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A Gaussian Mixture Framework for Cooperative Rehabilitation Therapy in Assistive Impedance-based Tasks

Mahdi Maaref[1], Alireza Rezazadeh[2], Kimia Shamaei[1], Mahdi Tavakoli[1], *Member, IEEE*

***Abstract*—Rehabilitation robots can aid patients to practice activities of daily living in order to enhance muscle strength and recover motor functions. In this paper, we focus on robot-assisted rehabilitation for cooperative therapy tasks that elicit impedance-based behaviors from the patient. For instance, if the rehabilitation robot is controlled to behave as a self-closing door and if pulling this simulated door open is the therapy task the patient needs to complete, the patient's hand should display a minimum required impedance to complete the task. When a patient is unable to complete the task, determining the minimum assistance to be provided to the patient by the rehabilitation robot such that the task can be accomplished is of interest. In this paper, we compare the impedance behavior of a therapist in multiple trials of the task with that of the patient using a Learning from Demonstration (LfD) technique that utilizes Gaussian mixture models. First and during the demonstration phase, the therapist performs the tasks individually so that the robot gains insight into how a healthy person would perform the task. Next and during the reproduction phase, the robot will cooperate with the patient in the therapist's absence and provide him/her with adaptive external assistance on a patient-specific and as-needed basis so that the task can be completed. To encourage active participation, provision of assistance to the patient is coupled to the variability observed in the therapist's behavior across various trials of the task. Therefore, the presented framework transfers the constraints and underlying characteristics of a given impedance-based task to the rehabilitation robot leading to cooperative interaction between the robot and the patient where the robot provides just-enough assistance. Experimental results involving 1D and 2D impedance-based tasks show that the proposed framework effectively provides the patient with assistance as needed during cooperative therapy.**

Keywords: Robotic Rehabilitation, Assistive Therapy, Assist-as-needed Therapy, learning from demonstration, Gaussian mixture, impedance-based task.

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[1]Mahdi Maaref (Corresponding Author), Kimia Shamaei, Mahdi Tavakoli (Principal Investigator) are with the Department of Electrical and Computer Engineering, University of Alberta, AB, Canada T6G 2V4. E-mail: maaref, shamaei, mahdi.tavakoli@ualberta.ca

[2] Alireza Rezazadeh is with the Shahid Beheshti University, Velenjak, Tehran, Iran. E-mail: A-Rezazade@sbu.ac.ir.

I. Introduction

There has been a rise in the number of disabled people mainly due to an increase in the aging population [1]. There has also been an increase in the rate of incidence of stroke, which is one of the main causes of disabilities worldwide [2]. Stroke patients may lose all or part of their abilities and are usually not be able to perform their daily living activities alone. Therapy exercises are highly recommended for people with disabilities in order to regain their strengths [3]. However, physical therapy resources are limited, leading to reductions in the duration of rehabilitation activities and an increase in the workload of therapists [4]. robotic Rehabilitation is one of the promising technologies to provide efficient, optimal and affordable means of rehabilitation and reduce the burden on the healthcare system [5], [6].

In this paper, we are interested in robot-assisted rehabilitation for cooperative therapy tasks that elicit an impedance-based behavior from the patient. In this context, the role of the therapist is to teach the robot how the task is performed normally (demonstration phase, also called learning phase). In the next step, the robot will help reproduce the performance of the task by the patient through assisting him/her as needed (reproduction phase). Fig. 1 illustrates the overall idea of presented method, which has two distinct phases. The following discussion is framed around a cooperative therapy task involving opening a self-closing door, but is not limited to it. Assume the rehabilitation robot is controlled to behave similar to a self-closing door. During the first phase ("demonstration phase"), the therapist interacts

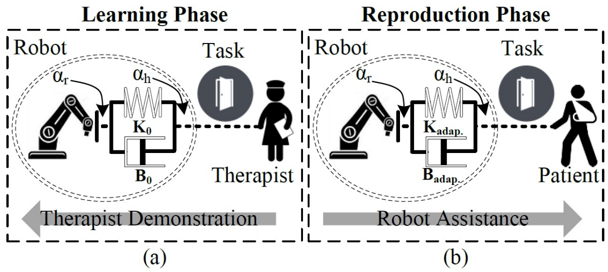

Figure 1: The proposed Learning from Demonstration scheme in a robotic rehabilitation system: (a) Learning phase: In this phase, the robot interacts with the therapist and the task is demonstrated to the robot. The direction of the arrow demonstrates that the information and specifications of the task is transferred from therapist to the robot. In the proposed framework, the robot does not need to learn how the therapist assists the patient; it is sufficient for it to learn how the therapist performs the task ideally. (b) Reproduction phase: In this phase, the robot interacts with the patient. As it is shown, the robot's internal controller coefficients are now $K_{adap.}$ and $B_{adap.}$ which represents the adaptive control of the robot in this phase. Also, the direction of the arrow is changed, which demonstrates the direction of the assistance from the robot to the patient. In this phase, the robot analyses the behavior of the patient precisely and compares it with that of the therapist. Therefore, it is able to estimate the assistance as needed adaptively.

with the robot in order to open the simulated door and complete the task (Fig. 1 (a)). Based on logged robot-therapist interaction data, the underlying specifications and constraints of this impedance-based task are statistically analyzed and learned using Gaussian mixture models. During the next step, when the therapist no longer intervenes (Fig. 1(b)), the system adaptively determines on a patient-specific basis the minimum assistance that must be provided for completion of the task by the patient.

The approach in this paper provides adaptive assistance to a given patient in performance of tasks based on the *performance differential* the robotic system observes between the behavior of the therapist in the demonstrations phase and that of the patient in the reproduction phase in a novel Learning from Demonstration (LfD) framework [7], [8]. The proposed framework is very useful in clinical settings where the therapist has knowledge of the task to be achieved but not the ability to accordingly reprogram/reconfigure the robot. This is because the proposed paradigm allows for teaching a rehabilitation robot the behavior expected from a perfect user (therapist) in performing the task by physically demonstrating it rather than explicitly programming the robot through machine commands.

The paper is structured as follows: In section II, related research on robotic rehabilitation systems is surveyed and the main contributions of the proposed framework are explained. In section III, the dynamic model of interaction between the robot's end-effector and the patient's arm is introduced. In Section IV, the proposed learning model for impedance-based tasks, which spans the 3D space of time, position and force, is presented. In Section V, the reproduction paradigm is developed. The proposed strategy is experimentally tested in Section VI. Finally, Section VII summarizes the main findings of this paper and lists future work.

II. Related Works

Research on Assist as Needed (AAN) robot-assisted therapy is motivated by the fact that passive assistance does not promote motor recovery as well as adaptive assistance [9].

Pehlivan et al. [10], [11] suggested using adaptive control as opposed to impedance control. They used Gaussian Radial Basis functions (RBF) to model the ability and effort of the patient. The use of an adaptive controller is meant to provide the torque needed to complete the trajectory with an increased mechanical compliance and lower feedback gains. Similarly, Wolbrecht et al. [12] modeled the patient's muscle abilities and tone through a Lyapunov-based control framework. Within the framework and as part of their adaptive controller, their assistance scheme also implemented a Gaussian RBF. Furthermore, they proposed a forgetting factor algorithm to adjust the robot torque input according to the patient's performance and to prevent the patient from letting the robot take control. In the work of Vergaro et al. [13], a combination of impedance and adaptive control was implemented. An impedance control scheme is used to provide an attractive component directed from the patient's hand to the target, a viscous component to dampen high-frequency oscillations of small magnitude, and a virtual wall to tunnel the shape of the trajectory. The adaptive controller adjusts the amplitude of the force field according to the patient's performance.

In contrast to the work in which the patient is provided with adaptive assistance, we have administered the assistance using the LfD technique during the cooperative performance of the task. The benefits of introducing LfD to robotic rehabilitation therapy can be evaluated from two different perspectives. First, in rehabilitation robotics, the required programming of robots can be cumbersome. The formulation of robot actions for the activities of daily living (one example of which is door opening) is demanding because it strongly depends on the environment, which is unstructured and may change significantly from one case to the other (in the self-closing door opening example, each door comes with a different spring stiffness). Since daily living activities can robustly be performed by a normal human being without any difficulty, the most promising way to alleviate the problem mentioned above is to take advantage of the human experience and skill in performing daily living activities and to transfer them to the robot via LfD without the need for re-programming the robot every time [14].

Another benefit of using LfD in rehabilitation robotics is in capturing and using the behaviour of the therapist during variable training for the patient. In [15], it is shown that providing variability in training enhances motor function recovery. It is discussed how fixed trajectory training strategies drive the spinal cord into a state of learned helplessness. In contrast to other statistical approaches such as RBF, our proposed GMM approach has the ability to combine probability distribution and approximate multivariate functions with multiple variables across different trials of the task [7]. So, it directly acts as powerful method for serving variable training to patients. We measure this variability by capturing and learning from the behavior of a therapist in daily living activities in an LfD context. Adaptive assistance methods available in the literature do not feature this strength of our method either.

III. Robot/Human Interaction Model

In all robotic rehabilitation settings, the robot has physical interaction with an environment, which can be the therapist or the patient or both, through its end-effector. This interaction can be in the form of kinematic constraints or dynamic contacts between the robot and the environment. For rehabilitation cooperative tasks, using traditional position control for the robot cannot be useful because the environment with which the robot interacts is a human, which is very hard to model. Therefore, various dynamic models of robot motion and physical interaction between the end-effector and the environment have been proposed in previous studies such as impedance control techniques [16], [17] as well as dynamic movement primitives, which is a very popular framework for encoding movement trajectories [18]. In [19], a perfect nonlinear dynamic coupling is presented for modeling the robot behaviour in contact with the environment. This model supposes that the end-effector of the robot can be treated equivalently to a single unit mass moving in space. It also emphasizes that the mass is driven by a virtual spring-damper system. The physical interaction between the robot and the therapist or the patient is modeled by set of virtual spring-damper couplers connected between the two [20]. Based on this, we model the interactions that should happen for a cooperative rehabilitation as shown in Fig. 1. In [21], a comparison between the above-mentioned model and other popular dynamic frameworks has been done with regard to the response to perturbations.

The robot controller ensures that the dynamics of the robot interacts with the robot's environment through a virtual mass-spring-damper system. In other words, the control law ensures that

$$f_{sensor} = \Lambda(\ddot{a}_r - \ddot{a}_h) + \Psi(\dot{a}_r - \dot{a}_h) + \Gamma(a_r - a_h) \quad (1)$$

in which a_h is the position of the human hand (either the therapist or the patient) that is interacting with the robot, and a_r is the robot position. Also, f_{sensor} is the environment generalized force applied to the robot, which can be measured by a force sensor attached to the robot end-effector. Λ , Ψ and Γ denote the inertia, damping and stiffness matrices of the desired mass-spring-damper system, respectively. It is important to mention that different simplifications have been done to (1). For instance, in [22], it is assumed that Λ can be assumed to be zero, where the resultant spring-damper system is similar to a PD controller. In [23] and [24], (1) is simplified as follows to describe the robot's desired motion during interaction with the environment:

$$\ddot{a}_r = \Gamma(a_h - a_r) - \Psi.\dot{a}_r + f_{sensor} \quad (2)$$

Here, an interaction between a human and a robot is assumed as a perfect nonlinear dynamic coupling. Since the robot end effector in modeled to be equivalent to a single unit mass moving in the Cartesian space, the basic characteristics of cooperative rehabilitation tasks involve not only the path to be followed, but also the proper force interaction pattern between the two as they move. Therefore, in the learning phase, the path of the virtual spring which represents the cooperative behaviour of the therapist is estimated. It is important to mention that force and position data and the first and second time derivatives of position are captured by means of sensors directly during the learning phase. For calculating the virtual spring path, we use pre-selected values for the damping and stiffness matrices. Although there is not any direct guiding principle for this pre-selection and the matrices are selected experimentally, by changing them, we can tune the difficulty of the task.

IV. Demonstrating the task

In the context of human-robot interaction, the goal of the demonstration phase (also called the learning phase)

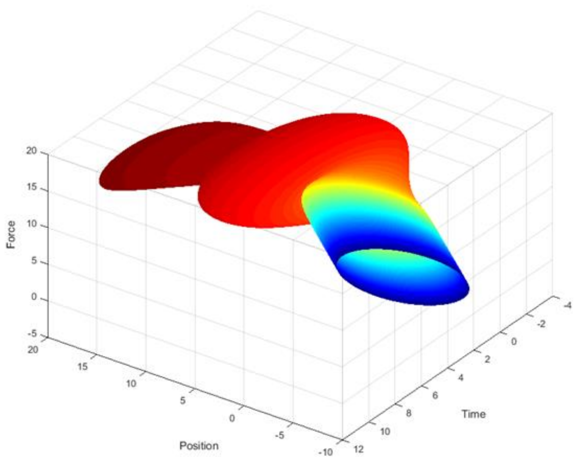

Figure 2: Encoding of the demonstrated behaviour by the therapist. The experiment portrays a scenario simulating a Door Opening Task.This figure illustrates the 3D representation of the trajectory required for successful performance of the task, which consists of time, force and position data. As it can be seen, there is variability in the therapist's behavior across various trials of the task. For instance, the start and stop points for the door are consistent across the trials while the trajectory to get from the start point to the stop point varies.

is to model the human behavior, which is captured during the demonstration of a task and to determine underlying specifications and constraints of the interaction. In cooperative robotic rehabilitation applications, the therapist demonstrates the ideal performance of the task to the robot and lets it analyze the behavior displayed by the therapist, possibly using statistical approaches such as what follows.

Using statistical approaches such as Gaussian Mixture Model (GMM) for finding a generalized model of demonstrated behavior involving motion, force or impedance has been reported in [25]. A GMM is a probabilistic model obtained as a mixture of a finite number of Gaussian distributions with unknown parameters, which is able to model the variability of the human's actions in performing the task across various trials. In most cases, capturing the human's motion sequences requires using more than one Gaussian for modeling the task and this is why GMM has to be used. Compared to simple averaging of the human motion sequences, Gaussian modeling requires a much smaller number of demonstrated motion sequences in order to retrieve smooth trajectories that are useful for provision of assistant by the robot to the patient.

In this paper, we present a learning model that is capable of characterizing cooperative impedance-based rehabilitation tasks such that the position-force profile of human/robot interaction can be captured. The robot first learns the impedance-based behaviour [26] of a healthy human (the therapist) during task performance in the demonstration phase. Later and in the reproduction phase, the robot uses the above-obtained ideal trajectory to obtain the movements the patient should have. The robot then assists the patient by compensating for the differences between the patient's ideal movements (i.e., those consistent with the therapist's behavior) and the patient's actual movements.

Throughout this paper we use r,h and t to indicate robot force interaction variables, human(the therapist or the patient) position variables and time variables respectively. Also for indicating the robot-side and human-side and time vectors, we add superscript of r,h and t to the vector. Formally, the learned trajectory consists of a p-dimensional robot-side column vector $\beta^r \in \mathbb{R}^p$, a q-dimensional human-side column vector $\beta^h \in \mathbb{R}^q$, and a one-dimensional time variable $\beta^t \in \mathbb{R}^1$. It is important to mention that in humanoid robots with multiple degrees of freedom, it is possible that the motions of two or more joints results in a smaller number of interaction forces with the human (asymmetry of joint and Cartesian spaces). In such robots, the length of human-side vectors and the robot-side vector can be different. Based on the sampling rate used in the system for data capture, each demonstration of the task by the therapist would generate the dataset $\beta = \{\beta_n^r, \beta_n^h, \beta_n^t\}_{n=1}^N$. If the therapist performs M demonstrations to teach the robot the underlying features of the given task, the whole dataset will be $\beta = \{\{\beta_{nm}^r, \beta_{nm}^h, \beta_{nm}^t\}_{n=1}^N\}_{m=1}^M$; this set has $\Psi = N.M.(p+q+1)$ members.

A GMM can be rewritten as

$$f_{R,H,T}\left(r,h,t\right) = \sum_{i=1}^{K} \pi_i \mathcal{N}(r,h,t|\mu_i,\Sigma_i) \tag{3}$$

in which

$$\mathcal{N}\left(r,h,t \mid \mu_i,\Sigma_i\right) = \frac{1}{\sqrt{\left(2\pi\right)^{p+q+1}\left|\Sigma_i\right|}} e^{-\frac{1}{2}\left(\left(\beta-\mu_i\right)^T {\Sigma_i}^{-1}\left(\beta-\mu_i\right)\right)} \tag{4}$$

As it is mentioned, β is the data of position, force and time captured by the robot during the learning phase as $\beta = \{\{\beta_{nm}^r, \beta_{nm}^h, \beta_{nm}^t\}_{n=1}^N\}_{m=1}^M$. (4) give the probability density function (pdf) of the multivariate (p+q+1)-dimensional Gaussian. The parameters of such a model include the number of mixture components K, the prior weights π_i, the means μ_i and the covariance matrix Σ_i. The joint pdf extracts the features of the rehabilitation task based on the following matrices:

$$\mu_i = \begin{bmatrix} \mu_i^r \\ \mu_i^h \\ \mu_i^t \end{bmatrix}, \quad \begin{array}{r} \mu_i^r \in \mathbb{R}^p \\ \mu_i^h \in \mathbb{R}^q \\ \mu_i^t \in \mathbb{R}^1 \end{array} \tag{5}$$

$$\Sigma_i = \begin{bmatrix} \Sigma_i^{rr} & \Sigma_i^{rh} & \Sigma_i^{rt} \\ \Sigma_i^{hr} & \Sigma_i^{hh} & \Sigma_i^{ht} \\ \Sigma_i^{tr} & \Sigma_i^{th} & \Sigma_i^{tt} \end{bmatrix}$$

In other words, the significance of μ_i and Σ_i in the context of the rehabilitation task is to determine the mean and variance of the interaction force between the robot end effector and the therapist/patient in different positions and times of the performance of the task. Thus, the GMM model is $(\theta_1, \theta_2, \dots , \theta_k)$ where $\theta_i = \{\pi_i \ , \mu_i \ , \ \Sigma_i\}$ are the model parameters to be found subject to the constraint

$$\sum_{i=1}^{K} \pi_i = 1 \ , \ \forall i \ : \pi_i \geq 0$$

and collected input/output dataset $\beta = \{\{\beta_{nm}^r, \beta_{nm}^h, \beta_{nm}^t\}_{n=1}^N\}_{m=1}^M$. The mixture model is trained by means of the Expectation-Maximization (EM) algorithm [27].

For the purpose of reconstructing a general form for human/robot interaction in the rehabilitation task, we apply Gaussian Mixture Regression (GMR). GMR allows to extract a single generalized model made up from the set of input/output pairs (dataset) that were used to train the GMM. The generalized model encapsulates all of the essential features of the dataset and can predict the outputs using new inputs that are not necessarily in the dataset.

Note that the concept of GMR with two different input vectors had been previously presented in [26]. However, for the first time we will derive the mathematical formulation for such a regression in this paper. This will allow us to establish a position-force-time model to explain the underlying specifications and constraints of an impedance-based task. We start by considering a classical result of the multivariate Gaussian density, which is that when partitioning the joint density into a conditional one, the conditional density is also multivariate Gaussian. Theorem 1 calculates the mean and variance of the resulting conditional density.

Theorem 1: If $\begin{bmatrix} \beta^r \\ \beta^h \\ \beta^t \end{bmatrix} \sim \mathcal{N}_{p+q+1}(\mu, \ \Sigma)$, where μ and Σ are defined as in (5), the Gaussian conditional density of $f_{R|H,T}(r|H = h, T = t)$ has a mean that is a linear function of h and t, and also has a constant variance. Mathematically,

$$f_{R|H,T}(r|H = h, T = t) \sim \mathcal{N}_{p+q+1}\left(\hat{\mu}, \ \widehat{\Sigma}\right)$$

where $\hat{\mu} = \mathcal{A}h + \mathcal{B}t + \mathcal{C}$ and $\widehat{\Sigma} = \mathcal{D}$ with $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$ being constant matrices. This theorem is proved in the appendix.

While Theorem 1 talks about one Gaussian distribution, it can be extended to a mixture of several Gaussian distributions. In fact, for K Gaussians, the GMR is formulated as

$$\begin{aligned} & f_{R|H,T}(r|H = h, T = t) = \\ & \sum_{i=1}^{K} \varpi_i . \mathcal{N}_{p+q+1}\left(r|h, t, \hat{\mu}_i, \hat{\Sigma}_i\right) \end{aligned} \tag{6}$$

in which ϖ_i is the probability that the i-th Gaussian distribution correspond to the inputs h and t. Mathematically,

$$\varpi_i = \mathcal{P}(i \mid h, t) = \frac{\mathcal{P}(i)\mathcal{P}(h, t|i)}{\sum_{j=1}^{K} \mathcal{P}(j)\mathcal{P}(h, t|j)} \tag{7}$$

which yields

$$\varpi_i = \frac{\pi_i \mathcal{N}(h, t \mid \mu_i, \Sigma_i)}{\sum_{j=1}^{K} \pi_j \mathcal{N}(h, t \mid \mu_j, \Sigma_j)} \tag{8}$$

where μ_i *and* Σ_i were defined in (5). So, for a mixture of K components, the conditional expectation and covariance of the output vector can be approximated by the multi model distribution with following parameters: [7]

$$\begin{aligned} \hat{\mu} &= \sum_{i=1}^{K} \varpi_i . \hat{\mu}_i \\ \hat{\Sigma} &= \sum_{i=1}^{K} \varpi_i{}^2 (\hat{\mu}_i^2 + \hat{\Sigma}_i) - (\sum_{i=1}^{K} \varpi_i . \hat{\mu}_i)^2 \end{aligned} \tag{9}$$

Fig. 2 depicts the probabilistic encoding of the demonstration of a task in the force-position-time reference system after applying Gaussian process (GMM and GMR) to the dataset. Gaussian mean and variance vectors over the executed repetitions are used for tracing the 3D mesh. As it can be seen in Fig. 2, the essential features and constraints of a given task can be extracted from such a 3D mesh. For instance, observing the variance values along the trajectory (as shown by the wider and narrower parts of the mesh) can inform about how differently the same task was done by a human (e.g., the therapist) across different trials.

The overall idea of the demonstration phase in the presented framework is illustrated in Fig. 3. As it is

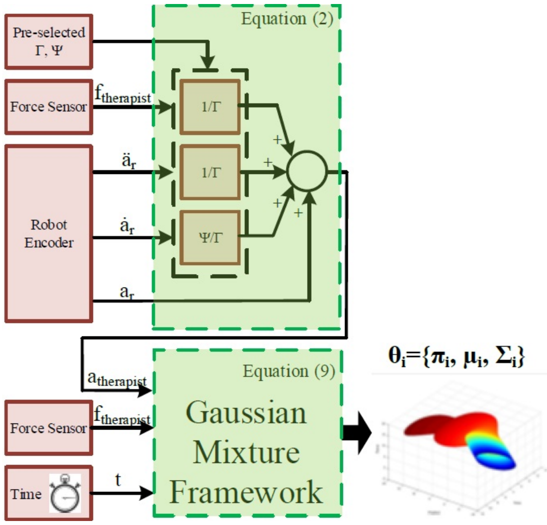

Figure 3: Block Diagram in the learning phase. (Top): (2) is used to determine the therapist's arm position during each of trials in the demonstration phase. The data of force sensor and robot encoder are used. For damping and stiffness matrices, we use pre-selected values. (Bottom): (9) is used to derive the GMR. Time data is also used. The result of the demonstration phase is the GMR parameters and a 3D mesh.

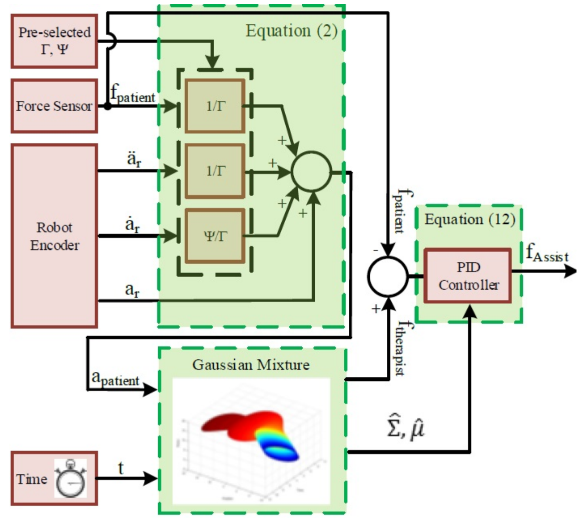

Figure 4: Block Diagram in the reproduction phase. (2) is used to determine the patient's arm position. Using the 3D mesh obtained in the demonstration phase, we can compare the force which was exerted by the therapist during the demonstration with that of the patient during the reproduction and calculate the required external assistance force.

shown in Fig. 3 (top), (2) is used to determine the therapist's arm position during each of m trials in the demonstration phase. n data points captured from both the force sensor and robot encoders are stored. We use pre-selected values for damping and stiffness matrices Γ and Ψ in (2). After calculating the position of the therapist's hand in each of the m trials through (2), the model presented in (9) is used to derive the GMR based on the data set $\beta = \{\{\beta^r_{nm}, \beta^h_{nm}, \beta^t_{nm}\}^N_{n=1}\}^M_{m=1}$. As it can be seen from Fig. 3 (bottom), the data of force sensor and time is also needed to calculate the GMR. The result of the demonstration phase as shown in Fig. 3 (bottom) is the the GMR parameters $\theta_i = \{\pi_i, \mu_i, \Sigma_i\}$ and a 3D mesh such as the one in Fig. 2.

V. Reproduction of the Task

When a patient is unable to complete a task for which the performance expected from a perfect user was learned by the robot in the previous section, determining the assistance to be provided to the patient may be done from a task reproduction perspective. Reproduction is based on comparing the performance of a therapist in demonstration of the task with that of the patient. Based on this *performance differential*, the LfD framework helps to determine the required assistance on a patient-specific basis for the task to be completed.

To calculate the performance differential, the GMR in (9) interrogates the previously-demonstrated behavior of the therapist to provide the expected β^r (such as the force interaction) for the present β^h (such as the position of the arm) and β^t (time). In the conventional LfD framework, where success in performing the task is the key, exact task reproduction is the focus of the robotic system. However, in the presented framework, applying the assistance as needed is of importance to encourage the participation between the patient and the robot.

The overall idea of the reproduction phase in the presented framework is shown in Fig. 4. As it is shown, (2) is used to determine the patient's arm position a_{patient}. Using the 3D mesh obtained in the demonstration phase, or equivalently using (9), we can compare the force which was exerted by the therapist during the demonstration with that of the patient during the reproduction (live data of the force sensor) and calculate the required external assistance force.

In order to calculate the therapeutic assistance to the patient, we use a PID control law as follows:

$$f_{assist}(t) = K.e(t) + I.\int e(t)\mathrm{d}t + D.\frac{\mathrm{d}e(t)}{\mathrm{d}t} \quad (10)$$

where

$$e(t) = f_{therapist} - f_{patient} \quad (11)$$

Here, $f_{therapist}$ is the therapist's (demonstrated) behaviour using the model presented in (9) and $f_{patient}$ shows the patient's (reproduced) behaviour during the task performance.

The proportional term produces a therapeutic assistance that is proportional to the error between the patient's performance and the *average* demonstrated therapeutic performance. A high proportional gain results in a large assistance being provided for a given error. The integral term provides assistance that is proportional to not only the magnitude of the error but also the duration of the error. This term accelerates the convergence of the patient's performance towards the demonstrated therapeutic performance and theoretically ensures zero error at the end of the task. Note that in the presented control law, the integral term suffers from accumulated errors, which make this term very large. A well-known technique known as integral anti-windup can be used to reset the integral value. The derivative term reacts in prediction to future errors and is meant to slow down the rate of change of the assistance provided to the patient. Note that since the integral term responds to the errors accumulated from the past, it can lead to so much therapeutic assistance that causes an overshoot in the performance of the patient compared to the demonstrated therapeutic performance.

In the above PID law for provision of therapeutic assistance to the patient, it is possible to adaptively change the P, I and D gains such that the patient is actively engaged in the task, that assistance is provided to the patient only as needed and when needed, and that the variability in the therapist's demonstrated performance of the task in the learning phase is taken into account. This can happen if the PID gains are chosen as

$$\begin{aligned} K(t) &= K_0 - K'(\Sigma^2(t) - |e(t)|) \\ I(t) &= I_0 - I'(\Sigma^2(t) - |e(t)|) \\ D(t) &= D_0 - D'(\Sigma^2(t) - |e(t)|) \end{aligned} \tag{12}$$

where $\Sigma^2(t)$ refers to the variability in the therapist's demonstrated performance of the task in the learning phase, which was calculated via GMR in (9). If the patient is close to the therapist's average demonstrated performance, then $e(t)$ is close to zero and minimum assistance gains are employed (e.g., $K(t) < K_0$). If the patient deviates from the above but is still within the range of the performances demonstrated by the therapist, then the assistance gains are increased (e.g., $K(t) = K_0$ when the patient is following the extreme performances demonstrated by the therapist). If the patient deviates significantly from the performances demonstrated by the therapist, the assistance gains are further increased to bring the patient in line (e.g., $K(t) > K_0$). The above is a therapy model in which the provision of assistance is coupled to the variability observed in the therapist's behaviour across various trials of the task. This will lead to encouraging free participation of the patient as therapeutic intervention (assistance) is more tightly enforced where there is low variability in the therapist's behaviour in different trials of the task.

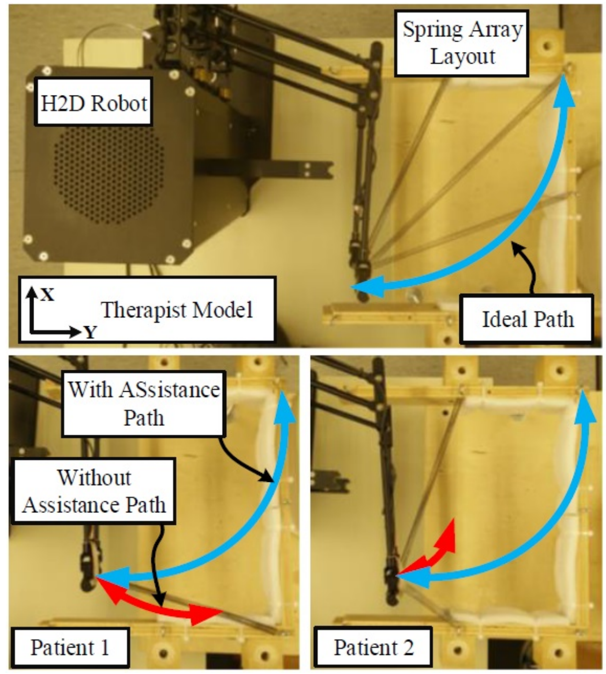

Figure 5: (top) Learning phase for experimental setup using the HD2 robot and the symmetrical springs that simulate the behaviour of a healthy person (therapist). In this phase, the symmetrical springs move the robot's end effector (door handle) in the blue arrow's path and simulate the successful performance of the door opening task. So, the specifications and constraints of this performance are transferred to the robot. (bottom left) In the reproduction phase, we disconnect one of the springs to simulate a patient with disability. The red arrow demonstrates the path of the movement without external assistance coming from the robot. By generating adaptive assistance by the robot, the non-symmetrical spring (patient) is can perform the task successfully and move through the blue arrow's path. (bottom right) Reproduction phase for patient 2 (corresponding to a different layout of the non-symmetrical springs).

VI. Discussion and Results

A. Experimental Setup

The experimental results for showing the effectiveness of the presented LfD-based cooperative rehabilitation

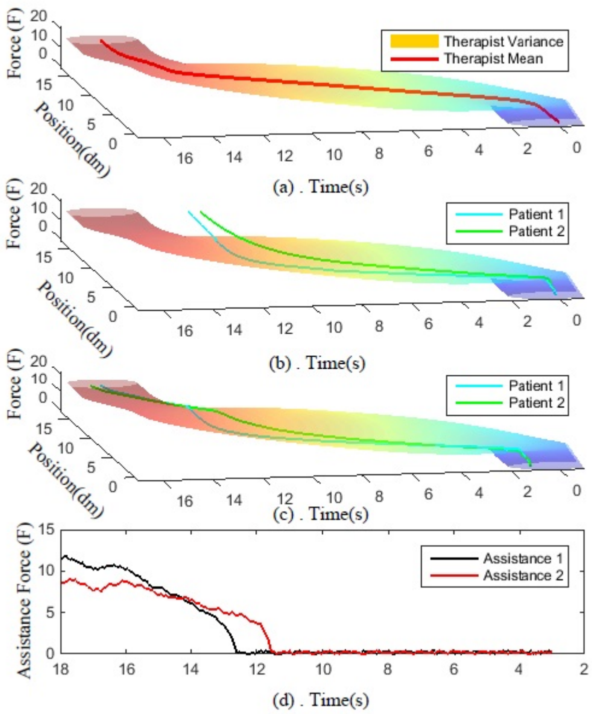

Figure 6: The proposed method in the door opening task. (a) The projection of the demonstrated mesh of therapist's behaviour in the force-position coordinates. The data relating to the mean and the variance of the therapist across various trials of the task is used for extracting essential features of the task. (b) The data of the patient's performance without therapeutic assistance from the robot. The blue and green lines show a large deviation when the patient performs the task without receiving assistance from the robot. (c) By using the previously presented assistance model, the robot provides external assistance to the patient so that the task can be done successfully. (d) The black and red lines show the magnitude of the assistance for both patients. As it can be seen, the assistance is provided more strictly when the patient starts to deviate from the range of performances demonstrated by the therapist while it is minimal otherwise.

system is provided in this section for both learning and reproducing phases. Two different sets of experiments have been conducted for the purpose of evaluating the performance of the proposed framework.

The apparatus used includes an HD2 Haptic Device (Quanser Inc., Markham, Ontario, Canada) that provides 6 degrees of freedom (DOF) of position sensing and 6-DOF force feedback. The device has a parallel mechanism that is highly back drivable and has negligible friction. Since the device uses capstan drives, the perceived endpoint inertia is low while the device has a rigid structure.

Two different tasks are implemented in this paper to show that the proposed approach is not limited to 1-DOF tasks. First, a 1-DOF task simulating door opening is conducted and next a 2-DOF task simulating cutting a cake is presented. For the door opening task, the humans (therapist and patient) were simulated with a spring array system. However, in the second task, the authors personally mimicked the roles of the therapist and the patient. In both tasks, the robot simulates the impedances one feels when either opening a self-closing door or cutting a cake; this is an appealing aspect of the proposed framework because it gives the flexibility to implement various activities of daily living using the same robot, in a repeatable manner and cost effectively.

B. *Task one: Opening the door*

Being a learning from demonstration test, this experiment consists of the two different phases shown in Fig. 1. In both phases, a 2-DOF spring array is connected to the HD2 robot end-effector as shown in Fig. 5. This spring array emulates both a therapist in the learning phase, and a patient affected by muscle impairment in the reproduction phase. In the first phase, where a therapist is to demonstrate the task performance to the robot, the array's springs are configured symmetrically around the line of motion to be taken by the robot end-effector as the task progresses. In the second phase, where a patient is to try to move the robot in order to perform the task, some of the array's springs are removed such that the remaining springs are no longer symmetric relative to the line of motion such that motor deficiency is simulated. We employ two asymmetric configurations of the springs in order to model two different patients with different motor capabilities. The symmetric (i.e., therapist) and the two asymmetric (i.e., patients) configurations of the mass-spring array are shown in Fig.5.

For this experiment, we first control the HD2 device to behave as the self-closing door. For pulling this simulated door open, the human user's hand should display a minimum required impedance. A total of 5 demonstrations are carried out, each lasting around 16 seconds and therefore resulting in around 1600 sample points (100 Hz is the data logging rate). For the demonstration, the therapist-emulating, symmetric spring array is connected to the robot end-effector. To move towards its equilibrium point, the spring array pulls the simulated door (robot end-effector), thus opening the door. A virtual damper is implemented in the robot controller in order to complement the physical spring array (which inevitably has some mass) to more closely

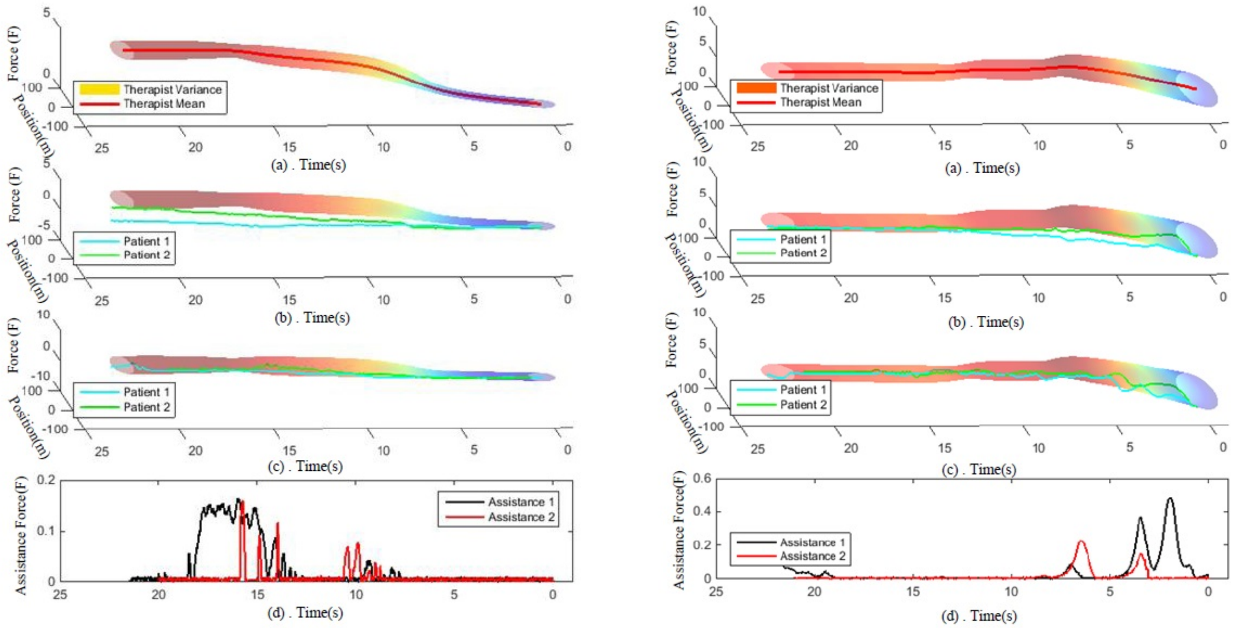

Figure 7: The proposed method in cutting the cake task. This task is performed in two DOFs. Therefore, the left and right figures represent the z direction (up-down cutting) and the x-y plane (side to side cutting), respectively. (a) The mean and the variance of the therapist's behavior across various trials. (b) The blue and green lines show the performance of the two patients in the absence of therapeutic assistance from robot. This figure shows a large deviation with respect to the therapist's behavior. (c) The blue and green lines show the performance of the patients in the presence of the therapeutic assistance from robot using the previously presented assistance model. This figure shows that the task can be done successfully. (d) The magnitudes of the external assistance for both patients. Just like the door opening task, the assistance is provided more strictly when the patient starts to deviate from the range of performances demonstrated by the therapist while it is minimal otherwise.

model the human, which is considered a mass-spring-damper system. And again, variability is introduced in the therapist's demonstrated performance by changing this virtual damping parameter randomly.

Since the door has a specific impedance, the patient arm should show a minimum required impedance to open the door. In Fig. 6(a), the red line represents the the therapist's demonstrated performance. In contrast to this, in Fig. 6(b), the blue and green line show a large deviation when a patient performs the task without receiving assistance from the robot. By using the previously presented assistance model, the robot provides assistance to the patient in the form of robot-generated forces so that the task can be done successfully; see Fig. 6(c). The magnitude of the assistance is also shown in Fig. 6(d). As it can be seen, more assistance is provided when the patient starts to deviate from the range of performances demonstrated by the therapist while it is minimal otherwise. The experiments show that as a patient gets tired or the task difficulty increases, more assistance need to be provided to the patient. [1]

C. Task two: Cutting the cake

This experiment also consists of the two different phases shown in Fig. 1 for demonstration and reproduction. The authors simulate the role of the therapist and the patient by grabbing the robot end-effector in the demonstration and reproduction phases, respectively. In the first phase, the person playing the therapist's role demonstrates the task performance to the robot, which consists of lowering the robot end-effector (simulating the knife) in the z direction to a desired depth followed by moving it on an arc in the $x-y$ plane (simulating the cake cutting). In the second phase, the two people playing the patient's role begin to move the robot in order to perform the task but do not exert the required forces for lowering the knife and/or cutting through the simulated cake.

[1]This paper has supplementary downloadable material available at https://youtu.be/QJkwJd1PAiE showing the experiments in the paper.

For this experiment, we will have the HD2 device controlled to demonstrate the resistance one feels when cutting through a cake in up-down and side-to-side directions. For doing this task, the human user's hand should display a minimum required impedance. A total of 25 demonstrations are carried out, each lasting around 6 seconds and therefore resulting in around 2500 sample points (100 Hz is the data logging rate). The results in Fig. 7(left) correspond to the z direction while those in Fig. 7(right) correspond to the $x-y$ plane. In Fig. 7(a), the red line represents the the therapist's demonstrated performance. In contrast to this, in Fig. 7(b), the blue and green line show a large deviation when each of the two patients perform the task without receiving assistance from the robot. By using the previously presented assistance model, the robot provides assistance to the patients in the form of robot-generated forces so that the task can be done successfully; see Fig. 7(c). The magnitude of the assistance is also shown in 7(d). As it can be seen, the assistance is provided more strictly when the patient starts to deviate from the range of performances demonstrated by the therapist while it is minimal otherwise. This shows that the patient is only assisted as needed to promote free and active participation of the patient.

VII. Conclusion

In this paper, a new framework has been proposed for assist-as-needed rehabilitation therapy in impedance-based tasks, which involve the coordination of time, position and force. The main contribution of the work is to develop a Learning from Demonstration based technique for the provision of assistance to the patient in impedance-based tasks. In this technique, first the robot learns how a healthy person (a therapist) perform the given task and then adaptively interacts with the patient to provide just enough assistance so that the patient can perform the task successfully. The provision of assistance to the patient is also coupled to the variability observed in the therapist's behavior. This is called assist-as-needed therapy, which is desirable because it encourages active participation of the patient. The validity of the proposed approach has been shown using experiments involving 1-DOF and 2-DOF activities of daily living. In the future, we will bring the proposed paradigm to a clinical setting for patient studies.

Appendix

Proof of Theorem 1:

In order to calculate mean $\hat{\mu} = E\left(r|H=h, T=t\right)$, let us define $z = r + \mathcal{M}h + \mathcal{N}t$. So,

$$\hat{\mu} = E\left(z - \mathcal{M}h - \mathcal{N}t \mid H=h, T=t\right)$$

$$= E\left(z \mid H=h, T=t\right) - \mathcal{M}\left(h \mid H=h, T=t\right) -$$

$$\mathcal{N}\left(t \mid H=h, T=t\right)$$

$$= E\left(z \mid H=h, T=t\right) - \mathcal{M}h - \mathcal{N}t$$

In [28], it is proven that joint normal distributions are independent if they are uncorrelated. Since Z, H and T are jointly normal, if we make them uncorrelated, they will be independent. So, we need to find $\mathcal{M}$ and $\mathcal{N}$ such that

$$\begin{cases} cov\left(Z,H\right) = 0 \\ cov\left(Z,T\right) = 0 \end{cases}$$

This yields

$$\begin{cases} cov\left(R,H\right) + \mathcal{M}.cov\left(H,H\right) + \mathcal{N}.\ cov\left(T,H\right) = 0 \\ cov\left(R,T\right) + \mathcal{M}.cov\left(H,T\right) + \mathcal{N}.\ cov\left(T,T\right) = 0 \end{cases}$$

As it was stated before, the covariance between R and H (or R and T) appears in the covariance matrix in (5), therefore we can write:

$$\begin{cases} \Sigma_{rh} + \mathcal{M}.\Sigma_{hh} + \mathcal{N}.\ \Sigma_{th} = 0 \\ \Sigma_{rt} + \mathcal{M}.\Sigma_{ht} + \mathcal{N}.\ \Sigma_{tt} = 0 \end{cases}$$

This consists of $p+q+1$ equations and unknowns. So by solving it, we have

$$\mathcal{N} = \left(\Sigma_{rt} - \Sigma_{rh}{\Sigma_{hh}}^{-1}\Sigma_{ht}\right)\left(\Sigma_{th}{\Sigma_{hh}}^{-1}\Sigma_{ht} - \Sigma_{tt}\right)^{-1}$$

$$\mathcal{M} = \left(\Sigma_{rh} - \Sigma_{rt}{\Sigma_{tt}}^{-1}\Sigma_{th}\right)\left(\Sigma_{ht}{\Sigma_{tt}}^{-1}\Sigma_{th} - \Sigma_{hh}\right)^{-1}$$

Therefore, Z is independent from both H and T. So,

$$E\left(z \mid H=h, T=t\right) = E\left(z\right) = \mu^r + \mathcal{M}\mu^h + \mathcal{N}\mu^t$$

This yields $\hat{\mu} = \mu^r + \mathcal{M}\mu^h + \mathcal{N}\mu^t - \mathcal{M}h - \mathcal{N}t$. Therefore:

$$\mathcal{A} = -\mathcal{M}$$

$$\mathcal{B} = -\mathcal{N}$$

$$\mathcal{C} = \mu^r + \mathcal{M}\mu^h + \mathcal{N}\mu^t$$

For calculating the variance $\widehat{\Sigma} = var(r|H=h\ ,\ T=t)$, again by defining z and calculating $\mathcal{M}$ and $\mathcal{N}$, we have

$$\widehat{\Sigma} = var\left(z - \mathcal{M}h - \mathcal{N}t \mid H=h\ ,\ T=t\right)$$

$$= var\left(Z \mid H=h\ ,\ T=t\right) +$$

$$\mathcal{M}.var\left(H \mid H=h\ ,\ T=t\right).\acute{\mathcal{M}} +$$

$$\mathcal{N}.var\left(T \mid H=h\ ,\ T=t\right).\acute{\mathcal{N}} -$$

$$\mathcal{M}.cov\left(Z,H\right) - cov\left(Z,H\right).\acute{\mathcal{M}} -$$

$$\mathcal{N}.cov\left(Z,T\right) - cov\left(Z,T\right).\acute{\mathcal{N}} +$$

$$\mathcal{M}.cov\left(H,T\right).\acute{\mathcal{N}} + \mathcal{N}.cov\left(T,H\right).\acute{\mathcal{M}}$$

Previously, we proved that Z is independent from both H and T, so:

$$\widehat{\Sigma} = var\left(z\right) + \mathcal{M}\Sigma_{ht}\acute{\mathcal{N}} + \ \mathcal{N}\Sigma_{th}\acute{\mathcal{M}}$$

or equivalently

$$\widehat{\Sigma} = \Sigma_{rr} + \mathcal{M}\Sigma_{hh}\acute{\mathcal{M}} \ + \mathcal{N}\Sigma_{tt}\acute{\mathcal{N}} \ + \mathcal{M}\Sigma_{ht}\acute{\mathcal{N}} + \mathcal{N}\Sigma_{th}\acute{\mathcal{M}}$$

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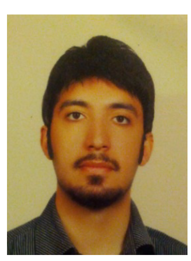

Mahdi Maaref received the B.E degree from the University of Tehran in 2008 and M.Sc. degree at the same University in 2011 and His PhD degree from Shahid Beheshti University in 2016 all in Electrical Engineering. He is currently a visiting research collaborator at the University of Alberta. His main interest is on Digital Control System Design with emphasis on Tele Operation Robotics.

Mahdi Tavakoli is an Associate Professor in the Department of Electrical and Computer Engineering, University of Alberta, Canada. He received his BSc and MSc degrees in Electrical Engineering from Ferdowsi University and K.N. Toosi University, Iran, in 1996 and 1999, respectively. He received his PhD degree in Electrical and Computer Engineering from the University of Western Ontario, Canada, in 2005. In 2006, he was a post-doctoral researcher at Canadian Surgical Technologies and Advanced Robotics (CSTAR), Canada. In 2007-2008, he was an NSERC Post-Doctoral Fellow at Harvard University, USA. Dr. Tavakoli's research interests broadly involve the areas of robotics and systems control. Specifically, his research focuses on haptics and teleoperation control, medical robotics, and image-guided surgery. Dr. Tavakoli is the lead author of Haptics for Teleoperated Surgical.

Alireza Rezazadeh received the B.S., M.S., and Ph.D. degrees from Tehran University in 1991, 1993, and 2000, respectively, all in electrical engineering. He is currently an Associate Professor with Shahid Beheshti University (SBU), Tehran, Iran. His research interests are control systems, power system optimization, and swarm intelligent computation.

Kimia Shamaei received the B.S. and M.S. degrees in electrical engineering from University of Tehran, Tehran, Iran, in 2010 and 2013. Currently, she is a PhD student at University of California, Riverside. Her current research interest is on Pattern Recognition, Estimation theory with emphasis on intelligent transportation systems.