University of Alberta

Stability of Multilateral Haptic Teleoperation Systems

by

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A thesis submitted to the Faculty of Graduate Studies and Research
in partial fulfillment of the requirements for the degree of

Master of Science
in
Control Systems

Department of Electrical and Computer Engineering

Spring 2013
Edmonton, Alberta

This thesis is dedicated to Nicole, Andreas, Luciano, and Norma.

Abstract

Multilateral systems involving haptic information sharing between several users have recently found interesting applications in cooperative haptic teleoperation and haptic-assisted training. It is intuitively understood that some tasks are performed more effectively with two hands or through collaboration than one hand or individual operation. By using multiple user interfaces ("masters") and one remote robot ("slave") or more, multilateral tele-cooperation systems enable haptic information sharing and collaboration in performing a task in a remote environment between multiple users. Despite the aforementioned benefits, research in this area is still in its initial stage. In fact, the only multilateral system that has been thoroughly investigated is the most basic one: the bilateral teleoperation system involving teleoperation between one master and one slave.

As with any other robotic system, stability of multilateral haptic teleoperation systems is of paramount importance. Study of stability of such systems must consider the fact that the human users are part of the closed-loop system and thus affect the stability. However, to model the human operator is practically impossible, imposing great difficulties in the system's stability analysis. This thesis presents a novel criterion to study the stability of multilateral teleoperation systems based on passivity. This criterion provides researchers with an analytical, closed-form, necessary and sufficient condition to investigate the stability of multilateral haptic teleoperation systems. The thesis also proposes a numerical method for investigation of absolute stability of trilateral teleoperators.

Acknowledgments

I would like to express my deepest gratitude to my supervisor Dr. Mahdi Tavakoli for giving me the opportunity to work in this fascinating research. I would like to thank him for his excellent supervision, his encouragement and his invaluable support throughout my years as a junior researcher. Our weekly long engaging discussions made my research experience enjoyable and productive, and were instrumental for the successful outcome of my thesis.

I would like to thank my lab mates in the Telerobotic and Biorobotic Systems group for providing the friendly warm environment much needed when conducting research. I am especially grateful to Ali Jazajery and Jian Li for the theoretical and technical discussions we had which benefited my investigation.
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I also own my gratitude to my wife Nicole for her constant support and encouragement through the duration of this research.

I would like to extend my thanks to the Northern Alberta Institute of Technology (NAIT), my full-time workplace, for having given me the opportunity to embark in this journey. I would like to make a special reference to Andy van der Veen, my supervisor at NAIT, who was always eager to facilitate the time for me to be able to advance with my research.

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Introduction

1.1 Motivation

One of the five senses the human has is the sense of touch. It is that sense that allows us to explore and manipulate an object by feeling it and sensing its roughness, size, stiffness, etc. When an object we intend to manipulate is not physically reachable, we use tools as extensions to our arms. Now, imagine that the extension tool is capable of recreating for us the sense of touch. In that case, we are able to manipulate remote objects and “feel” as if we are in direct contact with them. The described scenario is realized by haptic teleoperation systems. These systems are made up of one (or more) human operator(s) couple to one (or more) master robot(s) in order to control the movement of a remote slave to perform a task on a remote environment.

The key motivation for this research is to establish a criterion for investigating the stability of multilateral haptic teleoperation systems, which can be modeled as *n*-port networks. The realization of a *teleoperator* involve one or more *master* robots (i.e., user interfaces), one or more *slave* robots (i.e., remote robots), control units, and communication channels between the masters and the slaves. A *multilateral teleoperation system* is formed once the above teleoperator is coupled to human operators in one end and to external environments in the other end; naturally, human operators are coupled to the masters while the environments interact with the slaves. The multilateral teleoperation system is said to provide haptic feedback if all of the slave/environment interaction forces are reflected back to the human operators via the masters.

Figure 1.1 shows a multilateral *teleoperation system* made up of *n* robots. One potential scenario for Figure 1.1 is that *n*-1 master's robots are sharing the execution of a task in a remote environment by collaboratively controlling the movement of a slave robot [5], [6], [7], [8], [9]. In Figure 1.1, each human operator/master interaction is denoted by F_{hi}, $i=1,\ldots,n-1$, and the slave/environment interaction is denoted by F_e. Also, V_{hi}, V_e, F_{cmi}, and F_{cs} are the masters' and the slave's velocities and control signals, respectively. Impedances Z_{hi} and Z_e denote the dynamic characteristics of the human operators and the remote environment, respectively. Z_{mi} and Z_s denote the linear impedances of the masters and the slave, respectively. Moreover, F_{hi}* and F_e* are the operators' and the environment's exogenous input forces.

A valid question one can ask is "why do we need stability criteria for multilateral teleoperation systems?" The answer is that stability criteria can give researchers formal and accurate information on the trade-offs between performance and stability of the multilateral teleoperation system. For a better understanding of this statement, consider a teleoperation task involving flipping the three-way switch shown in Figure 1.2. Assume that the human operator has been asked to move the switch from state 1 to state 2 but not to state 3.

The teleoperation system should exhibit a sufficiently satisfactory performance so that the human operator can flip the switch by teleoperation of the slave robot through the master robot; for this, the slave robot's overshoot should be no more than the position difference between states 2 and 3. In this example, it is evident that master-slave position error, which is a measure of teleoperation system performance, directly affects the performance of the task by the operator. To achieve a small enough master-slave position error, the slave's position controller gains have to be selected large. However, selecting too large a controller gain risks making the system non-passive or even unstable [38], [39], [40]. *The upper limit on the controller gains before stability is lost is what can be determined using the passivity and absolute stability criteria developed in this thesis*. The theoretical passivity and absolute stability criteria developed in this thesis are, therefore, valuable results that allow for obtaining maximum performance in the stable region.

In practice, the upper limit imposed on the control gain for ensuring stability may restrict the performance to the extent that task performance is severely undermined. For example, in the same 3-way switch task, one may find that the highest slave's controller gain for which the system remains stable is still not high enough to complete the task successfully (especially if the switch is sticky and the position difference between states 2 and 3 is small) even though the same task is done readily under direct touch. Therefore, it is also informative to study if successful completion of this task is possible at all and this study can be facilitated using the passivity and absolute stability criteria proposed in this thesis.

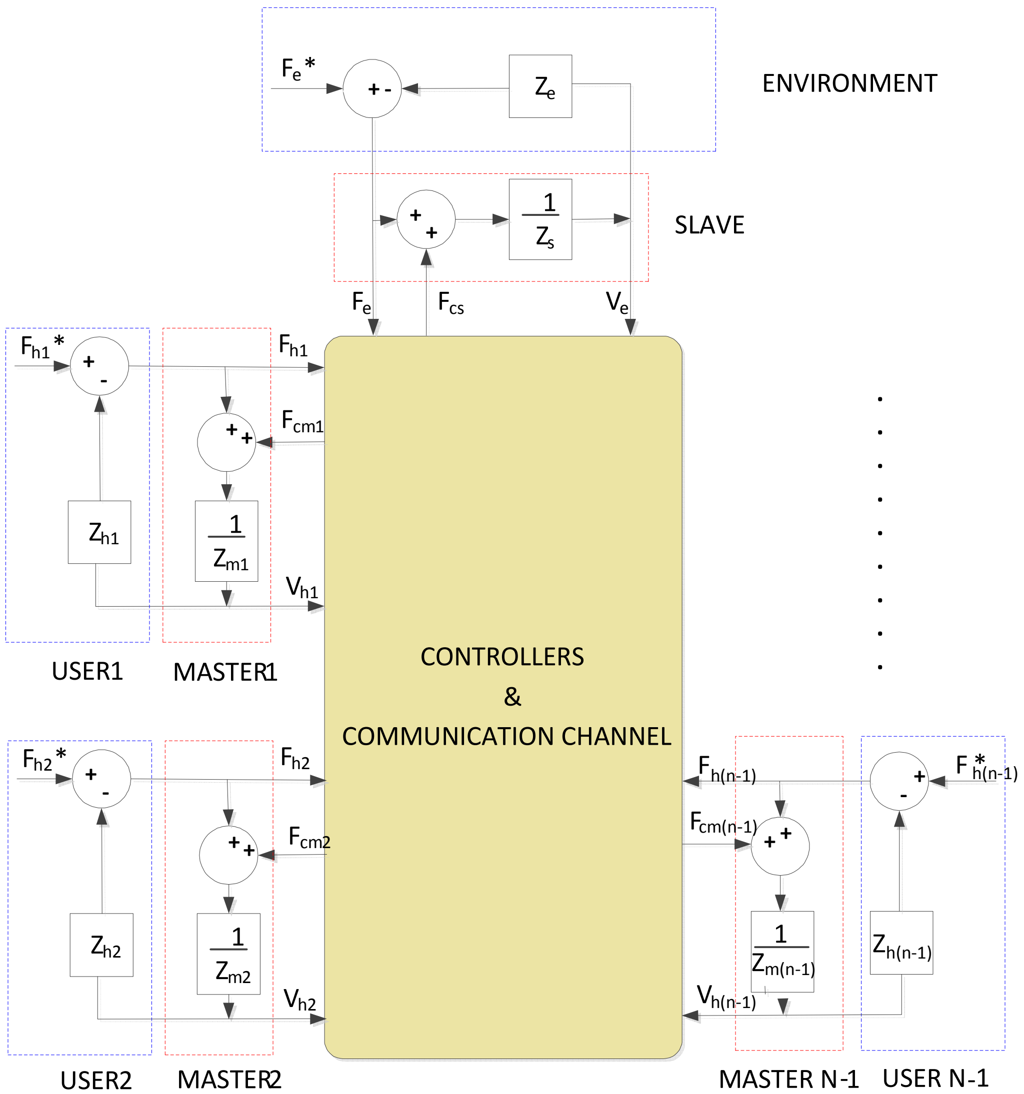

Figure 1.1. A multilateral haptic teleoperation system consisting of *n*-1 master robots and one slave robot.

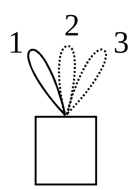

Figure 1.2. A three-way switch.

The central problem when studying the stability of teleoperation systems is that human operators and environments are part of the closed-loop system and thus their models are necessary for stability analysis. In practice, however, such models are next to impossible to acquire. For instance, the dynamics of a human operator changes according to the task at hand [3], [4].

For the simplest case – a bilateral haptic teleoperation system (Figure 1.3) that can be modeled as a 2-port network (Figure 1.4) – there exist well-known methods to investigate the stability. Such a study of stability is valid when the 2-port network is connected to *unknown* terminations (human operator and environment) that are passive. These methods are known as Llewellyn's absolute stability criterion and Raisbeck's passivity criterion [11]. We will describe these criteria later in this chapter. A method to study the stability of multilateral teleoperation systems beyond the bilateral case, which is the subject of this thesis, is still in demand.

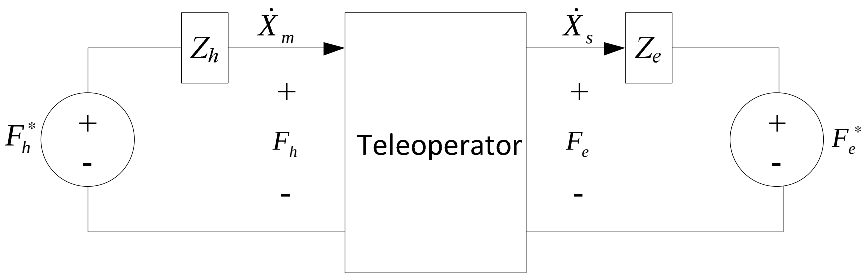

Figure 1.3. A bilateral teleoperation system comprising a human operator, a teleoperator (consisting of a master, a slave, controllers, and a communication channel), and an environment.

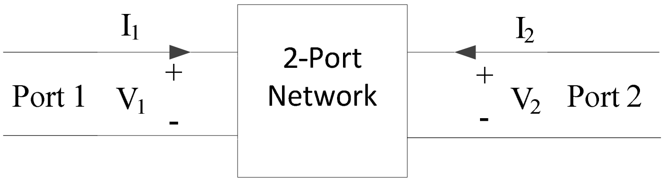

Figure 1.4. A 2-port network.

1.2 Emerging Applications for Multilateral Teleoperation Systems

Multilateral teleoperation systems beyond the bilateral one can offer greater advantages: They can be used to haptically train people in performing remote tasks, they can increase task efficiency where it helps to use two hands instead of one, they can help to perform a task in cooperation among several human operators, etc.

Multilateral teleoperation systems can be categorized as single-master/multi-slave (SMMS), multi-master/single-slave (MMSS), and multi-master/multi-slave (MMMS) systems. All of them are subjects of intense research nowadays due to their potential applications. Next, some interesting applications of such systems are listed.

Single-master/multi-slave (SMMS). In this configuration, an operator coupled to a single master controls multiple slave robots. Applications of such systems include multiple slave robots capable of performing cooperative manipulation and grasping of a common object [19], semi-autonomous teleoperation on remote or inaccessible environments [20], formation control of mobile robots teleoperated by a human operator [21], and haptic-assisted micromanipulation with improved human operability [22]. In the SMMS configuration, the slave robots are designed with systems that avoid collisions among them.

Multi-master/single-slave (MMSS). In this configuration, multiple human teleoperators control a single slave robot to perform a task on a remote environment. Applications of this configuration include tele-rehabilitation [23] and surgical training where an experienced surgeon mentors a trainee surgeon through shared control of a surgical robot [24].

Multi-master/multi-slave (MMMS). In these systems, multiple masters control multiple slave robots. Applications of MMMS can be found in teleoperation tasks performed in large-scale environments where the overall system is made up of individual subsystems working in tele-cooperation [25], remote pick-and-place tasks [26], and shipping of hazardous materials, surveillance sensor networks, and rescue [27].

1.3 Literature Survey

While the applications of multilateral haptic systems are expanding rapidly, a question we have to ask is, what stability criteria are there in order to investigate the stability of multilateral teleoperation systems? The stability analysis of an n-lateral teleoperation system is equivalent to that of an n-port network, where n can be equal or greater than 2. The key difference between a multilateral teleoperation system (with $n > 2$) and a bilateral teleoperation system (with $n = 2$) is that the former cannot be modeled as a 2-port network (note: a good critical review for absolute stability of 2-port networks is given in [12]). Consequently, conventional theories for absolute stability or passivity analysis of 2-port networks can be applied to bilateral teleoperation systems but not to multilateral teleoperation systems with $n > 2$. Unfortunately, existing research on this topic has not given satisfactory results from a practical perspective.

In finding conditions for stability of n-port networks, a researcher will face the decision of whether to use the theory of passivity to establish a criterion for stability or to find an absolute stability condition. Note that, as elaborated later, passivity implies absolute stability in the sense that if an n-port network is passive, then it is also absolutely stable.

The few attempts to find a criterion for absolute stability of n-port networks can be categorized based on whether they involve conditions on immittance parameters or scattering parameters. The following are the existing criteria for passivity and absolute stability of *n*-port networks, to the best of the author's knowledge.

Passivity criteria for *n*-ports

In 1954, Raisbeck wrote a paper proposing a general definition of passivity of a network [29]. His definition is considered general because it goes beyond "realizable networks" and assumes neither rationality nor reciprocity. Raisbeck only presented a criterion limited to the investigation of passivity of 2-port networks known as Raisbeck's passivity criterion. He did not extend this criterion for the general case of *n*-port networks where *n* can be an integer larger than 2.

In 1959, Youla et al. published the first formal justification of the passivity definition for *n*-port networks based on Raisbeck's general passivity definition (with minor differences) [31]. The paper presented a rigorous theory of passive LTI *n*-port networks but is fairly involved and stops short of proposing a passivity criterion.

Another interesting work on the passivity of *n*-port networks has been presented by Wyatt et al. [32]. This paper is another rigorous attempt to present a definition for passivity of *n*-ports departing from energy considerations. It goes to the level of expressing necessary and sufficient conditions for passivity of several classes of *n*-ports such as resistive *n*-ports, capacitive/inductive *n*-ports, linear *n*-ports (using state space representation), etc. Like the previous case, this paper stops short of proposing a passivity criterion for *n*-port networks.

In [33], Anderson and Spong utilized concepts from network theory and introduced a tool for checking the passivity of an *n*-port network based on the singular value of the scattering matrix of the network. They showed that a network is passive if and only if the norm of its scattering operator is less than or equal to one. The scattering operator S is defined as

$$F - v = S(F + v) \tag{1.1}$$

and maps effort plus flow into effort minus flow. In (1.1), F is the effort measured across the network's ports and v is the flow entering the network's ports. In relation to haptic teleoperation systems, the effort variable is equivalent to force and the flow variable is equivalent to velocity. In relation to electrical networks, effort is equivalent to voltage and flow is equivalent to current.

In the Laplace domain, (1.1) becomes

$$F(s) - V(s) = S(s)(F(s) + V(s)) \tag{1.2}$$

According to [33], the n-port network is passive if and only if

$$\|S\|_{\infty} \leq 1 \tag{1.3}$$

This is equivalent to

$$\sup_{\omega} \lambda^{1/2}(S^{*}(j\omega)S(j\omega)) \leq 1 \tag{1.4}$$

where λ denotes the eigenvalue of a square matrix, * denotes the complex conjugate transpose, and ω is the frequency. Condition (1.4) is difficult to verify in the general case especially without knowledge of the model parameters for the robots and the controllers, making it not suitable for control synthesis.

Absolute stability criteria for *n*-ports

For absolute stability analysis of n-port networks, researchers have used techniques involving impedance parameters, scattering parameters, and graphical analysis as shown in [13], [34], [35], and [36]. All proposed methods have issues; some are too complex in order to have a practical application and some others make oversimplifications.

In [16], Ku proposes two methods to study the stability of non-reciprocal n-port networks using impedance parameters. In the first method, he studies the stability of the nonreciprocal *n*-port by finding a reciprocal *n*-port network which has the same stability characteristics as the nonreciprocal n-port network in question. This approach is limited to nonreciprocal n-port networks that do have an equivalent reciprocal network and have a certain structure. The second approach uses the impedance matrix of a nonreciprocal 3-port network and reduces it into a 2-port network by terminating the 3rd port in a fixed reactance. It is obvious that by terminating any 3rd port in a fixed reactance, the stability conditions are the same as stability conditions of 2-port networks. This approach works as follows: Let port 2 be terminated in a fixed reactance and find stability conditions between port 1 and port 3. Then, let port 3 be terminated in a fixed reactance and find stability conditions between port 1 and port 2. In this case, for different values of fixed reactances, a family of circles is obtained (by using the bilinear transformation of the input impedance) whose characteristics define the region of stability. The fixed reactance can take on different values, which make for a long and iterative process. Thus, the analysis in [16] is considerably involved.

In [35], Boehm and Albright presented a solution for the case of 3-port networks. The stability analysis is done by using scattering parameters and investigating the absolute stability of 2-port networks resulting from terminating the 3-port network to a fixed termination at the 3rd port. This makes the method rather complex since for each termination, three stability conditions involving its reflection coefficient need to be considered; consequently, the necessary conditions for stability of 3-ports result in a total of nine equations.

Graphical solutions have also been proposed based on reflection coefficients, scattering parameters, and stability plots using the Smith Chart. In [36], Tan presents a simplified graphical analysis based on 2-port stability criterion using a single parameter μ. The criterion establishes that a 2-port network is absolutely stable if and only if the geometrically derived parameter μ satisfies

$$\mu = \frac{1 - |s_{11}|^2}{|s_{22} - \Delta s_{11}^*||s_{12}s_{21}|} \geq 1 \qquad (1.5)$$

where s_{ij} are the elements of the scattering matrix S and Δ is the determinant of S. Tan postulates that a 3-port network arbitrarily terminated at one of its ports can be reduced to a 2-port network whose absolute stability can be investigated by using (1.5). Condition (1.5) results in stability plots that are mapped into the Smith Chart. Like previous cases, this method does not propose a general solution for absolute stability of *n*-port networks; on the contrary, by restricting one of the ports to an arbitrary value, it transforms the 3-port network into many 2-port networks.

In conclusion, tools known so far to evaluate the passivity and absolute stability of *n*-port networks are still in their infancy and more research has to be done in order to find a complete analytical solution to the problem.

1.4 Contribution of the Thesis

The contributions of this thesis are twofold. First, it presents a closed-form and practically-useful criterion for passivity of *n*-port networks (*n* equal or greater than 2), which can be used to investigate the stability of multilateral haptic teleoperation systems. Second, it gives a procedure for direct investigation of absolute stability of trilateral haptic teleoperation systems. In this case, an analytical expression needed to assess the absolute stability of three port networks is derived. The numerical evaluation of such an expression gives information regarding the stability of trilateral haptic teleoperation systems.

1.5 Thesis Synopsis and Organization

The following is a summary of each chapter of the thesis.

Chapter 2 presents an overview of passivity and absolute stability of 2-port networks. The intention of this chapter is to familiarize the reader with the most important characteristics and issues of 2-port network stability analysis. By studying the work done so far for the case of bilateral systems (2-port networks), the reader will gain the necessary knowledge required to understand the complexity of multilateral haptic systems beyond the bilateral one. This knowledge will be crucial and necessary for the study of *n*-port network stability.

In Chapter 3, a novel method to investigate the passivity of *n*-port networks, based on immittance parameters of the network, is presented. The method is given as a closed-form criterion for passivity of *n*-port networks and can be used to investigate the stability of multilateral haptic teleoperation systems. This criterion, which is necessary and sufficient for passivity of an *n*-port network, imposes *n* conditions on the immittance parameters of the network and another set of *n* conditions on the residues of the immittance parameters at their imaginary-axis poles.

Chapter 4 discusses a method for investigation of absolute stability of 3-port networks. The chapter departs from the fundamental definition of absolute stability as it applies to 3-port networks and arrives at an expression that can be evaluated iteratively and numerically for assessment of stability of trilateral haptic teleoperation systems.

In chapter 5, the passivity and absolute stability of a dual-user haptic system for control of a single teleoperated robot (i.e., a trilateral haptic teleoperation system) are investigated through simulations in order to verify the findings in Chapters 3 and 4. Chapter 6 presents the conclusions of this research as well as directions for future research.

Chapter 2

Passivity and Absolute Stability of Bilateral Teleoperators

2.1 The Bilateral Teleoperation System and the 2-port Network Representation

2-port networks are overwhelmingly the method of choice for modeling a bilateral teleoperation system, which consists of a slave robot and a master user interface. The human operator controls the slave and is provided with haptic feedback concerning slave/environment contact forces through the master. Figure 2.1 shows the equivalent electrical circuit representation of a bilateral teleoperation system. Usually, only the linear dynamics of the master and slave are considered as in

$$f_m + f_h = M_m \ddot{x}_m \,, \qquad f_s - f_e = M_s \ddot{x}_s \tag{2.1}$$

In the above, the hand/master interaction is denoted by f_h and the slave/environment interaction is denoted by f_e. Also, M_m, M_s, x_m, x_s, f_m, and f_s are the master's and the slave's inertias, positions, and control signals, respectively. In Fig 2.1, impedances Z_h and Z_e denote the dynamic characteristics of the human operator's hand and the remote environment, respectively. Moreover, F_h^* and F_e^* are the operator's and environment's exogenous input forces, which are independent of the teleoperation system behavior [1].

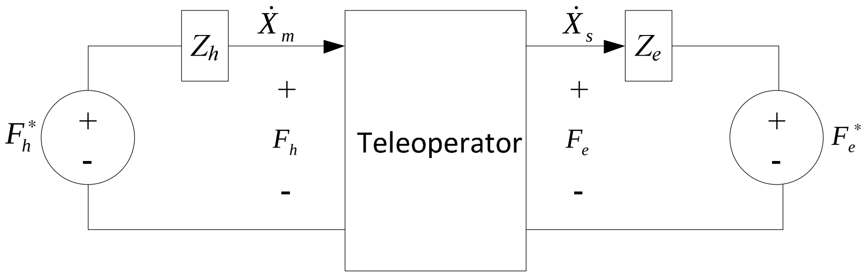

Figure 2.1. A 2-port network model of a bilateral teleoperation system.

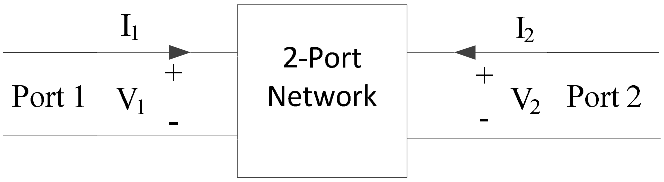

Figure 2.2. A general 2-port network.

Figure 2.2 shows a general 2-port network, in which the pair (I_1, V_1) is the input current and voltage and the pair (I_2, V_2) is the output current and voltage. Depending on which combination of these four quantities (I_1, I_2, V_1, and V_2) are chosen as independent and dependent variable pairs, six different ways for modeling the 2-port network exist. Table 2.1 shows these six possible representations of a 2-port network [11].

Table 2.1. Different representations of a 2-port network.

Independent Variables	**Dependent Variables**	**Parameter Type**
I_1, I_2	V_1, V_2	Open-circuit impedances (z)
V_1, V_2	I_1, I_2	Short-circuit admittances (y)
I_1, V_2	V_1, I_2	Hybrid parameters (h)
V_1, I_2	I_1, V_2	Inverse hybrid parameters (g)
V_2, I_2	V_1, I_1	Chain parameters (A, B, C, D)
V_1, I_1	V_2, I_2	Inverse chain parameters ($\mathscr{A}$, $\mathscr{B}$, $\mathscr{C}$, $\mathscr{D}$)

For instance, using the impedance parameters, the 2-port network can be modeled as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{2.2}$$

Accordingly, the impedance model of the bilateral teleoperation system in Figure 1 is given by [15]

$$\begin{bmatrix} F_h \\ F_e \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} \dot{X}_m \\ -\dot{X}_s \end{bmatrix} \tag{2.3}$$

2.2 Criterion for Absolute Stability and Passivity of a 2-port Network with Unknown Terminations

2.2.1 Preliminaries

Closed-loop stability is crucial for safe teleoperation. For instance, when a surgeon remotely guides a robot to operate on a patient, stability is of ultimate importance. For the analysis of closed-loop stability of a teleoperation system, according to Figure 2.1, the knowledge of the human operator and the environment dynamics are needed in addition to that of the teleoperation system's immittance parameters *(z, y, h,* or *g*). In practice, however, the models of the human operator and the environment are usually unknown, uncertain, and/or time-varying. This makes it impossible to use conventional techniques to investigate the closed-loop stability of a teleoperation system.

Assuming that $Z_h(s)$ and $Z_e(s)$ in Figure 2.1 are passive, we might be able to draw stability conditions that are independent of the human operator and the environment. Two well-known methods have been developed to investigate the stability of a 2-port network that is connected to unknown terminations. These methods are known as Llewellyn's absolute stability criterion and Raisbeck's passivity criterion. Both criteria work under the assumption that both the operator and the environment are passive. The following definitions are needed before presenting these criteria.

Definition: Passivity *[11]*

A 2-port network is passive if, for all excitations, the total energy delivered to the network at its input and output ports is non-negative. Hence, passivity is a property of the 2-port network which establishes that it cannot deliver more energy than what is delivered to it. Assuming that the 2-port network has zero energy stored at time t = 0, the network is said to be passive if it satisfies

$$E(t) = \int_0^t (i_1(\tau)v_1(\tau) + i_2(\tau)v_2(\tau))\, d\tau \geq 0 \tag{2.4}$$

where $i_i(t)$ and $v_i(t)$ are the instantaneous values of the current and voltages at port i with i=*1,2*, and $E(t)$ represents the total energy exchange for the 2-port network.

Definition: Activity *[11]*

If a network is not passive, then it is active.

Definition: Positive realness *[11]*

A rational function $F(s)$ is positive real if and only if, in addition to being real for real s, it meets the following conditions:

- **a.** $F(s)$ has no poles neither zeros in the right half plane (RHP),
- **b.** Any poles of $F(s)$ on the imaginary axis are simple with real and non-negative residues, and
- **c.** $\Re e\{F(j\omega)\} \geq 0, \quad \forall\, \omega.$

Definition: Absolute stability

The following definitions are equivalent:

A. A 2-port network is absolutely stable if it remains stable under all possible uncoupled *passive terminations.*

B. A 2-port network is absolutely stable if, when connected to a passive termination at one of its ports, the other port will display a passive behavior.

C. A 2-port network is absolutely stable if the port currents are zero at all real frequencies for all passive terminations.

Definition: Potential instability *[11]*

A 2-port network is potentially unstable if it is possible to find uncoupled passive terminations that, when connected to the network, produce an unstable system. If a 2-port network is not absolutely stable, then it is potentially unstable.

Theorem: Equivalence between positive realness and passivity for LTI systems *[29]*

Consider a linear time invariant system *H* defined by $Hx = h * x$, where h has a Laplace transform that has no poles in the right half plane. System *H* is passive if and only if $\Re e\{\widehat{H}(j\omega)\} \geq 0$, for all real frequencies ω, where $\widehat{H}(j\omega)$ is the Fourier transform of $h(t)$.

This theorem establishes that an LTI system is passive if and only if its transfer function is a positive real function. This theorem is stated for a 1-port network. Its extension to *n*-port networks is presented later in Chapter 3.

2.2.2 Relationship between input/output impedance and absolute stability

What follows is a simple proof of the assertion that a 2-port network is absolutely stable if its input and output impedance for passive terminations z_2 and z_1 are positive real functions. Figure 2.3 shows a 2-port network driven by a voltage source V_s and with terminations z_2 and z_1. The system can be represented by the following equation

$$\begin{bmatrix} V_s \\ 0 \end{bmatrix} = \begin{bmatrix} z_{11} + z_1 & z_{12} \\ z_{21} & z_{22} + z_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{2.5}$$

The transfer function for this system is the ratio of response to excitation and is given as

$$\frac{I_2}{V_s} = \frac{z_{21}}{(z_{11} + z_1)(z_{22} + z_2) - z_{12}z_{21}} \tag{2.6}$$

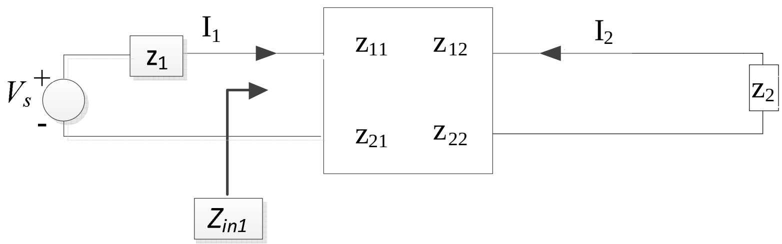

Figure 2.3. A double terminated 2-port network.

For the system to be stable, the transfer function cannot have poles in the right half of the complex frequency plane. The poles of the double terminated 2-port network are the roots of the characteristic equation

$$(z_{11} + z_1)(z_{22} + z_2) - z_{12}z_{21} = 0 \tag{2.7}$$

With some manipulations, equation (2.7) can be written in any of the following two equivalent forms

$$z_1 + Z_{in1} = 0 \qquad \text{or} \qquad z_2 + Z_{in2} = 0 \tag{2.8}$$

with Z_{in1} and Z_{in2} given as:

$$Z_{in1} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + z_2} = \frac{z_{11}z_2 + z_{11}z_{22} - z_{12}z_{21}}{z_{22} + z_2} \tag{2.9}$$

$$Z_{in2} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + z_1} = \frac{z_{22}z_1 + z_{11}z_{22} - z_{12}z_{21}}{z_{11} + z_1} \tag{2.10}$$

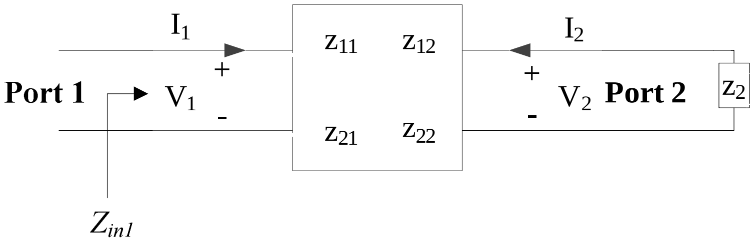

Figure 2.4. A nonreciprocal 2-port network terminated at port 2.

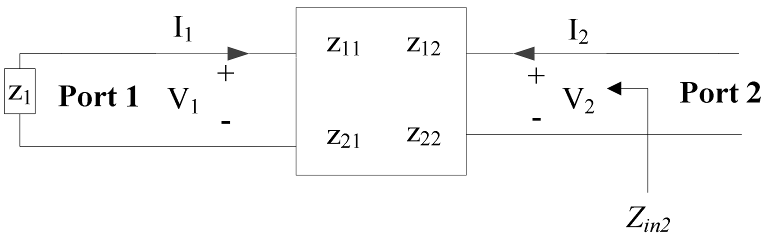

Figure 2.5. A nonreciprocal 2-port network terminated at port 1.

On the other hand, If *Z(s)* is any arbitrary positive real function, then *Z(s)* has to satisfy the following two conditions

$$Z(s) \text{ is real when } s \text{ is real}$$

$$Re[Z(s)] \geq 0 \quad \text{for} \quad Re[s] \geq 0 \tag{2.11}$$

As previously stated, a positive real function cannot have poles or zeros in the right half of the s-plane; poles or zeros along the imaginary axis are allowed only if they are simple. Next, the argument for positive realness of Z_{in1} is presented (the same argument applies to positive realness of Z_{in2}).

The first equation in (2.8) represents another way of writing the system's characteristic equation, thus

$$z_1 + Z_{in1} = 0 \quad \triangleq \quad (z_{11} + z_1)(z_{22} + z_2) - z_{12}z_{21} = 0 \qquad (2.12)$$

If z_1 is passive then it is positive real. If Z_{in1} is also positive real, meaning that $Re(Z_{in1}) \geq 0$, then $z_1 + Z_{in1}$, which represents the total impedance of port 1 loop, is positive real as well; thus, $z_1 + Z_{in1}$ cannot have zeros in the right half plane. Zeros of $z_1 + Z_{in1}$ are the zeros of the characteristic equation (2.7), which in turn are the poles of the system's transfer function given in (2.6). In conclusion, the 2-port network is absolutely stable if the input impedance Z_{in1} is a positive real function. The same proof can be applied for the case of Z_{in2} ■

2.2.3 Llewellyn's absolute stability criterion

If *p* represents any of the four immittance parameters (*z*, *y*, *h*, *g*) of a 2-port network, the criterion establishes that the network is absolutely stable if and only if [11]:

1. p_{11} and p_{22} have no poles in the right-half plane (RHP),
2. Any poles of p_{11} and p_{22} on the imaginary axis are simple with real and positive residues,
3. For all real values of frequencies ω, we have
   $\Re e(p_{11}) \geq 0$
   $\Re e(p_{22}) \geq 0$
   $$2\Re e(p_{11})\Re e(p_{22}) - \Re e(p_{12}p_{21}) - |p_{12}p_{21}| \geq 0 \qquad (2.13)$$

where $\Re e(*)$ denotes the real part of a complex number.

If any of the above conditions is not satisfied, then the 2-port network is potentially unstable. As mentioned before, the advantage of using absolute stability in haptic teleoperation systems is that models of the operator and environment will not be needed for stability analysis.

Proof of Llewellyn's criterion for absolute stability:

A simple proof of Llewellyn's criterion for absolute stability can be offered by using the properties of bilinear transformations [34]. Recall that, by definition, a 2-port network is absolutely stable if its input (and output) impedance for any passive load (and source) impedance is a positive real function. The following derivation applies to any of the four immittance parameters (z, *y*, *h*, *g*) of a nonreciprocal 2-port network.

Using impedance parameters (which is one the four possible choices of immittance parameters), the relation between voltages and current in the 2-port network of Figure 2.6 can be written as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{2.14}$$

where all the impedance parameters are complex quantities of the form

$$\mathrm{z}_{ij} = r_{ij} + jx_{ij} \tag{2.15}$$

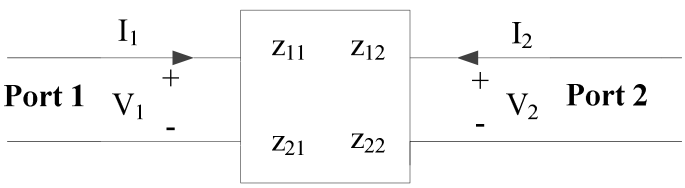

Figure 2.6. A general nonreciprocal 2-port network.

Equations (2.9) and (2.10) from the Section 2.2.2 show that $Z_{in1}$and Z_{in2} are bilinear transformations of the terminations z_2 and z_1, respectively. A bilinear transformation transforms circles into circles with straight lines as limiting cases

[16]. The borderline of passivity in the z_2 complex plane is the jω-axis meaning that any impedance to the right of the jω-axis is passive and any impedance to the left is non-passive (active).

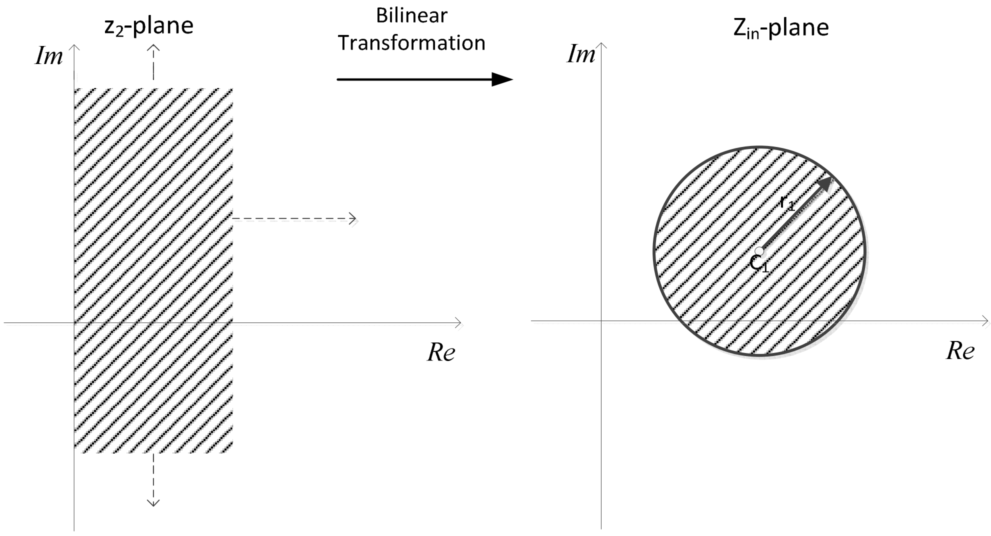

Figure 2.7. The input impedance Z_{in1} as a bilinear transformation of z_2.

In Figure 2.7, for all passive z_2, Z_{in1} is transformed into a circle centered at C_1 with radius $r_1$where

$$C_1 = z_{11} - \frac{z_{12}z_{21}}{2\Re e(z_{22})} \tag{2.16}$$

and

$$r_1 = \frac{|z_{12}z_{21}|}{2\Re e(z_{22})} \tag{2.17}$$

In Section 2.2.1, we established that a 2-port network is absolutely stable if the input impedance Z_{in1} (and Z_{in2}) is a positive real function. This fact is represented by the two inequalities (2.18) and (2.19).

$$\Re e(Z_{in1}) \geq 0 \quad \text{for all passive } z_2 \tag{2.18}$$

and

$$\Re e(Z_{in2}) \geq 0 \quad \text{for all passive } z_1 \tag{2.19}$$

For the nonreciprocal 2-port network to be absolutely stable, condition (2.18) and condition (2.19) must be satisfied.

Condition (2.18) means that

$$\Re e(C_1) - r_1 \geq 0 \tag{2.20}$$

Substituting (2.16) and (2.17) into the condition (2.20) yields

$$\frac{2\Re e(z_{11})\Re e(z_{22}) - \Re e(z_{12}z_{21}) - |z_{12}z_{21}|}{2\Re e(z_{22})} \geq 0 \tag{2.21}$$

Also for all passive z_1, Z_{in2} is transformed into a circle centered at $C_2$with radius r_2. Following the same procedure as for Z_{in1}, condition (2.19) yields

$$\frac{2\Re e(z_{11})\Re e(z_{22}) - \Re e(z_{12}z_{21}) - |z_{12}z_{21}|}{2\Re e(z_{11})} \geq 0 \tag{2.22}$$

Furthermore,

$$\Re e(z_{11}) \geq 0 \tag{2.23}$$

and

$$\Re e(z_{22}) \geq 0 \tag{2.24}$$

are necessary conditions for the stability since they represent conditions (2.18) and (2.19) under open circuit terminations.

From (2.21) to (2.24), we have

$$\Re e(z_{11}) \geq 0$$

$$\Re e(z_{22}) \geq 0$$

$$2\Re e(z_{11})\Re e(z_{22}) - \Re e(z_{12}z_{21}) - |z_{12}z_{21}| \geq 0 \qquad (2.25)$$

which is Llewellyn's criterion for absolute stability [18].

2.2.4 Raisbeck's passivity criterion

The necessary and sufficient conditions for passivity of a 2-port network with the immittance parameter p are [11]:

1. The p-parameters have no RHP poles.

2. Any poles of the p-parameters on the imaginary axis are simple, and the residues of the p-parameters at these poles satisfy the following conditions:
   If k_{ij} denotes the residue of p_{ij} and k_{ij}^* is the complex conjugate of k_{ji}, then

$$k_{11} \geq 0$$

$$k_{22} \geq 0$$

$$k_{11}k_{22} - k_{12}k_{21} \geq 0 \quad with \;\; k_{21} = k_{12}^* \qquad (2.26)$$

3. The real and imaginary part of the p-parameters satisfy the following conditions for all real frequencies ω

$$\Re e(p_{11}) \geq 0$$

$$\Re e(p_{22}) \geq 0$$

$$4\Re e(p_{11})\Re e(p_{22}) - \left(\Re e(p_{12}) + \Re e(p_{21})\right)^2 - \left(\mathcal{I}m(p_{12}) - \mathcal{I}m(p_{21})\right)^2 \geq 0 \qquad (2.27)$$

where $\mathcal{I}m(*)$ denotes the imaginary part of a complex expression.

Proof of Raisbeck's passivity criterion of 2-port networks will not be shown here. Instead, a full proof of passivity of *n*-port networks as an extension of Raisbeck's criterion will be presented in the next chapter. ■

2.2.5 Comparison of stability vs. passivity: The stability–activity diagram

As mentioned previously, the advantage of using passivity in haptic teleoperation systems is that models of the operator and environment are not needed for stability analysis. However, stability conditions drawn from passivity are conservatives compared to absolute stability. There is one case in which the passivity and absolute stability criteria overlap. It is when the 2-port network representing the teleoperator is reciprocal. By definition, a 2-port network is said to be reciprocal if the ratio of response to excitation is invariant to an interchange of the locations of the excitation and the response. In terms of z-parameters, a 2-port network is reciprocal if $z_{12} = z_{21}$.

A comparison between the aforementioned criteria for passivity and absolute stability of 2-port networks shows that Raisbeck's passivity criterion implies condition 1, condition 2, and the first two sub-conditions of condition 3 of Llewellyn's absolute stability criterion. The difference between the two criteria indeed lies in the last sub-condition of conditions 3. For absolute stability, this condition can be written as [10]

$$\frac{r_{121}}{\sqrt{r_{11}r_{22}}} \leq 1 \tag{2.28}$$

where r_{121} is the real part of $\sqrt{z_{12}z_{21}}$ while r_{11} and r_{22} are the real part of z_{11} and z_{22}, respectively. Moreover, for passivity, the last condition can be written as

$$\frac{r_{121}^2}{r_{11}r_{22}} + \frac{(|z_{12}| - |z_{21}|)^2}{4r_{11}r_{22}} \leq 1 \qquad (2.29)$$

Obviously, absolute stability and passivity coincide when $z_{12} = z_{21}$.

Equations (2.28) and (2.29) can be represented in the stability-activity diagram shown in Figure 2.8 by plotting $\frac{||z_{12}|-|z_{21}||}{2\sqrt{r_{11}r_{22}}}$ vs. $\frac{r_{121}}{\sqrt{r_{11}r_{22}}}$. The graph shows the boundary between the regions of passivity and activity is the first quadrant of a circle of unit radius. The boundary between the regions of absolute stability and potential instability is represented by the vertical line at $\frac{r_{121}}{\sqrt{r_{11}r_{22}}} = 1$. It is observed from the graph that the condition for passivity implies the condition for absolute stability; however, the condition for absolute stability does not necessarily imply the condition for passivity. Consequently, passive networks are absolutely stable but not all absolutely stable networks are passive.

By using this graph, one can easily determine the stability of a bilateral teleoperator. The immittance matrix representing the bilateral teleoperator in question will be a function of frequency ($s = j\omega$). Thus, by running ω from 0 to ∞ a curve can be plotted on the stability-activity diagram. If the curve is completely inside the passive region, the teleoperator is passive and, therefore, absolutely stable. If the curve lies anywhere inside the absolutely stable region, the teleoperator is absolutely stable but not necessarily passive. If any point of the curve lies to the right of the vertical line $\frac{r_{121}}{\sqrt{r_{11}r_{22}}} = 1$, the teleoperator is potentially unstable, meaning that there is at least one combination of passive environment and passive operator for which the system is unstable.

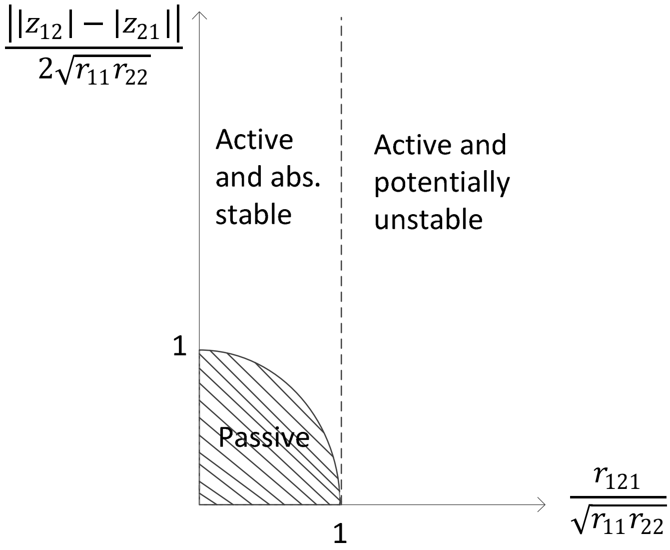

Figure 2.8. The stability-activity diagram.

2.3 Transparency of a Bilateral Teleoperation System

Besides stability, a main goal of teleoperation control is transparency. Although not a subject of this research, a brief description of transparency is provided next. Transparency is the ability of a teleoperation system to present the undistorted dynamics of the remote environment to the human operator [2], and requires the master and the slave positions and interactions to match regardless of the operator and environment dynamics. Mathematically, it can be expressed as

$$f_h = f_e, \quad x_m = x_s \tag{2.30}$$

The hybrid representation of the teleoperator shown in Figure 2.1 is given by

$$\begin{bmatrix} F_h \\ -\dot{X}_s \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} \dot{X}_m \\ F_e \end{bmatrix} \tag{2.31}$$

Using hybrid parameters, full transparency is guaranteed if

$$h_{11} = 0,\ \ h_{12} = 1,\ \ h_{21} = -1,\ \ h_{22} = 0 \tag{2.32}$$

where the H-matrix components h_{ij} can be interpreted as

$h_{11} = F_h/\dot{X}_m\big|_{F_e=0}$: input impedance when the slave is in free motion

$h_{12} = F_h/F_e|_{\dot{X}_{m=0}}$: force tracking when the master is in locked motion

$h_{21} = -\dot{X}_s/\dot{X}_m\big|_{F_e=0}$: - velocity tracking when the slave is in free space

$h_{22} = -\dot{X}_s/F_e\big|_{\dot{X}_m=0}$: output admittance when the master is in locked motion

Chapter 3

Passivity of Multilateral Teleoperators

3.1 Introduction

An *n*-port network can be defined as a network containing *n* pairs of terminals for external connections. Each pair of terminals represents a port to which an external network can be connected (Figure 3.1). The external behavior of the *n*-port network can be determined if all the I_i currents and V_i voltages are known. If for any given port the product of current and voltage is positive, then power is entering that port.

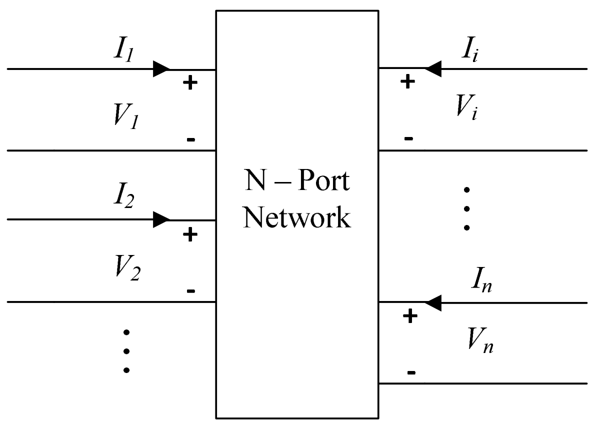

Figure 3.1. A general *n*-port network.

As a natural extension from 2-ports, passivity of an *n*-port network is a sufficient condition for the stability of the network when coupled to passive termination. An attractive feature of passivity is that it applies to both linear and non-linear networks and it is based on simple energy concepts. In this chapter, the necessary and sufficient conditions for passivity of an *n*-port network are presented.

3.2 Passivity Conditions for Linear *n*-port Networks

By analogy with the case of 2-port networks, we define an *n*-port network to be passive if, for all excitations, the total energy exchange at the network's input and output ports is non-negative. Assuming that the 2-port network has zero energy stored at time t = 0, this passivity definition is expressed as

$$E(t) = \int_0^t \left(i_1(\tau)v_1(\tau) + i_2(\tau)v_2(\tau) + \ldots + i_n(\tau)v_n(\tau)\right) d\tau \geq 0 \qquad (3.1)$$

where $E(t)$ is the total energy delivered to the *n*-port network.

Based on (3.1) for the case of $n = 2$, Raisbeck found the necessary and sufficient conditions for passivity of 2-port networks [29]. We recall here that in the case of linear and time invariant networks, passivity and positive realness are equivalent, which explains why Raisbeck arrived to the conclusion that a necessary and sufficient condition for a network to be passive is that its impedance function has to be a positive real function.

The *n*-port network passivity theorem that we propose later in this chapter holds for **any of the four immittance parameters**, yet for brevity it is written only in terms of impedance parameters. The proof for the theorem is given in Section 3.3. Using the impedance parameters of the *n*-port network, the relation in the *s*-domain between voltages and currents is given by

$$\begin{bmatrix} V_1(s) \\ V_2(s) \\ \vdots \\ V_n(s) \end{bmatrix} = \begin{bmatrix} z_{11}(s) & z_{12}(s) & \cdots & z_{1n}(s) \\ z_{21}(s) & z_{22}(s) & \cdots & z_{2n}(s) \\ \vdots & \vdots & \ddots & \vdots \\ z_{n1}(s) & z_{n2}(s) & \cdots & z_{nn}(s) \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \\ \vdots \\ I_n(s) \end{bmatrix} \tag{3.2}$$

which can be compactly described as $\boldsymbol{V} = \boldsymbol{ZI}$ (note that $\boldsymbol{I}$ is the vector of current and not the identity matrix). In the proof of this theorem, we will need the following definitions.

Definition: Hermitian matrix

A Hermitian matrix $\boldsymbol{H}$ is a square matrix with complex elements h_{ij} for which the following property holds: $h_{ij} = h_{ji}^*$. Consequently, a Hermitian matrix $\boldsymbol{H}$ is equivalent to its own conjugate transpose.

The eigenvalues of a Hermitian matrix are always real-valued. Another important attribute of a Hermitian matrix $\boldsymbol{H}$ is that it is always possible to find a square unitary matrix $\boldsymbol{U}$ (i.e., $\boldsymbol{U}^*\boldsymbol{U}$ is the identity matrix) such that $\boldsymbol{U}^*\boldsymbol{HU}$ is a diagonal matrix with the eigenvalues of $\boldsymbol{H}$ on its diagonal. Hence, it is always possible to diagonalize a Hermitian matrix.

Definition: Hermitian form

A Hermitian form is an expression of the form $\sum h_{ij} a_j\, a_i^*$ in which the coefficients h_{ij} are the complex elements of a Hermitian matrix $\boldsymbol{H}$.

Definition: Reduced row-echelon form

A reduced row-echelon form is a matrix form that has the following properties:

- The first nonzero number in a row is a 1 (leading 1).
- All rows made up entirely of zeros are grouped together at the bottom of the matrix.
- The leading 1 in the lower row occurs farther to the right than the leading

1 in the higher row.

- Each column that contains a leading 1 has zeros everywhere else.

Theorem 1: Passivity of an n-port network.

The necessary and sufficient conditions for passivity of an *n*-port network are

A. The *z*-parameters have no RHP poles.

B. Any poles of the *z*-parameters on the imaginary axis are simple, and the residues k_{ij} of the *z*-parameters at these poles satisfy the following conditions:

1. $k_{ii} \geq 0 \qquad\qquad i = 1, 2, \ldots, n$

2. $\dfrac{k_{11}k_{22} - k_{12}k_{21}}{k_{11}} \geq 0$

3. $\dfrac{k_{11}k_{33} - k_{12}k_{21}}{k_{11}} - \dfrac{(k_{11}k_{23} - k_{21}k_{13})(k_{11}k_{32} - k_{31}k_{12})}{k_{11}(k_{11}k_{22} - k_{12}k_{21})} \geq 0$

$\vdots$

$\vdots$

$n.\ k_{nn} - \sum_{i=1}^{n-1} |u_{in}|^2 k'_{ii} \geq 0 \qquad \forall\, u_{ij}$ with $i \leq j$ (3.3)

where

- k_{ij} denotes the residue of z_{ij} .
- The terms u_{ij} are the elements of an upper triangular matrix $\boldsymbol{U}$ used to diagonalize the residues matrix $\boldsymbol{K}$ according to $\boldsymbol{U}^*\boldsymbol{K}'\boldsymbol{U} = \boldsymbol{K}$, with $\boldsymbol{U}^*$being equal to the transpose complex conjugate of $\boldsymbol{U}$.
- The coefficients k'_{ii} are the elements of the diagonal matrix $\boldsymbol{K}'$.

C. The complex z'-parameters satisfy the following conditions for all real frequencies ω

1. $z'_{ii} \geq 0 \qquad\qquad i = 1, 2, \ldots, n$

2. $\dfrac{z'_{11}z'_{22} - z'_{12}z'_{21}}{z'_{11}} \geq 0$

3. $\dfrac{z'_{11}z'_{33} - z'_{13}z'_{31}}{z'_{11}} - \dfrac{(z'_{11}z'_{23} - z'_{21}z'_{13})(z'_{11}z'_{32} - z'_{31}z'_{12})}{z'_{11}(z'_{11}z'_{22} - z'_{12}z'_{21})} \geq 0$

⋮

⋮

$n.\;\; z'_{nn} - \sum_{i=1}^{n-1} |w_{in}|^2 z''_{ii} \geq 0 \qquad \forall\, w_{ij} \text{ with } i \leq j \qquad (3.4)$

where

- $z'_{ij} = \frac{1}{2}\left(z_{ij} + z^*_{ji}\right)$ are the elements of the matrix $\boldsymbol{Z}'$.
- The terms w_{ij} are the elements of an upper triangular matrix $\boldsymbol{W}$ used to diagonalize the matrix $\boldsymbol{Z}'$ according to $\boldsymbol{W}^*\boldsymbol{Z}''\boldsymbol{W} = \boldsymbol{Z}'$, with $\boldsymbol{W}^*$equal to the transpose complex conjugate of $\boldsymbol{W}$.
- The elements z''_{ii} are the entries of the main diagonal matrix $\boldsymbol{Z}''$.

Note: $\boldsymbol{Z}$ is the impedance matrix representing the *n*-port network. $\boldsymbol{Z}'$ is equal to half the sum of $\boldsymbol{Z}$ and the transpose of its complex conjugate. ■

3.3 Proof of Theorem 1

Previously in Chapter 2 it was shown that for LTI systems, passivity and positive realness of the network's transfer function are equivalent. Hence, for the simple case of a 1-port network ($n = 1$) the energy requirement in (3.1) in the *s*-domain is equivalent to

$$\Re e\{Z(s)\} \geq 0 \quad \text{for} \quad \Re e\{s\} \geq 0 \tag{3.5}$$

where $\Re e\{\}$ denotes the real part and $Z(s)$ represents the input impedance of the 1-port network. $Z(s)$ can be expressed as

$$Z(s) = \frac{V(s)}{I(s)} \tag{3.6}$$

where $V(s)$ is the voltage across the 1-port and $I(s)$ is the current flowing through the port. By manipulating (3.6) as

$$Z(s) = \frac{V(s)}{I(s)} = \frac{V(s)\, I^*(s)}{I(s)\, I^*(s)} = \frac{V(s)\, I^*(s)}{|I(s)|^2} \tag{3.7}$$

equation (3.5) is equivalent to

$$\Re e\{V(s)\, I^*(\mathrm{s})\} \geq 0 \quad \text{for} \quad \Re e\{s\} \geq 0 \tag{3.8}$$

where $I^*(\mathrm{s})$ is the complex conjugate of $I(s)$. Notice that $|I(s)|^2$ in the denominator of (3.7) is always positive.

By analogy with (3.8), (3.1) is equivalent to the following condition

$$\Re e\{V_1(s)I_1^* + V_2(s)I_2^* \cdots V_n(s)I_n^*\} \geq 0 \qquad \text{for} \quad \Re e\{s\} \geq 0 \tag{3.9}$$

Eliminating the voltages in (3.9) by using (3.2), we find that the *n*-port network passivity is equivalent to

$$\Re e\{F(s)\} \geq 0 \quad \text{for} \quad \Re e\{s\} \geq 0 \tag{3.10}$$

where

$$\Re e\{F(s)\} = \Re e\{z_{11}(s)I_1(s)I_1^*(s) + \cdots + z_{1n}(s)I_n(s)I_1^*(s) + z_{21}(s)I_1(s)I_2^*(s) + z_{2n}(s)I_n(s)I_2^*(s) + \cdots + z_{n1}(s)I_1(s)I_n^*(s) + \cdots + z_{nn}(s)I_n(s)I_n^*(s)\} \tag{3.11}$$

On the other hand, we know that the rational function $F(s)$ is positive real (i.e., (3.10) holds) if and only if, in addition to being real for real s, $F(s)$ meets the following conditions:

1. $F(s)$ has no poles in the right half plane (RHP)
2. Any poles of $F(s)$ on the imaginary axis are simple with real and non-negative residues
3. $\Re e\{F(j\omega)\} \geq 0 \quad \forall\, \omega$

For condition **1**, we require that none of the *z*-parameters of the *n*-port network have any poles in the RHP. To investigate condition **2**, assume that $F(s)$ has a simple pole at $s = j\omega_0$ with a residue k_0. Let $k_{11}, k_{12}, \cdots, k_{21} \cdots k_{nn}$ denote the residues of $z_{11}, z_{12}, \cdots, z_{21} \cdots z_{nn}$, respectively, at this pole. Expanding *F(s)* in a Laurent series about $s = j\omega_0$ and keeping only the dominant terms in the immediate neighborhood of the pole, we get

$$\frac{k_0}{s - j\omega_0} = \frac{k_{11}(j\omega_0)\, I_1(j\omega_0)\, I_1^*(j\omega_0)}{s - j\omega_0} + \cdots + \frac{k_{1n}(j\omega_0)\, I_n(j\omega_0)\, I_1^*(j\omega_0)}{s - j\omega_0} + \cdots + \frac{k_{n1}(j\omega_0)\, I_1(j\omega_0)\, I_n^*(j\omega_0)}{s - j\omega_0} + \cdots + \frac{k_{nn}(j\omega_0)\, I_n(j\omega_0)\, I_n^*(j\omega_0)}{s - j\omega_0} \tag{3.12}$$

which is equivalent to

$$k_0 = k_{11}(j\omega_0)\, I_1(j\omega_0)\, I_1^*(j\omega_0) + \cdots + k_{1n}(j\omega_0)\, I_n(j\omega_0)\, I_1^*(j\omega_0) + \cdots$$

$$+k_{n1}(j\omega_0)\, I_1(j\omega_0)\, I_n^*(j\omega_0) + \cdots + k_{nn}(j\omega_0)\, I_n(j\omega_0)\, I_n^*(j\omega_0) \qquad (3.13)$$

In (3.13), k_0 must be a real and non-negative number to satisfy condition **2**. Terms k_{ii} for $i = 1, 2, \dots, n$ are real and positive since the impedances z_{ii} are positive real functions. Also, $I_i(j\omega_0)\, I_i^*(j\omega_0)$ is real and positive. Note that in the pairs $k_{ij}(j\omega_0)\, I_j(j\omega_0)\, I_i^*(j\omega_0) + k_{ji}(j\omega_0)\, I_i(j\omega_0)\, I_j^*(j\omega_0)$, since $I_j(j\omega_0)\, I_i^*(j\omega_0)$ and $I_i(j\omega_0)\, I_j^*(j\omega_0)$ are complex conjugates, k_{ij} and k_{ji} are also complex conjugates.

Since the right side of (3.13) is a Hermitian form (with $h_{ij} = k_{ij}$), it can be diagonalized with respect to the Hermitian matrix with coefficients k_{ij}. To do so, (3.13) can be written in matrix form as

$$k_0 = \begin{bmatrix} I_1^* & I_2^* & \cdots & I_n^* \end{bmatrix} \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & \cdots & k_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ k_{n1} & k_{n2} & \cdots & k_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = \boldsymbol{I}^* \boldsymbol{K} \boldsymbol{I} \qquad (3.14)$$

The $\boldsymbol{K}$-matrix is diagonalizable and we want to find a linear transformation $\boldsymbol{U}^* \boldsymbol{K}' \boldsymbol{U} = \boldsymbol{K}$ where $\boldsymbol{K}'$ is a diagonal matrix, $\boldsymbol{U}$ is an upper triangular matrix, and $\boldsymbol{U}^*$ (the transpose complex conjugate of $\boldsymbol{U}$) is a lower triangular matrix.

$$\begin{bmatrix} u_{11}^* & 0 & 0 & \cdots & 0 \\ u_{12}^* & u_{22}^* & 0 & \cdots & 0 \\ u_{13}^* & u_{23}^* & u_{33}^* & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ u_{1n}^* & u_{2n}^* & u_{3n}^* & \cdots & u_{nn}^* \end{bmatrix} \begin{bmatrix} k_{11}' & 0 & 0 & \cdots & 0 \\ 0 & k_{22}' & 0 & \cdots & 0 \\ 0 & 0 & k_{33}' & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & k_{nn}' \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ 0 & u_{22} & u_{23} & \cdots & u_{2n} \\ 0 & 0 & u_{33} & \ddots & u_{3n} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & u_{nn} \end{bmatrix}$$

$$= \begin{bmatrix} k_{11} & k_{12} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & k_{23} & \cdots & k_{2n} \\ k_{31} & k_{32} & k_{33} & \ddots & k_{3n} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ k_{n1} & k_{n2} & k_{n3} & \cdots & k_{nn} \end{bmatrix} \tag{3.15}$$

which represents the system $\boldsymbol{U}^*\boldsymbol{K}'\boldsymbol{U} = \boldsymbol{K}$. Solving for $\boldsymbol{K}'$ and $\boldsymbol{U}$ will lead us to expressions for each k'_{ij} as a function of k_{ij} elements. The solution will follow.

The left hand side of system (3.15) can be written as

$$\begin{bmatrix} |u_{11}|^2 k'_{11} & u^*_{11}k'_{11}u_{12} & u^*_{11}k'_{11}u_{13} & \cdots & u^*_{11}k'_{11}u_{1n} \\ u^*_{12}k'_{11}u_{11} & |u_{12}|^2k'_{11} + |u_{22}|^2k'_{22} & u^*_{12}k'_{11}u_{13} + u^*_{22}k'_{22}u_{23} & \cdots & \sum_{i=1}^{n} u^*_{i2}k'_{ii}u_{in} \\ u^*_{13}k'_{11}u_{11} & u^*_{13}k'_{11}u_{12} + u^*_{23}k'_{22}u_{22} & |u_{13}|^2k'_{11} + |u_{23}|^2k'_{22} + |u_{33}|^2k'_{33} & \cdots & \sum_{i=1}^{n} u^*_{i3}k'_{ii}u_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u^*_{1n}k'_{11}u_{11} & \sum_{i=1}^{n} u^*_{in}k'_{ii}u_{i2} & \sum_{i=1}^{n} u^*_{in}k'_{ii}u_{i3} & \cdots & \sum_{i=1}^{n} |u_{in}|^2k'_{ii} \end{bmatrix} \tag{3.16}$$

for all u_{ij} and u^*_{ij} with $i \le j$. In (3.16), we used $u^*_{ij}\, u_{ij} = |u_{ij}|^{\,2}$.

Remarks: In (3.16), in elements u_{ij}, the first sub-index i represents the row-location and the second sub index j represents the column-location. However, in elements u^*_{ij}, the first sub-index i represents the column-location and the second sub-index j represents the row-location.

Equation (3.15) is equivalent to the following system of equations:

$$|u_{11}|^2 k'_{11} = k_{11}$$

$$u^*_{11}u_{12}k'_{11} = k_{12}$$

$$u_{11}^{*}u_{13}k'_{11} = k_{13}$$

$$u_{12}^{*}u_{11}k'_{11} = k_{21}$$

$$|u_{12}|^2 k'_{11} + u_{22}^2 k'_{22} = k_{22}$$

$$\vdots$$

$$|u_{13}|^2 k'_{11} + |u_{23}|^2 k'_{22} + u_{33}^2 k'_{33} = k_{33}$$

$$\vdots$$

$$\sum_{i=1}^{n} |u_{in}|^2 k'_{ii} = k_{nn} \qquad \forall\, u_{ij} \text{ and } u_{ij}^{*} \text{ with } i \le j \tag{3.17}$$

Solution to the system of equation (3.17) is straightforward:

$$k'_{11} = \frac{k_{11}}{|u_{11}|^2}$$

$$k'_{22} = \frac{k_{11}k_{22} - k_{12}k_{21}}{|u_{22}|^2 k_{11}}$$

$$k'_{33} = \frac{k_{11}k_{33} - k_{12}k_{21}}{|u_{33}|^2 k_{11}} - \frac{(k_{11}k_{23} - k_{21}k_{13})(k_{11}k_{32} - k_{31}k_{12})}{|u_{33}|^2 k_{11}(k_{11}k_{22} - k_{12}k_{21})}$$

$$\vdots$$

$$k'_{nn} = k_{nn} - \sum_{i=1}^{n-1} |u_{in}|^2 k'_{ii} \qquad \forall\, u_{ij} \text{ with } i \le j \tag{3.18}$$

Now, (3.14) can be rewritten as

$$k_0 = \boldsymbol{I}^{*}\boldsymbol{K}\boldsymbol{I} = \boldsymbol{I}^{*}\boldsymbol{U}^{*}\boldsymbol{K}'\boldsymbol{U}\boldsymbol{I} = (\boldsymbol{U}\boldsymbol{I})^{*}\boldsymbol{K}'(\boldsymbol{U}\boldsymbol{I}) \tag{3.19}$$

implying that k_0 will be non-negative and equivalently condition **B** in the theorem holds iff k'_{ii} in (3.18) are all non-negative. The expressions on the right hand side of (3.18) are all divided by coefficients of the form $|u_{ii}|^2$. Those coefficients are clearly positive and, hence, conditions $k'_{ii} \geq 0$ become

1. $k_{ii} \geq 0 \qquad\qquad i = 1, 2, \ldots, n$
2. $\dfrac{k_{11}k_{22} - k_{12}k_{21}}{k_{11}} \geq 0$
3. $\dfrac{k_{11}k_{33} - k_{12}k_{21}}{k_{11}} - \dfrac{(k_{11}k_{23} - k_{21}k_{13})(k_{11}k_{32} - k_{31}k_{12})}{k_{11}(k_{11}k_{22} - k_{12}k_{21})} \geq 0$

$\vdots$

$\vdots$

$n.$ $k_{nn} - \sum_{i=1}^{n-1} |u_{in}|^2 k'_{ii} \geq 0 \qquad \forall\, u_{ij}$ with $i \leq j$ (3.20)

Therefore, it is established that condition 2 holds iff (3.3) holds.

By representing the $\boldsymbol{U}$ matrix in the reduced row-echelon form ($u_{ij} = 1,\ \forall\ i = j$) the calculations of conditions **C** are greatly simplified. System (3.20) shows the reduced row-echelon equivalent of system (3.15).

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ u^*_{12} & 1 & 0 & \cdots & 0 \\ u^*_{13} & u^*_{23} & 1 & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ u^*_{1n} & u^*_{2n} & u^*_{3n} & \cdots & 1 \end{bmatrix} \begin{bmatrix} k'_{11} & 0 & 0 & \cdots & 0 \\ 0 & k'_{22} & 0 & \cdots & 0 \\ 0 & 0 & k'_{33} & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & k'_{nn} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} & \cdots & u_{1n} \\ 0 & 1 & u_{23} & \cdots & u_{2n} \\ 0 & 0 & 1 & \ddots & u_{3n} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$= \begin{bmatrix} k_{11} & k_{12} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & k_{23} & \cdots & k_{2n} \\ k_{31} & k_{32} & k_{33} & \ddots & k_{3n} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ k_{n1} & k_{n2} & k_{n3} & \cdots & k_{nn} \end{bmatrix} \qquad (3.21)$$

where $u_{ij} = u^*_{ji} = 1,\ \forall\ i = j$.

Regarding condition **3**, the real part of $F(j\omega)$ can be obtained from

$$\Re e\{F(j\omega)\} = \frac{1}{2}[F(j\omega) + F^*(j\omega)] \qquad (3.22)$$

where $F(j\omega)$ is given as

$$\begin{aligned} F(j\omega) = {} & z_{11}(j\omega)I_1(j\omega)I_1^*(j\omega) + \cdots + z_{1n}(j\omega)I_n(j\omega)I_1^*(j\omega) \\ & + z_{21}(j\omega)I_1(j\omega)I_2^*(j\omega) + \cdots + z_{2n}(j\omega)I_n(j\omega)I_2^*(j\omega) \\ & + \cdots + \\ & \vdots \\ & + z_{n1}(j\omega)I_1(j\omega)I_n^*(j\omega) + \cdots + z_{nn}(j\omega)I_n(s)I_n^*(j\omega) \qquad (3.23) \end{aligned}$$

and $F^*(j\omega)$ is given as

$$\begin{aligned} F^*(j\omega) = {} & z_{11}^*(j\omega)I_1^*(j\omega)I_1(j\omega) + \cdots + z_{1n}^*(j\omega)I_n^*(j\omega)I_1(j\omega) \\ & + z_{21}^*(j\omega)I_1^*(j\omega)I_2(j\omega) + \cdots + z_{2n}^*(j\omega)I_n^*(j\omega)I_2(j\omega) \\ & + \cdots + \\ & \vdots \\ & + z_{n1}^*(j\omega)I_1^*(j\omega)I_n(j\omega) + \cdots + z_{nn}^*(j\omega)I_n^*(j\omega)I_n(j\omega) \qquad (3.24) \end{aligned}$$

Substituting (3.23) and (3.24) in (3.22) we have

$$\begin{aligned} \Re e\{F(j\omega)\} = {} & \frac{1}{2}[z_{11}(j\omega)I_1(j\omega)I_1^*(j\omega) + \cdots + z_{1n}(j\omega)I_n(j\omega)I_1^*(j\omega) \\ & + z_{21}(j\omega)I_1(j\omega)I_2^*(j\omega) + \cdots + z_{2n}(j\omega)I_n(j\omega)I_2^*(j\omega) \\ & + \cdots + \end{aligned}$$

$$\vdots$$

$$+ z_{n1}(j\omega)I_1(j\omega)I_n^*(j\omega) + \cdots + z_{nn}(j\omega)I_n(s)I_n^*(j\omega)$$

$$+ z_{11}^*(j\omega)I_1^*(j\omega)I_1(j\omega) + \cdots + z_{1n}^*(j\omega)I_n^*(j\omega)I_1(j\omega)$$

$$+ z_{21}^*(j\omega)I_1^*(j\omega)I_2(j\omega) + \cdots + z_{2n}^*(j\omega)I_n^*(j\omega)I_2(j\omega)$$

$$+ \cdots +$$

$$\vdots$$

$$+ z_{n1}^*(j\omega)I_1^*(j\omega)I_n(j\omega) + \cdots + z_{nn}^*(j\omega)I_n^*(j\omega)I_n(j\omega)] \tag{3.25}$$

By using $z'_{ij} = \frac{1}{2}\left[z_{ij} + z_{ji}^*\right]$, $\Re e\{F(j\omega)\}$ can be written as

$$\Re e\{F(j\omega)\} = z'_{11}(j\omega)I_1(j\omega)I_1^*(j\omega) + \cdots + z'_{1n}(j\omega)I_n(j\omega)I_1^*(j\omega)$$

$$+ z'_{21}(j\omega)I_1(j\omega)I_2^*(j\omega) + \cdots + z'_{2n}(j\omega)I_n(j\omega)I_2^*(j\omega)$$

$$+ \cdots +$$

$$\vdots$$

$$+ z'_{n1}(j\omega)I_1(j\omega)I_n^*(j\omega) + \cdots + z'_{nn}(j\omega)I_n(s)I_n^*(j\omega) \tag{3.26}$$

or equivalently as

$$\Re e\{F(j\omega)\} = \boldsymbol{I}^* \, \boldsymbol{Z}' \, \boldsymbol{I} \tag{3.27}$$

where

$$\boldsymbol{Z}' = \begin{bmatrix} z'_{11}(j\omega) & z'_{12}(j\omega) & \cdots & z'_{1n}(j\omega) \\ z'_{21}(j\omega) & z'_{22}(j\omega) & \cdots & z'_{2n}(j\omega) \\ \vdots & \vdots & \ddots & \vdots \\ z'_{n1}(j\omega) & z'_{n2}(j\omega) & \cdots & z'_{nn}(j\omega) \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} z_{11}(j\omega) + z_{11}^*(j\omega) & z_{12}(j\omega) + z_{21}^*(j\omega) & \cdots & z_{1n}(j\omega) + z_{n1}^*(j\omega) \\ z_{21}(j\omega) + z_{12}^*(j\omega) & z_{22}(j\omega) + z_{22}^*(j\omega) & \cdots & z_{2n}(j\omega) + z_{n2}^*(j\omega) \\ \vdots & \vdots & \ddots & \vdots \\ z_{n1}(j\omega) + z_{1n}^*(j\omega) & z_{n2}(j\omega) + z_{2n}^*(j\omega) & \cdots & z_{nn}(j\omega) + z_{nn}^*(j\omega) \end{bmatrix} \tag{3.28}$$

In general, the *z*-parameters have complex values, i.e., $z_{ij} = r_{ij} + jx_{ij}$ where r_{ij} is the real part and x_{ij} is the imaginary part of z_{ij} .

It is easy to see that (3.26) is a Hermitian form. Using a procedure similar to (3.14)-(3.19), which was for the residue matrix, the matrix $\boldsymbol{Z}'$ can be expressed as $\boldsymbol{Z}' = \boldsymbol{W}^* \, \boldsymbol{Z}'' \, \boldsymbol{W}$ where $\boldsymbol{Z}''$ is a diagonal matrix and $\boldsymbol{W}$ is an upper triangular matrix. The reduced row-echelon system is

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ w_{12}^* & 1 & 0 & \cdots & 0 \\ w_{13}^* & w_{23}^* & 1 & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ w_{1n}^* & w_{2n}^* & w_{3n}^* & \cdots & 1 \end{bmatrix} \begin{bmatrix} z_{11}'' & 0 & 0 & \cdots & 0 \\ 0 & z_{22}'' & 0 & \cdots & 0 \\ 0 & 0 & z_{33}'' & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & z_{nn}'' \end{bmatrix} \begin{bmatrix} 1 & w_{12} & w_{13} & \cdots & w_{1n} \\ 0 & 1 & w_{23} & \cdots & w_{2n} \\ 0 & 0 & 1 & \ddots & w_{3n} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$= \begin{bmatrix} z_{11}' & z_{12}' & z_{13}' & \cdots & z_{1n}' \\ z_{21}' & z_{22}' & z_{23}' & \cdots & z_{2n}' \\ z_{31}' & z_{32}' & z_{33}' & \ddots & z_{3n}' \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ z_{n1}' & z_{n2}' & z_{n3}' & \cdots & z_{nn}' \end{bmatrix} \tag{3.29}$$

The solution to (3.29) is

$$z_{11}'' = z_{11}'$$

$$z_{22}'' = \frac{z_{11}' z_{22}' - z_{12}' z_{21}'}{z_{11}'}$$

$$z''_{33} = \frac{z'_{11}z'_{33} - z'_{13}z'_{31}}{z'_{11}} - \frac{(z'_{11}z'_{23} - z'_{21}z'_{13})(z'_{11}z'_{32} - z'_{31}z'_{12})}{z'_{11}(z'_{11}z'_{22} - z'_{12}z'_{21})}$$

$\vdots$

$\vdots$

$$z''_{nn} = z'_{nn} - \sum_{i=1}^{n-1} |w_{in}|^2 z''_{ii} \qquad w_{ij} \text{ with } i \leq j \tag{3.30}$$

Now, (3.27) can be rewritten as

$$\Re e\{F(j\omega)\} = \boldsymbol{I}^* \, \boldsymbol{Z}' \, \boldsymbol{I} = \boldsymbol{I}^* \, \boldsymbol{W}^* \, \boldsymbol{Z}'' \, \boldsymbol{W}\boldsymbol{I} = (\boldsymbol{W}\boldsymbol{I})^* \boldsymbol{Z}'' (\boldsymbol{W}\boldsymbol{I}) \tag{3.31}$$

Therefore, $\Re e\{F(j\omega)\} \geq 0, \forall\, \omega$ (i.e., condition 3) holds iff the z''-parameters in (3.30) are non-negative (this also implies $z'_{22} \geq 0, z'_{33} \geq 0, \cdots, z'_{nn} \geq 0$). Therefore, condition 3 holds iff (3.4) holds.

In summary, conditions **A**, **B** and **C** are necessary and sufficient for (3.10) or equivalently (3.9), which defines the *n*-port network passivity. This concludes the proof. ■

Remarks: In Chapter 2, it was stated that for the case of LTI systems, positive real transfer functions represent passive systems. Hence, we could also present another proof of Theorem 1 by using the definition of positive real transfer functions as it is shown below.

Definition: Positive real transfer function [37]

Consider an n-port network with an $n \times n$ proper transfer function matrix $\boldsymbol{G}(s)$. $\boldsymbol{G}(s)$ is positive real if

- Poles of all elements of $\boldsymbol{G}(s)$ are in $\Re e\{s\} \leq 0$
- For all real ω for which $j\omega$ is not a pole of any element of $\boldsymbol{G}(s)$, the matrix $\boldsymbol{G}(j\omega) + \boldsymbol{G}^{\boldsymbol{T}}(-j\omega)$ is positive semidefinite.
- Any pure imaginary pole $j\omega$ of any element of $\boldsymbol{G}(s)$ is a simple pole and the residue matrix $\lim_{s\to j\omega}(s - j\omega)\boldsymbol{G}(s)$ is positive semidefinite Hermitian.

In the above conditions, $\boldsymbol{G}^{\boldsymbol{T}}(-j\omega)$ denotes the transpose complex conjugate of $\boldsymbol{G}(j\omega)$.

In our proof, we used $\boldsymbol{V} = \boldsymbol{ZI}$ which is a compact form of system (3.2). Clearly the transfer function $\boldsymbol{G}(s)$ is the $n \times n$ impedance matrix $\boldsymbol{Z}$**.** The second condition above, i.e., $\boldsymbol{G}(j\omega) + \boldsymbol{G}^{\boldsymbol{T}}(-j\omega)$ being positive semidefinite, is the same as (3.27) being positive semidefinite. The third condition above leads to the conditions for the residues k_{ij}.

3.4 Case Study: Passivity Conditions of a 2-port Network

In this section, the special case of passivity of 2-port networks is considered. We will proceed the same way as we did when we arrived at the passivity Theorem 1. The result will be compared with the well-known Raisbeck' Criterion. The network has been modeled using the impedance parameters as shown in (3.32).

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{3.32}$$

Assuming that the z-parameters have no RHP poles, we move into the analysis of the residues for which Equation (3.15) has to be solved for the case of $n = 2$. The system is represented below:

$$\begin{bmatrix} 1 & 0 \\ u_{12}^* & 1 \end{bmatrix} \begin{bmatrix} k'_{11} & 0 \\ 0 & k'_{22} \end{bmatrix} \begin{bmatrix} 1 & u_{12} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \tag{3.33}$$

It is easy to see that solving (3.33) results in

$$u_{12} = k_{12}/k_{11} \quad \text{and} \quad u_{12}^* = k_{21}/k_{11}$$

and

$$k'_{11} = k_{11} \geq 0$$
$$k'_{22} = \frac{k_{11}k_{22} - k_{12}k_{21}}{k_{11}} \geq 0 \tag{3.34}$$

Therefore, it is straightforward that condition **B** in Theorem 1 is same as condition 2 in Raisbeck's criterion. Also, in the following we show that solving (3.29) for *n* = 2 results in condition 3 in the Raisbeck's criterion.

Writing z_{ij} as $r_{ij} + jx_{ij}$ where r_{ij} is the real part and x_{ij} is the imaginary part of z_{ij}, we have

$$\begin{bmatrix} z'_{11}(j\omega) & z'_{12}(j\omega) \\ z'_{21}(j\omega) & z'_{22}(j\omega) \end{bmatrix} = \begin{bmatrix} r_{11} & \frac{1}{2}(r_{12} + r_{21}) + \frac{j}{2}(x_{12} - x_{21}) \\ \frac{1}{2}(r_{12} + r_{21}) - \frac{j}{2}(x_{12} - x_{21}) & r_{22} \end{bmatrix} \tag{3.35}$$

Using (3.29) for the case of $n = 2$ the following system is formed:

$$\begin{bmatrix} 1 & 0 \\ w_{12}^* & 1 \end{bmatrix} \begin{bmatrix} z''_{11} & 0 \\ 0 & z''_{22} \end{bmatrix} \begin{bmatrix} 1 & w_{12} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} z'_{11} & z'_{12} \\ z'_{21} & z'_{22} \end{bmatrix} \tag{3.36}$$

Solving (3.36) results in:

$$w_{12} = z'_{12}/z'_{11} \qquad \text{and} \quad w_{12}^* = z'_{21}/z'_{11}.$$

and

$$z''_{11} = z'_{11} \geq 0$$
$$z''_{22} = \frac{z'_{11}z'_{22} - z'_{12}z'_{21}}{z'_{11}} \geq 0 \qquad (3.37)$$

By using $z'_{ij} = \frac{1}{2}\left(z_{ij} + z^*_{ji}\right)$, the second condition in (3.37) reduces to:

$$4r_{11}r_{22} - (r_{12} + r_{21})^2 - (x_{12} - x_{21})^2 \geq 0 \qquad (3.38)$$

with $r_{ij=} \Re e(z_{ij})$ and $x_{ij} = \Im m(z_{ij})$ with $i, j = 1, 2$

Inequality (3.37) is the same as the last of condition 3 in Raisbeck's criterion with $r_{ij=} \Re e(p_{ij})$ and $x_{ij} = \Im m(p_{ij})$.

We conclude that for 2-ports, by using similar procedure as the one used for finding conditions of passivity of n-port networks, the final result is the same as Raisbeck's criterion. In the future, one does not have to go through all these calculations; on the contrary, we have presented Theorem 1 which allows for direct investigation of passivity of *n*-port networks where *n* can be any positive integer number equal or larger than 2.

Chapter 4

Absolute Stability of Trilateral Teleoperators

4.1 Introduction

A 3-port network can be defined as a network containing 3 pairs of terminals for external connections. Each pair of terminals represents a port to which an external network can be connected (Figure 4.1). 3-port networks can be used to model trilateral haptic system used in applications such as dual-user haptic teleoperation (two masters and one slave robots) and triple-user collaborative haptic virtual environments in which three users perform a task together. As it has been previously stated, for closed-loop stability analysis of such systems, the model of the human operator and environment is required in addition to the model of the teleoperation system *immitance* parameters *(z, y, h,* or *g)*. In practice, the human operator and environment models are usually unknown, uncertain, and/or time-varying. Hence, the discussion of stability of 3-port networks is done under the assumption that the human operator and environment terminations are passive but otherwise arbitrary.

This chapter discusses a method for direct investigation of absolute stability of 3-port networks. The derivation of the method is based on the fundamental definition of the absolute stability of 3-port networks from which an analytical stability condition is derived. Numerical evaluation of this condition for a given network can be used for evaluation of its stability.

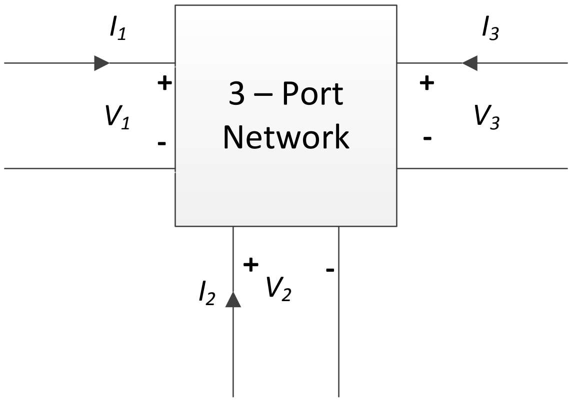

Figure 4.1. A 3-port network.

4.2 Absolute Stability Condition for 3-port Networks

In this section, a step-by-step method for analysis of absolute stability of a 3-port network is introduced. Any of the four immitance parameters *(z, y, h,* or *g)* can be used and, therefore, without loss of generality, the z-parameters have been chosen. Using the impedance parameters, the 3-port network can be modeled as

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & z_{13} \\ z_{21} & z_{22} & z_{23} \\ z_{31} & z_{32} & z_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \tag{4.1}$$

As it was explained in Chapter 2 (although for the case of 2-port network), a 2-port network is absolutely stable if the input impedance Z_{in}, when one of the ports is connected to a passive termination, is a positive real function. Hence, we will look for conditions of absolute stability of a 3-port network by investigating the positive realness of its input impedance when two of the ports are connected to passive terminations.

By leaving port 1 open (arbitrary choice) and terminating ports 2 and 3 to passive terminations z_2 and z_3 respectively, the input impedance looking into port 1 is the ratio between the voltage across that port (V_1) and the current flowing through that port (I_1):

$$Z_{in} = \frac{V_1}{I_1} \tag{4.2}$$

From (4.1) we have

$$V_1 = z_{11}I_1 + z_{12}I_2 + z_{13}I_3 \tag{4.3}$$

and

$$I_2 = \frac{-V_2}{z_2} \quad \text{and} \quad I_3 = \frac{-V_3}{z_3} \tag{4.4}$$

Substituting (4.4) in (4.3) and after some manipulations, an expression for Z_{in} can be found as shown below:

$$\begin{aligned} Z_{in} = z_{11} - \frac{z_{12}}{z_2}\frac{1}{E}\left\{\left(1 + \frac{z_{33}}{z_3}\right)z_{21} - \frac{z_{23}}{z_3}z_{31}\right\} \\ - \frac{z_{13}}{z_3}\frac{1}{E}\left\{\left(1 + \frac{z_{22}}{z_2}\right)z_{31} - \frac{z_{32}}{z_2}z_{21}\right\} \end{aligned} \tag{4.5}$$

where E is a function of the network impedance and the termination impedances z_2 and z_3:

$$E = 1 + \frac{z_{33}}{z_3} + \frac{z_{22}}{z_2} + \frac{z_{22}z_{33} - z_{23}z_{32}}{z_2 z_3} \tag{4.6}$$

Equation (4.5) can be written as

$$Z_{in} = \frac{\alpha z_3 + \beta z_2 + \theta}{z_2 z_3 + \alpha' z_2 + \beta' z_3 + \gamma} + z_{11} \tag{4.7}$$

with $\alpha, \beta, \gamma, \theta, \alpha',$ and β' being dependent only on the 3-port network impedances z_{ij}:

$\alpha = -z_{12} z_{21}$

$\beta = -z_{13} z_{31}$

$\gamma = z_{22} z_{33} - z_{23} z_{32}$

$\theta = z_{12} z_{31} z_{23} + z_{21} z_{13} z_{32} - z_{12} z_{21} z_{33} - z_{13} z_{31} z_{22}$

$\alpha' = z_{33}$

$\beta' = z_{22}$

Equation (4.7) indicates that Z_{in} is a bilinear transformation of z_3 (or z_2). This fact can become clearer if we divide the numerator and denominator of (4.7) by $z_2 + \beta'$ in order to have the standard expression of a bilinear transformation. Hence, (4.7) becomes

$$Z_{in} = \frac{\left(\frac{\alpha}{z_2 + \beta'}\right) z_3 + \left(\frac{\beta z_2 + \theta}{z_2 + \beta'}\right)}{z_3 + \left(\frac{\alpha' z_2 + \gamma}{z_2 + \beta'}\right)} + z_{11} \tag{4.8}$$

By choosing $A = \left(\frac{\alpha}{z_2+\beta'}\right)$, $B = \left(\frac{\beta z_2+\theta}{z_2+\beta'}\right)$, and $D = \left(\frac{\alpha' z_2+\gamma}{z_2+\beta'}\right)$,

Equation (4.8) becomes

$$Z_{in} = \frac{Az_3 + B}{z_3 + D} + z_{11} \tag{4.9}$$

Notice that in (4.9), the coefficients $A, B,$ and D are all function of both the 3-port network impedance parameters z_{ij} and the termination impedance z_2.

By extending the definition of absolute stability in Chapter 2 to the case of 3-port networks, a 3-port network is absolutely stable if Z_{in} is a positive real function for all passive terminations z_2 and z_3. Then, the stability of the 3-port network is equivalent to $\Re e\{Z_{in}\} \geq 0$ for all passive terminations z_2 and z_3.

The input impedance Z_{in} is clearly a complex quantity and as such it can be graphed in the complex plane. Recall that Z_{in} as given in (4.9) is a bilinear transformation of the termination impedance z_3. Also notice that Z_{in} is dependent on z_2 and this will be considered later in our analysis.

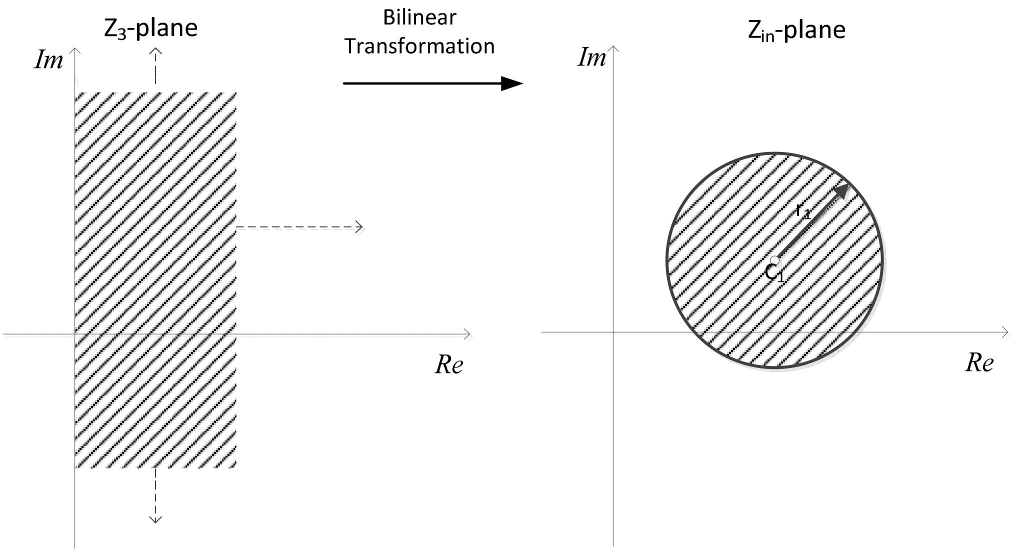

Figure 4.2. The input impedance Z_{in} as a bilinear transformation of z_3.

Hence, for all passive terminations z_3 (i.e., for all impedances z_3 in the right half of the complex plane), Z_{in} is transformed into a circle – see Figure 4.2. The first term in the right hand side of (4.9) represents this circle while the second term (the impedance parameter z_{11}) represents a shift of the circle. Figure 4.2 shows the circle without the shift caused by z_{11}.

With a circle with centre C_1 and radius r_1 being the result of the bilinear transformation $\frac{Az_3+B}{z_3+D}$ for all passive terminations z_3, it can be shown that

$$C_1 = A - \frac{AD - B}{2\Re e\{D\}} \tag{4.10}$$

and

$$r_1 = \frac{|AD - B|}{2|\Re e\{D\}|} \tag{4.11}$$

For the nonreciprocal 3-port network defined in (4.1) to be absolutely stable, condition $\Re e\{Z_{in}\} \geq 0$ must be satisfied, which means that we need to have

$$\Re e(C_1) - r_1 + \Re e\{z_{11}\} \geq 0 \tag{4.12}$$

Equation (4.12) shows that the stability condition $\Re e\{Z_{in}\} \geq 0$, which represents the entire Right Half Plane (RHP) in the impedance complex plane, simply requires the disk with centre C_1 and radius r_1 to be entirely in the RHP. According to Equation (4.12), for the 3-port network to be absolutely stable the sum of the centre C_1 plus z_{11} both projected onto the real axes minus the radius of the disk has to result in a positive real number, which in turns means that every point of the shifted disk has to be in the RHP. The term $\Re e\{z_{11}\}$ in (4.12) can be any real number (negative, zero, or positive). A graphical interpretation of Equation (4.12) is shown in Fig 4.3, assuming $\Re e\{z_{11}\}$ positive.

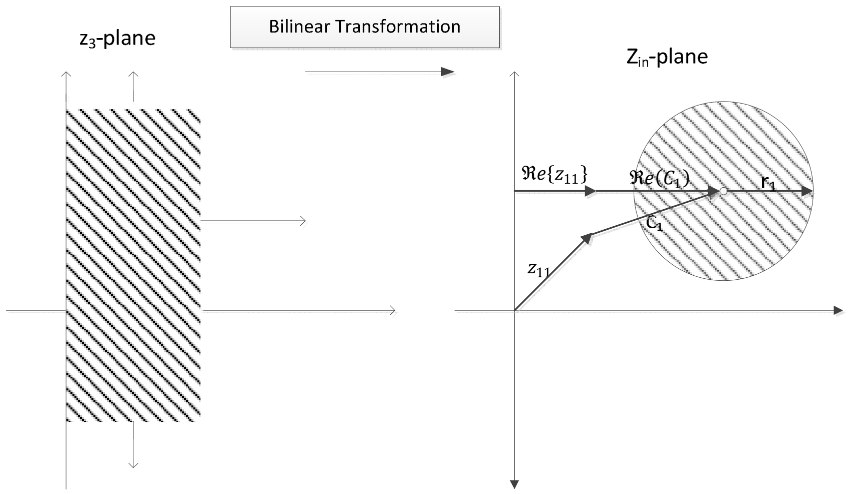

Figure 4.3. The input impedance Z_{in} as a bilinear transformation of z_3 and shifted by the impedance parameter z_{11}.

Substituting (4.10) and (4.11) into condition (4.12) yields

$$\Re e\left\{A - \frac{AD - B}{2\Re e\{D\}}\right\} - \frac{|AD - B|}{2|\Re e\{D\}|} + \Re e\{z_{11}\} \geq 0 \tag{4.13}$$

Equation (4.13) is the final condition for absolute stability of a 3-port network. This condition can be used for analysis of stability of trilateral haptic systems, in which case the numerical evaluation of either Equation (4.13) or Equation (4.9) will tell if the system is stable or not for a given controller.

4.3 Procedure for Numerical Evaluation of the Stability Condition for Trilateral Systems

Equation (4.9) shows Z_{in} as a bilinear transformation in the termination impedance z_3. The coefficients $A = \left(\frac{\alpha}{z_2+\beta'}\right)$, $B = \left(\frac{\beta z_2+\theta}{z_2+\beta'}\right)$, and $D = \left(\frac{\alpha' z_2+\gamma}{z_2+\beta'}\right)$ are also bilinear transformations but in the termination impedance z_2. Consequently, (4.9) represents three bilinear transformations of z_2, nested inside a bilinear transformation of z_3. The following pseudo-code is a procedure that can be used for numerical evaluation of the proposed absolute stability condition of a 3-port network. The nested loops of the pseudo-code come as a direct consequence of the nested nature of Equation (4.9).

Pseudo-Code

for $\omega = 0 \rightarrow \infty$ {

compute $\{\alpha, \beta, \gamma, \theta, \alpha', \beta'\}$ *(complex numbers)*

for all passive z_2 *find circles* $\{A, B, D\}$ *(in terms of radii and centres)*

for each point in circle A {

for each point in circle B {

for each point in circle D {

If $\Re e\left\{A - \frac{AD-B}{2\Re e\{D\}}\right\} - \frac{|AD-B|}{2|\Re e\{D\}|} + \Re e\{z_{11}\} < 0$,

then network is not absolutely stable.

}

}

}

}

The above procedure was tested for two different cases as discussed below.

<u>Case 1: An absolutely stable trilateral network</u>

Youla proved that, for reciprocal n-port networks, strict passivity and absolute stability are the same [28]. In terms of *z*-parameters, a 3-port network is reciprocal if $z_{ij} = z_{ji}$ for i and $j = 1, 2, 3$. Hence, if Z represents the impedance matrix of a reciprocal trilateral network, then the network is absolutely stable if and only if it is strictly passive.

Consider the matrix Z shown in (4.14), which represents the impedance matrix of a reciprocal 3-port network.

$$\boldsymbol{Z} = \begin{bmatrix} 3 & 1+s & 3-2s \\ 1+s & 1 & 2 \\ 3-2s & 2 & 8 \end{bmatrix} \tag{4.14}$$

where $s = j\omega$. Note that $Z(j\omega) + Z^T(-j\omega)$ is positive definite because all the principal minors of $Z(j\omega) + Z^T(-j\omega)$ are positive real numbers (6, 8, and 56).

$$\boldsymbol{Z}(j\omega) + \boldsymbol{Z}^{\boldsymbol{T}}(-j\omega) = \begin{bmatrix} 6 & 2 & 6 \\ 2 & 2 & 4 \\ 6 & 4 & 16 \end{bmatrix} \tag{4.15}$$

The matrix $\boldsymbol{Z}$ is strictly passive and, due to its symmetry, absolutely stable. Now, to test the absolute stability of the 3-port network via Equation (4.12), a MATLAB code can be implemented to plot Z_{in} at different frequencies according to the previously-described pseudo code. We compute Z_{in} at three different frequencies $= \{0.1\frac{rad}{s}, 1\frac{rad}{s}, 10\frac{rad}{s}\}$ and for a rather large number of points inside each of the *A*, *B*, and *D* circles. In Figure 4.4, Figure 4.5, and Figure 4.6, plots of Z_{in} at the three different frequencies show that the real parts of Z_{in} are always in the right half plane.

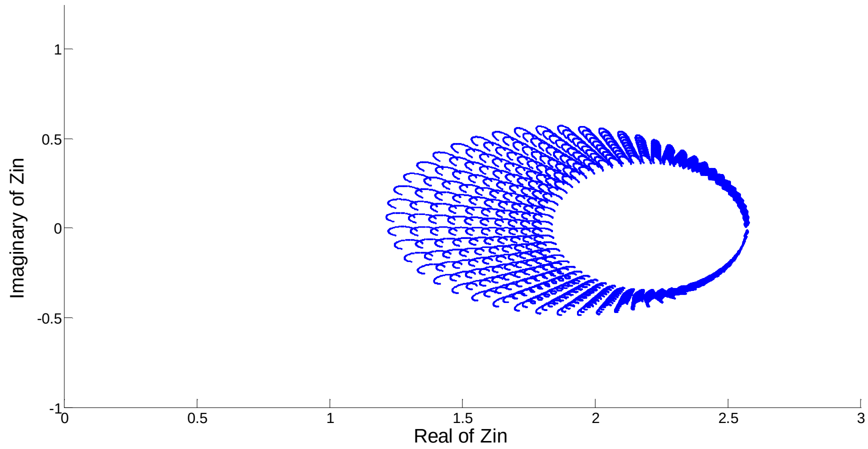

Figure 4.4. Plot of Z_{in} of an absolutely stable 3-port network at $\omega = 0.1 \frac{rad}{s}$.

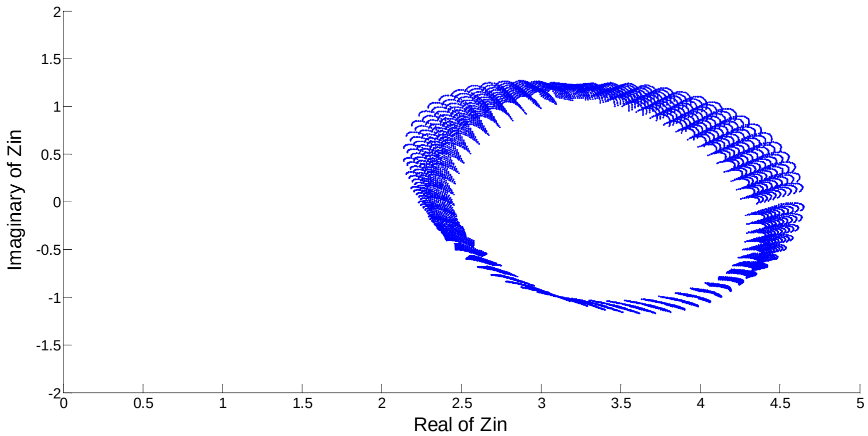

Figure 4.5. Plot of Z_{in} of an absolutely stable 3-port network at $\omega = 1 \frac{rad}{s}$.

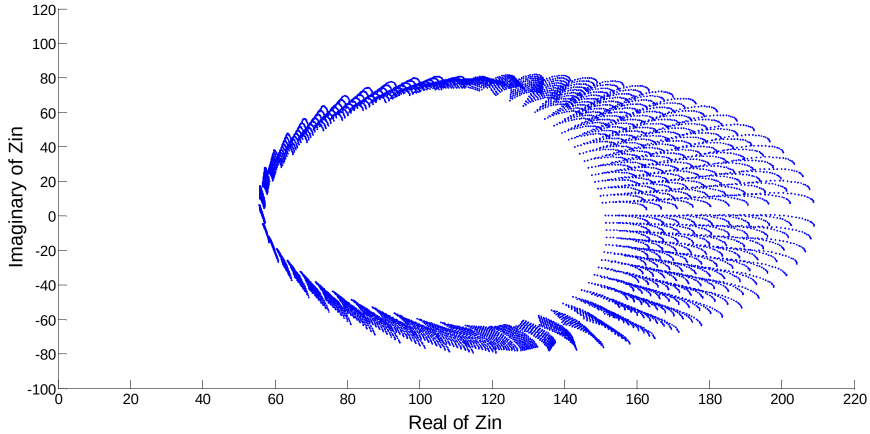

Figure 4.6. Plot of Z_{in} of an absolutely stable 3-port network at $\omega = 10\,\frac{rad}{s}$

Case 2: A potentially unstable trilateral network

The impedance matrix of a potentially unstable reciprocal trilateral network is shown below:

$$\boldsymbol{Z} = \begin{bmatrix} 0.3 & 1+s & 3-2s \\ 1+s & 1 & 2 \\ 3-2s & 2 & 8 \end{bmatrix} \tag{4.16}$$

where $s = j\omega$.

We note that

$$\boldsymbol{Z}(j\omega) + \boldsymbol{Z}^{\boldsymbol{T}}(-j\omega) = \begin{bmatrix} 0.6 & 2 & 6 \\ 2 & 2 & 4 \\ 6 & 4 & 16 \end{bmatrix} \tag{4.17}$$

has principal minors of 0.6, -2.8, and -30.4, making the matrix $\boldsymbol{Z}$ not positive real and, therefore, not passive. Due to the symmetry of $\boldsymbol{Z}$, it is therefore potentially unstable. On the other hand, we compute $\Re e\{Z_{in}\}$ at two different frequencies

$\omega = \{0.1\frac{rad}{s}, 1\frac{rad}{s}\}$. Plots of Z_{in} at $\omega = 0.1\frac{rad}{s}$ show that all real parts of Z_{in} are negatives. For $\omega = 1\frac{rad}{s}$, the real parts of Z_{in} are located in both the left and the right half plane.

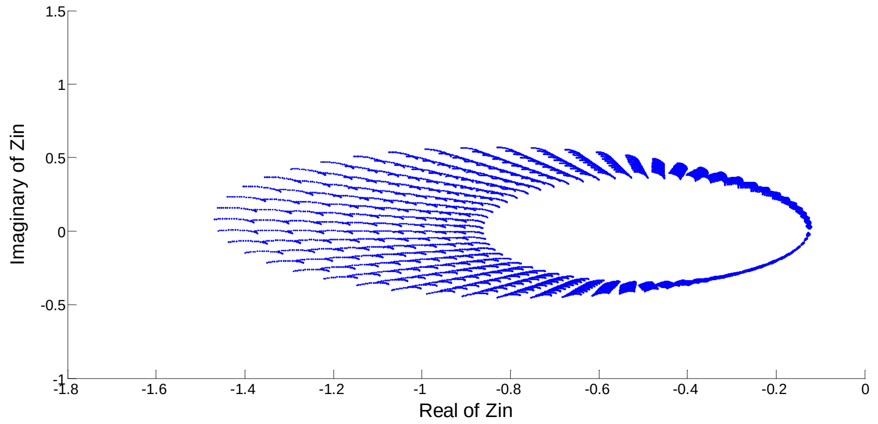

Figure 4.7. Plot of Z_{in} of a potentially unstable 3-port network at $\omega = 0.1\frac{rad}{s}$.

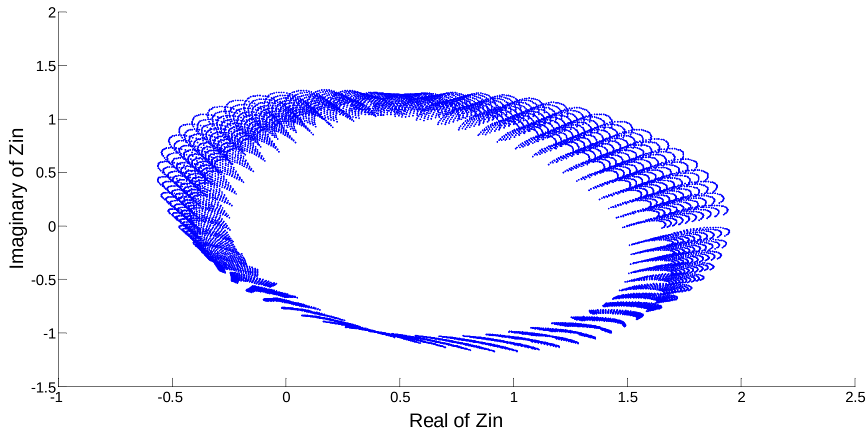

Figure 4.8. Plot of Z_{in} of a potentially unstable 3-port network at $\omega = 1\frac{rad}{s}$.

Previously, it was mentioned that in the case of a reciprocal network, absolute stability and strict passivity are equivalent. This fact allowed us to validate Equation (4.13) as a condition for absolute stability of trilateral networks. It is now possible to use the proposed condition for analysis of general non-reciprocal trilateral networks.

Chapter 5

Application of Passivity and Absolute Stability Criteria to a Dual-User Haptic Teleoperation System

5.1 Introduction

In this chapter, the Passivity Criterion proposed in Chapter 3 and the Absolute Stability Condition proposed in Chapter 4 will be used in order to find passivity conditions and evaluate the stability of a trilateral haptic teleoperation system. For both cases, the 3-port network is represented by its impedance matrix. This 3-port network is a dual-user haptic teleoperation system, in which two master robots for two operators share the control of one slave robot to perform a task in a remote environment. This configuration has many real-world applications such as training a trainee to do a task under haptic guidance from a mentor. In Section 5.2, the impedance matrix of the dual-user haptic teleoperation system is found by using the so-called four-channel multilateral shared control architecture proposed in [14]. Section 5.3 is devoted to finding passivity conditions of such a trilateral haptic system. Lastly, Section 5.4 is concerned with simulations of both passivity and absolute stability of the dual-user haptic teleoperation system.

5.2 A Dual-User Shared Haptic Control Teleoperation System

In a dual-user haptic teleoperation system, the goal is that two users coupled to two master robots (one user per one master robot) collaboratively control a slave robot to perform a task in a remote environment. As shown in [14], the desired position and force for each robot are weighted sums of positions and forces of the other two robots, with the weights being determined by a parameter α whose value ranges from 0 to 1 – see Figure 5.1. For instance, if $\alpha = 1$, the slave robot will be fully controlled by User 1 and User 2 only receives large force feedback urging him/her to follow User 1's motions. On the other hand, the same parameter α can be given a value of 0, in which case the slave robot is fully controlled by User 2, allowing User 1 to assess the skill level of User 2 by feeling the reflected forces. Lastly, if $0 < \alpha < 1$, then the two users collaborate and each contributes to the position command while receiving some force feedback. This provides "hand-over-hand" training using haptic assistance.

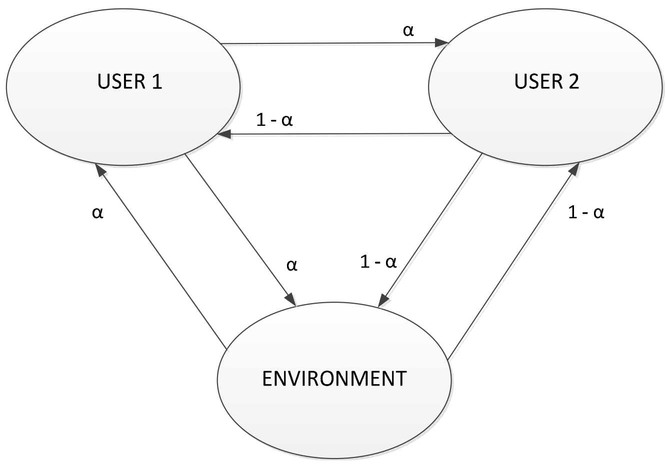

Figure 5.1. A dual-user haptic teleoperation system.

Consider the four-channel multilateral shared control architecture given in [14] and depicted in Figure 5.2. Under the assumption that each user is interfaced with his/her master robot and the slave is in contact with the environment, the dynamics of the two masters and slave can be model in frequency domain as

$$Z_{m1}V_{h1} = F_{h1} + F_{cm1}$$

$$Z_{m2}V_{h2} = F_{h2} + F_{cm2}$$

$$Z_sV_e = F_e + F_{cs} \tag{5.1}$$

In (5.1), $Z_{m1} = M_{m1}s$, $Z_{m2} = M_{m2}s$ and $Z_s = M_s s$ are the models of the two masters and the single slave, respectively. Also, F_{h1}, F_{h2} and F_e are the contact forces between each master and its human operator, and between the slave and its environment, respectively. Lastly, V_{h1}, V_{h2}, and V_e are the velocities of the two users and the environment respectively. In Fig 5.2, F_{h1}^*, F_{h2}^*, and F_e^* are the two operator’s and environment’s exogenous input forces, which are independent of the teleoperation system behavior [1].

The controller outputs in the 4-channel architecture are

$$F_{cm1} = -C_{m1}V_{h1} - C_{4m1}V_{h1d} + C_{6m1}F_{h1} - C_{2m1}F_{h1d}$$

$$F_{cm2} = -C_{m2}V_{h2} - C_{4m2}V_{h2d} + C_{6m2}F_{h2} - C_{2m2}F_{h2d}$$

$$F_{cs} = -C_sV_e + C_1V_{ed} + C_5F_e + C_3F_{ed} \tag{5.2}$$

for $i = 1, 2$. C_{mi} and C_s are local position controllers, and C_{6mi} and C_5 are local force controllers for the two masters and the slave, respectively. Also, the controllers C_1, C_{4i} are position compensators similar to C_s and C_{mi}, respectively. C_{2mi} and C_3 are feedforward force terms for the two masters and the slave, respectively. Lastly, V_{hid} and V_{ed} are the desired positions, and F_{hid} and F_{ed} are the desired forces for the two masters and the slave, respectively.

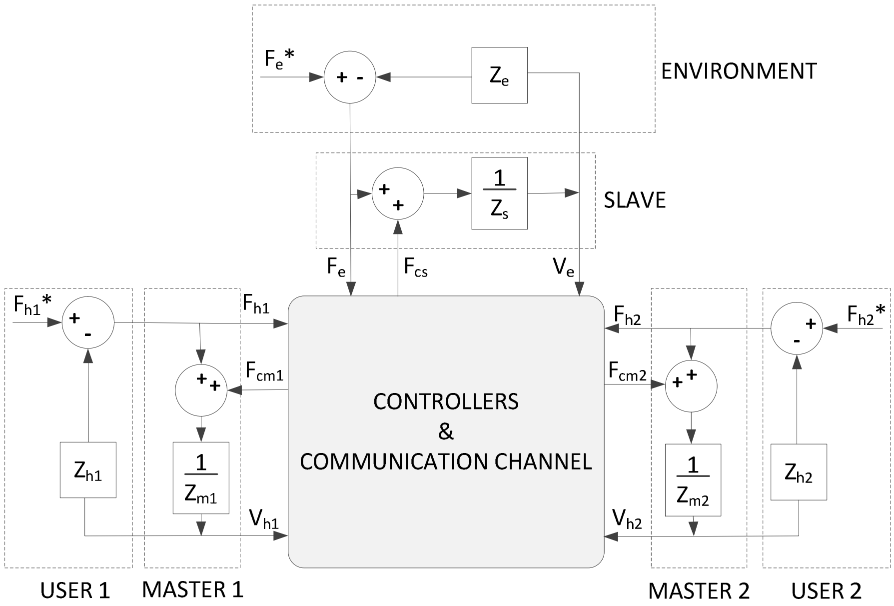

Figure 5.2. A dual-user haptic teleoperation system under four-channel control.

As mentioned before, in this 3-robot shared control architecture, the desired velocity and force of each robot is a function of the velocities and forces of the other two robots, as the following set of equations state:

$$
\begin{aligned}
V_{h1d} &= \alpha V_e + (1-\alpha)\, V_{h2} \\
V_{h2d} &= (1-\alpha)V_e + \alpha\, V_{h1} \\
V_{ed} &= \alpha V_{h1} + (1-\alpha)\, V_{h2} \\
F_{h1d} &= \alpha F_e + (1-\alpha)F_{h2} \\
F_{h2d} &= (1-\alpha)F_e + \alpha\, F_{h1} \\
F_{ed} &= \alpha F_{h1} + (1-\alpha)\, F_{h2}
\end{aligned}
\tag{5.3}
$$

where $\alpha \epsilon [0, 1]$ is the weight parameter specifying the relative authority that each operator has over the slave and the corresponding share of force feedback he/she receives.

Position-error based (PEB) control is a special case of dual-user shared control architecture, which does not need any force sensor measurements. The PEB controller works by minimizing the difference between the weighted master and slave positions, thus reflecting a force related to this difference to each user once the slave makes contact with an object. In the PEB control architecture the following choices are made: $C_3 = C_5 = C_{2m1} = C_{2m2} = C_{6m1} = C_{6m2} = 0$. Also, for good position tracking the common choice is $C_1 = C_s$, $C_{4m1} = -C_{m1}$ and $C_{4m2} = -C_{m2}$. Here, we have

$$C_{m1} = \frac{K_{pm1} + K_{vm1}s}{s}$$
$$C_{m2} = \frac{K_{pm2} + K_{vm2}s}{s}$$
$$C_s = \frac{K_{ps} + K_{vs}s}{s} \tag{5.4}$$

By using (5.1), (5.2), (5.3), and (5.4), the impedance matrix of the closed-loop multilateral system in

$$\begin{bmatrix} F_{h1} \\ F_{h2} \\ F_e \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & z_{13} \\ z_{21} & z_{22} & z_{23} \\ z_{31} & z_{32} & z_{33} \end{bmatrix} \begin{bmatrix} V_{h1} \\ V_{h1} \\ V_e \end{bmatrix} \tag{5.5}$$

is found as

$z_{11} = \left(M_{m1}s^2 + K_{vm1}s + K_{pm1}\right)/s$

$z_{12} = \left(-K_{vm1}s + K_{pm1}\alpha + K_{vm1}s\alpha - K_{pm1}\right)/s$

$z_{13} = \left(-K_{vm1}s\alpha - K_{pm1}\alpha\right)/s$

$$z_{21} = \left(-K_{vm2}s\alpha - K_{pm2}\alpha\right)/s$$

$$z_{22} = \left(M_{m2}s^2 + K_{vm2}s + K_{pm2}\right)/s$$

$$z_{23} = \left(-K_{vm2}s + K_{pm2}\alpha + K_{vm2}s\alpha - K_{pm2}\right)/s$$

$$z_{31} = -\left(K_{vs}s\alpha + K_{ps}\alpha\right)/s$$

$$z_{32} = \left(-K_{vs}s + K_{ps}\alpha + K_{vs}s\alpha - K_{ps}\right)/s$$

$$z_{33} = \left(M_s s^2 + K_{vs}s + K_{ps}\right)/s \tag{5.6}$$

5.3 Applying Passivity Criterion to the Dual-User Shared Haptic Control Teleoperation System

The passivity criterion of *n*-port networks formulated in Chapter 3 reduces to the following conditions for the case of a 3-port network:

A. The *z*-parameters have no RHP poles.

B. Any poles of the *z*-parameters on the imaginary axis are simple, and the residues k_{ij} of the *z*-parameters at these poles satisfy the following conditions:

1. $k_{ii} \geq 0 \qquad\qquad i = 1, 2, 3$
2. $\dfrac{k_{11}k_{22} - k_{12}k_{21}}{k_{11}} \geq 0$
3. $\dfrac{k_{11}k_{33} - k_{12}k_{21}}{k_{11}} - \dfrac{(k_{11}k_{23} - k_{21}k_{13})(k_{11}k_{32} - k_{31}k_{12})}{k_{11}(k_{11}k_{22} - k_{12}k_{21})} \geq 0 \qquad (5.7)$

C. The complex z'-parameters satisfy the following conditions for all real frequencies ω

1. $z'_{ii} \geq 0 \qquad\qquad\qquad i = 1, 2, 3$

2. $\dfrac{z'_{11}z'_{22} - z'_{12}z'_{21}}{z'_{11}} \geq 0$

3. $\dfrac{z'_{11}z'_{33} - z'_{13}z'_{31}}{z'_{11}} - \dfrac{(z'_{11}z'_{23} - z'_{21}z'_{13})(z'_{11}z'_{32} - z'_{31}z'_{12})}{z'_{11}(z'_{11}z'_{22} - z'_{12}z'_{21})} \geq 0 \qquad (5.8)$

where $z'_{ij} = \frac{1}{2}\left(z_{ij} + z^{*}_{ji}\right)$.

Analysis of (5.6) shows that all the elements of the 3-port network impedance matrix have only a simple pole on the imaginary axis, thus fulfilling condition **A**. Analysis of the residues (condition **B**) leads to the following conditions:

$$k_{11} = K_{pm1} \geq 0 \tag{5.9}$$

$$k_{22} = K_{pm2} \geq 0 \tag{5.10}$$

$$k_{33} = K_{ps} \geq 0 \tag{5.11}$$

$$\frac{k_{11}k_{22} - k_{12}k_{21}}{k_{11}} = (1 - \alpha + \alpha^2)K_{pm1}K_{pm2} \geq 0 \tag{5.12}$$

$$\frac{k_{11}k_{33} - k_{12}k_{21}}{k_{11}} - \frac{(k_{11}k_{23} - k_{21}k_{13})(k_{11}k_{32} - k_{31}k_{12})}{k_{11}(k_{11}k_{22} - k_{12}k_{21})} = 0 \tag{5.13}$$

The inequality (5.12) always holds as $(1 - \alpha + \alpha^2) > 0$ for all $\alpha \in [0, 1]$.

Analysis of the impedance matrix according to Condition **C** leads to the following conditions on the controllers' gains:

$$K_{vm1} \geq 0 \tag{5.14}$$

$$K_{vm2} \geq 0 \tag{5.15}$$

$$K_{vs} \geq 0 \tag{5.16}$$

$$4K_{vm1}K_{vm2} - (K_{vm1} - \alpha K_{vm1} + \alpha K_{vm2})^2$$
$$- \frac{\left(K_{pm1} - \alpha K_{pm1} + \alpha K_{pm2}\right)^2}{\omega^2} \geq 0 \tag{5.17}$$

Condition (5.17) will be fulfilled for all real frequencies ω if the gains of the PD controllers satisfy:

$$\frac{K_{pm1}}{K_{pm2}} = \frac{\alpha}{1-\alpha} \tag{5.18}$$

$$4K_{vm1}K_{vm2} - (K_{vm1} - \alpha K_{vm1} + \alpha K_{vm2})^2 \geq 0 \tag{5.19}$$

Using (5.18) in the last condition of **C** (condition 3 of (5.8)), we get the following inequality:

$$\frac{-1}{\omega^2}\left\{\frac{\left(K_{pm1} - K_{ps}\right)^2[K_{vm1}(1-\alpha)^2(2-\alpha) + K_{vm2}\alpha^2(1+\alpha)]}{2\alpha} + \frac{(1-2\alpha)^2K_{vm1}}{\alpha^2}\right.$$
$$\left.+ \frac{\left({K_{pm1}}^2 - {K_{ps}}^2\right)(1-2\alpha)(1-\alpha)[\alpha^2K_{vm2} + (\alpha+2)K_{vm1}]}{2\alpha}\right\}$$
$$+\{(1+\alpha)(2-\alpha)K_{vm1}K_{vm2}K_{vs} - \alpha^2(2-\alpha)K_{vm2}K_{vs}(K_{vm2} + K_{vs})$$
$$-(1-\alpha+\alpha^2)K_{vm1}K_{vm2}[(1-\alpha)K_{vm1} + \alpha K_{vm2}]$$
$$-(1-\alpha)^2(1+\alpha)K_{vm1}K_{vs}(K_{vm1} + K_{vs})\} \geq 0 \tag{5.20}$$

Equation (5.20) will be fulfilled for all real frequencies ω if the controller’s gains and parameter α satisfy the following conditions:

$$K_{pm1} = K_{pm2} = K_{ps}$$

$$K_{vm1} = K_{vm2} = K_{vs}$$
$$\alpha = {}^{1}\!/_{2} \tag{5.21}$$

As a conclusion, the dual-user haptic teleoperation system is passive if the set of equations (5.21) holds. Notice that (5.21) is a sufficient, frequency-independent, and compact condition for passivity of the PEB dual-user haptic teleoperation system described in Section 5.2.

5.4 Simulation Study: The Dual-User Shared Haptic Control Teleoperation System

5.4.1 Simulation study: Passivity conditions

In this section, the passivity conditions for the PEB dual-user haptic teleoperation system found in Section 5.3 will be verified via MATLAB/Simulink simulations. The simulation is done assuming no time delay in the communication channels between the three robots.

According to Equation (3.1) and assuming that the energy stored in the system for $t < 0$ is zero, the 3-port network is passive if and only if

$$E(t) = \int_0^t (i_1(\tau)v_1(\tau) + i_2(\tau)v_2(\tau) + i_3(\tau)v_3(\tau))\, d\tau \geq 0 \tag{5.22}$$

where $E(t)$ is the total energy delivered to the 3-port network. Such a *passivity observer* is incorporated in the simulations in order to evaluate (5.22).

In the simulations, all ports of the 3-port network are connected to the passive terminations with a transfer function $\frac{1}{1+s}$. An input F_{h1}^{*} in the form of a sine wave is applied by the Master 1's operator. The three robots are modeled by masses

$M_{m1} = 0.7$, $M_{m2} = 0.9$, and $M_s = 0.5$.

According to the previous section, the dual-user haptic teleoperation system is passive if the set of equations (5.21) holds. Table 5.1 shows two sets of controllers' gains used for these simulations, one set is in agreement with conditions given in (5.21), thus representing a passive trilateral system. The other set violates (5.21), representing a non-passive system. For all simulations, $= {}^{1}/_{2}$.

Table 5.1 Controllers' gains for (A) passive and (B) non-passive PEB trilateral system.

System	Master 1's controller	Master 2's controller	Slave's controller
(A) Passive	$K_{pm1} = 5$ $K_{vm1} = 10$	$K_{pm2} = 5$ $K_{vm2} = 10$	$K_{ps} = 5$ $K_{vs} = 10$
(B) Non-Passive	$K_{pm1} = 5$ $K_{vm1} = 10$	$K_{pm2} = 100$ $K_{vm2} = 10$	$K_{ps} = 5$ $K_{vs} = 10$

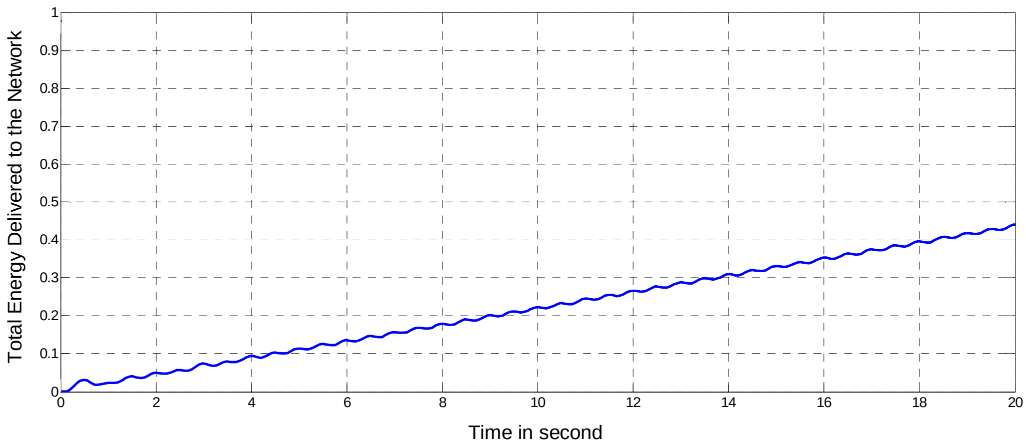

Figure 5.3. Passivity observer: Total energy delivered to a passive trilateral system.

Figure 5.3 shows that choosing the controllers' gains according to the conditions found in previous section results in a passive system (to which positive energy is delivered at all times). Figure 5.4 clearly shows that a violation of such conditions may result in a non-passive system (the energy delivered to the network is not always positive).

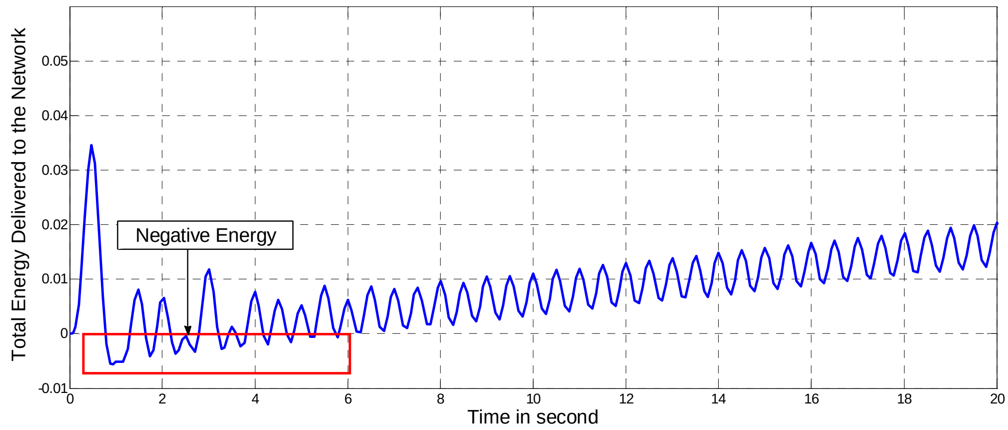

Figure 5.4. Passivity observer: Total energy delivered to a non-passive trilateral system.

5.4.2 Simulation study: Absolute stability condition

In this section, the stability condition found in chapter 4 will be used for stability analysis of the PEB dual-user haptic teleoperation system. Results from this analysis will be verified via MATLAB/Simulink simulations. The three robots are modeled by masses $M_{m1} = 0.7$, $M_{m2} = 0.9$, and $M_s = 0.5$, and $\alpha = {}^{1}\!/_{2}$.

Our research group has recently found sufficient absolute stability conditions for the PEB dual-user haptic teleoperation system. These frequency independent conditions are all functions of the controllers' gains as it is shown below:

$$K_{pmi} > 0 \tag{5.23}$$

$$K_{vmi} > 0 \tag{5.24}$$

$$K_{ps} > 0 \tag{5.25}$$

$$K_{vs} > 0 \tag{5.26}$$

$$\frac{K_{pm1}}{K_{vm1}} = \frac{K_{pm2}}{K_{vm2}} \tag{5.27}$$

$$5 - 2\sqrt{6} \leq \frac{K_{pm1}}{K_{vm1}} \frac{K_{vs}}{K_{ps}} \leq 5 + 2\sqrt{6} \tag{5.28}$$

In the simulation study, we use the stability condition presented in Chapter 4 for the analysis of two different cases: One case in which the controllers' gains meet the conditions (5.23)-(5.28) thus resulting in an absolutely stable system, and another case in which the controllers' gains were chosen so as to have a potentially unstable system. Values for the controllers' gains are shown in Table 5.2.

Table 5.2 Controllers' gains for (A) absolutely stable and (B) potentially unstable PEB trilateral teleoperator.

System	**Master 1's controller**	**Master 2's controller**	**Slave's controller**
(A) Absolutely Stable	$K_{pm1} = 2$ $K_{vm1} = 10$	$K_{pm2} = 20$ $K_{vm2} = 100$	$K_{ps} = 50$ $K_{vs} = 120$
(B) Potentially Unstable	$K_{pm1} = 10$ $K_{vm1} = 15$	$K_{pm2} = 130$ $K_{vm2} = 30$	$K_{ps} = 8$ $K_{vs} = 70$

A MATLAB code was written to test $\Re e\{Z_{in}\} \geq 0$ according to (4.13). An explanation of results will follow.

The system's impedance matrix (5.6) developed in Section 5.2 was evaluated by using the controllers' gains given in Table 5.2. Figure 5.5 and Figure 5.6 show plots of Z_{in} for system (A) of Table 5.2, which is absolutely stable as controller's gains meet conditions (5.23)-(5.28). Both plots (at two different frequencies) show that the real parts of Z_{in} are all in the right half plane, confirming that the trilateral teleoperator is indeed absolutely stable.

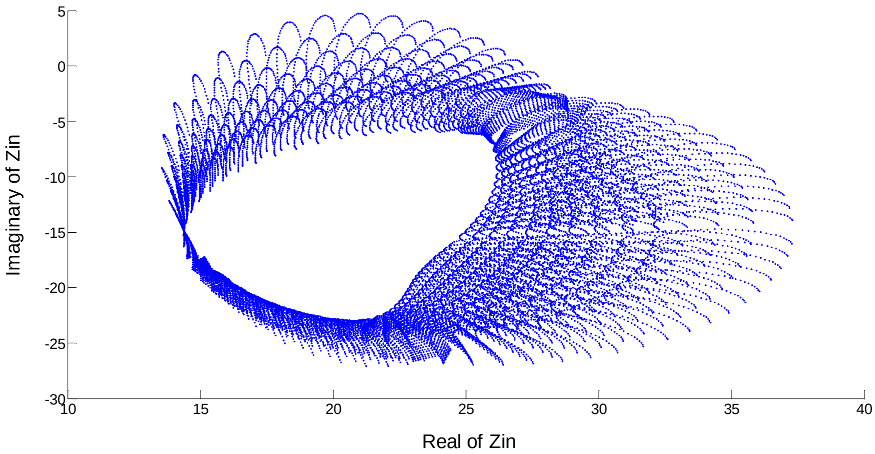

Figure 5.5. Plot of Z_{in} for an absolutely stable 3-port network at $\omega = 0.1\frac{rad}{s}$.

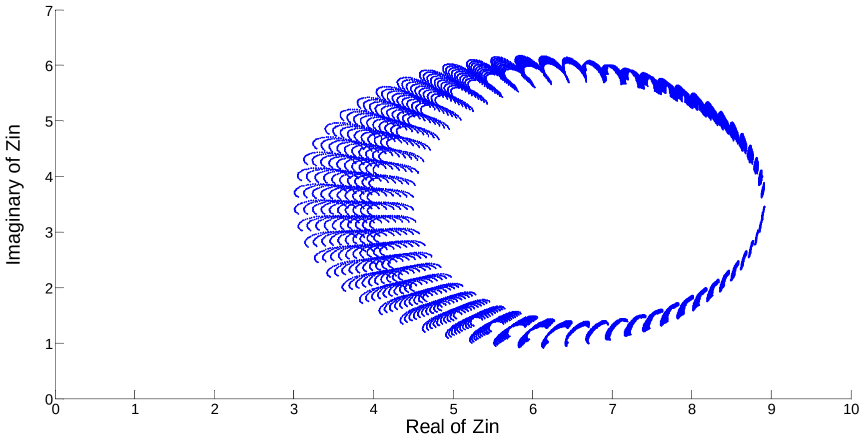

Figure 5.6. Plot of Z_{in} for an absolutely stable 3-port network at $\omega = 5\frac{rad}{s}$.

Figure 5.7 and Figure 5.8 show plots of Z_{in} for system (B) of Table 5.2, which is potentially unstable. Both plots (at two different frequencies) show that the real parts of Z_{in} are in both the right and left half plane, meaning that the trilateral teleoperator is potentially unstable.

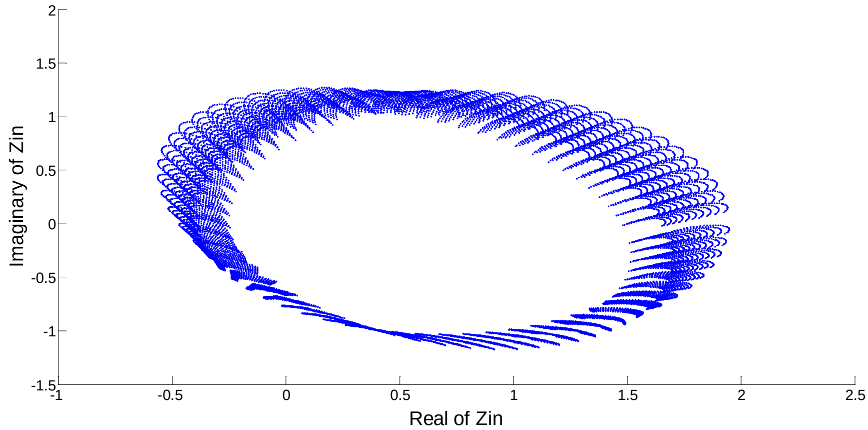

Figure 5.7. Plot of Z_{in} of a potentially unstable 3-port network at $\omega = 0.5\frac{rad}{s}$.

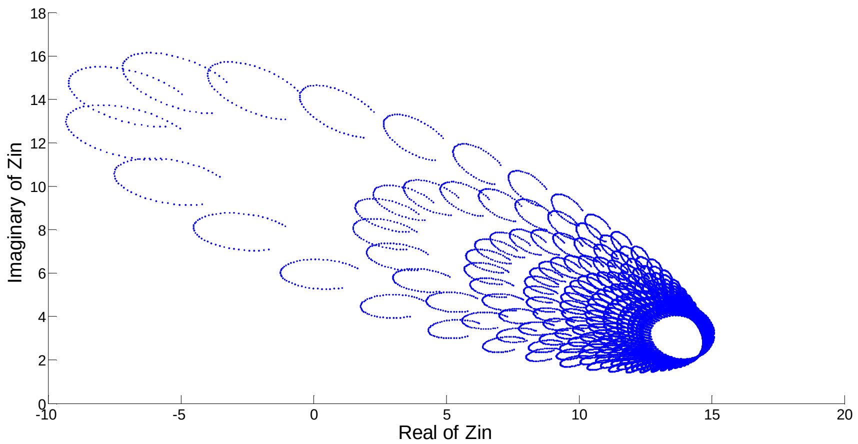

Figure 5.8. Plot of Z_{in} of a potentially unstable 3-port network at $\omega = 5\frac{rad}{s}$.

The last step is verification of the above result by simulation (via Simulink) of the system. In this simulation, port 2 and port 3 were terminated to passive loads with transfer functions $\frac{1}{1+s}$ while the input energy at port 1 of the trilateral teleoperator was monitored. For a fair comparison, the same controllers' gains given in Table 5.2 were used. According to the definition of absolute stability (Chapter 2), the trilateral system in question is absolutely stable if and only if at all times $t \geq 0$ we have [29]

$$E_1(t) = \int_0^t \left(i_1(\tau)v_1(\tau)\right) d\tau \geq 0 \tag{5.29}$$

where $E_1(t)$ represents the input energy at port 1.

Figure 5.9 is the plot representing the energy at port 1 for system (A) of Table 5.2. The plot shows that the energy is positive at all times $t \geq 0$, consequently it shows that the system is absolutely stable. This fact is consistent with the result found by applying our proposed absolute stability test to the same system.

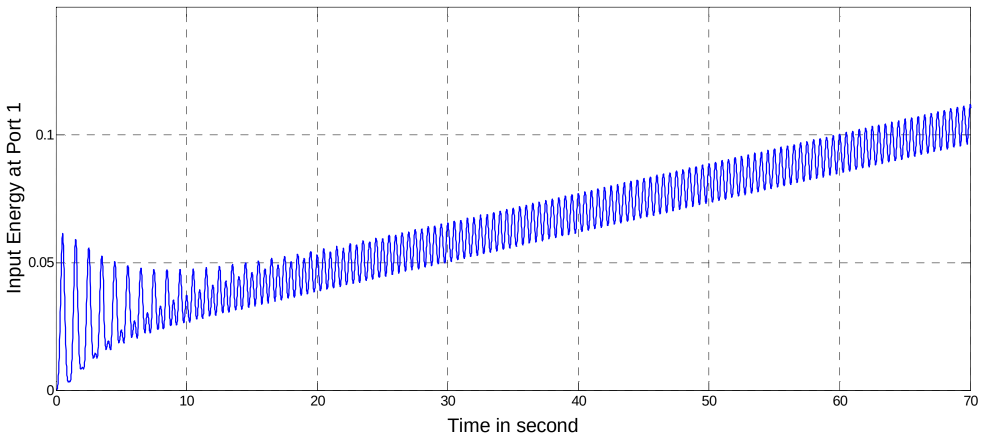

Figure 5.9. Input energy at port 1of an absolutely stable 3-port network.

Figure 5.10 is the plot representing the energy at port 1 for system (B) of Table 5.2. The plot shows that the energy is sometimes negative and consequently the system is potentially unstable. This result is also in agreement with the one found by applying the proposed absolute stability test to the same system (B).

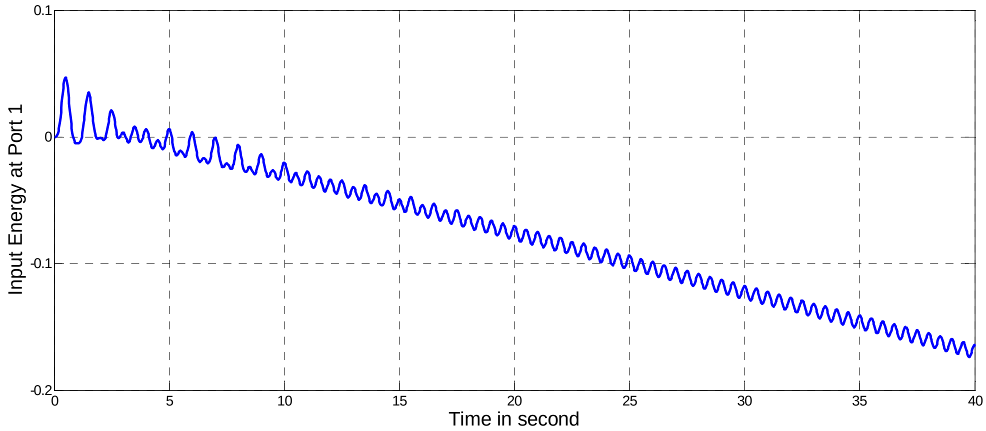

Figure 5.10. Input energy at port 1 of a potentially unstable 3-port network.

Overall, these simulation results are consistent with those found when using the proposed absolute stability test.

Chapter 6

Conclusions and Future Directions

6.1 Conclusions

This thesis presents two novel methods for stability analysis of n-port networks with passive terminations. The proposed methods can be used for either analysis and design (in the case of the proposed passivity criterion) or analysis (in the case of the proposed absolute stability test) of multilateral systems involving haptic information sharing between a number of users. The major contributions of the thesis are summarized below:

- A passivity theorem for investigation of passivity of n-port networks is proposed. The theorem gives the necessary and sufficient conditions for passivity of the n-port network based on the immittance parameters of the network. The use of immittance parameters is preferable compared to more complex techniques found in the literature, which are based on scattering parameters and reflection coefficients. Moreover, the literature has tried to investigate the passivity of 3-port networks by assuming one known/fixed termination, thus reducing the 3-port into a 2-port network whose passivity analysis has been known for a long time. In contrast, the closed-form conditions given in this thesis make it possible to investigate the passivity of n-port networks (thus not necessarily limited to n=3) directly and without resorting to using any known/fixed terminations, assuring a complete general solution to the problem.

- A mathematical expression for testing the absolute stability of 3-port networks is also proposed in this thesis. The method is based on evaluation of the real part of the driving point impedance of the 3-port network under investigation. The numerical evaluation should be done in a range of frequencies of interest. Due to the nature of the proposed technique, the evaluation of such expression is useful for stability analysis purposes only.

6.2 Future Directions

The following is a list of potential future work.

1. The passivity theorem and absolute stability condition given in this thesis have been developed in the frame of 1 degree of freedom (DOF) systems. A step forward would be its extension to 2- and 3-DOF systems.

2. The proposed stability condition is an excellent tool for analysis of stability of 3-port networks. The numerical evaluation of such condition can be used to determine frequency ranges over which the system is stable. Extension of this condition from 3-port networks to 4-port networks would be very beneficial.

3. A study to compare the level of conservatism of absolute stability versus passivity would be very beneficial. It is expected that, similar to bilateral teleoperation systems, the absolute stability criterion is less conservative than the passivity criterion and that the two criteria become the same when the trilateral system is represented by a reciprocal network.

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