Stability Analysis of Delayed 4-Channel Bilateral Teleoperation Systems

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Abstract

This paper studies the stability of a delayed 4-channel bilateral teleoperation system based on the passivity framework. Assuming that the operator and the environment are passive systems, the stability of the teleoperation system is reduced to ensuring the passivity of a master control unit (MCU), a slave control unit (SCU), and the time-delayed communication channel. Each of these three blocks is modeled as a 2×2 transfer function matrix and passified using our proposed approach in a multi-loop feedback (MLF) structure. We report conditions on the controllers of the 4-channel architecture that are sufficient for passivity of MCU and SCU. Simulation results confirm the validity of these conditions for the stability of the teleoperation system.

Index Terms: H.5.2 [Information Interfaces and Presentation]: User Interfaces—Haptic I/O; I.2.9 [Artificial Intelligence]: Problem Solving, Control Methods, and Search—Control theory

1 Introduction

In the design of teleoperation systems, ensuring stability in the presence of communication time delay is of vital importance. A teleoperation system consists of a two-port network representing a teleoperator (comprising a master robot, a slave robot, their controllers, and a communication channel) coupled to two one-port networks representing a human operator and an environment. A teleoperator's two-port network model can be an impedance matrix relating velocities to forces, a hybrid matrix relating a mixed force-velocity vector to another mixed force-velocity vector, or an admittance matrix relating forces to velocities [4, 9]. As discussed later, based on this modeling, two-port network theory can be used to analyze the teleoperator's passivity and, therefore, the teleoperation system's stability.

Passivity-based stability analysis of bilateral teleoperation systems was first introduced in [1] through scattering theory, and then presented in the wave variables framework in [8]. A review of time delay compensation techniques for teleoperation systems can be found in [2]. Assume that the human operator and the environment demonstrate passive behaviors. In [1, 8], the emphasis is on passifying the communication channel *assuming that the master and the slave when combined with their respective controllers are passive systems*. Although the master and the slave robots are always passive [6], there is no guarantee that the master control unit (MCU) and the slave control unit (SCU) are also passive. To the best of the authors' knowledge, the conditions under which the MCU and the SCU remain passive have not been reported before. In this paper, we derive conditions for passivity of the MCU and the SCU while passifying the delayed communication channel.

The wave variables for passifying a delayed communication channel were developed for a channel that can be modeled as a 2-port network [8]. In order to apply the wave variables method to a delayed 4-channel teleoperation system, in which the communication channel has 4 inputs and 4 outputs, similar to the practice in [3], a weighted sum of force and velocity at each side of the teleoperation system is sent through the channel making the communication channel appear as a 2-port network. However, this approach to delay compensation in the 4-channel teleoperation system introduces non-physical variables that complicate the teleoperator's passivity analysis. In this paper, without any need for physical interpretation of the signals that are involved in a delay-compensated 4-channel teleoperation system, a transfer matrix based approach to modeling and stability analysis is presented that is easier to follow compared to traditional two-port network based passivity analyses.

The rest of the paper is organized as follows. Section 2 provides mathematical preliminaries required for this paper. In Section 3, the 4-channel teleoperation system is modeled. In Section 4, the passification procedure for the delayed communication channel within the 4-channel architecture is presented. Section 5 finds a condition for stability of the delay-compensated 4-channel teleoperation system in terms of passivity of the MCU and the SCU. Simulation results are presented in Section 6 and Section 7 presents the conclusions.

2 Mathematical Preliminaries

Definition 1. *[4] The M-port network N_M shown in Fig. 1 with zero initial energy is passive if and only if*

$$\int_0^t \sum_{i=1}^{M} f_i(\tau)v_i(\tau)d\tau \geq 0, \quad \forall t \geq 0 \tag{1}$$

for all admissible forces f_i's and velocities v_i's.

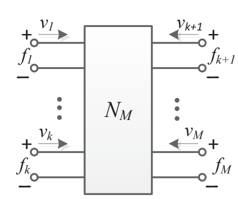

Figure 1: An M-port network.

Definition 2. *[7] The system*

$$\dot{x} = f(x,u) \tag{2}$$

$$y = h(x,u) \tag{3}$$

is said to be passive if there exists a continuously differentiable positive semidefinte function $V(x)$ (called storage function) such that

$$u^T y \geq \dot{V} = \frac{\partial V}{\partial x} f(x,u), \forall (x,u) \in R^n \times R \tag{4}$$

Moreover, it is said to be strictly passive if $u^T y > \dot{V}$.

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Lemma 1. *[7] The LTI minimal realization*

$$\dot{x} = Ax + Bu \quad (5)$$
$$y = Cx + Du \quad (6)$$

with $G(s) = C(sI - A)^{-1}B + D$ *is*

- *passive if* $G(s)$ *is positive real;*
- *strictly passive if* $G(s)$ *is strictly positive real.*

Definition 3. *[7] An* $n \times n$ *proper rational transfer function matrix* $G(s)$ *is positive real if*

- *poles of all elements of* $G(s)$ *are in* $Re[s] \leq 0$,
- *for all real* ω *for which* $j\omega$ *is not a pole of any element of* $G(s)$, *the matrix* $G(j\omega) + G^T(-j\omega)$ *is positive semidefinite, and*
- *any pure imaginary pole of* $j\omega$ *of any element of* $G(s)$ *is a simple pole and the residue matrix* $lim_{s\to j\omega}(s - j\omega)G(s)$ *is positive semidefinite Hermitian.*

Applying the Definition 3 and Lemma 1 to a 2×2 transfer matrix $G(s)$, which can represent a two-port network, leads to Raisbeck's passivity criterion (see Appendix A for details).

3 System Modeling

The LTI dynamics of the operator and the environment are assumed to be

$$F_h = F_h^* - Z_h V_m \quad (7)$$
$$F_e = F_e^* - Z_e V_s \quad (8)$$

where F_h and F_e[1] are the operator force applied to the master robot and the environment force applied to the slave robot, Z_h and Z_e are the operator and the environment impedances, V_m and V_s are the operator and the environment velocities, and F_h^* and F_e^* are the exogenous force inputs from the operator and the environment, respectively. The LTI models of the master and the slave robots are assumed to be

$$Z_m V_m = F_h + F_{cm} \quad (9)$$
$$Z_s V_s = F_e + F_{cs} \quad (10)$$

where $Z_m = M_m s$ and $Z_s = M_s s$ are the impedances of single-DOF master and salve robots, respectively. Also, F_{cm} and F_{cs} are the control signals for the master and the slave robots, respectively. In the 4-channel teleoperation architecture in Fig. 2, these control signals are

$$F_{cm} = -C_m V_m - C_4 V_{md} + C_6 F_h - C_2 F_{hd} \quad (11)$$
$$F_{cs} = -C_s V_s + C_1 V_{sd} + C_5 F_e + C_3 F_{ed} \quad (12)$$

where $C_m = (K_{dm} + \frac{K_{pm}}{s})$ and $C_s = (K_{ds} + \frac{K_{ps}}{s})$ represent local PD position controllers, C_6 and C_5 are local force controllers, C_2 and C_3 are force feedback and feed-forward controllers, and C_1 to C_4 are position compensators working in conjunction with C_s and C_m, V_{md} and V_{sd} are desired velocities, and F_{hd} and F_{ed} are desired forces for the master and the slave, respectively.

[1] In this paper F_e is considered as the environment force applied to the slave where in literature it is defined as the slave's force applied upon the environment. This change of notation is made to preserve symmetric structure of the 4-channel teleoperation system.

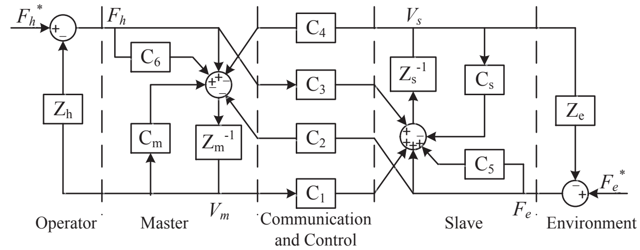

Figure 2: [6] 4-channel bilateral teleoperation system structure

4 Channel Delay Compensation in 4-channel Teleoperation

Inspired by [3], for compensating for the communication delay in a 4-channel teleoperation system, a weighted sum of force and velocity at each side of the teleoperation system must be considered as incoming signals such that the communication channel appears as a 2-port network system. Once we do so, the 4-channel teleoperation system in Fig. 2 is re-modeled as shown in Fig. 3. The 2×2 transfer matrix relating the outputs of the delayed communication channel to its inputs are

$$\begin{bmatrix} I_1 \\ -V_2 \end{bmatrix} = \begin{bmatrix} 0 & e^{-sT} \\ -e^{-sT} & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} \quad (13)$$

Based on Definition 3, the channel transfer matrix is not positive real and thus the delayed communication channel is not passive. Realizing a passive communication channel can be carried out through two methods. The first method is based on the wave variables formulation [8]. In Fig. 4, the outputs of the wave transformation block at the master side are

$$I_1 = bV_1 + \sqrt{2b}u_m; v_m = \sqrt{2b}V_1 + u_m \quad (14)$$

and at the slave side they are

$$-V_2 = -\frac{1}{b}(-I_2 + \sqrt{2b}v_s); u_s = \frac{1}{b}(\sqrt{2b}I_2 - bv_s) \quad (15)$$

Since $u_m(t) = u_s(t - T)$ and $v_s(t) = v_m(t - T)$, we get

$$I_1 = e^{-sT}I_2 - be^{-sT}V_2 + bV_1 \quad (16)$$
$$-V_2 = -e^{-sT}V_1 \frac{1}{b} e^{-sT} I_1 + \frac{1}{b} I_2 \quad (17)$$

This will change the channel transfer function matrix from (13) to

$$C(s) = \begin{bmatrix} b\frac{e^{sT} - e^{-sT}}{(e^{sT} + e^{-sT})} & \frac{2}{(e^{sT} + e^{-sT})} \\ -\frac{2}{(e^{sT} + e^{-sT})} & \frac{e^{sT} - e^{-sT}}{b(e^{sT} + e^{-sT})} \end{bmatrix} \quad (18)$$

which is positive real according to Definition 3, and therefore is passive. The second method follows a purely transfer function based approach. The objective here is to change the original delayed channel (13) to become a passive transfer matrix. Thus, the problem boils down to solving the following equations for unknown coefficients a_1, a_2, b_1, b_2, c_1 and c_2, so that the resulting transfer matrix satisfies the positive-realness requirements.

$$I_1 = aV_1 + be^{-sT}(-V_2) + ce^{-sT}I_2 \quad (19)$$
$$-V_2 = a^{'}I_2 + b^{'}e^{-sT}V_1 + c^{'}e^{-sT}I_1 \quad (20)$$

Note that if $T = 0$, we need to get $I_1 = I_2$ and $V_2 = V_1$. To satisfy this, it is necessary that

$$a = b, c = 1, a^{'} = -c^{'}, b^{'} = -1$$

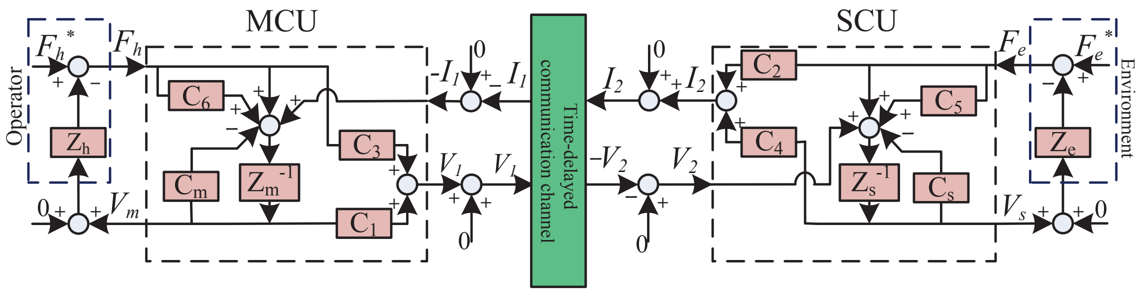

Figure 3: A 4-channel bilateral teleoperation system in which the communication channel has been re-modeled as a two-port network

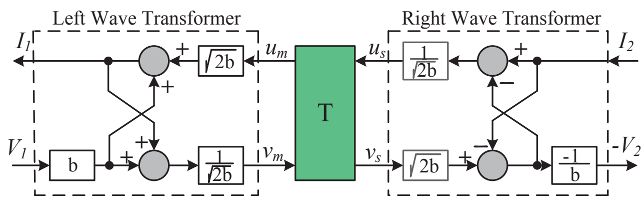

Figure 4: Delay-compensated communication channel.

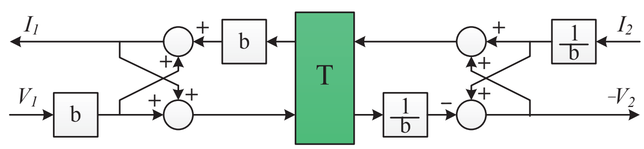

Figure 5: Another delay compensated communication channel.

Substituting these in (19)-(20), the new channel will be positive real if

$$c' = \frac{-1}{b}$$

Thus, the new channel (19)-(20) will have only one free parameter b. Finally, the transfer matrix of this delay-compensated channel will turn out to be the same as (18), which meets the positive-realness conditions according to Niemeyer01092004 3 and is passive.

5 Stabilization of a Delay-Compensated 4-channel Teleoperation System

A mathematically involved and intractable approach to stabilizing the delayed 4-channel teleoperation system is to consider the teleoperator as a whole

$$\begin{bmatrix} V_m \\ V_s \end{bmatrix} = G_{total} \begin{bmatrix} F_h \\ F_e \end{bmatrix} \tag{21}$$

where $G_{total}(s)$ is to be passified through the design of the controllers. However, this transfer matrix is far too complicated to analyze for passivity. In the context of bilateral teleoperation systems in the presence of constant time-delay, passivity-based stability methods are attempt to passify the communication channel *assuming that both the MCU and the SCU are passive*. However, the passivity of the MCU and the SCU needs to be guaranteed via proper control. Here, it is shown that there are certain conditions involving the values of the gains of the master and slave controllers in order for the passivity of the MCU and SCU to be guaranteed.

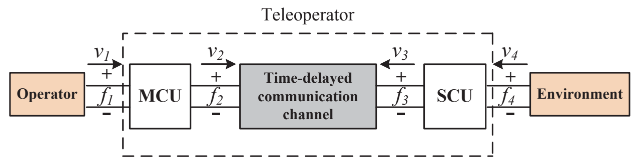

Figure 6: 2PN structure; passivity of the human operator, MCU, time-delayed communication channel, SCU, and environment is sufficient for passivity (and stability) of the teleoperation system.

Three different approaches to the passivity analysis are presented below. The first two approaches, the 2-port network (2PN) and single-loop feedback (SLF) structures, are previously introduced in literature and we show that they cannot stabilize the 4-channel teleoperation system while using the third approach, multi-loop feedback (MLF) structure that is proposed in this paper, the stability of the system will be guaranteed.

5.1 Approach 1: 2-Port Network (2PN) structure

Theorem 5.1. *The teleoperator, i.e., the teleoperation system excluding the operator and the environment as shown in Fig. 6, is passive if the MCU, the time delayed communication channel and the SCU are passive.*

Proof. If the MCU, the time delayed communication channel and the SCU are passive. Based on Definition 1

$$\int f_1 v_1 + \int f_2(-v_2) \geq 0 \tag{22}$$

$$\int f_2 v_2 + \int f_3 v_3 \geq 0 \tag{23}$$

$$\int f_3(-v_3) + \int f_4 v_4 \geq 0 \tag{24}$$

The sum of (22) - (24) gives

$$\int f_1 V_1 + \int f_4 V_4 \geq 0 \tag{25}$$

which implies the passivity of the teleoperator. □

The problem faced in practice with the 2PN structure in the context of 4-channel teleoperation is that since a weighted sum of both force and velocity are exchanged between the MCU, the channel, and the SCU, physical interpretation of these signals is not easy and writing the two-port network models based on immitance parameters is rather difficult. It is preferable to pursue a solely transfer function based approach that deals with system inputs and outputs regardless of their physical interpretation (or lack thereof).

5.2 Approach 2: Single-Loop Feedback (SLF) structure

In this section, the stability analysis method proposed in [3], the SLF structure, is further investigated. As seen in Fig. 3, we have

$$\frac{F_h(1+C_6)}{Z_{tm}} - \frac{I_1}{Z_{tm}} = V_m \qquad \frac{F_e(1+C_5)}{Z_{ts}} + \frac{V_2}{Z_{ts}} = V_s \tag{26}$$

$$C_3F_h + C_1V_m = V_1 \qquad C_2F_e + C_4V_s = I_2 \tag{27}$$

where $Z_{tm} = Z_m + C_m$ and $Z_{ts} = Z_s + C_s$. Manipulating (26)-(27) will result in the single-loop feedback structure equivalent of the teleoperation system shown in Fig. 7, where $C(s)$ is the transfer matrix (18) of the delay-compensated communication channel and

$$G(s) = \begin{bmatrix} C_1Z_{tm}^{-1} & 0 \\ 0 & C_4Z_{ts}^{-1} \end{bmatrix} \tag{28}$$

Theorem 5.2. *Assume that a teleoperation system is coupled to an environment and a human operator that are passive but otherwise arbitrary. If $G(s)$ is strictly positive real, then for inputs with finite norms, the outputs in the system shown in Fig. 7 will have finite norms. (Note that the delay has been compensated for in the channel)*

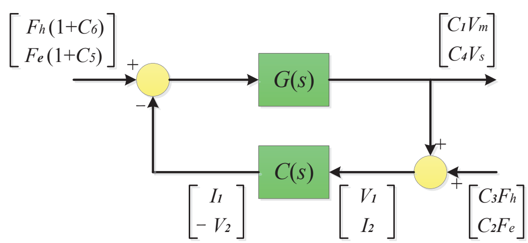

Figure 7: SLF structure; equivalent single-loop feedback structure of the 4-channel teleoperator (i.e., not including the human operator and the environment).

A major drawback of SLF is that it guarantees input-output stability from the inputs involving F_h and F_e to the outputs involving V_m and V_s. However, we know that there are two more feedback loops involving Z_h and Z_e in a teleoperation system (not shown in Fig. 7) that can affect the stability of the closed-loop system. In other words, only F_h^* and F_e^* are true inputs to the system. Incorporating the dynamics of the human operator and the environment into Fig. 7 results in a complicated structure that cannot easily be analyzed for passivity using the tools listed in Section 2.

5.3 Approach 3: Multi-Loop Feedback (MLF) structure

MLF structure, which is the proposed approach of this paper will enable us to study the passivity of the 4-channel teleoperation system easily compared to 2-PN or SLF structures. The 4-channel bilateral teleoperation system in Fig. 3 can be represented through an MLF structure as shown in Fig. 8, where $e_1 = V_m$, $e_2 = \begin{pmatrix} F_h \\ -I_1 \end{pmatrix}$, $e_3 = \begin{pmatrix} V_1 \\ I_2 \end{pmatrix}$, $e_4 = \begin{pmatrix} V_2 \\ F_e \end{pmatrix}$, $e_5 = V_s$, $u_1 = 0$, $u_2 = \begin{pmatrix} F_h^* \\ 0 \end{pmatrix}$, $u_3 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $u_4 = \begin{pmatrix} 0 \\ F_e^* \end{pmatrix}$, $u_5 = 0$, $y_1 = V_mZ_h$, $y_2 = \begin{pmatrix} V_m \\ V_1 \end{pmatrix}$, $y_3 = \begin{pmatrix} I_1 \\ -V_2 \end{pmatrix}$, $y_4 = \begin{pmatrix} I_2 \\ V_s \end{pmatrix}$ and $y_5 = V_sZ_e$.

Theorem 5.3. *The system in Fig. 8 is passive if Z_e, SCU, time-delayed communication channel, MCU, and Z_h blocks are passive.*

Proof. Let $V_1(x_1), V_2(x_2), V_3(x_3), V_4(x_4)$ and $V_5(x_5)$ be the storage functions of Z_e, the SCU, the time-delayed communication channel,

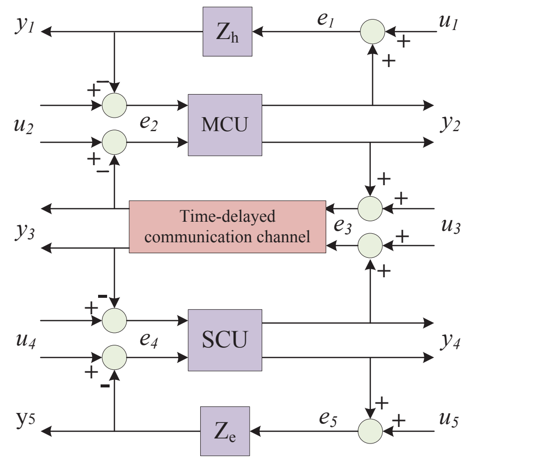

Figure 8: MLF structure; Equivalent multi-loop feedback structure of the 4-channel teleoperation system.

the MCU and Z_h. Assume the initial stored energy in each of these systems is zero. Based on Definition 2

$$e_i^T y_i \geq \dot{V}_i \tag{29}$$

Fom the feedback loops in Fig. 8, it can be seen that

$$e_1 = u_1 + [1 \quad 0]y_2 \tag{30}$$

$$e_2 = \begin{bmatrix} [1 \quad 0]u_2 - y_1 \\ [0 \quad 1]u_2 - [1 \quad 0]y_3 \end{bmatrix} \tag{31}$$

$$e_3 = \begin{bmatrix} [1 \quad 0]u_3 + [0 \quad 1]y_2 \\ [0 \quad 1]u_3 + [1 \quad 0]y_4 \end{bmatrix} \tag{32}$$

$$e_4 = \begin{bmatrix} [1 \quad 0]u_4 - [0 \quad 1]y_3 \\ [0 \quad 1]u_4 - y_5 \end{bmatrix} \tag{33}$$

$$e_5 = u_5 + [0 \quad 1]y_4 \tag{34}$$

Therefore, it is easy to show that

$$e_1^Ty_1 + e_2^Ty_2 + e_3^Ty_3 + e_4^Ty_4 + e_5^Ty_5 = u_1^Ty_1 + u_2^Ty_2 + u_3^Ty_3 + u_4^Ty_4 + u_5^Ty_5$$

For the entire teleoperation system, let us define $u = [u_1 \quad u_2 \quad u_3 \quad u_4 \quad u_5]^{\mathrm{T}}$, $y = [y_1 \quad y_2 \quad y_3 \quad y_4 \quad y_5]^{\mathrm{T}}$ Thus,

$$u^Ty = u_1^Ty_1 + u_2^Ty_2 + u_3^Ty_3 + u_4^Ty_4 + u_5^Ty_5 \geq \dot{V}_1 + \dot{V}_2 + \dot{V}_3 + \dot{V}_4 + \dot{V}_5$$

Taking $V(x) = V_1(x_1) + V_2(x_2) + V_3(x_3) + V_4(x_4) + V_5(x_5)$, we obtain

$$u^Ty \geq \dot{V} \tag{35}$$

This concludes the proof. □

5.3.1 Passification of MCU and SCU

With Z_h and Z_e assumed passive, because of Theorem 5.3, the teleoperation system stabilization problem will be reduced to passifying the communication channel, the MCU and the SCU. As explained in Section 4, the communication channel can be passified using either of the methods that were described, i.e., the wave variables method in Fig. 4 or the transfer matrix based method in Fig. 5. For ensuring passivity of the MCU and the SCU, controllers need to be designed such that they meet the definition of positive realness.

Relating inputs to corresponding outputs, the transfer matrices of the MCU and the SCU satisfy

$$\begin{bmatrix} V_m \\ V_1 \end{bmatrix} = G_{MCU} \begin{bmatrix} F_h \\ -I_1 \end{bmatrix} \quad , \quad \begin{bmatrix} I_2 \\ V_s \end{bmatrix} = G_{SCU} \begin{bmatrix} V_2 \\ F_e \end{bmatrix} \tag{36}$$

where

$$G_{MCU} = \begin{bmatrix} \frac{1+C_6}{Z_m+C_m} & \frac{1}{Z_m+C_m} \\ C_3 + \frac{C_1(1+C_6)}{Z_m+C_m} & \frac{C_1}{Z_m+C_m} \end{bmatrix} \tag{37}$$

$$G_{SCU} = \begin{bmatrix} \frac{C_4}{Z_s+C_s} & C_2 + \frac{C_4(1+C_5)}{Z_s+C_s} \\ \frac{1}{Z_s+C_s} & \frac{1+C_5}{Z_s+C_s} \end{bmatrix} \tag{38}$$

Assuming that C_2, C_3, C_5, C_6 (but not C_1 and C_4) are scalar gains and C_1 to C_6 are to be designed, applying Theorem 5.3 leads to

$$(1+C_6)K_{dm}\omega^2 > 0, \quad (1+C_5)K_{ds}\omega^2 > 0 \tag{39}$$

$$\Re[C_1(j\omega)((K_{pm} - M_m\omega^2)j - K_{dm}\omega)] > 0 \tag{40}$$

$$\Re[C_4(j\omega)((K_{ps} - M_s\omega^2)j - K_{ds}\omega)] > 0 \tag{41}$$

$$\begin{aligned} &[(K_{pm} - M_m\omega^2) - C_3K_M](1+C_6)K_{dm}[(2\Re C_1)(K_{pm} - M_m\omega^2) \\ &+ (2\Im C_1)K_{dm}\omega] - [(K_{pm} - M_m\omega^2) + C_3K_M]^2 - (1+C_6)^2((\Re C_1)^2 \\ &+ (\Im C_1)^2)K_M + K_{dm}^2\omega^2 - 2(1+C_6)(\Im C_1)(K_{pm} - M_m\omega^2)K_{dm}\omega \\ &+ 2(1+C_6)(\Re C_1)K_{dm}^2\omega^2 \geq 0 \end{aligned} \tag{42}$$

$$\begin{aligned} &[(K_{ps} - M_s\omega^2) - C_2K_S](1+C_5)K_{ds}[(2\Re C_4)(K_{ps} - M_s\omega^2) \\ &+ (2\Im C_4)K_{ds}\omega - [(K_{ps} - M_s\omega^2) + C_2K_S]^2 - (1+C_5)^2((\Re C_4)^2] \\ &+ (\Im C_4)^2)K_S + K_{ds}^2\omega^2 - 2(1+C_5)(\Im C_4)(K_{ps} - M_s\omega^2)K_{ds}\omega \\ &+ 2(1+C_5)(\Re C_4)K_{ds}^2\omega^2 \geq 0 \end{aligned} \tag{43}$$

where $K_M = (K_{pm} - M_m\omega^2)^2 + K_{dm}^2\omega^2$ and $K_S = (K_{ps} - M_s\omega^2)^2 + K_{ds}^2\omega^2$. Inequalities (39)-(43) are general sufficient conditions for ensuring passivity of the delayed 4-channel teleoperation system in the sense that any given set of controllers ($C_1, \cdots, C_6$ and proportional-derivative C_m and C_s) for any given system (M_m and M_s) can be checked for passivity, and are reported for the first time in this paper.

The transparency conditions for the 4-channel teleoperation system (without delay) shown in Fig. 2 are

$$C_1 = C_s + Z_s, C_4 = -(C_m + Z_m), C_2 = 1 + C_6, C_3 = -(1 + C_5) \tag{44}$$

It is easy to see that (44) and (39)-(43) are incompatible. This means that using the controllers (44), which guarantee transparency under zero delay, may not result in a stable teleoperation system when there exists time delay.

Though the controllers (44) could not ensure passivity, let us assume that C_1 and C_4 have similar structures to $C_s + Z_s$ and $C_m + Z_m$, respectively. This results in PID-like controllers $C_1 = K_{m1}s + \frac{K_{p1}}{s} + K_{d1}$ and $C_4 = K_{m4}s + \frac{K_{p4}}{s} + K_{d4}$ with free parameters K_{mi}, K_{pi}, K_{di}, $i = 1, 4$. Applying Definition 3, we get the following conditions to ensure the stability of the 4-channel delayed teleoperation system:

(A)	M_m	0.3	K_{pm}	100	K_{dm}	10	C_2	-2	C_6	1	C_4	$0.3s+10+100/s$
	M_s	0.6	K_{ps}	200	K_{ds}	20	C_3	-4	C_5	3	C_1	$0.6s+20+200/s$
(B)	M_m	0.3	K_{pm}	100	K_{dm}	10	C_2	-10	C_6	1	C_4	$0.3s+10+100/s$
	M_s	0.7	K_{ps}	300	K_{ds}	30	C_3	2	C_5	3	C_1	$0.35s+30+300/s$

Table 1: The masses and controllers gains used in the simulation. (A) passive and (B) non-passive.

$$K_{mi}, K_{pi}, K_{di} > 0, \quad for \quad i = 1, 4 \tag{45}$$

$$C_2 < 0, \quad C_3 < 0, \quad 1 + C_6 > 0, \quad 1 + C_5 > 0 \tag{46}$$

$$\frac{M_1}{M_m} = \frac{K_{p1}}{K_{pm}} = \frac{-C_3}{1+C_6} \tag{47}$$

$$\frac{M_2}{M_s} = \frac{K_{p2}}{K_{ps}} = \frac{-C_2}{1+C_5} \tag{48}$$

$$K_{dm}(2\sqrt{(-C_3)} - C_3) \geq (1+C_6)K_{d1} - 1 \tag{49}$$

$$K_{ds}(2\sqrt{(-C_2)} - C_2) \geq (1+C_5)K_{d4} - 1 \tag{50}$$

In the simulation study that follows, the above controllers for the 4-channel teleoperation system are used. However, as mentioned before, conditions (39)-(43) are general and can be tested for any given set of controllers.

Remark Fig. 9 depicts a 2-loop feedback variant of the 4-channel teleoperation system. It can be shown that it is passive if the time-delayed communication channel and G_{MS} are passive (note that since the operator and the environment are assumed passive, based on Definition 3 the matrix $\begin{bmatrix} Z_s & 0 \\ 0 & Z_e \end{bmatrix}$ will be passive as well). The combination of the MCU and SCU is given by

$$G_{MS} = \begin{bmatrix} \frac{1+C_6}{Z_{tm}} & 0 & \frac{1}{Z_{tm}} & 0 \\ 0 & \frac{1+C_5}{Z_{ts}} & 0 & \frac{1}{Z_{ts}} \\ C_3 + \frac{C_1(1+C_6)}{Z_{tm}} & 0 & \frac{C_1}{Z_{tm}} & 0 \\ 0 & C_2 + \frac{C_4(1+C_5)}{Z_{ts}} & 0 & \frac{C_4}{Z_{ts}} \end{bmatrix} \tag{51}$$

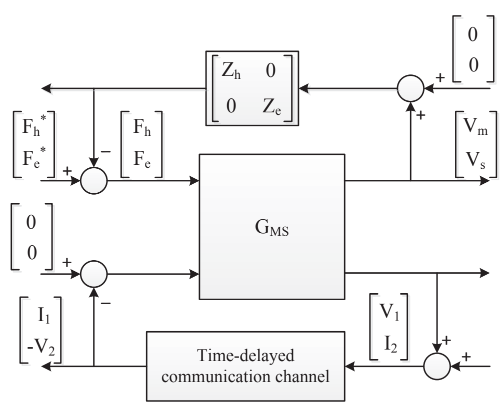

Figure 9: 2-loop structure of 4-channel teleoperation system

Again, the communication channel can be passified using either of the methods in Section 4. It is easy to show that the passivity of G_{MS} is equivalent to the passivity of *MCU* and *SCU* in the structure shown in Fig. 8.

6 Simulation Results

In this section, the passivity conditions (45)-(50) found in the previous section will be verified via simulations. For checking the passivity of the 4-channel teleoperation system, a passivity observer[10] has been incorporated into the simulation to calculate the dissipated energy in the teleoperator. This dissipated energy is given by

$$E_{dissipated} = \int_0^t F_h(\tau)V_m(\tau)d\tau + \int_0^t F_e(\tau)V_s(\tau)d\tau \geq 0 \quad (52)$$

The teleoperator is passive if this integral is non-negative at all times.

The 4-channel teleoperation system in Fig. 3 has been simulated in MATLAB/Simulink. The time delay in the communication channel is set to 0.5 sec. A pair of 1-DOF master and slave robots modeled by point masses is considered. Both the master and the slave are connected to LTI terminations with transfer functions $\frac{1}{s+1}$. This termination is passive as for $s = j\omega$ we have $Re(\frac{1}{s+1}) = \frac{1}{\omega^2+1} > 0$ when $\omega > 0$. A sine-wave (with unit amplitude and 1 Hz frequency) F_h^* is used.

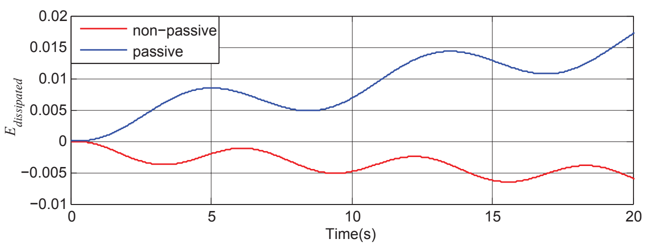

Figure 10: Passivity observer $E_{dissipated}$ used for the teleoeprator's passivity analysis

According to (45)-(50), it is expected that the passivity of the 4-channel teleoperator should depend on the controller gains. Fig. 10 shows that when the controllers gains are chosen to meet conditions (45)-(50), e.g., as listed in Table 1(A), the teleoperator is passive. However, for a set of masses and controllers gains that do not satisfy (45)-(50), e.g., as listed in Table 1(B), the teleoperator will become non-passive.

7 Conclusion and Future Work

In this paper, the stability of the 4-channel teleoperation system in the presence of time delay is studied. When delay exists in the communication channel, addressing the passivity (and stability) of the teleoperation system as a whole is too complicated and computationally intractable. Modifying the structure of the 4-channel teleoperation system as shown in Fig. 8 (or Fig. 9) will enable us to study the requirements of passivity on a modular and tractable basis. On the other hand, it was discussed that passifying the communication channel alone will not guarantee the stability of the entire teleoperation system. Assuming that both the operator and the environment are passive, controllers for the master and the slave still need to satisfy a set of requirements found in this paper. As for the future work, a similar passivity analysis will be applied to the 4-channel multilateral teleoperation systems in the presence of time delay.

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A Appendix

Raisbeck's passivity criterion [5]. The necessary and sufficient condition for passivity of a 2-port network with immitance parameters p are

1. The p-parameters have no RHP poles
2. Any poles of the p-parameters on the imaginary axis are simple, and the residues of the p-parameters at these poles satisfy the following conditions (k_{ij} denotes the residue of p_{ij} and k_{ij}^* is the complex conjugate of k_{ji}):

$$k_{11} \geq 0$$
$$k_{22} \geq 0$$
$$k_{11}k_{22} - k_{12}k_{21} \geq 0, \quad with \quad k_{21} = k_{12}^*$$

3. The real and imaginary part of the p parameters satisfy

$$\Re p_{11} \geq 0$$
$$\Re p_{22} \geq 0$$
$$4\Re p_{11}\Re p_{22} - (\Re p_{12} + \Re p_{21})^2 - (\Im p_{12} - \Im p_{21})^2 \geq 0$$

Proof.

$$G(j\omega) = \begin{bmatrix} p_{11}(j\omega) & p_{12}(j\omega) \\ p_{21}(j\omega) & p_{22}(j\omega) \end{bmatrix}$$

is positive real if

$$G(j\omega) + G^T(-j\omega) =$$
$$\begin{bmatrix} p_{11}(j\omega) + p_{11}(-j\omega) & p_{12}(j\omega) + p_{21}(-j\omega) \\ p_{21}(j\omega) + p_{12}(-j\omega) & p_{22}(j\omega) + p_{22}(-j\omega) \end{bmatrix} =$$
$$\begin{bmatrix} \Re P_{11} & ((\Re P_{12} + \Re P_{21}) + j(\Im P_{12} - \Im P_{21})) \\ ((\Re P_{12} + \Re P_{21}) - j(\Im P_{12} - \Im P_{21})) & 2\Re P_{22} \end{bmatrix}$$

is positive semidefinite. Since positive semi-definiteness is equivalent to having non-negative leading principle minors, the conditions in 3 will be met. Also the following matrix must be positive semidefinite hermitian. (k_{ij} denotes the residue of p_{ij} and k^*ij is the complex conjugate of k_{ij}):

$$lim_{s \to j\omega}(s - j\omega)G(s) = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

Applying the positive semidefinite hermitian conditions will leave us with the same conditions in 2. This concludes the proof. □

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