi_i = \begin{bmatrix}-X_i & -Y_i & -U_i\\ * & -Z_i & -V_i\\ * & * & -M_i\end{bmatrix}$$

$$\bar{\Pi} = \begin{bmatrix}\bar{\Psi}_{11} & \bar{\Psi}_{12} & P\bar{B} + \bar{A}^T\Psi\bar{B}\\ * & \bar{\Psi}_{22} & -U_2 + V_2^T + \kappa Y_2\\ * & * & \bar{\Psi}_{33}\end{bmatrix}$$

$$\bar{\Psi}_{11} = P\bar{A} + \bar{A}^TP + Q + \bar{A}^T\Psi\bar{A} + U_1^T + U_1 + \eta_m X_1$$

$$\bar{\Psi}_{12} = -U_1 + V_1^T + \eta_m Y_1$$

$$\bar{\Psi}_{22} = -Q - V_1^T - V_1 + U_2^T + U_2 + \eta_m Z_1 + \kappa X_2$$

$$\bar{\Psi}_{33} = \bar{B}^T\Psi\bar{B} - V_2^T - V_2 + \kappa Z_2. \quad (32)$$

From (31), we know that if

$$\bar{\Pi} < 0, \quad (33)$$

$$\Pi_i \leq 0, \qquad i = 1, 2 \quad (34)$$

then, we have $\dot{V}(t) < -\epsilon|\zeta(t)|^2$ for a sufficiently small positive constant ϵ, which means that system (16) is asymptotically stable.

On the other hand, if there exist matrices $P > 0$, $Q > 0$, $M_i > 0$, U_i, V_i, $i = 1, 2$, satisfying (19), which by the Schur complement [5] is equivalent to

$$\Pi + \eta_m \begin{bmatrix}U_1M_1^{-1}U_1^T & U_1M_1^{-1}V_1^T & 0 & 0\\ * & V_1M_1^{-1}V_1^T & 0 & 0\\ * & * & 0 & 0\\ * & * & * & 0\end{bmatrix} + \kappa \begin{bmatrix}0 & 0 & 0 & 0\\ * & U_2M_2^{-1}U_2^T & U_2M_2^{-1}V_2^T & 0\\ * & * & V_2M_2^{-1}V_2^T & 0\\ * & * & * & 0\end{bmatrix} < 0,$$

where

$$\Pi = \begin{bmatrix}\Psi_{11} & -U_1 + V_1^T & P\bar{B} + \bar{A}^T\Psi\bar{B} + \bar{C}^T\bar{D} & P\bar{E} + \bar{A}^T\Psi\bar{E}\\ * & \Psi_{22} & -U_2 + V_2^T & 0\\ * & * & \Psi_{33} & \bar{B}^T\Psi\bar{E}\\ * & * & * & -\gamma^2 I + \bar{E}^T\Psi\bar{E}\end{bmatrix}, \quad (35)$$

then, there must exist matrices $P > 0$, $Q > 0$, $M_i > 0$, U_i, V_i and matrices $\begin{bmatrix}X_i & Y_i\\ * & Z_i\end{bmatrix} \geq 0, i = 1, 2$, satisfying

$$\Pi + \eta_m \begin{bmatrix}X_1 & Y_1 & 0 & 0\\ * & Z_1 & 0 & 0\\ * & * & 0 & 0\\ * & * & * & 0\end{bmatrix} + \kappa \begin{bmatrix}0 & 0 & 0 & 0\\ * & X_2 & Y_2 & 0\\ * & * & Z_2 & 0\\ * & * & * & 0\end{bmatrix} < 0 \quad (36)$$

and

$$\begin{bmatrix}U_iM_i^{-1}U_i^T & U_iM_i^{-1}V_i^T\\ * & V_iM_i^{-1}V_i^T\end{bmatrix} \leq \begin{bmatrix}X_i & Y_i\\ * & Z_i\end{bmatrix}, \quad i = 1, 2. \quad (37)$$

It is noted that (36) implies (33), and (37) is equivalent to (34) via a Schur complement operation. Therefore, system (16) is asymptotically stable, and thus, the augmented closed-loop system in (8) is asymptotically stable.

Next, we will establish the $\mathscr{H}_\infty$ tracking performance for the augmented closed-loop system in (8). To this end, assume zero-initial conditions, and consider the Lyapunov–Krasovskii functional defined in (21). Then, along the solution of the augmented closed-loop system in (16), we have

$$\dot{V}_1(t) = 2x^T(t)P\left[\bar{A}\zeta(t) + \bar{B}\zeta(t-\eta_m-\eta(t)) + \bar{E}v(t)\right] \tag{38}$$

and $\dot{V}_2(t), \dot{V}_3(t)$ are given in (23) and (24). In addition, by following similar lines as earlier, we can obtain (27)–(30). Therefore, with (21), (23), (24), (27)–(30) and (38), we have

$$\dot{V}(t) \le \bar{\phi}^T(t)\tilde{\Pi}_1\bar{\phi}(t) + \int_{t-\eta_m}^{t}\phi_1^T(t,\alpha)\Pi_1\phi_1(t,\alpha)d\alpha + \int_{t-\eta_m-\eta(t)}^{t-\eta_m}\phi_2^T(t,\alpha)\Pi_2\phi_2(t,\alpha)d\alpha \tag{39}$$

where $\phi_i(t,\alpha)$ and Π_i are defined in (32)

$$\bar{\phi}(t) = \left[\zeta^T(t) \quad \zeta^T(t-\eta_m) \quad \zeta^T(t-\eta_m-\eta(t)) \quad v^T(t)\right]^T$$

$$\tilde{\Pi}_1 = \begin{bmatrix} \bar{\Psi}_{11} & \bar{\Psi}_{12} & P\bar{B}+\bar{A}^T\Psi\bar{B} & P\bar{E}+\bar{A}^T\Psi\bar{E} \\ * & \bar{\Psi}_{22} & -U_2+V_2^T+\kappa Y_2 & 0 \\ * & * & \bar{\Psi}_{33} & \bar{B}^T\Psi\bar{E} \\ * & * & * & \bar{E}^T\Psi\bar{E} \end{bmatrix}$$

with Ψ_{ii} and Ψ defined in (20), and $\bar{\Psi}_{ii}$ defined in (32).

Now consider the following index:

$$\mathscr{J} = \int_0^\infty \left[e^T(\theta)e(\theta) - \gamma^2 v^T(\theta)v(\theta)\right]d\theta. \tag{40}$$

Under zero-initial conditions, we have $V(0)=0$ and $V(\infty)\ge 0$, which leads to

$$\mathscr{J} = \int_0^\infty \left[e^T(t)e(t) - \gamma^2 v^T(t)v(t) + \dot{V}(t)\right]dt - V(\infty) \le \int_0^\infty \left[e^T(t)e(t) - \gamma^2 v^T(t)v(t) + \dot{V}(t)\right]dt. \tag{41}$$

By substituting (39) into (41) and by considering the second equation in (16), we have

$$e^T(t)e(t) - \gamma^2 v^T(t)v(t) + \dot{V}(t) \le \bar{\phi}^T(t)\tilde{\Pi}_2\bar{\phi}(t) + \int_{t-\eta_m}^{t}\phi_1^T(t,\alpha)\Pi_1\phi_1(t,\alpha)d\alpha + \int_{t-\eta_m-\eta(t)}^{t-\eta_m}\phi_2^T(t,\alpha)\Pi_2\phi_2(t,\alpha)d\alpha \tag{42}$$

where

$$\tilde{\Pi}_2 = \tilde{\Pi}_1 + \begin{bmatrix} \bar{C}^T\bar{C} & 0 & \bar{C}^T\bar{D} & 0 \\ * & 0 & 0 & 0 \\ * & * & \bar{D}^T\bar{D} & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix}$$

and Π_i is defined in (32). By using similar arguments as earlier, (19) guarantees $\tilde{\Pi}_2 < 0$ and $\Pi_i \le 0$, and thus, for all nonzero $v \in L_2[0,\infty)$, we have

$$z^T(t)z(t) - \gamma^2 v^T(t)v(t) + \dot{V}(t) < 0$$

which means $\mathscr{J} < 0$. Therefore, we can conclude from (40) that, for all nonzero $v \in L_2[0,\infty)$, we have $\|e\|_2 < \gamma\|v\|_2$, and the proof is completed. □

Remark 8: The conditions in Theorem 1 are LMIs over the matrix variables $P>0, Q>0, M_i>0, U_i, V_i, i=1,2$. These matrix variables can be computed with the help of standard numerical software (such as the Matlab LMI toolbox [13]).

Remark 9: In deriving the $\mathscr{H}_\infty$ output tracking performance conditions in Theorem 1, we have utilized some state-of-the-art techniques for sampled-data and time-delay systems. The sampled behavior is dealt with by an input-delay approach, and the transformed delay system is analyzed by a new Lyapunov functional plus free weighting matrix techniques. The most significant feature is that no model transformation has been performed to the delay system in (17), which is essentially different from the results obtained in [42] based on a descriptor model transformation. This helps us avoid using bounding techniques for seeking upper bounds of the inner product between two vectors. Similar ideas appear in [18] and [20], which have been shown to be potentially less conservative than those using the model transformation method.

Remark 10: If the lower bound of the network-induced delays is assumed to be zero, that is, $\eta_m = 0$, we can see from the aforesaid proof that the Lyapunov–Krasovskii functional in (21) reduces to

$$V(t) = V_1(t) + V_2(t)$$
$$V_1(t) = \zeta^T(t)P\zeta(t)$$
$$V_2(t) = \int_{-\kappa}^{0}\int_{t+\beta}^{t}\dot{\zeta}^T(\alpha)M\dot{\zeta}(\alpha)d\alpha d\beta.$$

Then, by following similar lines as in the aforesaid proof, we can obtain an $\mathscr{H}_\infty$ output tracking performance condition for the case $0 \le \eta_k \le \eta_M$, given in the following corollary.

Corollary 1: Consider the networked control system in Fig. 1, and suppose the network-induced delays satisfy $0 \le \eta_k \le \eta_M$. Given the matrices A, B, C, D, E, G, H and the controller gains K_1 and K_2, the augmented closed-loop system in (8) achieves the $\mathscr{H}_\infty$ output tracking performance γ if there exist matrices $P>0, M>0,$, satisfying

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & U & P\bar{E}+\bar{\kappa}\bar{A}^T M\bar{E} \\ * & \Omega_{22} & V & \bar{\kappa}\bar{B}^T M\bar{E} \\ * & * & -\bar{\kappa}^{-1}M & 0 \\ * & * & * & -\gamma^2 I + \bar{\kappa}\bar{E}^T M\bar{E} \end{bmatrix} < 0$$

where $\bar{\kappa}$ is given in (18) and

$$\Omega_{11} = P\bar{A} + \bar{A}^T P + \bar{\kappa}\bar{A}^T M\bar{A} + U + U^T + \bar{C}^T\bar{C}$$
$$\Omega_{12} = P\bar{B} + \bar{\kappa}\bar{A}^T M\bar{B} - U + V^T + \bar{C}^T\bar{D}$$
$$\Omega_{22} = \bar{\kappa}\bar{B}^T M\bar{B} - V - V^T + \bar{D}^T\bar{D}. \tag{43}$$

Now we use a numerical example to show the advantage by introducing the lower bound of transmission delays.

TABLE I
COMPARISON FOR DIFFERENT VALUES OF η_m

η_m (s)	0	0.05	0.1	0.15	0.2
$\gamma_{\min}$	3.9018	3.1017	2.5700	2.1922	1.9103

Example 1: Suppose the matrices A, B, C, D, E, G, H and the controller gains K_1 and K_2 in (9) are given by

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad E = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}$$
$$C = [\,1 \quad 0\,], \quad D = 0.5, \quad G = -1, \quad H = 0.5$$
$$K_1 = [\,-1 \quad 1\,], \quad K_2 = 1.$$

It is assumed that the network-induced delays η_k satisfy $\eta_m \leq \eta_k \leq 0.4$ s, the maximum number of data packet dropouts is 2, and the sampling period is 10 ms. Our purpose is to determine the minimum guaranteed $\mathscr{H}_\infty$ output tracking performances for different values of η_m.

When we do not consider the lower bound of the network-induced delays, that is, $\eta_m = 0$, by using Corollary 1 and Theorem 1 (assume η_m to be sufficiently small), the minimum guaranteed $\mathscr{H}_\infty$ output tracking performance obtained is $\gamma_{\min} = 3.9018$. However, if we assume $\eta_m = 0.1$ s, the minimum guaranteed $\mathscr{H}_\infty$ output tracking performance obtained is $\gamma_{\min} = 2.5700$, showing that considering the lower bound of the signal transmission delay gives rise to less conservative results. A more detailed comparison for different values of η_m is provided in Table I.

C. $\mathscr{H}_\infty$ Output Tracking Controller Design

In this section, we solve the problem of $\mathscr{H}_\infty$ output tracking controller design based on Theorem 1.

Proposition 1: Consider the networked control system in Fig. 1. There exists a state-feedback controller in the form of (3) such that the augmented closed-loop system in (8) achieves the $\mathscr{H}_\infty$ output tracking performance γ if there exist matrices $\bar{P} > 0, \bar{Q} > 0, \bar{M}_i > 0, \bar{U}_i, \bar{V}_i, i = 1, 2$, and $\bar{K}$ satisfying (44), shown at the bottom of the page, where

$$\tilde{B} = [\,B^T \quad 0\,]^T$$
$$\Theta_{11} = \bar{A}\bar{P} + \bar{P}\bar{A}^T + \bar{Q} + \bar{U}_1^T + \bar{U}_1$$
$$\Theta_{22} = -\bar{Q} - \bar{V}_1^T - \bar{V}_1 + \bar{U}_2^T + \bar{U}_2. \tag{45}$$

Moreover, if the earlier condition is feasible, the gain matrix of a desired controller in the form of (3) is given by

$$[\,K_1 \quad K_2\,] = \bar{K}\bar{P}^{-1}. \tag{46}$$

Proof: From Theorem 1, we know that there exists a state-feedback controller in the form of (3) such that the augmented closed-loop system in (8) achieves the $\mathscr{H}_\infty$ output tracking performance γ if there exist matrices $P > 0$, $Q > 0$, $M_i > 0$, U_i, $V_i, i = 1, 2$, satisfying (19). First by the Schur complement, (19) is equivalent to (47) shown at the bottom of the page.
where

$$\bar{\Theta}_{11} = P\bar{A} + \bar{A}^T P + Q + U_1^T + U_1$$
$$\bar{\Theta}_{22} = -Q - V_1^T - V_1 + U_2^T + U_2.$$

Performing a congruence transformation to (47) by diag$\{P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, I, I, I, I\}$, together with the change of matrix variables defined by

$$\bar{P} \triangleq P^{-1}, \quad \bar{M}_i \triangleq M_i^{-1} \quad \bar{K} \triangleq [\,K_1 \quad K_2\,]\,P^{-1},$$
$$\bar{Q} \triangleq P^{-1}QP^{-1}, \quad \bar{U}_i \triangleq P^{-1}U_iP^{-1}, \quad \bar{V}_i \triangleq P^{-1}V_iP^{-1}$$

we obtain (44), and the proposition is proved. □

The condition in Proposition 1 still cannot be implemented by using standard numerical software due to the existence

$$\begin{bmatrix} \Theta_{11} & -\bar{U}_1 + \bar{V}_1^T & \tilde{B}\bar{K} & \bar{U}_1 & 0 & \bar{E} & \bar{P}\bar{C}^T & \bar{P}\bar{A}^T & \bar{P}\bar{A}^T \\ * & \Theta_{22} & -\bar{U}_2 + \bar{V}_2^T & \bar{V}_1 & \bar{U}_2 & 0 & 0 & 0 & 0 \\ * & * & -\bar{V}_2^T - \bar{V}_2 & 0 & \bar{V}_2 & 0 & \bar{K}^T D^T & \bar{K}^T \tilde{B}^T & \bar{K}^T \tilde{B}^T \\ * & * & * & -\eta_m^{-1}\bar{P}\bar{M}_1^{-1}\bar{P} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\kappa^{-1}\bar{P}\bar{M}_2^{-1}\bar{P} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I & 0 & \bar{E}^T & \bar{E}^T \\ * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & -\eta_m^{-1}\bar{M}_1 & 0 \\ * & * & * & * & * & * & * & * & -\kappa^{-1}\bar{M}_2 \end{bmatrix} < 0 \tag{44}$$

$$\begin{bmatrix} \bar{\Theta}_{11} & -U_1 + V_1^T & P\bar{B} & U_1 & 0 & P\bar{E} & \bar{C}^T & \bar{A}^T & \bar{A}^T \\ * & \bar{\Theta}_{22} & -U_2 + V_2^T & V_1 & U_2 & 0 & 0 & 0 & 0 \\ * & * & -V_2^T - V_2 & 0 & V_2 & 0 & \bar{D}^T & \bar{B}^T & \bar{B}^T \\ * & * & * & -\eta_m^{-1}M_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\kappa^{-1}M_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I & 0 & \bar{E}^T & \bar{E}^T \\ * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & -\eta_m^{-1}M_1^{-1} & 0 \\ * & * & * & * & * & * & * & * & -\kappa^{-1}M_2^{-1} \end{bmatrix} < 0 \tag{47}$$

of the terms $\bar{P}\bar{M}_i^{-1}\bar{P}$. By noticing $\bar{M}_i > 0$, we have $(\bar{M}_i - \bar{P})\bar{M}_i^{-1}(\bar{M}_i - \bar{P}) \geq 0$, which is equivalent to

$$-\bar{P}\bar{M}_i^{-1}\bar{P} \leq \bar{M}_i - 2\bar{P}, \quad i = 1, 2. \tag{48}$$

By combining (44) and (48), we readily obtain the following theorem.

Theorem 2: Consider the networked control system in Fig. 1. There exists a state-feedback controller in the form of (3) such that the augmented closed-loop system in (8) achieves the $\mathscr{H}_\infty$ output tracking performance γ if there exist matrices $\bar{P} > 0$, $\bar{Q} > 0$, $\bar{M}_i > 0$, $\bar{U}_i$, $\bar{V}_i$, $i = 1, 2$, and $\bar{K}$ satisfying (49), shown at the bottom of the page, where Θ_{ii} is given in (45) and

$$\Upsilon_i = \bar{M}_i - 2\bar{P}, \quad i = 1, 2. \tag{50}$$

Moreover, if the aforesaid condition is feasible, the gain matrix of a desired controller in the form of (3) is given by (46).

Theorem 2 presents an LMI condition for the existence of desired state-feedback controllers based on the inequalities in (48). In the following, we present another approach to solve the condition in Proposition 1.

Now introduce additional matrix variables $N_i > 0$, and replace (44) with (51), shown at the bottom of the page.

$$N_i - \bar{P}\bar{M}_i^{-1}\bar{P} \leq 0, \quad i = 1, 2. \tag{52}$$

By Schur complement, (52) is equivalent to

$$\begin{bmatrix} -N_i^{-1} & \bar{P}^{-1} \\ * & -\bar{M}_i^{-1} \end{bmatrix} \leq 0, \quad i = 1, 2. \tag{53}$$

Then, we readily obtain the following theorem.

Theorem 3: Consider the networked control system in Fig. 1. There exists a state-feedback controller in the form of (3) such that the augmented closed-loop system in (8) achieves the $\mathscr{H}_\infty$ output tracking performance γ if there exist matrices $\bar{P} > 0$, $S > 0$, $\bar{Q} > 0$, $\bar{M}_i > 0$, $R_i > 0$, $N_i > 0$, $T_i > 0$, $\bar{U}_i$, $\bar{V}_i$, $i = 1, 2$, and $\bar{K}$ satisfying (51) and

$$\begin{bmatrix} -T_i & S \\ * & -R_i \end{bmatrix} \leq 0, \quad i = 1, 2 \tag{54}$$

$$\bar{P}S = I, \quad \bar{M}_i R_i = I, \quad N_i T_i = I, \quad i = 1, 2. \tag{55}$$

Moreover, if the aforesaid condition is feasible, the gain matrix of a desired controller in the form of (3) is given by (46).

The condition presented in Theorem 3 is equivalent to that in Proposition 1. It is noted that this condition is not a convex set due to the matrix equality constraints in (55). Several approaches have been proposed to solve such nonconvex feasibility problems, among which the cone complementarity linearization (CCL) method [12] is the most commonly used one (for instance, the CCL algorithm has been used for solving the controller design problems as well as model reduction problems [14], [15]). The basic idea in CCL algorithm is that if the LMI $\begin{bmatrix} P & I \\ I & L \end{bmatrix} \geq 0$ is feasible in the $n \times n$ matrix variables $L > 0$ and $P > 0$, then $\mathrm{tr}(PL) \geq n$, and $\mathrm{tr}(PL) = n$ if and only if $PL = I$.

Now using a cone complementarity method [12], we suggest the following nonlinear minimization problem involving LMI conditions instead of the original nonconvex feasibility problem formulated in Theorem 3.

Problem TCD (Tracking Controller Design):

$$\min \mathrm{tr}\left(\bar{P}S + \sum_{i=1}^{2}\left(\bar{M}_i R_i + N_i T_i\right)\right)$$

subject to (51), (54) and

$$\begin{bmatrix} \bar{P} & I \\ I & S \end{bmatrix} \geq 0, \quad \begin{bmatrix} \bar{M}_i & I \\ I & R_i \end{bmatrix} \geq 0, \quad \begin{bmatrix} N_i & I \\ I & T_i \end{bmatrix} \geq 0, \quad i = 1, 2.$$

$$\begin{bmatrix} \Theta_{11} & -\bar{U}_1 + \bar{V}_1^T & \tilde{B}\bar{K} & U_1 & 0 & \bar{E} & \bar{P}\bar{C}^T & \bar{P}\bar{A}^T & \bar{P}\bar{A}^T \\ * & \Theta_{22} & -\bar{U}_2 + \bar{V}_2^T & V_1 & U_2 & 0 & 0 & 0 & 0 \\ * & * & -\bar{V}_2^T - \bar{V}_2 & 0 & V_2 & 0 & \bar{K}^T D^T & \bar{K}^T \tilde{B}^T & \bar{K}^T \tilde{B}^T \\ * & * & * & \eta_m^{-1}\Upsilon_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \kappa^{-1}\Upsilon_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I & 0 & \bar{E}^T & \bar{E}^T \\ * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & -\eta_m^{-1}\bar{M}_1 & 0 \\ * & * & * & * & * & * & * & * & -\kappa^{-1}\bar{M}_2 \end{bmatrix} < 0 \tag{49}$$

$$\begin{bmatrix} \Theta_{11} & -\bar{U}_1 + \bar{V}_1^T & \tilde{B}\bar{K} & \bar{U}_1 & 0 & \bar{E} & \bar{P}\bar{C}^T & \bar{P}\bar{A}^T & \bar{P}\bar{A}^T \\ * & \Theta_{22} & -\bar{U}_2 + \bar{V}_2^T & \bar{V}_1 & \bar{U}_2 & 0 & 0 & 0 & 0 \\ * & * & -\bar{V}_2^T - \bar{V}_2 & 0 & \bar{V}_2 & 0 & \bar{K}^T D^T & \bar{K}^T \tilde{B}^T & \bar{K}^T \tilde{B}^T \\ * & * & * & -\eta_m^{-1} N_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\kappa^{-1} N_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I & 0 & \bar{E}^T & \bar{E}^T \\ * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & -\eta_m^{-1}\bar{M}_1 & 0 \\ * & * & * & * & * & * & * & * & -\kappa^{-1}\bar{M}_2 \end{bmatrix} < 0 \tag{51}$$

According to [12], if the solution of the aforesaid minimization problem is $5\,(n+r)$, that is

$$\min \operatorname{tr}\left(\bar{P}S+\sum_{i=1}^{2}\left(\bar{M}_iR_i+N_iT_i\right)\right)=5\,(n+r)$$

then the conditions in Theorem 3 are solvable. Although it is still not possible to always find the global optimal solution, the proposed nonlinear minimization problem is easier to solve than the original nonconvex feasibility problem. Algorithm 1 in [12] can be easily adapted to solve Problem TCD.

IV. Robust $\mathscr{H}_\infty$ Tracking Control Design

The main task of this section is to investigate the problem of robust $\mathscr{H}_\infty$ tracking control for systems with uncertain matrix data. Here, we shall consider two types of parameter uncertainties: norm-bounded uncertainty and polytopic uncertainty.

A. Norm-Bounded Uncertain Case

A popular way of dealing with deterministic uncertainty is to assume that the deviation of the system parameters from their nominal values is norm-bounded, which has been widely used in the robust control area [36], [37], [39]. Many practical systems possess parameter uncertainties that can be either exactly modeled or overbounded. In our case, we make the following assumption:

Assumption 1: Assume that the matrices A, B, C, D of the system in (1) have the following form

$$\begin{aligned} A&=A_0+\Delta A, \quad B=B_0+\Delta B\\ C&=C_0+\Delta C \quad D=D_0+\Delta D \end{aligned} \tag{56}$$

where A_0, B_0, C_0, D_0 are known constant matrices with appropriate dimensions. ΔA, ΔB, ΔC, ΔD are real-valued time-varying matrix functions representing norm-bounded parameter uncertainties satisfying

$$\begin{bmatrix} \Delta A & \Delta B \\ \Delta C & \Delta D \end{bmatrix}=\begin{bmatrix} F_1 \\ F_2 \end{bmatrix}\Delta\,(t)\,[\,J_1 \quad J_2\,]$$

where $\Delta\,(t)$ is a real uncertain matrix function with Lebesgue measurable elements satisfying $\Delta^T\,(t)\,\Delta\,(t)\le I$, and F_1, F_2, J_1, J_2 are known real constant matrices with appropriate dimensions. These matrices specify how the uncertain parameters in $\Delta\,(t)$ enter the nominal matrices A_0, B_0, C_0, D_0.

Before proceeding further, we give the following lemma which will be used later [40].

Lemma 1: Given appropriately dimensioned matrices Σ_1, Σ_2, Σ_3, with $\Sigma_1^T=\Sigma_1$, then

$$\Sigma_1+\Sigma_3\Delta\,(t)\,\Sigma_2+\Sigma_2^T\Delta^T\,(t)\,\Sigma_3^T<0 \tag{57}$$

holds for all $\Delta\,(t)$ satisfying $\Delta^T\,(t)\,\Delta\,(t)\le I$ if and only if for some $\epsilon>0$

$$\Sigma_1+\epsilon\Sigma_3\Sigma_3^T+\epsilon^{-1}\Sigma_2^T\Sigma_2<0.$$

Now we present the results on robust $\mathscr{H}_\infty$ tracking control for system (1) with norm-bounded uncertainty.

Corollary 2: Consider the networked control system in Fig. 1, and suppose the system in (1) contains norm-bounded uncertainty described in (56). There exists a state-feedback controller in the form of (3) such that the augmented closed-loop system in (8) achieves the $\mathscr{H}_\infty$ output tracking performance γ if there exist matrices $\bar{P}>0$, $\bar{Q}>0$, $\bar{M}_i>0$, $\bar{U}_i$, $\bar{V}_i$, $i=1,2$, $\bar{K}$, and scalar $\epsilon>0$ satisfying (58), as shown at the bottom of the page, where Θ_{22} is given in (45), $\bar{\Theta}_{44}$ and $\bar{\Theta}_{55}$ are given in (50), and

$$\begin{aligned} \Upsilon&=\bar{A}_0\bar{P}+\bar{P}\bar{A}_0^T+\bar{Q}+\bar{U}_1^T+\bar{U}_1+\epsilon\bar{F}_1\bar{F}_1^T\\ \bar{A}_0&=\begin{bmatrix} A_0 & 0\\ 0 & G\end{bmatrix},\quad \tilde{B}_0=\begin{bmatrix} B_0\\ 0\end{bmatrix},\quad \bar{C}_0=[\,C_0 \quad -H\,]\\ \bar{F}_1&=\begin{bmatrix} F_1\\ 0\end{bmatrix},\quad \bar{J}_1=[\,J_1 \quad 0\,]. \end{aligned} \tag{59}$$

Moreover, if the aforesaid condition is feasible, the gain matrix of a desired controller in the form of (3) is given by (46)

Proof: First, substituting the norm-bounded uncertain matrices A, B, C, D defined in (56) into (9) yields

$$\begin{aligned} \bar{A}&=\bar{A}_0+\bar{F}_1\Delta\,(t)\,\bar{J}_1,\quad \tilde{B}=\tilde{B}_0+\bar{F}_1\Delta\,(t)\,J_2\\ \bar{C}&=\bar{C}_0+F_2\Delta\,(t)\,\bar{J}_1,\quad D=D_0+F_2\Delta\,(t)\,J_2 \end{aligned} \tag{60}$$

where $\bar{A}_0$, $\tilde{B}_0$, $\bar{C}_0$, $\bar{F}_1$, $\bar{J}_1$ are defined in (59). Then, by substituting the matrices in (60) into (49), we have (57) with (*), as shown at the bottom of the next page.

$$\begin{bmatrix}
\Upsilon & -\bar{U}_1+\bar{V}_1^T & \tilde{B}_0\bar{K} & \bar{U}_1 & 0 & \bar{E} & \bar{P}\bar{C}_0^T+\epsilon\bar{F}_1F_2^T & \bar{P}\bar{A}_0^T+\epsilon\bar{F}_1\bar{F}_1^T & \bar{P}\bar{A}_0^T+\epsilon\bar{F}_1\bar{F}_1^T & \bar{P}\bar{J}_1^T\\
* & \Theta_{22} & -\bar{U}_2+\bar{V}_2^T & \bar{V}_1 & \bar{U}_2 & 0 & 0 & 0 & 0 & 0\\
* & * & -\bar{V}_2^T-\bar{V}_2 & 0 & \bar{V}_2 & 0 & \bar{K}^TD_0^T & \bar{K}^T\tilde{B}_0^T & \bar{K}^T\tilde{B}_0^T & \bar{K}^TJ_2^T\\
* & * & * & \bar{\Theta}_{44} & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & \bar{\Theta}_{55} & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & * & -\gamma^2 I & 0 & \bar{E}^T & \bar{E}^T & 0\\
* & * & * & * & * & * & -I+\epsilon F_2F_2^T & \epsilon F_2\bar{F}_1^T & \epsilon F_2\bar{F}_1^T & 0\\
* & * & * & * & * & * & * & -\eta_m^{-1}\bar{M}_1+\epsilon\bar{F}_1\bar{F}_1^T & \epsilon\bar{F}_1\bar{F}_1^T & 0\\
* & * & * & * & * & * & * & * & -\kappa^{-1}\bar{M}_2+\epsilon\bar{F}_1\bar{F}_1^T & 0\\
* & * & * & * & * & * & * & * & * & -\epsilon I
\end{bmatrix}<0 \tag{58}$$

By invoking Lemma 1 together with a Schur complement operation, (57) holds if and only if for some $\epsilon > 0$ (58) holds, and the proof is completed. □

B. Polytopic Uncertain Case

An alternative approach to characterize uncertain parameters is using the polytopic uncertainty description, where the system matrices are supposed to contain partially unknown parameters and they reside in a given polytope. The polytopic uncertainty has also been widely investigated in the literature [16], [29].

Assumption 2: The matrices A, B, C, D, E of the system in (1) contain partially unknown parameters. Assume that $\Omega \triangleq (A, B, C, D, E) \in \mathscr{R}$, where $\mathscr{R}$ is a given convex-bounded polyhedral domain described by s vertices

$$\mathscr{R} = \left\{ \Omega(\lambda) \middle| \Omega(\lambda) = \sum_{i=1}^{s} \lambda_i \Omega_i; \sum_{i=1}^{s} \lambda_i = 1, \lambda_i \geq 0 \right\} \tag{61}$$

where $\Omega_i = (A_i, B_i, C_i, D_i, E_i)$ denote the vertices of the polytope.

Since the LMI condition (49) in Theorem 2 is affine in the system matrices, this theorem can therefore be directly used for the robust $\mathscr{H}_\infty$ tracking control problem on the basis of quadratic stability notion. Then, we present the following corollary without proof.

Corollary 3: Consider the networked control system in Fig. 1, and suppose the system in (1) contains polytopic uncertainty described in (61). There exists a state-feedback controller in the form of (3) such that the augmented closed-loop system in (8) achieves the $\mathscr{H}_\infty$ output tracking performance γ if there exist matrices $\bar{P} > 0$, $\bar{Q} > 0$, $\bar{M}_i > 0$, $\bar{U}_i$, $\bar{V}_i$, $i = 1, 2$, and $\bar{K}$ satisfying (49) for $i = 1, \ldots, s$, where the matrices A, B, C, D, E are taken with A_i, B_i, C_i, D_i, E_i, respectively.

V. Illustrative Example

Example 2: Suppose the physical plant in Fig. 1 is a satellite system that appears in [4] and [13]. The satellite system consists of two rigid bodies joined by a flexible link. This link is modeled as a spring with torque constant k and viscous damping f. Denoting the yaw angles for the two bodies (the main body and the instrumentation module) by θ_1 and θ_2, the control torque by $u(t)$, the moments of inertia of the two bodies by J_1 and J_2, and the torque disturbance by $w(t)$, the dynamic equations are given by

$$J_1\ddot{\theta}_1(t) + f(\dot{\theta}_1(t) - \dot{\theta}_2(t)) + k((\theta_1(t) - \theta_2(t))) = u(t)$$
$$J_2\ddot{\theta}_2(t) + f(\dot{\theta}_1(t) - \dot{\theta}_2(t)) + k((\theta_1(t) - \theta_2(t))) = w(t).$$

When the output is the angular position $\theta_2(t)$, the state-space representation of the aforesaid equation is given by

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & J_1 & 0 \\ 0 & 0 & 0 & J_2 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1(t) \\ \dot{\theta}_2(t) \\ \ddot{\theta}_1(t) \\ \ddot{\theta}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k & k & -f & f \\ k & -k & f & -f \end{bmatrix}$$
$$\times \begin{bmatrix} \theta_1(t) \\ \theta_2(t) \\ \dot{\theta}_1(t) \\ \dot{\theta}_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} w(t)$$
$$y(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_1(t) \\ \theta_2(t) \\ \dot{\theta}_1(t) \\ \dot{\theta}_2(t) \end{bmatrix}.$$

Here, we choose $J_1 = J_2 = 1$, $k = 0.09$ and $f = 0.04$ (the values of k and f are chosen within their respective ranges). Suppose the reference model is given by

$$\dot{x}_r(t) = -x_r(t) + r(t)$$
$$y_r(t) = 0.5x_r(t). \tag{62}$$

Then, the corresponding matrices described in Section II are given by

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -0.3 & 0.3 & -0.004 & 0.004 \\ 0.3 & -0.3 & 0.004 & -0.004 \end{bmatrix}$$
$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$
$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}, \quad D = 0, \quad A_r = -1, \quad H = 0.5.$$

$$\Sigma_1 = \begin{bmatrix} \bar{\Upsilon} & -\bar{U}_1 + \bar{V}_1^T & \tilde{B}_0\bar{K} & U_1 & 0 & \bar{E} & \bar{P}\bar{C}_0^T & \bar{P}\bar{A}_0^T & \bar{P}\bar{A}_0^T \\ * & \Theta_{22} & -\bar{U}_2 + \bar{V}_2^T & V_1 & U_2 & 0 & 0 & 0 & 0 \\ * & * & -\bar{V}_2^T - \bar{V}_2 & 0 & V_2 & 0 & \bar{K}^T D_0^T & \bar{K}^T\tilde{B}_0^T & \bar{K}^T\tilde{B}_0^T \\ * & * & * & \eta_m^{-1}\Upsilon_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \kappa^{-1}\Upsilon_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\gamma^2 I & 0 & \bar{E}^T & \bar{E}^T \\ * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & -\eta_m^{-1}\bar{M}_1 & 0 \\ * & * & * & * & * & * & * & * & -\kappa^{-1}\bar{M}_2 \end{bmatrix} \tag{*}$$
$$\Sigma_2 = \begin{bmatrix} \bar{J}_1\bar{P} & 0 & J_2\bar{K} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\Sigma_3^T = \begin{bmatrix} \bar{F}_1^T & 0 & 0 & 0 & 0 & 0 & F_2^T & \bar{F}_1^T & \bar{F}_1^T \end{bmatrix}$$
$$\bar{\Upsilon} = \bar{A}_0\bar{P} + \bar{P}\bar{A}_0^T + \bar{Q} + \bar{U}_1^T + \bar{U}_1.$$

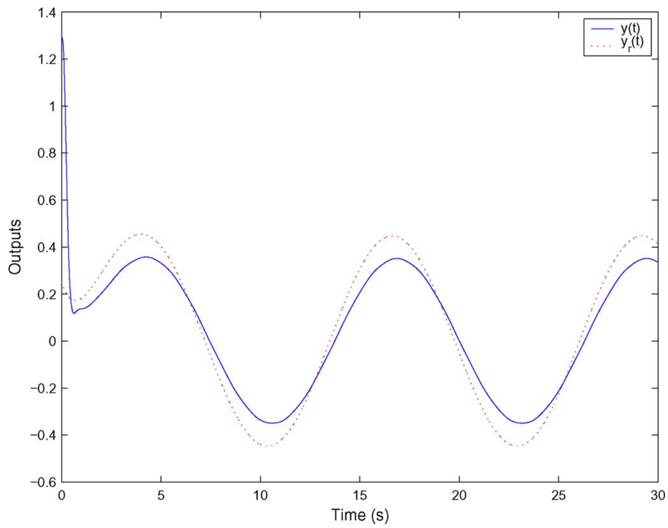

Fig. 2. Outputs $y(t)$ and $y_r\ (t)$ for inputs (63).

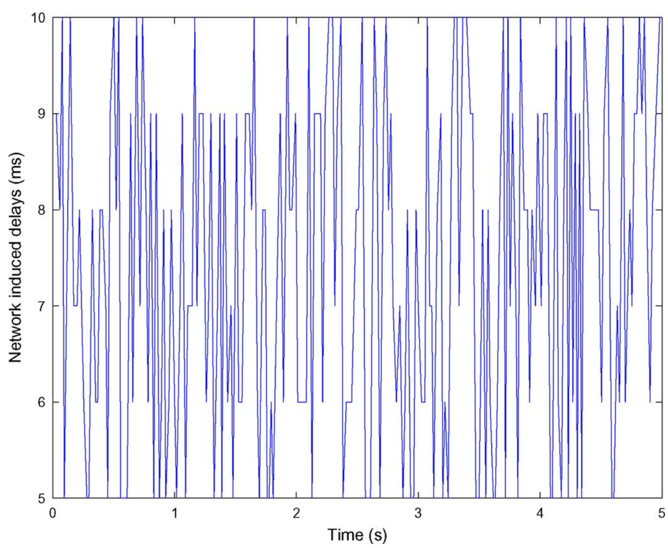

Fig. 3. Network-induced delays.

It is assumed that: the sampling period $h = 10$ ms; the network-induced delay bounds in (5) are given by $\eta_m = 5$ ms, and $\eta_M = 10$ ms; the maximum number of data packet dropouts $\bar{\delta} = 1$. Then, from (15) we have $\kappa = 25$ ms.

Our purpose is to design a state-feedback controller in the form of (3) such that the output $y(t)$ of the satellite system tracks the reference signal $y_r\ (t)$ generated by model (62) well in the $\mathscr{H}_\infty$ sense. By solving the LMIs in Theorem 2 utilizing the Matlab LMI Toolbox, we obtain the following gain matrices for the state-feedback controller in (3):

$$K_1 = [-41.56 \quad -17630.50 \quad -20.92 \quad -4256.35]$$

$$K_2 = 6917.26$$

and the obtained minimum guaranteed $\mathscr{H}_\infty$ tracking performance in terms of the feasibility of (49) is $\gamma^* = 0.1267$.

For simulation purposes, we assume

$$w(t) = 0.5 \sin 5t, \quad r(t) = \sin 0.5t. \tag{63}$$

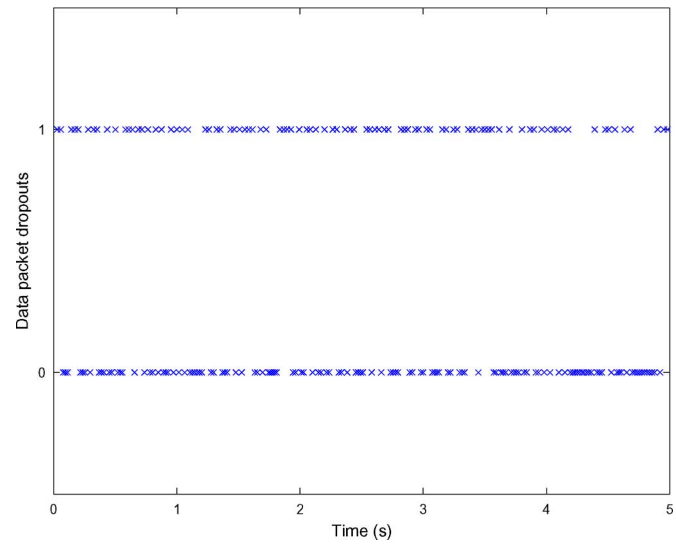

Fig. 4. Data packet dropouts.

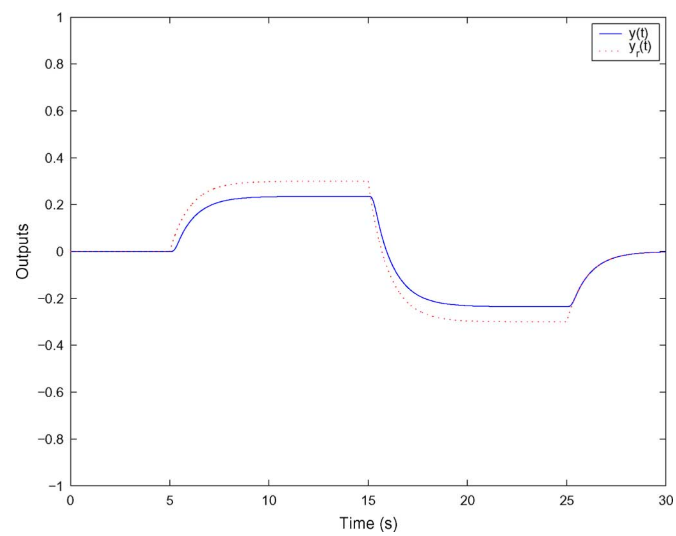

Fig. 5. Outputs $y(t)$ and $y_r\ (t)$ for inputs (64).

In addition, the initial condition of the satellite system is assumed to be $[-0.5 \quad 1.3 \quad 0.3 \quad -0.3]^T$ and the initial condition of the reference model is 0.5. The output $y(t)$ of the satellite system and $y_r\ (t)$ of the reference model are shown in Fig. 2, from which we can see that $y(t)$ tracks $y_r\ (t)$ well. In the simulation, the network-induced delays and the data packet dropouts are generated randomly according to the aforesaid assumption, and shown in Figs. 3 and 4. From Fig. 2, we can see that though the initial condition is nonzero, the tracking performance is pretty good.

Now, we take another set of input signals as follows:

$$w(t) = \begin{cases} 0.1 \sin 5t, & 8 \le t \le 22 \\ 0.0, & \text{otherwise} \end{cases}$$

$$r(t) = \begin{cases} 0.6, & 5 \le t \le 15 \\ -0.6, & 15 \le t \le 25 \\ 0.0, & \text{otherwise} \end{cases} \tag{64}$$

and the initial condition is assumed to be zero for both the satellite system and the reference model. The network-induced delays and data packet dropouts are the same as earlier. Fig. 5 depicts the outputs $y(t)$ of the satellite system and $y_r(t)$ of the reference model. By calculation, $\|v\|_2 = \|w\|_2 + \|r\|_2 = 0.2897$, $\|e\|_2 = \|y - y_r\|_2 = 2.6961$, which yields

$$\frac{\|e\|_2}{\|v\|_2} = 0.1075 < \gamma^* = 0.1267$$

showing the effectiveness of the $\mathscr{H}_\infty$ tracking controller design.

VI. Conclusion

This paper has investigated the problem of $\mathscr{H}_\infty$ output tracking for networked control systems. The problem is solved by using a sampled-data approach, which has taken both the network-induced delays and data packet dropouts into consideration. The network-induced delays are assumed to have both a lower bound and an upper bound, which is more general than those used in the literature. A new model based on the updating instants of the ZOH (instead of the sampling instants) has been formulated, and a state-feedback controller design procedure has been proposed, which guarantees the output of the closed-loop networked control system that tracks the output of a given reference model well in the $\mathscr{H}_\infty$ sense. The introduction of the lower bound of network-induced delays has been shown, via a numerical example, to be advantageous for reducing conservatism. The controller design method was further extended to more general cases, where the system matrices of the physical plant contain parameter uncertainties, represented in either polytopic or norm-bounded frameworks. An illustrative example has shown the usefulness and effectiveness of the proposed tracking controller design methods. Finally, it is worth mentioning that the proposed controller is a constant one, and more involved sampler and holder systems are expected to achieve better performance, as illustrated in [27], which deserve further study in the network environment.

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