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Position and Velocity Synchronization in Bilateral Teleoperation in Presence of Stochastic Disturbances in Control Inputs

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***Abstract*—The problem of synchronization of bilateral teleoperators in the presence of stochastic disturbances in control inputs is considered in this paper. It is clear that the mechanical systems are often subjected to random disturbances and it can influence the performance of the control system in an uncertain manner. To cope with this, the new adaptive controller is proposed. This technique uses the exponential practical stability concept which guarantees that the tracking error and its derivative converge to an arbitrarily small neighborhood of zero by appropriate tuning of the controller's parameters. It is noteworthy that, the proposed method does not need information, such as the physical parameters of the master and slave robots. Finally, the simulation results are given to show the effectiveness of proposed technique.**

I. Introduction

Nowadays the teleoperation systems are used significantly in various applications; for instance, telesurgery, underwater and space explorations.

The stability in teleoperation systems means bounded tracking error between the behavior of the master and slave robots and different control techniques have been suggested to examine the stability of the teleoperation systems. The passivity-based approaches are the most important methods to ensure the stability of teleoperation systems. In [1] it is shown that the interconnection of passive systems is passive which allows us to analyze the stability of complicated systems by considering each part of systems separately. In these approaches, scattering signals [2] and wave variable methods [3] are used to ensure the passivity of the communication channel. In [4], has been shown that the scattering wave variable method leads to a bounded steady-state error when there is a mismatch of initial position conditions between the master and slave robots and can not ensure the position tracking. To solve this, some PD-like schemes are developed in [5], [6]. In [7] the teleoperation system is formulated to an output synchronization problem and adaptive versions of this method have been developed in [8], [9] which can synchronize the master and slave positions and velocities under the dynamical uncertainties. In the recent years, also some intelligent methods have been developed to deal with this problem [10], [11]. However, in mentioned works the disturbances in the control inputs are neglected which can influence the performance of the teleoperation systems in an uncertain manner. This is an issue which is considered in this paper.

In some independent researches, it has been focused on stability analysis [12], [13], [14], [15] and control of mechanical systems [16], [17], [18], [19]. In [18] and [19] an adaptive controller for a class of stochastic Lagrangian control systems has been developed and this method has been extended to control the revolute joints robot manipulator in presence of stochastic disturbance in control input in [20]. These methods use the exponential practical stability concept to guarantee the stability of the system. In this paper, the idea in [20] is used to design the effective framework for the position and velocity synchronization in bilateral teleoperation systems in the presence of stochastic disturbances in control inputs. The proposed adaptive controller does not need any information on the parameters of the master and slave robot and just uses some well- known properties of the revolute joint manipulators [21].

The random disturbance is modeled as white noise with unknown power spectrum density(PSD). In this work, we restrict ourselves on passive bilateral teleoperation systems without time delay in the transmission channel. In the end, the simulation results are given to verify the efficacy of the innovative framework to synchronize the behavior of the master and slave robots.

II. Notation

The following notations are used in this paper.

- $\mathbb{R} := (-\infty, +\infty), \mathbb{R}_+ := [0, +\infty)$; $\mathbb{R}^n$ denotes the real n-dimensional space and $\mathbb{R}^{n\times r}$ denotes the real $n \times r$ matrix space.
- $\mathcal{C}^i(\mathbb{R}^n)$ denotes the set of all functions with continuous ith partial derivative on $\mathbb{R}^n$ and $\mathcal{C}^{2,1}(\mathbb{R}^n \times \mathbb{R}_+, \mathbb{R}_+)$

denotes the family of all non-negative functions $V(x,t)$ on $\mathbb{R}^n \times \mathbb{R}_+$ which are $\mathcal{C}^2$ in x and $\mathcal{C}^1$ in t and also κ represent the set of all functions $\mathbb{R}^n \times \mathbb{R}_+$ which are continuous, strictly increasing and vanishing at zero.
- For a vector x, x^T and $|x|$ denote the transpose of x and it's standard Euclidean norm respectively.
- For a matrix A, A^T and A^{-1} denote the transpose and inverse of A respectively and $\|A\|_F = \sqrt{Tr(A^TA)}$ represents the Frobenius norm where $Tr(.)$ denote the matrix trace.
- $E(.)$ denotes the mathematical expectation.
- Subscript i represents both master and slave robot

III. Preliminaries

A. Nonlinear stochastic dynamical system

Consider the nonlinear stochastic system which is described by following Itô equation:

$$dx(t) = f(x(t),t)dt + g(x(t),t)dW(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is state vector, functions $f : \mathbb{R}^n \times \mathbb{R}_+ \longrightarrow \mathbb{R}^n$ and $g : \mathbb{R}^n \times \mathbb{R}_+ \longrightarrow \mathbb{R}^{n\times r}$ are both piecewise continuous in $t \in \mathbb{R}_+$ and locally Lipschitz in $x \in \mathbb{R}^n$. $W(t)$ is an r-dimensional independent standard Wiener process(Brownian motion).

For stability analysis of system (1), in the following some useful definitions are given:

Definition 1 [18], [19]. If there exist positive constants λ and d and a function $\vartheta \in \kappa$ such that:

$$E|x(t)|^p \leq \vartheta(|x_0|)e^{-\lambda(t-t_0)} + d \qquad t \geq t_0, x_0 \in \mathbb{R}^n \tag{2}$$

System (1) is called to be p-th moment exponentially practically stable. In addition in the case of $p = 2$ it called exponentially practically stable in mean square.

Definition 2 [18], [19]. (Chebyshev's inequality)For a p-th moment exponentially practically stable system, probability of $|x(t)| > R$ is :

$$P\{|x(t)| > R\} \leq \frac{E|x(t)|^p}{R^p} \leq \frac{\vartheta(|x_0|) + d}{R^p} \tag{3}$$

Definition 3 [18], [19]. for $V(x,t) \in \mathcal{C}^{2,1}(\mathbb{R}^n \times \mathbb{R}_+, \mathbb{R}_+)$ the infinitesimal generator ($\mathcal{L}V$ operator) according to (1) is defined as:

$$\begin{aligned}\mathcal{L}V(x,t) \stackrel{\Delta}{=} &V_t(x,t) + V_x(x,t)f(x,t) \\ &+ \frac{1}{2}Tr\{g^T(x,t)V_{xx}(x,t)g(x,t)\}\end{aligned} \tag{4}$$

where $V_t = \dfrac{\partial V}{\partial t}$, $V_x = (\dfrac{\partial V}{\partial x_1}, \dfrac{\partial V}{\partial x_2}, \ldots, \dfrac{\partial V}{\partial x_n})$ and $V_{xx} = (\dfrac{\partial^2 V}{\partial x_i \partial x_j})_{n\times n}$.

A criterion for p-th moment exponential practical stability of system (1) is given in the following lemma[18], [19]:

Lemma 1. For system (1), if there exist positive constants k_i , $k_i^{'}$, p_i , $p_i^{'}$, c , d_c and a function $V(x,t) \in \mathcal{C}^{2,1}(\mathbb{R}^n \times \mathbb{R}_+, \mathbb{R}_+)$ such that:

$$\sum_{i=1}^{n} k_i|x_i|^{p_i} \leq V(x,t) \leq \sum_{i=1}^{n} k_i^{'}|x_i|^{p_i^{'}} \tag{5}$$

$$\mathcal{L}V(x,t) \leq -cV(x,t) + d_c \tag{6}$$

system is p-th moment exponentially practically stable where $p = min\{p_1, \cdots, p_n\}$ and $\vartheta(s)$, λ and d of Definition 1 can be calculated from following equations [18], [19]:

$$\vartheta(s) = n^{\frac{p}{2}} \sum_{i=1}^{n} (\frac{1}{k_i} \sum_{j=1}^{n} k_j{}' s^{p_j{}'})^{\frac{p}{p_i}} \tag{7a}$$

$$\lambda = \frac{cp}{max\{p_i\}} \tag{7b}$$

$$d = n^{\frac{p}{2}} \sum_{i=1}^{n} (\frac{d_c}{k_i c})^{\frac{p}{p_i}} \tag{7c}$$

B. Some useful inequalities

The following lemmas presents some inequalities which are used in this paper.

Lemma 2. For $\forall a, b \in \mathbb{R}$ and $\epsilon \in \mathbb{R}_+$ following inequalities hold:

$$a \leq \frac{1}{\epsilon}a^2 + \epsilon \tag{8a}$$

$$-ab \leq \frac{1}{2}a^2 + \frac{1}{2}b^2 \tag{8b}$$

$$(a \pm b)^2 \leq 2(a^2 + b^2) \tag{8c}$$

Lemma 3. (Cauchy-Schwarz inequality) for any vectors $x, y \in \mathbb{R}^n$:

$$|x^T y| \leq |x||y| \tag{9}$$

Lemma 4. (Young's inequality) for any vectors $x, y \in \mathbb{R}^n$, $p > 1$ and any scalar $\epsilon \in \mathbb{R}_+$:

$$x^T y \leq \frac{\epsilon^p}{p}|x|^p + \frac{1}{q\epsilon^q}|y|^q \tag{10}$$

where $q = \dfrac{p}{p-1}$.

Lemma 5. For matrices $A, B \in \mathbb{R}^{n\times n}$ and $x \in \mathbb{R}^n$:

$$\|AB\|_F \leq \|A\|_F\|B\|_F \tag{11a}$$

$$|Ax| \leq \|A\|_F|x| \tag{11b}$$

IV. System description

This section begins by introducing the general dynamical model for teleoperation system and some useful properties of this model, some the new variables are defined which enable us to use the stability theory which has been discussed in section III. Control scheme of the bilateral teleoperation with random disturbances has been presented in figure 1.

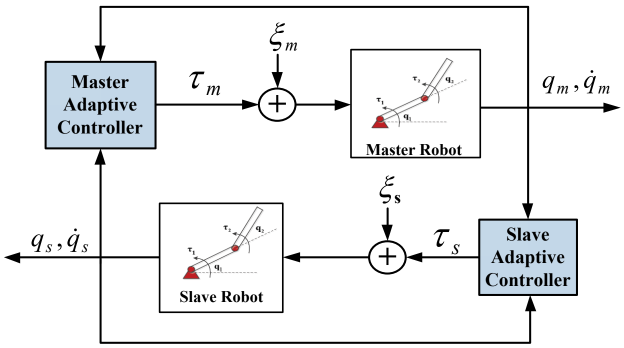

Fig. 1. Control scheme of the bilateral teleoperation with random disturbances

The master and slave are modeled as a pair of n-DOF revolute joints manipulators. The dynamical model of the teleoperation system is given by:

$$\begin{aligned} M_m(q_m)\ddot{q}_m + C_m(q_m,\dot{q}_m)\dot{q}_m + G_m(q_m) &= \tau_m + \xi_m \\ M_s(q_s)\ddot{q}_s + C_s(q_s,\dot{q}_s)\dot{q}_s + G_s(q_s) &= \tau_s + \xi_s \end{aligned} \quad (12)$$

where $q_i, \dot{q}_i, \ddot{q}_i \in \mathbb{R}^n$ are joint positions, velocities and accelerations. $\tau_i \in \mathbb{R}^n$ represent the control signals and $\xi_i \in \mathbb{R}^n$ depict the random disturbance signals(modeled by white noise with unknown PSD). $M_i(q_i) \in \mathbb{R}^{n\times n}$ are inertia matrices; $C_i(q_i,\dot{q}_i) \in \mathbb{R}^{n\times n}$ are the Coriolis centrifugal effects and $G_i(q)_i \in \mathbb{R}^n$ represents the gravitational forces. The dynamical model of revolute joints manipulators has the following well-known properties [21].

P1. The inertia matrices are lower and upper bounded:

$$0 < k_{m_i} I \le M_i(q_i) \le k_{M_i} I < \infty \quad (13)$$

where $M_i(q_i)$ are symmetric and positive definite and I is identity matrix of appropriate dimension.

P2. Boundedness of coriolis forces:

$$\forall q_i, \dot{q}_i \in \mathbb{R}^n \quad \exists k_{c_i} \in \mathbb{R}_+ \qquad |C_i(q_i,\dot{q}_i)\dot{q}_i| \le k_{c_i}|\dot{q}_i|^2 \quad (14)$$

P3. Boundedness of gravitational forces:

$$\forall q_i \in \mathbb{R}^n \quad \exists k_{g_i} \in \mathbb{R}_+ \qquad |G_i(q_i)| \le k_{g_i} \quad (15)$$

For robot manipulators with revolute joints k_{m_i}, k_{M_i} ,k_{c_i} and k_{g_i} are constant.

A. Find Ito Equation

To write (12) in form of (1), $x^T = (q_m^T, \dot{q}_m^T, q_s^T, \dot{q}_s^T)$ is considered as a state vector and ξ is replaced by $\frac{dB}{dt}$ [14], where B is an r-dimensional independent Wiener process. In [22] it has also been shown that, if we consider the power spectral density(PSD) of white noise is $\frac{1}{2\pi}\Sigma$, it is equal to $dB = \Sigma dW$ which leads to the following $It\hat{o}$ form:

$$\begin{aligned} dq_m &= \dot{q}_m dt \\ d\dot{q}_m &= M_m^{-1}(\tau_m - C_m\dot{q}_m - G_m)dt + M_m^{-1}\Sigma_m dW_m \\ dq_s &= \dot{q}_s dt \\ d\dot{q}_s &= M_s^{-1}(\tau_s - C_s\dot{q}_s - G_s)dt + M_s^{-1}\Sigma_s dW_s \end{aligned} \quad (16)$$

where W_i are r-dimensional independent standard Wiener processes and $\Sigma_i \in \mathbb{R}^{r\times r}$ are unknown positive matrices. In this paper we assume that $r = n$.

To use the stability analysis which has been discussed in section III, new variables e_1 and e_2 are defined as follows:

$$e_1(t) \triangleq q_m(t) - q_s(t) \quad (17a)$$

$$e_2(t) \triangleq \dot{e_1}(t) + c_1 e_1(t) \quad (17b)$$

where $c_1 > 0$ is a design parameters. Combining (17a) and (17b) with (16) gives:

$$\begin{aligned} de_1 &= (e_2 - c_1 e_1)dt \\ de_2 &= \{-M_m^{-1}(C_m\dot{q}_m + G_m) + M_s^{-1}(C_s\dot{q}_s + G_s) \\ &\quad M_m^{-1}\tau_m - M_s^{-1}\tau_s + c_1 e_2 - c_1^2 e_1\}dt \\ &\quad + [M_m^{-1}\Sigma_m \vdots M_s^{-1}\Sigma_s]dW \end{aligned} \quad (18)$$

where $W^T = [W_m^T, W_s^T]$.

B. Assumptions

To design the adaptive controller, following assumptions are considered [20]:

$$0 \le \eta_{i_1}\phi_{i_1}(q_i) \le M_i^{-1}(q_i) \le \eta_{i_2}\phi_{i_2}(q_i) \quad (19a)$$

$$|C_i(q_i,\dot{q}_i)\dot{q}_i + G_i(q_i)|^2 \le \eta_{i_3}\phi_{i_3}(q_i,\dot{q}_i) \quad (19b)$$

where η_{i_j}, $j = 1, 2, 3$ and ϕ_{i_j} are unknown positive parameters and known smooth nonnegative functions respectively. The η_i and μ_i are also defined as follows [20]:

$$\eta_i \triangleq max\ \{\eta_{i_3}, \|\Sigma_i\|_F^2\} \times \eta_{i_2}^2 \quad (20a)$$

$$\mu_i \triangleq \frac{1}{\eta_{i_1}} \quad (20b)$$

It is also useful to define $\hat{\eta}_i$ and $\hat{\mu}_i$ as estimates of η_i and μ_i with the estimation errors $\tilde{\eta}_i = \hat{\eta}_i - \eta_i$ and $\tilde{\mu}_i = \hat{\mu}_i - \mu_i$ respectively.

By considering the properties of the robot according to **P**1,**P**2 and **P**3 in Section IV, η_i and ϕ_i can be considered as $\eta_{i_1} = \frac{1}{k_{M_i}}$, $\eta_{i_2} = \frac{1}{k_{m_i}}$, $\eta_{i_3} = k_{c_i}^2 + k_{g_i}^2$, $\phi_{i_1} = \phi_{i_2} = I$ and $\phi_{i_3} = 2(1 + |\dot{q}_i|^4)$, since the upper bound of $|C_i(q_i,\dot{q}_i)\dot{q}_i + G_i(q_i)|^2$ can be calculate by using the inequality (8c) [20]:

$$\begin{aligned} |C_i(q_i,\dot{q}_i)\dot{q}_i + G_i(q_i)|^2 &\le 2(|C_i(q_i,\dot{q}_i)\dot{q}_i|^2 + |G_i(q_i)|^2) \\ &\le 2(k_{c_i}^2 + k_{g_i}^2)(1 + |\dot{q}_i|^4) \end{aligned} \quad (21)$$

V. Proposed Adaptive Controller

The system (18) with following control inputs is exponentially practically stable in mean square.

$$\tau_m = -\phi_{m_1}^{-1} e_2 \hat{\mu}_m \bar{\tau}_m \quad (22a)$$

$$\tau_s = \phi_{s_1}^{-1} e_2 \hat{\mu}_s \bar{\tau}_s \quad (22b)$$

where $\bar{\tau}_i = \beta_i\hat{\eta}_i + k_i$ and k_i is constant and β_i is a function of ϕ_i which will be introduced in the next section. The adaptive laws are proposed as:

$$\dot{\hat{\mu}}_i = \gamma_{i_1}\bar{\tau}_i(e_2^T e_2)^2 - \sigma_{i_1}\hat{\mu}_i \quad (23a)$$

$$\dot{\hat{\eta}}_i = \gamma_{i_2}\beta_i(e_2^T e_2)^2 - \sigma_{i_2}\hat{\eta}_i \quad (23b)$$

where γ_{i_1} , γ_{i_2}, σ_{i_1} and σ_{i_2} are design parameters.

VI. Stability Analysis

A. Some Necessary Precalculations

Before stability analysis, some necessary calculations are given which will be used in the stability analysis. For simplicity in writing we define $\Lambda_i \triangleq M_i^{-1}\{C_i\dot{q}_i + G_i\}$.

Considering assumptions in (19a), (19b) and using (20a) and Lemma 5 one can find:

$$|\Lambda_i|^2 \leq \| -M_i^{-1}\|_F^2 |C_i\dot{q}_i + G_i|^2 \leq \eta_i \Phi_i \tag{24}$$

where $\Phi_i = \|\phi_{i_2}\|_F^2 \phi_{i_3}$ and:

$$|\Lambda_s - \Lambda_m|^2 \leq (|\Lambda_s| + |\Lambda_m|)^2 \leq 2\sum_{i=m,s} \eta_i \Phi_i \tag{25}$$

and:

$$\|M_i^{-1}\Sigma_i\|_F^2 \leq \|M_i^{-1}\|_F^2 \|\Sigma_i\|_F^2 \leq \eta_i \|\phi_{i_2}\|_F^2 \tag{26}$$

B. Proof

Consider the Lyapunov function:

$$\begin{aligned} V =& \frac{1}{4}(e_1^T e_1)^2 + \frac{1}{4}(e_2^T e_2)^2 \\ &+ \sum_{i=m,s}\{\frac{1}{2\gamma_{i_1}}\mu_i^{-1}\tilde{\mu}_i^2 + \frac{1}{2\gamma_{i_2}}\tilde{\eta}_i^2\} \end{aligned} \tag{27}$$

The infinitesimal generator of system (18) regarding the Lyapunov function (27) is:

$$\begin{aligned} \mathcal{L}V =& -c_1 e_1^T e_1\, e_1^T e_1 + e_1^T e_1\, e_1^T e_2 - c_1^2 e_2^T e_2\, e_2^T e_1 \\ &+ e_2^T e_2\, e_2^T\{\Lambda_s - \Lambda_m\} + c_1 e_2^T e_2\, e_2^T e_2 \\ &+ e_2^T e_2\, e_2^T (M_m^{-1}\tau_m - M_s^{-1}\tau_s) \\ &+ \frac{1}{2}\sum_{i=m,s} Tr\{\Sigma_i^T M_i^{-1}(2e_2e_2^T + e_2^T e_2 I)M_i^{-1}\Sigma_i\} \\ &+ \sum_{i=m,s}\frac{1}{\gamma_{i_1}}\mu_i^{-1}\tilde{\mu}_i\dot{\hat{\mu}}_i + \sum_{i=m,s}\frac{1}{\gamma_{i_2}}\tilde{\eta}_i\dot{\hat{\eta}}_i \end{aligned} \tag{28}$$

To use the result of Lemma 1 we consider each component of (28) separately(ith component of $\mathcal{L}V$ is shown by $\mathcal{L}V_i$). Clearly $\mathcal{L}V_1 = -c_1(e_1^T e_1)^2$ and $\mathcal{L}V_5 = c_1(e_2^T e_2)^2$. Considering Youngs inequality in lemma 4, choosing $x = e_1^T e_1 e_1^T$ and $y = e_2$ and $p = \frac{4}{3}$ leads to the following inequality for $\mathcal{L}V_2$:

$$\mathcal{L}V_2 \leq \frac{c_1}{4}(e_1^T e_1)^2 + \frac{27}{4c_1^3}(e_2^T e_2)^2 \tag{29}$$

Similarly, $\mathcal{L}V_3 \leq \frac{c_1}{4}(e_1^T e_1)^2 + \frac{3c_1^{\frac{7}{3}}}{4}(e_2^T e_2)^2$. For $\mathcal{L}V_4$, by using Young's inequality in Lemma 4 with $x = 1$, $y = e_2^T\{\Lambda_s - \Lambda_m\}$, $p = 2$ and Cauchy-Schwarz inequality in Lemma 3 and then (25) and (8a),

$$\begin{aligned} \mathcal{L}V_4 =& |e_2|^2\{\frac{1}{4} + |e_2|^2|\Lambda_s - \Lambda_m|^2\} \\ \leq& (\frac{1}{4\epsilon_1} + \sum_{i=m,s}\eta_i\Phi_i)(e_2^T e_2)^2 + \frac{\epsilon_1}{4} \end{aligned} \tag{30}$$

where $\epsilon_1 > 0$ is a design parameter. Sixth part of $\mathcal{L}V$ by substituting input torques from 22a and (22b) and then using assumption 1 in (19a) leads to:

$$\begin{aligned} \mathcal{L}V_6 &= -e_2^T e_2\, e_2^T (M_m^{-1}\phi_{m_1}^{-1}e_2\hat{\mu}_m\bar{\tau}_m + M_s^{-1}\phi_{s_1}^{-1}e_2\hat{\mu}_s\bar{\tau}_s) \\ &\leq -e_2^T e_2\, e_2^T e_2(\mu_m^{-1}\hat{\mu}_m\bar{\tau}_m + \mu_s^{-1}\hat{\mu}_s\bar{\tau}_s) \\ &= -(e_2^T e_2)^2\sum_{i=m,s}\{\bar{\tau}_i + \mu_i^{-1}\tilde{\mu}_i\bar{\tau}_i\} \end{aligned} \tag{31}$$

By using the definition of matrix trace and (26) then (8a) one can obtain:

$$\begin{aligned} \mathcal{L}V_7 &\leq \frac{3}{2}|e_2|^2\sum_{i=m,s}\|M_i^{-1}\Sigma_i\|_F^2 \\ &\leq \frac{3}{2}(\frac{1}{\epsilon_2}|e_2|^4 + \epsilon_2)\sum_{i=m,s}\|\phi_{i_2}\|_F^2\eta_i \\ &= \frac{3}{2\epsilon_2}(e_2^T e_2)^2\sum_{i=m,s}\|\phi_{i_2}\|_F^2\eta_i + \frac{3\epsilon_2}{2}\sum_{i=m,s}\|\phi_{i_2}\|_F^2\eta_i \end{aligned} \tag{32}$$

where $\epsilon_2 > 0$ is a design parameter. By substituting $\dot{\hat{\mu}}$ from adaptive laws (23a) and then $\hat{\mu} = \mu + \tilde{\mu}$ in $\mathcal{L}V_8$ and then (8b):

$$\begin{aligned} \mathcal{L}V_8 =& (e_2^T e_2)^2\sum_{i=m,s}\mu_i^{-1}\tilde{\mu}_i\bar{\tau}_i \\ &- \sum_{i=m,s}\{\frac{1}{\gamma_{i_1}}\sigma_{i_1}\mu_i^{-1}\tilde{\mu}_i^2 + \frac{1}{\gamma_{i_1}}\sigma_{i_1}\mu_i^{-1}\tilde{\mu}_i\mu_i\} \\ \leq& (e_2^T e_2)^2\sum_{i=m,s}\mu_i^{-1}\tilde{\mu}_i\bar{\tau}_i \\ &- \sum_{i=m,s}\{\frac{1}{2\gamma_{i_1}}\sigma_{i_1}\mu_i^{-1}\tilde{\mu}_i^2 + \frac{1}{2\gamma_{i_1}}\sigma_{i_1}\mu_i\} \end{aligned}$$

Similarly:

$$\begin{aligned} \mathcal{L}V_9 \leq& (e_2^T e_2)^2\sum_{i=m,s}\beta_i\tilde{\eta}_i \\ &- \sum_{i=m,s}\{\frac{1}{2\gamma_{i_2}}\sigma_{i_2}\tilde{\eta}_i^2 + \frac{1}{2\gamma_{i_2}}\sigma_{i_2}\eta_i^2\} \end{aligned} \tag{33}$$

Substituting $\mathcal{L}V_i$ inequalities, the following inequality can be found for $\mathcal{L}V$:

$$\begin{aligned} \mathcal{L}V \leq& -\frac{c_1}{2}(e_1^T e_1)^2 \\ &+ (\bar{d} + \sum_{i=m,s}\{(\Phi_i + \Psi_i)\eta_i + \beta_i\tilde{\eta}_i - \bar{\tau}_i\})(e_2^T e_2)^2 \\ &- \sum_{i=m,s}\{\frac{1}{2\gamma_{i_1}}\sigma_{i_1}\mu_i^{-1}\tilde{\mu}_i^2 + \frac{1}{2\gamma_{i_2}}\sigma_{i_2}\tilde{\eta}_i^2\} + d_c \end{aligned} \tag{34}$$

where $\Psi_i = \frac{3}{2\epsilon_2}\|\phi_{i_2}\|_F^2$. Defining the $\bar{d}$ and d_c as,

$$\bar{d} = \frac{27}{4c_1^3} + \frac{3}{4}c_1^{\frac{7}{3}} + \frac{1}{4\varepsilon_1} + c_1 \tag{35a}$$

$$\begin{aligned} d_c =& \frac{\epsilon_1}{4} + \frac{3\epsilon_2}{2}\sum_{i=m,s}\|\phi_{i_2}\|_F^2\eta_i \\ &+ \sum_{i=m,s}\{\frac{1}{2\gamma_{i_1}}\sigma_{i_1}\mu_i + \frac{1}{2\gamma i_2}\sigma_{i_2}\eta_i^2\} \end{aligned} \tag{35b}$$

and considering $\beta_i = \Phi_i + \Psi_i$ and $\bar{\tau}_i = \beta_i \hat{\eta}_i + \frac{\bar{d}}{2} + \frac{c_2}{4}$, $\mathcal{L}V$ leads to:

$$\begin{aligned}\mathcal{L}V \leq & -\frac{c_1}{2}(e_1^T e_1)^2 - \frac{c_2}{2}(e_2^T e_2)^2 \\ & - \sum_{i=m,s}\{\frac{1}{2\gamma_{i_1}}\sigma_{i_1}\mu_i^{-1}\tilde{\mu}_i^2 + \frac{1}{2\gamma_{i_2}}\sigma_{i_2}\tilde{\eta}_i^2\} + d_c \end{aligned} \quad (36)$$

with choosing $c = min\{2c_1, 2c_2, \sigma_1, \sigma_2\}$ in (36) and considering (27), $\mathcal{L}V \leq -cV + d_c$ holds. Hence according to Lemma 1, the system (18) is exponentially practically stable in mean square and $e_i(t), i = 1, 2$ satisfy the following inequality:

$$\lim_{t\to\infty} E|e_i(t)|^2 \leq (\frac{4d_c}{c})^{\frac{1}{2}} \quad (37)$$

Using (17b), and defining the $e(t) = e_1(t)$ as a tracking error, following results are straightforward.

$$\begin{aligned}\lim_{t\to\infty} E|e(t)|^2 \leq & (\frac{4d_c}{c})^{\frac{1}{2}} \\ \lim_{t\to\infty} E|\dot{e}(t)|^2 \leq & 2(1 + c_1^2)(\frac{4d_c}{c})^{\frac{1}{2}}\end{aligned} \quad (38)$$

According to Chebyshev's inequality from Definition 2 and considering (38), for any $\epsilon \geq 0$ and $\epsilon_0 \geq 0$, there exists $T > 0$ such that when $t > T$,

$$\begin{aligned}P\{|e(t)| > \epsilon\} \leq & \frac{1}{\epsilon^2}(\epsilon_0 + (\frac{4d_c}{c})^{\frac{1}{2}}) \leq \epsilon' \\ P\{|\dot{e}(t)| > \epsilon\} \leq & \frac{1}{\epsilon^2}(\epsilon_0 + 2(1 + c_1^2)(\frac{4d_c}{c})^{\frac{1}{2}}) \leq \epsilon'\end{aligned} \quad (39)$$

By considering $c = min\{2c_1, 2c_2, \sigma_1, \sigma_2\}$ and d_c from (35b), it is obvious that ϵ' can be made small enough by choosing ϵ_1, ϵ_2 small enough and γ_1, γ_2 large enough, and also they are independent from $c_1, c_2, \sigma_1, \sigma_2$.

It is important to note that choosing the design parameters is a trade-off between the tracking error and allowable control effort.

VII. Simulation

In this section, a simulation study is exhibited to illustrate the effectiveness of the proposed synchronization scheme. In this simulation, the model of Pelican robot is used for both master and slave. This robot is a 2-DOF and fixed base manipulator which is developed at CICESE, robotic lab [21]. The elements of inertia matrix are $M_{11} = m_1 l_{c_1}^2 + m_2(l_1^2 + l_{c_2}^2 + 2l_1 l_{c_2} c_2) + I_1 + I_2$, $M_{12} = M_{21} = m_2(l_{c_2}^2 + l_1 l_{c_2} c_2) + I_2$ and $M_{22} = m_2 l_{c_2}^2 + I_2$; the elements of the Coriolis and centrifugal matrix are $C_{11} = -h\dot{q}_2$, $C_{12} = -h(\dot{q}_1 + \dot{q}_2)$, $C_{21} = h\dot{q}_1$ and $C_{22} = 0$ and also the elements of the gravity vector are $G_1 = (m_1 l_{c_1} + m_2 l_1) g s_1 + m_2 l_{c_2} g s_{12}$ and $G_2 = m_2 l_{c_2} g s_{12}$. In which $h = m_2 l_1 l_{c_2} s_2$ and s_1, s_2, c_1 and s_{12} denote $sin(q_1), sin(q_2), cos(q_1)$ and $sin(q_1 + q_2)$ respectively. The description of physical parameters of Pelican robot is given in Table. I. Also, Σ_i which is related to PSD of disturbance for both master and slave side are $\Sigma_i = 0.01I$

The initial condition of master and slave robots are $q_m(0) = [\frac{\pi}{3}, \frac{\pi}{4}]^T$ and $q_s(0) = [\frac{\pi}{4}, -\frac{\pi}{6}]^T$ all of them with unit rad and $\dot{q}_i(0) = [0, 0]^T$ rad/s. The initial value of estimates of adaptive laws are $\hat{\eta}_m = \hat{\mu}_m = 0.1$ and $\hat{\eta}_s = \hat{\mu}_s = 0.2$. The design parameters are also selected as $c_1 = 1.5, c_2 = 1$, $\epsilon_1 = \epsilon_2 = 0.001$, $\gamma_{i_1} = 2, \gamma_{i_2} = 4, \sigma_{i_1} = 0.1$ and $\sigma_{i_2} = 0.3$. The simulation results are given in Figs. 2-9 which shows the validity of the proposed technique.

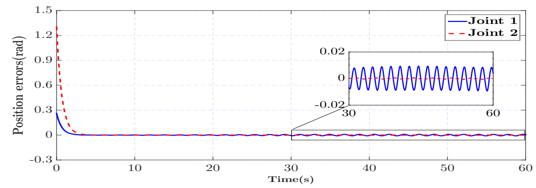

Fig. 2. The position error

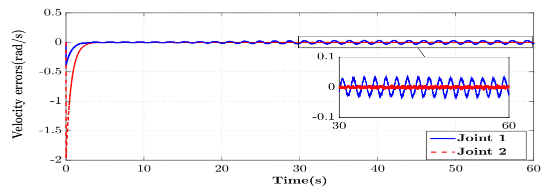

Fig. 3. The velocity error

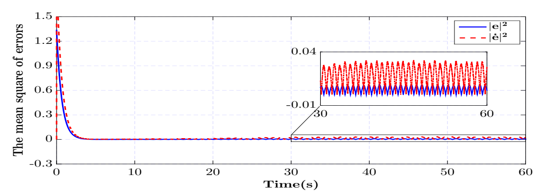

Fig. 4. The mean square of errors

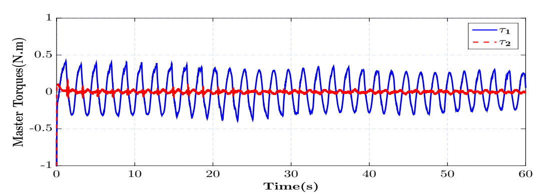

Fig. 5. The control signals(master)

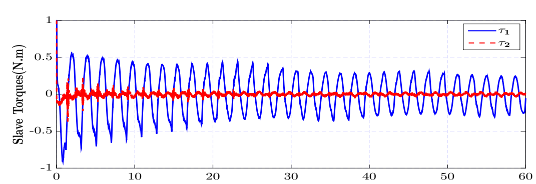

Fig. 6. The control signals(slave)

VIII. Conclusion

In this paper, a new architecture for position and velocity synchronization in bilateral teleoperation system has been

TABLE I
SIMULATION PARAMETERS

Parameter	Description	Unit	Master	Slave	From view of controllers
l_1	Length of link 1	m	0.13	0.26	Unknown
l_{c_1}	Distance to the center of mass (Link 1)	m	0.0492	0.0983	Unknown
m_1	Mass of link 1	kg	3.26	6.5225	Unknown
I_1	Inertia rel. to center of mass (Link 1)	$kg.m^2$	0.0152	0.1213	Unknown
l_2	Length of link 2	m	0.13	0.26	Unknown
l_{c_2}	Distance to the center of mass (Link 2)	m	0.01145	0.0229	Unknown
m_2	Mass of link 2	kg	1.0229	2.0458	Unknown
I_2	Inertia rel. to center of mass (Link 2)	$kg.m^2$	0.0014	0.0116	Unknown
g	Gravity acceleration	m/s^2	9.81	9.81	Unknown

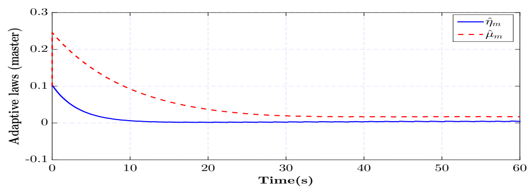

Fig. 7. The adaptive laws(master)

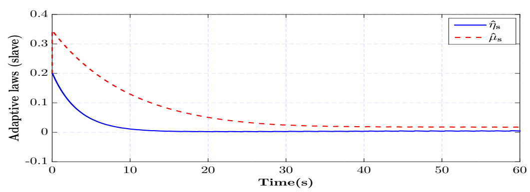

Fig. 8. The adaptive laws(slave)

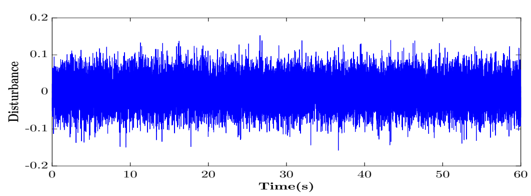

Fig. 9. Sample of exerted disturbances

developed. The stability analysis guarantees the state and velocity synchronization of the master/slave robot in the presence of stochastic disturbances in control inputs. The analytical study shows that the error and time derivatives of error can converge to an arbitrarily small neighborhood of zero by tuning the controller parameters. The proposed adaptive controller doesn't need any information of physical parameters of robots and just use the well-known properties of revolute joints manipulator. The simulation results have been exhibited which approved the theoretical results.

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