

This paper appears in IET Control Theory & Applications, vol. 5, no. 18, pp. 2063-2074, Dec 2011.

Disturbance Observer Based Control of Nonlinear Haptic Teleoperation Systems

A. Mohammadi, M. Tavakoli, H. J. Marquez

alireza3@ualberta.ca tavakoli@ece.ualberta.ca marquez@ece.ualberta.ca

Department of Electrical & Computer Engineering, University of Alberta, Edmonton, Alberta, Canada T6G 2V4

Abstract

Teleoperation systems are subject to different types of disturbances. Such disturbances, when unaccounted for, may cause poor performance and even instability of the teleoperation system. This paper presents a novel nonlinear bilateral control scheme using the concept of *disturbance observer based control* for nonlinear teleoperation systems. Lumping the effects of dynamic uncertainties and external disturbances into a single disturbance term enables us to design a disturbance observer in order to suppress these disturbances and alleviate their adverse effects on the teleoperation system. A disturbance observer based control law is proposed for nonlinear teleoperation systems which will guarantee global asymptotic force tracking and global exponential position and disturbance tracking when the bilateral teleoperation system is experiencing slow-varying disturbances. In the case of fast-varying disturbances, the tracking errors are shown to be globally uniformly ultimately bounded, with an ultimate bound that can be made as small as desired using the design parameters. Simulations are presented to show the effectiveness of the proposed approach.

Index Terms

Haptic teleoperation, 4-channel control, disturbance observer (DOB), dynamic uncertainties, transparency

I. INTRODUCTION

Teleoperation involves indirect performance of a task in a remote environment and is used to extend a person's sensing and manipulation capability to a remote location [1].

Every teleoperation system consists of a master robot (user interface) and a slave robot, which exchange different types of information such as force, position, and visual and auditory data via a communication channel. The master interacts with the human operator and the slave interacts with the remote environment. The human operator applies position commands to the master robot. The slave robot responds to the received position commands from the communication channel and moves to the desired position in the remote environment. If force feedback from the slave side to the master side is present, the system is called a bilateral teleoperation system to distinguish it from a unilateral teleoperation system, in which no force is reflected to the user. A bilateral teleoperation system is said to be transparent if the slave robot follows the position of the master robot and the master robot faithfully displays the slave-environment contact force to the human operator.

Considering the master and the slave robots to have linear models, various control schemes have been suggested for teleoperation systems in the literature [2]. The most successful control scheme in achieving a fully transparent teleoperation system is the 4-channel architecture [3], [4], [5]. In spite of its good performance, the 4-channel control scheme is mostly suitable for fixed linear models. Physical robots, however, are nonlinear systems subject to different uncertainties and disturbances, such as joint frictions, unknown payloads, etc. [6], [7].

Several solutions, based on nonlinear adaptive control schemes, have been proposed for nonlinear teleoperation systems subject to uncertainties. Considering the master and the slave robots to have uncertain nonlinear models, position and force tracking were achieved by ignoring the remote environment's and the human operator's uncertainties in the adaptation laws in [8], [9], and [10]. Later, adaptive schemes were developed for uncertain nonlinear master and slave robots and uncertain linear operator and environment models in [11], [12], [13] and [14]. However, these adaptive schemes suffer from considering the uncertainties to be parametric and assuming that the model of the uncertainties are available, i.e., they are mainly meant for the control of teleoperation systems with structured uncertainties.

Lumping the effect of all dynamic uncertainties and external disturbances into a virtual disturbance term is the key idea behind our disturbance observer based control scheme. Similar schemes have been used in applications such as control of direct drive motors, mechatronics system control [15], [16], [17], independent robot joint control [18], robot joint friction compensation [19], [20], and sensorless torque control of robotic manipulators [21], [22].

A nonlinear disturbance observer for robotic manipulators subject to disturbances was proposed in [23]. However, the closed-loop stability of the overall system including the disturbance observer and the controller was not investigated. The investigation of the stability and performance of a master-slave teleoperation system under disturbance observer based control is even more challenging and not studied either. A disturbance observer based controller has been designed for bilateral teleoperation systems in [24]. In this work, however, each joint of the master and the slave robots is treated as an independent single-input-single-output system and, thus, several disturbance observer based controllers are designed for these linear time-invariant (LTI) systems. This is in contrast to the fact that dynamics of robotic systems are highly nonlinear and involve couplings among the manipulator's various degrees of freedom. Our proposed approach accounts for such nonlinear and coupled dynamic characteristics of teleoperators.

This paper for the first time addresses the problem of disturbance observer based control of nonlinear teleoperation systems subject to dynamic uncertainties and disturbances. A disturbance observer based control law will be proposed and incorporated into the framework of the 4-channel teleoperation architecture. Under the proposed control law, full transparency and exponential disturbance and position tracking are achieved under slow-varying disturbances. In the case of fast-varying disturbances, the proposed control scheme guarantees global uniform ultimate boundedness of the tracking errors.

The organization of this paper is as follows. Section II introduces the nonlinear model of the teleoperation systems and 4-channel bilateral control architecture. Section III introduces the concept of disturbance observer based control and proposes a novel disturbance observer based controller for nonlinear teleoperation systems subject to dynamic uncertainties and disturbances. Stability and transparency of the closed-loop system are discussed in this section and it is shown that the proposed disturbance observer based controller achieves stability and full transparency when the teleoperation system is subject to slow-varying disturbances. Under fast-varying disturbances, it is shown that the proposed control scheme achieves stability while tracking errors are ultimately bounded. Finally, simulations in section IV show the efficiency of the proposed control scheme as compared with the case where no disturbance observer is employed.

II. NONLINEAR MODEL OF A TELEOPERATION SYSTEM

The dynamical models investigated in this paper and our proposed control law in section III will be described in the Cartesian space. This enables us to make a teleoperation system stable and transparent without requiring the master and the slave robots to have similar kinematics and dynamics.

A. Models of a teleoperation system in the joint and Cartesian spaces

The following dynamic equation governs the motion of an n -degree-of-freedom (DOF) robot in the joint space [25]:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{F}(\dot{\mathbf{q}}) = \boldsymbol{\tau} + \boldsymbol{\tau}_{ext} \quad (1)$$

where

$$\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{V}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) \quad (2)$$

and $\mathbf{q} \in R^{n \times 1}$ is the vector of joint positions, $\mathbf{M}(\mathbf{q}) \in R^{n \times n}$ is the inertia matrix, $\mathbf{F}(\dot{\mathbf{q}})$ is the vector of friction torques acting on the joints, $\mathbf{V}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} \in R^{n \times 1}$ is the vector of Coriolis and centrifugal forces, $\mathbf{G}(\mathbf{q}) \in R^{n \times 1}$ is the gravity vector, $\boldsymbol{\tau} \in R^{n \times 1}$ is the vector of the control torques applied to the joints, and $\boldsymbol{\tau}_{ext} \in R^{n \times 1}$ is the vector of the external disturbances exerted on the joints. Assume that $\hat{\mathbf{M}}(\mathbf{q})$ and $\hat{\mathbf{N}}(\mathbf{q}, \dot{\mathbf{q}})$ are the approximations of $\mathbf{M}(\mathbf{q})$ and $\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}})$ and $\Delta\mathbf{M}$ and $\Delta\mathbf{N}$ are the corresponding additive uncertain terms present in the model of the robot. That is to say

$$\mathbf{M}(\mathbf{q}) = \hat{\mathbf{M}}(\mathbf{q}) + \Delta\mathbf{M} \quad (3)$$

$$\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) = \hat{\mathbf{N}}(\mathbf{q}, \dot{\mathbf{q}}) + \Delta\mathbf{N} \quad (4)$$

Also assume that we have no prior information about the uncertain terms $\Delta\mathbf{M}$ and $\Delta\mathbf{N}$, the friction vector \mathbf{F} , and the vector of external disturbances $\boldsymbol{\tau}_{ext}$. Now, take

$$\boldsymbol{\tau}_d = \boldsymbol{\tau}_{ext} - \Delta\mathbf{M}\ddot{\mathbf{q}} - \Delta\mathbf{N} - \mathbf{F}(\dot{\mathbf{q}}) \quad (5)$$

By this definition, we lump the effect of all dynamic uncertainties, joint frictions and external disturbances into a single disturbance vector, i.e., τ_d . From (1), we get

$$\hat{\mathbf{M}}(\mathbf{q})\ddot{\mathbf{q}} + \hat{\mathbf{N}}(\mathbf{q}, \dot{\mathbf{q}}) = \boldsymbol{\tau} + \boldsymbol{\tau}_d \quad (6)$$

Now, define $\mathbf{J}(\mathbf{q})$ as the Jacobian matrix of the robot and \mathbf{x} as the position/orientation (pose) vector of the robot's end-effector in the Cartesian space. We assume that \mathbf{J} is of full column rank, i.e., the robot is not at a singularity and $\mathbf{J}_* = \mathbf{J}^T \mathbf{J}$ is invertible. Using $\dot{\mathbf{x}} = \mathbf{J}\dot{\mathbf{q}}$ and $\ddot{\mathbf{x}} = \dot{\mathbf{J}}\dot{\mathbf{q}} + \mathbf{J}\ddot{\mathbf{q}}$ in (6), we get the dynamic equation (7), which governs the motion of the end-effector of the robot in the Cartesian space.

$$\hat{\mathbf{M}}_x(\mathbf{q})\ddot{\mathbf{x}} + \hat{\mathbf{N}}_x(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{f} + \mathbf{d} \quad (7)$$

where,

$$\hat{\mathbf{M}}_x(\mathbf{q}) = \mathbf{J}\mathbf{J}_*^{-1}\hat{\mathbf{M}}(\mathbf{q})\mathbf{J}_*^{-1}\mathbf{J}^T \quad (8)$$

$$\hat{\mathbf{N}}_x(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{J}\mathbf{J}_*^{-1}\hat{\mathbf{N}}(\mathbf{q}, \dot{\mathbf{q}}) - \hat{\mathbf{M}}_x(\mathbf{q})\dot{\mathbf{J}}\mathbf{J}_*^{-1}\mathbf{J}^T\dot{\mathbf{x}} \quad (9)$$

and

$$\mathbf{f} = \mathbf{J}\mathbf{J}_*^{-1}\boldsymbol{\tau} \quad (10)$$

$$\mathbf{d} = \mathbf{J}\mathbf{J}_*^{-1}\boldsymbol{\tau}_d \quad (11)$$

Similar to a single robot, the dynamic equations describing the motions of the end-effectors of the master and the slave robots with n DOFs, which are interacting with the human operator and the remote environment, in the presence of dynamic uncertainties, external disturbances and joint frictions can be written as

$$\hat{\mathbf{M}}_{xm}(\mathbf{q}_m)\ddot{\mathbf{x}}_m + \hat{\mathbf{N}}_{xm}(\mathbf{q}_m, \dot{\mathbf{q}}_m) = \mathbf{f}_m + \mathbf{f}_h + \mathbf{d}_m \quad (12)$$

$$\hat{\mathbf{M}}_{xs}(\mathbf{q}_s)\ddot{\mathbf{x}}_s + \hat{\mathbf{N}}_{xs}(\mathbf{q}_s, \dot{\mathbf{q}}_s) = \mathbf{f}_s - \mathbf{f}_e + \mathbf{d}_s \quad (13)$$

where $\mathbf{q}_m, \mathbf{q}_s, \mathbf{x}_m, \mathbf{x}_s, \hat{\mathbf{M}}_{xm}(\mathbf{q}_m), \hat{\mathbf{M}}_{xs}(\mathbf{q}_s), \hat{\mathbf{N}}_{xm}(\mathbf{q}_m, \dot{\mathbf{q}}_m)$ and $\hat{\mathbf{N}}_{xs}(\mathbf{q}_s, \dot{\mathbf{q}}_s)$ are defined as in (7) and subscripts m and s refer to the master and the slave, respectively. Also, $\mathbf{f}_m, \mathbf{f}_s \in R^{6 \times 1}$ are the (equivalent) control forces applied to the master and the slave end-effectors in the Cartesian space, $\mathbf{f}_h, \mathbf{f}_e \in R^{6 \times 1}$ are the forces exerted to the master and the slave end-effectors by the human operator and the remote environment, and $\mathbf{d}_m, \mathbf{d}_s \in R^{6 \times 1}$ are the (equivalent) disturbance forces exerted to the master and the slave end-effectors in the Cartesian space. These disturbance forces represent the lumped effect of all dynamic uncertainties, frictions and external disturbances exerted to the master and the slave robots in the Cartesian space.

B. 4-channel control architecture

The 4-channel control architecture is shown in Figure 1 with disturbance observers incorporated into it. Note that the conventional 4-channel architecture does not have any disturbance observers. When no disturbance observer is used in the 4-channel control architecture, we have $\hat{\mathbf{d}}_m = 0$ and $\hat{\mathbf{d}}_s = 0$. Exogenous signals \mathbf{f}_h^* and \mathbf{f}_e^* are exerted by the human operator and the remote environment, respectively. The signals $\mathbf{x}_m, \mathbf{x}_s, \mathbf{f}_h, \mathbf{f}_e, \mathbf{f}_m, \mathbf{f}_s, \mathbf{d}_m$ and \mathbf{d}_s are as defined in (12)–(13). Position information is exchanged between the master and the slave via the position channels C_1 and C_4 . Force information is exchanged through the force channels C_2 and C_3 . In addition, C_m and C_s are local master and slave (position) controllers. Lastly, C_5 and C_6 provide the master and the slave with local force feedback from the human operator and the remote environment, respectively. In the conventional design of 4-channel controllers, it is assumed that $\mathbf{d}_m = \mathbf{0}$ and $\mathbf{d}_s = \mathbf{0}$. We are, however, going to deal with these disturbances in this paper.

Every control scheme designed for a teleoperation system should be able to satisfy two main requirements: closed-loop stability and transparency [3]. In a fully transparent teleoperation system we have $\mathbf{x}_s = \mathbf{x}_m$ and $\mathbf{f}_h = \mathbf{f}_e$. If the master and the slave can be modeled by LTI impedances¹ $Z_m(s)$ and $Z_s(s)$, the 4-channel teleoperation system of Figure 1, when no disturbance observer is incorporated into it, becomes fully transparent in the absence of disturbances and delays if the controllers are chosen as [5]

¹Causal dynamic operator which maps position/velocity to force.

$$\begin{aligned}
\mathbf{C}_1 &= \mathbf{Z}_s + \mathbf{C}_s \\
\mathbf{C}_2 &= \mathbf{I} + \mathbf{C}_6 \\
\mathbf{C}_3 &= \mathbf{I} + \mathbf{C}_5 \\
\mathbf{C}_4 &= -\mathbf{Z}_m - \mathbf{C}_m
\end{aligned} \tag{14}$$

III. DISTURBANCE OBSERVER BASED CONTROL OF TELEOPERATION SYSTEMS

In this section, first the concept of disturbance observer based control is introduced. Next, control laws are developed based on this concept and incorporated into the 4-channel architecture for a nonlinear teleoperation system. Then, the stability and transparency achieved by the proposed control law are investigated.

A. Disturbance observer based control concept

Consider (7), which describes a single robot dynamics in the Cartesian space. When there is no uncertainty in the model of the robot and no disturbances are exerted to the robot, we have $\mathbf{d} = \mathbf{0}$ and the control law \mathbf{f} can be designed in a way as if there exist no disturbances. In the presence of uncertainties and disturbances, the previous control law \mathbf{f} fails to perform properly. The idea behind employing a disturbance observer is to estimate the lumped disturbance, including the effect of the dynamic uncertainties, friction and external disturbances, that is exerted to the robotic manipulator and then to feed it back to the input of the robot. If the disturbance vector is exactly estimated, i.e., $\hat{\mathbf{d}} = \mathbf{d}$, disturbances will be cancelled out in the closed-loop equation and it seems as if we are dealing with a robot with known dynamics, for which we can easily design the controller.

B. Proposed teleoperation control laws

Extending the idea of disturbance observer based control of a single robot to a master-slave teleoperation system, we will design a disturbance observer for each of the master and the slave robots in order to estimate and cancel out the disturbances. Figure 1 depicts the proposed 4-channel architecture with disturbance observers incorporated into it.

Assume that the master and the slave disturbances are estimated to be $\hat{\mathbf{d}}_m$ and $\hat{\mathbf{d}}_s$, respectively. Later, we will introduce the disturbance observers that can provide us with these estimations. We propose the following nonlinear control laws for the master and slave robots described by (12) and (13), respectively:

$$\begin{aligned} \mathbf{f}_m &= \hat{\mathbf{M}}_{xm}(\mathbf{q}_m)[-C_m \mathbf{x}_m - C_2 \mathbf{f}_e - C_4 \mathbf{x}_s + C_6 \mathbf{f}_h + \mathbf{f}_h] \\ &\quad + \hat{\mathbf{N}}_{xm}(\mathbf{q}_m, \dot{\mathbf{q}}_m) - \mathbf{f}_h - \hat{\mathbf{d}}_m \end{aligned} \quad (15)$$

$$\begin{aligned} \mathbf{f}_s &= \hat{\mathbf{M}}_{xs}(\mathbf{q}_s)[-C_s \mathbf{x}_s + C_1 \mathbf{x}_m + C_3 \mathbf{f}_h - C_5 \mathbf{f}_e - \mathbf{f}_e] \\ &\quad + \hat{\mathbf{N}}_{xs}(\mathbf{q}_s, \dot{\mathbf{q}}_s) + \mathbf{f}_e - \hat{\mathbf{d}}_s \end{aligned} \quad (16)$$

where C_m , C_s , C_1 , \dots , and C_6 are some LTI controllers used in the above nonlinear control laws. Note the use of disturbance estimates $\hat{\mathbf{d}}_m$ and $\hat{\mathbf{d}}_s$ in the proposed control laws.

The disturbance observer based control laws (15) and (16), when applied to the master and slave described by (12)–(13), result in the following closed-loop equations for the two robots:

$$\begin{aligned} \ddot{\mathbf{x}}_m &= -C_m \mathbf{x}_m - C_2 \mathbf{f}_e - C_4 \mathbf{x}_s + C_6 \mathbf{f}_h \\ &\quad + \mathbf{f}_h + \hat{\mathbf{M}}_{xm}^{-1}(\mathbf{q}_m) \Delta \mathbf{d}_m \end{aligned} \quad (17)$$

$$\begin{aligned} \ddot{\mathbf{x}}_s &= -C_s \mathbf{x}_s + C_1 \mathbf{x}_m + C_3 \mathbf{f}_h - C_5 \mathbf{f}_e \\ &\quad - \mathbf{f}_e + \hat{\mathbf{M}}_{xs}^{-1}(\mathbf{q}_s) \Delta \mathbf{d}_s \end{aligned} \quad (18)$$

where $\Delta \mathbf{d}_m = \mathbf{d}_m - \hat{\mathbf{d}}_m$ and $\Delta \mathbf{d}_s = \mathbf{d}_s - \hat{\mathbf{d}}_s$ are the master and the slave disturbance estimation errors, respectively.

Remark 1. When $\Delta \mathbf{d}_m = \mathbf{0}$ and $\Delta \mathbf{d}_s = \mathbf{0}$, i.e., under ideal disturbance tracking, (17) and (18) describe an n -DOF conventional 4-channel teleoperation system without disturbances with the master and slave robots represented by identity inertia matrices, i.e., $\mathbf{Z}_m(s) = s^2 \mathbf{I}$ and $\mathbf{Z}_s(s) = s^2 \mathbf{I}$.

◇

Let us choose the master and the slave local position controllers in (15) and (16) to be of

proportional-derivative type:

$$\begin{aligned} \mathbf{C}_m &= \mathbf{K}_{mv}s + \mathbf{K}_{mp} \\ \mathbf{C}_s &= \mathbf{K}_{sv}s + \mathbf{K}_{sp} \end{aligned} \quad (19)$$

where \mathbf{K}_{mv} , \mathbf{K}_{mp} , \mathbf{K}_{sv} and \mathbf{K}_{sp} are constant gain matrices. Also, let us choose the force reflection gains in (15) and (16) to be

$$\begin{aligned} \mathbf{C}_2 &= \mathbf{C}_{mf} \\ \mathbf{C}_3 &= \mathbf{C}_{sf} \end{aligned} \quad (20)$$

where \mathbf{C}_{mf} and \mathbf{C}_{sf} are constant force reflection gain matrices. We choose the other controllers in (15) and (16) to satisfy the full transparency conditions listed in (14), i.e., we choose

$$\begin{aligned} \mathbf{C}_1 &= s^2\mathbf{I} + \mathbf{K}_{sv}s + \mathbf{K}_{sp} \\ \mathbf{C}_4 &= -(s^2\mathbf{I} + \mathbf{K}_{mv}s + \mathbf{K}_{mp}) \\ \mathbf{C}_5 &= \mathbf{C}_{sf} - \mathbf{I} \\ \mathbf{C}_6 &= \mathbf{C}_{mf} - \mathbf{I} \end{aligned} \quad (21)$$

Remark 2. In order to implement \mathbf{C}_1 and \mathbf{C}_4 in (21), we need to measure or compute the acceleration of the master and the slave robots. We can omit the acceleration terms if good low-frequency transparency is enough in the desired application. However, requiring good transparency over both low and high frequencies justifies using accelerometers [3]. Alternatively, one may employ precise numerical differentiation techniques, which are robust to measurement errors and input noises, in order to obtain acceleration and velocity signals from position measurement [26], [27].

◇

Using (19), (20) and (21) in (15)–(16) results in the control laws

$$\begin{aligned}\mathbf{f}_m &= \hat{\mathbf{M}}_{xm}(\mathbf{q}_m)[\ddot{\mathbf{x}}_s - \mathbf{K}_{mv}\Delta\dot{\mathbf{x}} - \mathbf{K}_{mp}\Delta\mathbf{x} + \mathbf{C}_{mf}\Delta\mathbf{f}] \\ &\quad + \hat{\mathbf{N}}_{xm}(\mathbf{q}_m, \dot{\mathbf{q}}_m) - \mathbf{f}_h - \hat{\mathbf{d}}_m\end{aligned}\quad (22)$$

$$\begin{aligned}\mathbf{f}_s &= \hat{\mathbf{M}}_{xs}(\mathbf{q}_s)[\ddot{\mathbf{x}}_m + \mathbf{K}_{sv}\Delta\dot{\mathbf{x}} + \mathbf{K}_{sp}\Delta\mathbf{x} + \mathbf{C}_{sf}\Delta\mathbf{f}] \\ &\quad + \hat{\mathbf{N}}_{xs}(\mathbf{q}_s, \dot{\mathbf{q}}_s) + \mathbf{f}_e - \hat{\mathbf{d}}_s\end{aligned}\quad (23)$$

where $\Delta\mathbf{x} = \mathbf{x}_m - \mathbf{x}_s$ is the position tracking error and $\Delta\mathbf{f} = \mathbf{f}_h - \mathbf{f}_e$ is the force tracking error. The master and the slave closed-loop dynamics (17) and (18) are reduced to

$$\Delta\ddot{\mathbf{x}} = -\mathbf{K}_{mv}\Delta\dot{\mathbf{x}} - \mathbf{K}_{mp}\Delta\mathbf{x} + \mathbf{C}_{mf}\Delta\mathbf{f} + \hat{\mathbf{M}}_{xm}^{-1}(\mathbf{q}_m)\Delta\mathbf{d}_m\quad (24)$$

$$\Delta\ddot{\mathbf{x}} = \mathbf{K}_{sv}\Delta\dot{\mathbf{x}} + \mathbf{K}_{sp}\Delta\mathbf{x} - \mathbf{C}_{sf}\Delta\mathbf{f} - \hat{\mathbf{M}}_{xs}^{-1}(\mathbf{q}_s)\Delta\mathbf{d}_s\quad (25)$$

Assume that matrices \mathbf{C}_{mf} , \mathbf{C}_{sf} and $\mathbf{C}_{mf}^{-1} + \mathbf{C}_{sf}^{-1}$ are invertible. Multiplying (24) by \mathbf{C}_{mf}^{-1} and (25) by \mathbf{C}_{sf}^{-1} and adding them together, we can find the dynamic equation governing the position tracking error

$$\Delta\ddot{\mathbf{x}} + \mathbf{K}_v\Delta\dot{\mathbf{x}} + \mathbf{K}_p\Delta\mathbf{x} = \Psi_{xm}(\mathbf{q}_m)\Delta\mathbf{d}_m - \Psi_{xs}(\mathbf{q}_s)\Delta\mathbf{d}_s\quad (26)$$

where

$$\mathbf{K}_v = (\mathbf{C}_{mf}^{-1} + \mathbf{C}_{sf}^{-1})^{-1}(\mathbf{C}_{sf}^{-1}\mathbf{K}_{sv} - \mathbf{C}_{mf}^{-1}\mathbf{K}_{mv})\quad (27)$$

$$\mathbf{K}_p = (\mathbf{C}_{mf}^{-1} + \mathbf{C}_{sf}^{-1})^{-1}(\mathbf{C}_{sf}^{-1}\mathbf{K}_{sp} - \mathbf{C}_{mf}^{-1}\mathbf{K}_{mp})\quad (28)$$

$$\Psi_{xm}(\mathbf{q}_m) = (\mathbf{C}_{mf}^{-1} + \mathbf{C}_{sf}^{-1})^{-1}\mathbf{C}_{mf}^{-1}\hat{\mathbf{M}}_{xm}^{-1}(\mathbf{q}_m)\quad (29)$$

$$\Psi_{xs}(\mathbf{q}_s) = (\mathbf{C}_{mf}^{-1} + \mathbf{C}_{sf}^{-1})^{-1}\mathbf{C}_{sf}^{-1}\hat{\mathbf{M}}_{xs}^{-1}(\mathbf{q}_s)\quad (30)$$

Before introducing the disturbance observers to be used with the proposed control laws (22) and (23) in the next section, let us investigate the effect of disturbances on the teleoperation system when the control laws (22)–(23) employ no disturbance estimates, i.e., when $\hat{\mathbf{d}}_m = \mathbf{0}$ and $\hat{\mathbf{d}}_s = \mathbf{0}$. Applying the control laws (22)–(23) with no disturbance estimate terms to the master and slave robots (12)–(13) the following position tracking error dynamics is obtained:

$$\Delta\ddot{\mathbf{x}} + \mathbf{K}_v\Delta\dot{\mathbf{x}} + \mathbf{K}_p\Delta\mathbf{x} = \Psi_{xm}(\mathbf{q}_m)\mathbf{d}_m - \Psi_{xs}(\mathbf{q}_s)\mathbf{d}_s \quad (31)$$

Now, let us consider the following candidate Lyapunov function with \mathbf{K}_p , as defined in (28), being a symmetric matrix:

$$\Phi(\Delta\dot{\mathbf{x}}, \Delta\mathbf{x}) = \frac{1}{2}\Delta\dot{\mathbf{x}}^T\Delta\dot{\mathbf{x}} + \frac{1}{2}\Delta\mathbf{x}^T\mathbf{K}_p\Delta\mathbf{x} \quad (32)$$

Taking the time derivative of the above function we get

$$\begin{aligned} \dot{\Phi} &= \Delta\ddot{\mathbf{x}}^T\Delta\dot{\mathbf{x}} + \Delta\dot{\mathbf{x}}^T\mathbf{K}_p\Delta\mathbf{x} = -\Delta\dot{\mathbf{x}}^T\mathbf{K}_v\Delta\dot{\mathbf{x}} \\ &\quad + \Delta\dot{\mathbf{x}}^T[\Psi_{xm}(\mathbf{q}_m)\mathbf{d}_m - \Psi_{xs}(\mathbf{q}_s)\mathbf{d}_s] \end{aligned} \quad (33)$$

The presence of disturbance terms in (31) may cause poor position tracking or even instability of the system. For instance, special values of disturbances \mathbf{d}_m and \mathbf{d}_s are able to make $\dot{\Phi}$ positive in a neighbourhood of the origin and thus cause instability of the position tracking error according to the Chetaev Theorem (see for example Theorem 3.12 in [28]). This means that the teleoperation system lacks position tracking and is not transparent.

C. Proposed disturbance observers

In this section, we will design the disturbance observers in a way that full transparency and disturbance tracking are achieved under the control laws (22)–(23). Our proposed disturbance observers designed in the Cartesian space given the master and slave (approximate) dynamics (12)–(13) are

$$\begin{aligned}\dot{\hat{\mathbf{d}}}_m &= -\mathbf{L}_m \hat{\mathbf{d}}_m + \mathbf{L}_m [\hat{\mathbf{M}}_{xm}(\mathbf{q}_m) \ddot{\mathbf{x}}_m + \hat{\mathbf{N}}_{xm}(\mathbf{q}_m, \dot{\mathbf{q}}_m) \\ &\quad - \mathbf{f}_h - \mathbf{f}_m] + \Psi_{xm}^T(\mathbf{q}_m)(\Delta \dot{\mathbf{x}} + \gamma \Delta \mathbf{x})\end{aligned}\quad (34)$$

$$\begin{aligned}\dot{\hat{\mathbf{d}}}_s &= -\mathbf{L}_s \hat{\mathbf{d}}_s + \mathbf{L}_s [\hat{\mathbf{M}}_{xs}(\mathbf{q}_s) \ddot{\mathbf{x}}_s + \hat{\mathbf{N}}_{xs}(\mathbf{q}_s, \dot{\mathbf{q}}_s) \\ &\quad + \mathbf{f}_e - \mathbf{f}_s] + \Psi_{xs}^T(\mathbf{q}_s)(-\Delta \dot{\mathbf{x}} - \gamma \Delta \mathbf{x})\end{aligned}\quad (35)$$

where γ is an arbitrary positive constant. Also, \mathbf{L}_m and \mathbf{L}_s are constant gain matrices.

Note that in the 4-channel teleoperation control architecture, it is assumed that \mathbf{f}_h and \mathbf{f}_e are measured. The proposed nonlinear disturbance observers in (34) and (35) need acceleration measurements. As mentioned before, the need for full transparency in a wide frequency range justifies using accelerometers, which is dictated by the transparency conditions in (14) for use in (22) and (23) – the same measurements will be needed in (34) and (35). Equations (12) and (13), along with (34) and (35), result in the following disturbance estimation error dynamics:

$$\dot{\hat{\mathbf{d}}}_m = \mathbf{L}_m \Delta \mathbf{d}_m + \Psi_{xm}^T(\mathbf{q}_m)(\Delta \dot{\mathbf{x}} + \gamma \Delta \mathbf{x}) \quad (36)$$

$$\dot{\hat{\mathbf{d}}}_s = \mathbf{L}_s \Delta \mathbf{d}_s + \Psi_{xs}^T(\mathbf{q}_s)(-\Delta \dot{\mathbf{x}} - \gamma \Delta \mathbf{x}) \quad (37)$$

Therefore, the following disturbance estimation error dynamics result from (36) and (37) for the master and the slave, respectively:

$$\Delta \dot{\mathbf{d}}_m = \dot{\mathbf{d}}_m - \mathbf{L}_m \Delta \mathbf{d}_m - \Psi_{xm}^T(\mathbf{q}_m)(\Delta \dot{\mathbf{x}} + \gamma \Delta \mathbf{x}) \quad (38)$$

$$\Delta \dot{\mathbf{d}}_s = \dot{\mathbf{d}}_s - \mathbf{L}_s \Delta \mathbf{d}_s - \Psi_{xs}^T(\mathbf{q}_s)(-\Delta \dot{\mathbf{x}} - \gamma \Delta \mathbf{x}) \quad (39)$$

Remark 3. The terms $\Psi_{xm}^T(\mathbf{q}_m)(\Delta \dot{\mathbf{x}} + \gamma \Delta \mathbf{x})$ and $\Psi_{xs}^T(\mathbf{q}_s)(-\Delta \dot{\mathbf{x}} - \gamma \Delta \mathbf{x})$ in (34) and (35) do not exist in the nonlinear disturbance observer proposed by [23]. These new terms are employed in the disturbance observers in order to improve the performance of the teleoperation system.

◇

First, we will investigate the stability and transparency of the teleoperation system when it is

subject to slow-varying disturbances. We have the following theorem regarding the stability and transparency of the system under slow-varying disturbances.

Theorem 1. *Consider the teleoperation system subject to disturbances described by (12) and (13). The master and the slave disturbance observers are given in (34) and (35). Then the disturbance observer based control laws given in (22) and (23) guarantee global asymptotic stability of the disturbance tracking error, the position tracking error, and the force tracking error if the following conditions hold:*

- $\mathbf{L}_m = \mathbf{L}_m^T > \mathbf{0}$ and $\mathbf{L}_s = \mathbf{L}_s^T > \mathbf{0}$ are constant symmetric and positive definite matrices,
- \mathbf{K}_v given by (27) exists and is a constant symmetric and positive definite matrix satisfying $\mathbf{K}_v > \gamma \mathbf{I}$,
- \mathbf{K}_p given by (28) exists and is a constant symmetric and positive definite matrix,
- $\dot{\mathbf{d}}_m \approx \mathbf{0}$ and $\dot{\mathbf{d}}_s \approx \mathbf{0}$, i.e., the rates of change of disturbances acting on the master and the slave robots are negligible in comparison with the estimation error dynamics (36) and (37).

■

Proof: Under the control laws (22) and (23), and according to the fourth condition of the Theorem, the position tracking error dynamics and the disturbance tracking error dynamics of the master and the slave are given by (26) and (38)–(39) with $\dot{\mathbf{d}}_m = \mathbf{0}$ and $\dot{\mathbf{d}}_s = \mathbf{0}$, respectively. Let us consider the following candidate Lyapunov function:

$$\begin{aligned}
 V(\Delta\dot{\mathbf{x}}, \Delta\mathbf{x}, \Delta\mathbf{d}_m, \Delta\mathbf{d}_s) &= \frac{1}{2}(\Delta\dot{\mathbf{x}} + \gamma\Delta\mathbf{x})^T(\Delta\dot{\mathbf{x}} + \gamma\Delta\mathbf{x}) \\
 &+ \frac{1}{2}\Delta\mathbf{x}^T(\mathbf{K}_p + \gamma\mathbf{K}_v - \gamma^2\mathbf{I})\Delta\mathbf{x} + \frac{1}{2}\Delta\mathbf{d}_m^T\Delta\mathbf{d}_m \\
 &+ \frac{1}{2}\Delta\mathbf{d}_s^T\Delta\mathbf{d}_s
 \end{aligned} \tag{40}$$

Taking the time derivative of the above function, we get

$$\begin{aligned}\dot{V} &= (\Delta\dot{\mathbf{x}} + \gamma\Delta\mathbf{x})^T(\Delta\ddot{\mathbf{x}} + \gamma\Delta\dot{\mathbf{x}}) + \Delta\dot{\mathbf{x}}^T(\mathbf{K}_p + \gamma\mathbf{K}_v \\ &\quad - \gamma^2\mathbf{I})\Delta\mathbf{x} + \Delta\dot{\mathbf{d}}_m^T \Delta\mathbf{d}_m + \Delta\dot{\mathbf{d}}_s^T \Delta\mathbf{d}_s\end{aligned}\quad (41)$$

Using (26), (38) and (39) in the above equation, we get

$$\begin{aligned}\dot{V} &= (\Delta\dot{\mathbf{x}} + \gamma\Delta\mathbf{x})^T[-\mathbf{K}_v\Delta\dot{\mathbf{x}} - \mathbf{K}_p\Delta\mathbf{x} + \\ &\quad \Psi_{xm}(\mathbf{q}_m)\Delta\mathbf{d}_m - \Psi_{xs}(\mathbf{q}_s)\Delta\mathbf{d}_s + \gamma\Delta\dot{\mathbf{x}}] + \\ &\quad \Delta\dot{\mathbf{x}}^T(\mathbf{K}_p + \gamma\mathbf{K}_v - \gamma^2\mathbf{I})\Delta\mathbf{x} + \\ &\quad [-\mathbf{L}_m\Delta\mathbf{d}_m - \Psi_{xm}^T(\mathbf{q}_m)(\Delta\dot{\mathbf{x}} + \gamma\Delta\mathbf{x})]^T \Delta\mathbf{d}_m + \\ &\quad [-\mathbf{L}_s\Delta\mathbf{d}_s + \Psi_{xs}^T(\mathbf{q}_s)(\Delta\dot{\mathbf{x}} + \gamma\Delta\mathbf{x})]^T \Delta\mathbf{d}_s \\ &= -\Delta\dot{\mathbf{x}}^T(\mathbf{K}_v - \gamma\mathbf{I})\Delta\dot{\mathbf{x}} - \gamma\Delta\mathbf{x}^T\mathbf{K}_p\Delta\mathbf{x} - \\ &\quad \Delta\mathbf{d}_m^T\mathbf{L}_m\Delta\mathbf{d}_m - \Delta\mathbf{d}_s^T\mathbf{L}_s\Delta\mathbf{d}_s\end{aligned}\quad (42)$$

According to (40) and the second condition of the theorem, the Lyapunov function V is positive definite in the entire state space $[\Delta\dot{\mathbf{x}}^T, \Delta\mathbf{x}^T, \Delta\mathbf{d}_m^T, \Delta\mathbf{d}_s^T]^T$ and is radially unbounded. The first three conditions of the theorem guarantee that \dot{V} is negative definite in the entire state space. Therefore, the velocity, position and disturbance tracking errors are globally asymptotically stable. Since $\lim_{t \rightarrow \infty} \Delta\dot{\mathbf{x}} = \mathbf{0}$, $\lim_{t \rightarrow \infty} \Delta\mathbf{x} = \mathbf{0}$, $\lim_{t \rightarrow \infty} \Delta\mathbf{d}_m = \mathbf{0}$ and $\lim_{t \rightarrow \infty} \Delta\mathbf{d}_s = \mathbf{0}$ and according to (26), we have $\lim_{t \rightarrow \infty} \Delta\ddot{\mathbf{x}} = \mathbf{0}$. Finally, (24) and (25) result in $\lim_{t \rightarrow \infty} \Delta\mathbf{f} = \mathbf{0}$. This concludes the proof. \blacksquare

The transparency achieved by the proposed control laws in (15) and (16) is a result of exchanging both position and force information between the master and the slave sides. This requires that we use all the four channels C_1 to C_4 in Figure 1 in order to transfer position and force data.

The next theorem states that the disturbance tracking and the position tracking errors of the teleoperation system, subject to slow-varying disturbances, can converge exponentially to zero under certain conditions.

Theorem 2. Consider the teleoperation system subject to disturbances described by (12) and (13). The master and the slave disturbance observers are given in (34) and (35). Under the control laws (22)–(23), the disturbance tracking and position tracking errors converge exponentially to zero provided that the conditions of Theorem 1 hold and $\gamma \leq \sqrt{\lambda_{\max}(\mathbf{K}_p + \gamma\mathbf{K}_v)}$ where $\lambda_{\max}(\cdot)$ represents the maximum eigenvalue of a matrix. ■

Proof: Consider the Lyapunov function proposed in the proof of Theorem 1. Let us define $\mathbf{e} = [\Delta\dot{\mathbf{x}}^T, \Delta\mathbf{x}^T, \Delta\mathbf{d}_m^T, \Delta\mathbf{d}_s^T]^T$. According to Rayleigh Inequality (see for example Theorem 2.5 in [28]), the Schwarz Inequality and from (40)

$$\begin{aligned} V(\mathbf{e}) &\leq \frac{1}{2}\|\Delta\dot{\mathbf{x}}\|_2^2 + \gamma\|\Delta\dot{\mathbf{x}}\|_2\|\Delta\mathbf{x}\|_2 + \\ &\quad \frac{1}{2}\lambda_{\max}(\mathbf{K}_p + \gamma\mathbf{K}_v)\|\Delta\mathbf{x}\|_2^2 + \\ &\quad \frac{1}{2}\|\Delta\mathbf{d}_m\|_2^2 + \frac{1}{2}\|\Delta\mathbf{d}_s\|_2^2 \end{aligned} \quad (43)$$

Note that if $\gamma \leq \sqrt{\lambda_{\max}(\mathbf{K}_p + \gamma\mathbf{K}_v)}$, we will have

$$\begin{aligned} &\frac{1}{2}(\|\Delta\dot{\mathbf{x}}\|_2 - \sqrt{\lambda_{\max}(\mathbf{K}_p + \gamma\mathbf{K}_v)}\|\Delta\mathbf{x}\|_2)^2 \\ &+ (-\gamma + \sqrt{\lambda_{\max}(\mathbf{K}_p + \gamma\mathbf{K}_v)})\|\Delta\dot{\mathbf{x}}\|_2\|\Delta\mathbf{x}\|_2 \geq 0 \\ &\Rightarrow \|\Delta\dot{\mathbf{x}}\|_2^2 + \lambda_{\max}(\mathbf{K}_p + \gamma\mathbf{K}_v)\|\Delta\mathbf{x}\|_2^2 \geq \\ &\quad \frac{1}{2}\|\Delta\dot{\mathbf{x}}\|_2^2 + \gamma\|\Delta\dot{\mathbf{x}}\|_2\|\Delta\mathbf{x}\|_2 + \frac{1}{2}\lambda_{\max}(\mathbf{K}_p + \gamma\mathbf{K}_v)\|\Delta\mathbf{x}\|_2^2 \end{aligned} \quad (44)$$

Using (44) in (43), we have

$$V(\mathbf{e}) \leq \mathbf{e}^T \mathbf{\Gamma}_1 \mathbf{e} \quad (45)$$

where $\mathbf{\Gamma}_1 = \text{diag}\{\mathbf{I}, \lambda_{\max}(\mathbf{K}_p + \gamma\mathbf{K}_v)\mathbf{I}, \frac{1}{2}\mathbf{I}, \frac{1}{2}\mathbf{I}\}$ represents a block-diagonal matrix with matrices

\mathbf{I} , $\lambda_{max}(\mathbf{K}_p + \gamma\mathbf{K}_v)\mathbf{I}$ and $\frac{1}{2}\mathbf{I}$ on its diagonals.

Taking the time derivative of the candidate Lyapunov function and similar to the calculations done in the proof of the Theorem 1, we get:

$$\dot{V} = -\mathbf{e}^T \underbrace{\text{diag}\{\mathbf{K}_v - \gamma\mathbf{I}, \gamma\mathbf{K}_p, \mathbf{L}_m, \mathbf{L}_s\}}_{\mathbf{\Gamma}_2} \mathbf{e} \quad (46)$$

According to Rayleigh inequality and equations (45) and (46), we have the following upper bound on the above positive definite candidate Lyapunov function and its time derivative:

$$V(\mathbf{e}) \leq \lambda_{max}(\mathbf{\Gamma}_1) \|\mathbf{e}\|_2^2 \quad (47)$$

and

$$\dot{V}(\mathbf{e}) \leq -\lambda_{min}(\mathbf{\Gamma}_2) \|\mathbf{e}\|_2^2 \quad (48)$$

where $\lambda_{min}(\cdot)$ represents the minimum eigenvalue of a given matrix. Therefore, we have

$$\begin{aligned} \dot{V}(\mathbf{e}) &\leq -\frac{\lambda_{min}(\mathbf{\Gamma}_2)}{\lambda_{max}(\mathbf{\Gamma}_1)} V(\mathbf{e}) \\ \Rightarrow V(\mathbf{e}) &\leq V(\mathbf{e}_0) \exp\left[-\frac{\lambda_{min}(\mathbf{\Gamma}_2)}{\lambda_{max}(\mathbf{\Gamma}_1)} t\right] \end{aligned} \quad (49)$$

The above inequality implies exponential convergence of position and disturbance tracking errors to the origin with a minimum rate determined by $-\frac{\lambda_{min}(\mathbf{\Gamma}_2)}{\lambda_{max}(\mathbf{\Gamma}_1)}$. ■

Remark 5. As it is shown in Theorem 2, the minimum exponential rate of convergence for the disturbance tracking and the position tracking errors to the origin is equal to $\frac{\lambda_{min}(\mathbf{\Gamma}_2)}{\lambda_{max}(\mathbf{\Gamma}_1)}$. If we define

$$\kappa_1 = \lambda_{\max}(\mathbf{\Gamma}_1) = \max\{1, \lambda_{\max}(\mathbf{K}_p + \gamma\mathbf{K}_v)\} \quad (50)$$

$$\begin{aligned} \kappa_2 = \lambda_{\min}(\mathbf{\Gamma}_2) = \min\{ & \lambda_{\min}(\mathbf{K}_v - \gamma\mathbf{I}), \\ & \gamma\lambda_{\min}(\mathbf{K}_p), \lambda_{\min}(\mathbf{L}_m), \lambda_{\min}(\mathbf{L}_s)\} \end{aligned} \quad (51)$$

One can simply determine the minimum rate of convergence by computing $\frac{\kappa_2}{\kappa_1}$.

◇

Now, we consider the case when the rate of change of disturbances is not negligible in comparison with the disturbance observer dynamics. In particular, we show that if the rate of change of the disturbances is bounded, the tracking errors will be globally uniformly ultimately bounded and the ultimate bound can be made as small as desired by using the design parameters. We claim that

Theorem 3. *Consider the teleoperation system subject to disturbances described by (12) and (13). The master and the slave disturbance observers are given in (34) and (35). Under the control laws (22)–(23), the tracking errors are globally uniformly ultimately bounded provided that*

- *The first three conditions of the Theorem 1 hold,*
- *The rate of change of disturbances is bounded, i.e., $\exists \zeta_m > 0, \zeta_s > 0$ such that $\|\dot{\mathbf{d}}_m(t)\|_2 < \zeta_m$ and $\|\dot{\mathbf{d}}_s(t)\|_2 < \zeta_s$ for $\forall t > 0$.*

■

Proof: Again, consider the Lyapunov function given in (40). We define the tracking error vector \mathbf{e} as in the proof of Theorem 2. Also, define κ_1 and κ_2 as in (50)–(51). Taking the time derivative of the candidate Lyapunov function and from (38)–(39), we get:

$$\dot{V} = -\mathbf{e}^T \mathbf{\Gamma}_2 \mathbf{e} + \dot{\mathbf{d}}_m^T \Delta \mathbf{d}_m + \dot{\mathbf{d}}_s^T \Delta \mathbf{d}_s \quad (52)$$

where $\mathbf{\Gamma}_2$ is defined in (46). Since $\|\dot{\mathbf{d}}_m(t)\|_2 < \zeta_m$ and $\|\dot{\mathbf{d}}_s(t)\|_2 < \zeta_s$ and according to Rayleigh

Inequality and Schwarz Inequality, we have

$$\dot{V} \leq -\kappa_2 \|\mathbf{e}(t)\|_2^2 + \zeta_m \|\Delta \mathbf{d}_m(t)\|_2 + \zeta_s \|\Delta \mathbf{d}_s(t)\|_2 \quad (53)$$

We define $\zeta = 2 \max(\zeta_m, \zeta_s)$. Since $\|\Delta \mathbf{d}_m(t)\|_2 \leq \|\mathbf{e}(t)\|_2$ and $\|\Delta \mathbf{d}_s(t)\|_2 \leq \|\mathbf{e}(t)\|_2$, we have

$$\begin{aligned} \dot{V} &\leq -\kappa_2 \|\mathbf{e}\|_2^2 + \zeta \|\mathbf{e}\|_2 \\ &= -\kappa_2(1 - \theta) \|\mathbf{e}\|_2^2 - \kappa_2 \theta \|\mathbf{e}\|_2^2 + \zeta \|\mathbf{e}\|_2 \end{aligned} \quad (54)$$

where $\theta \in (0, 1)$. Therefore, we have

$$\dot{V} \leq -\kappa_2(1 - \theta) \|\mathbf{e}\|_2^2, \quad \forall \|\mathbf{e}\|_2 \geq \frac{\zeta}{\theta \kappa_2} \quad (55)$$

As $V(\mathbf{e})$ is continuous, positive definite and radially unbounded, using Lemma 3.5 in [29] there exist class K_∞ functions ² $\alpha_1(\cdot)$ and $\alpha_2(\cdot)$ such that $\alpha_1(\|\mathbf{e}\|_2) \leq V(\mathbf{e}) \leq \alpha_2(\|\mathbf{e}\|_2)$. Using Corollary 5.1 in [29], we conclude that the tracking error is globally uniformly ultimately bounded with an ultimate bound determined by $\alpha_1^{-1}(\alpha_2(\frac{\zeta}{\theta \kappa_2}))$, i.e., there exists $T > 0$ such that $\|\mathbf{e}(t)\|_2 \leq \alpha_1^{-1}(\alpha_2(\frac{\zeta}{\theta \kappa_2}))$ for $\forall \mathbf{e}(0)$ and $\forall t \geq T$. ■

Therefore, the disturbance observer based control laws (22)–(23), with the disturbance observers given in (34)–(35), improve the performance of the teleoperation system in the presence of fast-varying disturbances by making the position and force tracking errors ultimately bounded. As it was shown in the proof of Theorem 3 the tracking error ultimate bound can be made as small as desired if $\kappa_2 = \min\{\lambda_{\min}(\mathbf{K}_v - \gamma \mathbf{I}), \gamma \lambda_{\min}(\mathbf{K}_p), \lambda_{\min}(\mathbf{L}_m), \lambda_{\min}(\mathbf{L}_s)\}$ is chosen to be sufficiently large, i.e., if the controller and the disturbance observer gains are chosen to be sufficiently large.

²A continuous function $\alpha(\cdot) : R^+ \rightarrow R^+$ is said to be in the class K_∞ if: (i) $\alpha(0) = 0$, (ii) it is strictly increasing, and (iii) $\alpha(r) \rightarrow \infty$ if $r \rightarrow \infty$.

IV. SIMULATION STUDY

In this section, computer simulations will illustrate effectiveness of the proposed control scheme. Both the master and the slave robots are considered to be planar two-link manipulators with revolute joints. The schematic diagram of the teleoperation system is shown in Figure 2.

The Cartesian dynamics of the manipulators are [25]

$$\mathbf{M}_x(\mathbf{q}) = \begin{bmatrix} m_2 + \frac{m_1}{s_2^2} & 0 \\ 0 & m_2 \end{bmatrix}$$

$$\mathbf{V}_x(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} V_{x_1}(\mathbf{q}, \dot{\mathbf{q}}) \\ V_{x_2}(\mathbf{q}, \dot{\mathbf{q}}) \end{bmatrix}$$

$$\mathbf{G}_x(\mathbf{q}) = \begin{bmatrix} m_1 g \frac{c_1}{s_2} + m_2 g s_{12} \\ m_2 g c_{12} \end{bmatrix}$$

where

$$\begin{aligned} V_{x_1} &= -(m_2 l_1 c_2 + m_2 l_2) \dot{q}_1^2 - m_2 l_2 \dot{q}_2^2 - \\ &\quad (2m_2 l_2 + m_2 l_1 c_2 + m_1 l_1 \frac{c_2}{s_2^2}) \dot{q}_1 \dot{q}_2 \\ V_{x_2} &= m_2 l_1 s_2 \dot{q}_1^2 + l_1 m_2 s_2 \dot{q}_1 \dot{q}_2 \end{aligned}$$

also, the forward kinematics and the Jacobian matrix are

$$\mathbf{h}(\mathbf{q}) = \begin{bmatrix} l_1 c_1 + l_2 c_{12} \\ l_1 s_1 + l_2 s_{12} \end{bmatrix}$$

$$\mathbf{J}(\mathbf{q}) = \begin{bmatrix} l_1 s_2 & 0 \\ l_1 c_2 + l_2 & l_2 \end{bmatrix}$$

where l_1 and l_2 are the lengths of the links, and m_1 and m_2 are the point masses of the links.

Also, we have $s_1 = \sin(q_1)$, $s_2 = \sin(q_2)$, $c_1 = \cos(q_1)$, $c_2 = \cos(q_2)$, $s_{12} = \sin(q_1 + q_2)$ and $c_{12} = \cos(q_1 + q_2)$.

In this simulation study, the remote environment and the human operator's hand are modeled as second-order LTI systems. We take

$$\begin{aligned}\mathbf{f}_h &= \mathbf{f}_h^* - (\mathbf{M}_h \ddot{\mathbf{x}}_m + \mathbf{B}_h \dot{\mathbf{x}}_m + \mathbf{K}_h \mathbf{x}_m) \\ \mathbf{f}_e &= \mathbf{f}_e^* + \mathbf{M}_e \ddot{\mathbf{x}}_s + \mathbf{B}_e \dot{\mathbf{x}}_s + \mathbf{K}_e \mathbf{x}_s\end{aligned}$$

where $\mathbf{M}_h = m_h \mathbf{I}$, $\mathbf{M}_e = m_e \mathbf{I}$, $\mathbf{B}_h = b_h \mathbf{I}$, $\mathbf{B}_e = b_e \mathbf{I}$, $\mathbf{K}_h = k_h \mathbf{I}$ and $\mathbf{K}_e = k_e \mathbf{I}$ are the mass, damping, and stiffness coefficients of the environment and the human operator's hand, and \mathbf{I} is the identity matrix. Also, \mathbf{f}_h^* and \mathbf{f}_e^* represent the human and the environment exogenous forces, respectively. The overall human hand parameters have been measured in several papers such as [11], [30] and [31]. In the simulations, the human hand parameters are chosen as in [30]. The remote environment is modeled by dampers and springs. These lead us to:

$$\begin{aligned}m_h &= 11.6 \text{ kg}, \quad b_h = 17 \text{ Nsm}^{-1}, \quad k_h = 243 \text{ Nm}^{-1} \\ b_e &= 5 \text{ Nsm}^{-1}, \quad k_e = 1 \text{ Nm}^{-1}\end{aligned}$$

The friction torques acting on the joints of the robots are generated based on the model in [32]. For the i -th joint of the robot, $i = 1, 2$, we have the frictions modeled as

$$\begin{aligned}\tau_{i \text{ friction}} &= F_{ci} \text{sgn}(\dot{q}_i) \left[1 - \exp\left(\frac{-\dot{q}_i^2}{v_{si}^2}\right) \right] \\ &\quad + F_{si} \text{sgn}(\dot{q}_i) \exp\left(\frac{-\dot{q}_i^2}{v_{si}^2}\right) + F_{vi} \dot{q}_i\end{aligned}$$

where F_{ci} , F_{si} , F_{vi} are the Coulomb, static, and viscous friction coefficients, respectively. The parameter v_{si} is the Stribeck parameter. In the simulations, the friction coefficients and the Stribeck parameter for the master and the slave are chosen as follows [7]:

$$F_{ci} = 0.49, F_{si} = 3.5, F_{vi} = 0.15, v_{si} = 0.189$$

$$i = 1, 2$$

We take the actual parameter values of the master and the slave robots to be

$$m_{1m} = 2.3kg, m_{2m} = 2.3kg, l_{1m} = 0.5m, l_{2m} = 0.5m$$

$$m_{1s} = 1.5kg, m_{2s} = 1.5kg, l_{1s} = 0.5m, l_{2s} = 0.5m$$

Assuming a maximum of $\pm 25\%$ uncertainty in these parameters, we take the approximate values of these master and the slave parameters to be

$$\hat{m}_{1m} = 2.82kg, \hat{m}_{2m} = 2.36kg, \hat{l}_{1m} = 0.62m, \hat{l}_{2m} = 0.56m$$

$$\hat{m}_{1s} = 1.16kg, \hat{m}_{2s} = 1.29kg, \hat{l}_{1s} = 0.38m, \hat{l}_{2s} = 0.52m$$

The controllers C_m, C_s, C_1, \dots , and C_6 are chosen as in (19), (20) and (21). The controllers and disturbance observer gains and initial conditions are chosen as

$$\gamma = 1$$

$$\mathbf{K}_{mv} = 50\mathbf{I} \quad , \quad \mathbf{K}_{sv} = 50\mathbf{I}$$

$$\mathbf{K}_{mp} = 50\mathbf{I} \quad , \quad \mathbf{K}_{sp} = 50\mathbf{I}$$

$$\mathbf{C}_{mf} = \mathbf{I} \quad , \quad \mathbf{C}_{sf} = \mathbf{I}$$

$$\mathbf{L}_m = 50\mathbf{I} \quad , \quad \mathbf{L}_s = 50\mathbf{I}$$

$$\hat{\mathbf{d}}_{0m} = \mathbf{0} \quad , \quad \hat{\mathbf{d}}_{0s} = \mathbf{0}$$

$$\Rightarrow$$

$$\mathbf{K}_v = 25\mathbf{I} \quad , \quad \mathbf{K}_p = 25\mathbf{I}$$

The above gains satisfy the conditions in the Theorem 1 and the Theorem 2. Note that the master and the slave controller and observer gains have been chosen to be equal– this choice is not necessary but is one that results in identical closed-loop master and slave dynamics (24) and

(25) when $\Delta \mathbf{d}_m = \mathbf{0}$ and $\Delta \mathbf{d}_s = \mathbf{0}$.

We choose different values for initial joint positions of the master and the slave while assuming that both robots are initially at rest. We take the initial joint position vectors to be

$$\begin{aligned}\mathbf{q}_{0m} &= [30^\circ, 45^\circ]^T \\ \mathbf{q}_{0s} &= [0, 22.5^\circ]^T\end{aligned}$$

We take $\mathbf{f}_e^* = [0, 0]^T$ and assume that the human operator moves the master end-effector such that the second joint of the master robot moves from 45° to 81.5° while the position of the first joint of the master robot is fixed. The slave end-effector should follow the position of the master end-effector. We assume that an external payload, with a mass equal to 0.5^{kg} , is connected to the end-effector of the slave. Also, we assume that a sinusoidal disturbance torque equal to $\tau_d = \sin(4\pi t)$ is exerted to the second joint of the slave robot. Sinusoidal disturbances are exerted to the joints of a robot in order to examine the efficiency of fault tolerant control schemes (see, for example, [33]). These external disturbances are exerted to the slave robot in addition to the disturbances due to joint frictions and dynamic uncertainties.

Figures 3 and 4 show the position tracking responses of the teleoperation system in presence and absence of the disturbance observers. Because of the sinusoidal disturbance acting on the second joint of the slave robot, the slave robot end-effector payload, the friction forces and the dynamic uncertainties present in the master and the slave, the control law without using disturbance observers fails to achieve good position tracking. As it can be observed from Figures 3 and 4, perfect position tracking has been achieved under the disturbance observer based control scheme while there are offsets in the position tracking error under the conventional control scheme.

Figure 5 shows the force tracking responses of the teleoperation system in presence and absence of the disturbance observers. The force tracking of the teleoperation system under the disturbance observer based control scheme has been significantly improved in comparison with the conventional scheme. Figures 6 and 7 show the disturbance tracking of the disturbance observer at the master and the slave sides. As it can be observed from these figures, the disturbances are not constant at the slave side. However, the estimated disturbances tend to

follow the actual disturbances with a bounded tracking error (see Theorem 3).

V. CONCLUSION

In this paper, a novel control scheme has been proposed for nonlinear teleoperation systems subject to dynamic uncertainties and disturbances. The designed nonlinear disturbance observer based controller is incorporated into the 4-channel bilateral teleoperation control framework. The proposed control scheme is able to decrease the adverse effects of the disturbances on the stability and transparency of the teleoperation system. Full transparency in terms of asymptotic convergence of the position tracking and force tracking errors to zero, and exponential convergence of disturbance tracking and position tracking errors to zero are achieved under slow-varying disturbances. In this case, the minimum exponential convergence rate can be determined by the proposed controller parameters. When the teleoperation system is subject to fast-varying disturbances, the proposed control scheme improves transparency and guarantees uniform ultimate boundedness of the teleoperation system tracking errors. Computer simulations are done to show the effectiveness of the proposed approach.

ACKNOWLEDGMENTS

This work was supported by the Natural Sciences and Engineering Research Council of Canada (NSERC).

REFERENCES

- [1] Sheridan, T. B.: 'Telerobotics', *Automatica*, 1989, 25, (4), pp. 487-507
- [2] Salcudean, S.: 'Control for teleoperation and haptic interfaces', in Siciliano, B. and Valavanis, K. P. (Eds.): 'Control Problems in Robotics and Automation' (New York : Springer-Verlag, 1998, vol. LNCIS 230), pp. 51-66
- [3] Lawrence, D. A.: 'Stability and transparency in bilateral teleoperation', *IEEE Trans. Robot. Autom.*, 1993, 9, pp. 624-637
- [4] Hashtrudi-Zaad, K., and Salcudean S. E.: 'Transparency in time delay systems and the effect of local force feedback for transparent teleoperation', *IEEE Trans. Robot. Autom.*, 2002, 18, (1), pp. 108-114
- [5] Tavakoli, M., Aziminejad, A., Patel, R. V., and Moallem, M.: 'High-fidelity bilateral teleoperation systems and the effect of multimodal haptics', *IEEE Trans. Syst., Man Cybern. B, bern.*, 2007, 37, (6), pp. 1512-1528
- [6] Mahvash, M., and Okamura, A.: 'Friction compensation for enhancing transparency of a teleoperator with compliant transmission', *IEEE Trans. Robot.*, 2007, 23, (6), pp. 1240-1246
- [7] Hung, N. V. Q., Tuan, H. D., Narikiyo, T., and Apkarian, P.: 'Adaptive Control for Nonlinearly Parameterized Uncertainties in Robot Manipulators', *IEEE Trans. Cont. Syst. Technol.*, 2008, 16, (3), pp. 458-468

- [8] Ryu, J. H., and Kwon, D. S.: 'A novel adaptive bilateral control scheme using similar closed-loop dynamic characteristics of master/slave manipulators', *J. Robot. Syst.*, 2001, 18, (9), pp. 533-543
- [9] Hung, N. V. Q., Narikiyo, T., and Tuan, H. D.: 'Nonlinear adaptive control of master-slave system in teleoperation', *Control Eng. Practice*, 2003, 11, (1), pp. 1-10
- [10] Chopra, N., Spong, M. W., and Lozano, R.: 'Synchronization of bilateral teleoperators with time delay', *Automatica*, 2008, 44, (8), pp. 2142-2148
- [11] Zhu, W. H., and Salcudean, S. E.: 'Stability guaranteed teleoperation: An adaptive motion/force control approach', *IEEE Trans. Autom. Control*, 2000, 45, (11), pp. 1951-1969
- [12] Sirouspour, S., and Setoodeh, P.: 'Adaptive nonlinear teleoperation control in multi-master/multi-slave environments'. *Proc. IEEE Int. Conf. Control Applicat.*, Toronto, Canada, Aug 2005, pp. 1263-1268
- [13] Malysz, P., and Sirouspour, S.: 'Nonlinear and filtered force/position mappings in bilateral teleoperation with application to enhanced stiffness discrimination', *IEEE Trans. Robot.*, 2009, 25, (5), pp. 1134-1149
- [14] Liu, X., Tavakoli, M., and Huang, Q.: 'Nonlinear adaptive bilateral control of teleoperation systems with uncertain dynamics and kinematics'. *Proc. IEEE/RSJ Int. Conf. Intell. Robots Syst.*, Taipei, Taiwan, Oct 2010, pp. 4244-4249
- [15] Komada, S., Ishida, M., Ohnishi, K., and Hori, T.: 'Disturbance observer-based motion control of direct drive motors', *IEEE Trans. Energy Convers.*, 1991, 6, (3), pp. 553-559
- [16] Ohnishi, K., Shibata, M., and Murakami, T.: 'Motion control for advanced mechatronics', *IEEE/ASME Trans. Mechatron.*, 1996, 1, (1), pp. 56-67
- [17] Liu, C. S., and Peng, H.: 'Disturbance observer based tracking control', *J. Dynamic Syst. Measurement and Control-Trans. of The ASME*, 2000, 122, (2), pp. 332-335
- [18] Eom, K. S., Suh, I. H., and Chung, W. K.: 'Disturbance observer based path tracking control of robot manipulator considering torque saturation'. *Proc. Int. Conf. Advanced Robot.*, Monterey, CA, Jul 1997, pp. 651-657
- [19] Sankaranarayanan, S., and Khorrami, F.: 'Model independent friction compensation'. *Proc. American Control Conf.*, Philadelphia, PA, Jun 1998, pp. 463-467
- [20] Zhang, G. G., and Furusho, J.: 'Control of robot arms using joint torque sensors'. *Proc. IEEE Int. Conf. Robot. Autom.*, Albuquerque, NM, Apr 1997, pp. 3148-3153
- [21] Komada, S., and Ohnishi, K.: 'Force feedback-control of robot manipulator by the acceleration tracing orientation method', *IEEE Trans. Ind. Electron.*, 1990, 37, (1), pp. 6-12
- [22] Murakami, T., Yu, F. M., and Ohnishi, K.: 'Torque sensorless control in multidegree-of-freedom manipulator', *IEEE Trans. Ind. Electron.*, 1993, 40, (2), pp. 259-265
- [23] Chen, W. H., Ballance, D. J., Gawthrop, P. J., and O'Reilly, J.: 'A nonlinear disturbance observer for robotic manipulators', *IEEE Trans. Ind. Electron.*, 2000, 47, (4), pp. 932-938
- [24] Matsumoto, Y., Katsura, S., and Ohnishi, K.: 'An analysis and design of bilateral control based on disturbance observer'. *Proc. IEEE Int. Conf. Ind. Technol.*, Maribor, Slovenia, Dec 2003, pp. 802-807
- [25] Craig, J. J.: 'Introduction to Robotics: Mechanics and Control' (Pearson Prentice Hall, 3rd edn. 2005)
- [26] Levant, A.: 'Robust exact differentiation via sliding mode technique', *Automatica*, 1998, 34, (3), pp. 379-384
- [27] Levant, A.: 'Higher order sliding: differentiation and black-box control'. *Proc. IEEE Conf. Decision and Control*, Sydney, Australia, Dec 2000, pp. 1703-1708
- [28] Marquez, H. J.: 'Nonlinear Control Systems' (John Wiley & Sons, Inc., 1st edn. 2003)
- [29] Khalil, H. K.: 'Nonlinear Systems' (Prentice Hall, 2nd edn. 1996)

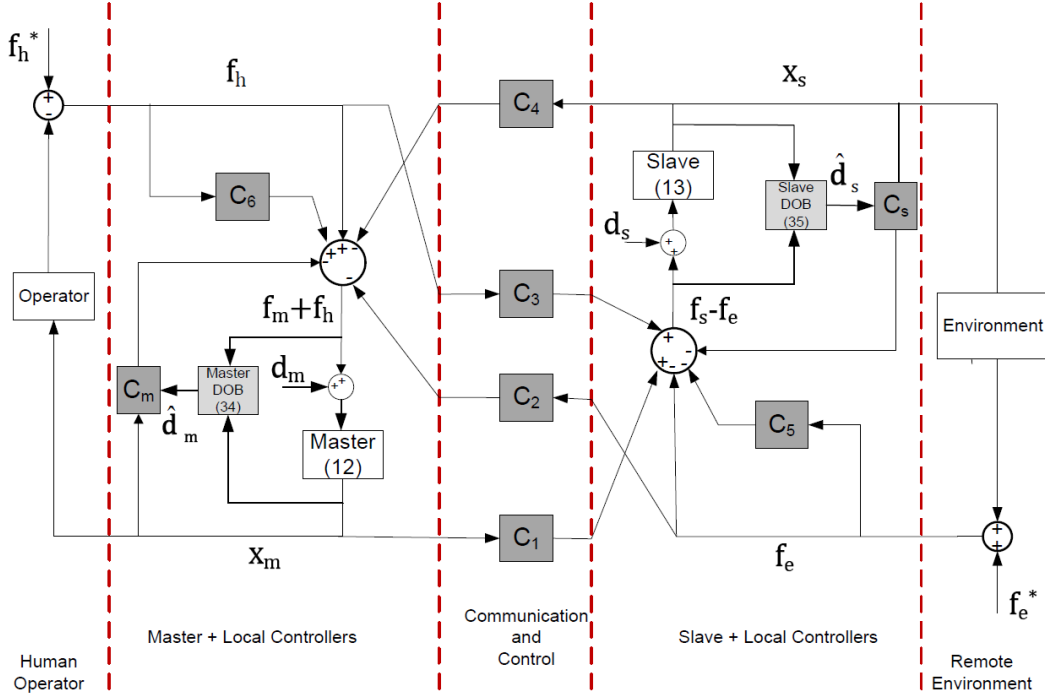


Fig. 1. Disturbance observer based 4-channel teleoperation control system.

[30] Kosuge, K., Fujisawa, Y., and Fukuda, T.: 'Control of mechanical system with man-machine interaction'. Proc. IEEE/RSJ Int. Conf. Intell. Robots Syst., Raleigh, USA, Jul 1992, pp. 87-92

[31] Speich, J. E., Shao, L., and Goldfarb, M.: 'Modeling the human hand as it interacts with a telemanipulation system', Mechatronics, 2005, 15, (9), pp. 1127-1142

[32] Armstrong-Hélouvy, B., Dupont, P., and Canudas de Wit, C.: 'A survey of models, analysis tools and compensation methods for the control of machines with friction', Automatica, 1994, 30, (7), pp. 1083-1138

[33] Bonivento, C., Gentili, L., and Paoli, A.: 'Internal model based fault tolerant control of a robot manipulator'. Proc. IEEE Conf. Decision and Control, Paradise Island, Bahamas, Dec 2004, pp. 5260-5265

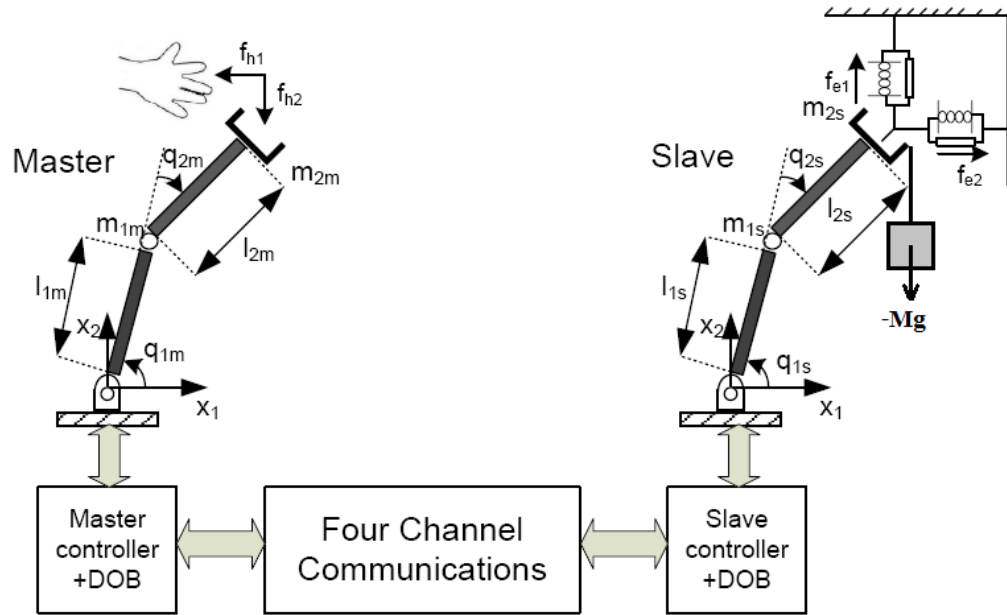


Fig. 2. Schematic diagram of the teleoperation system used in simulation.

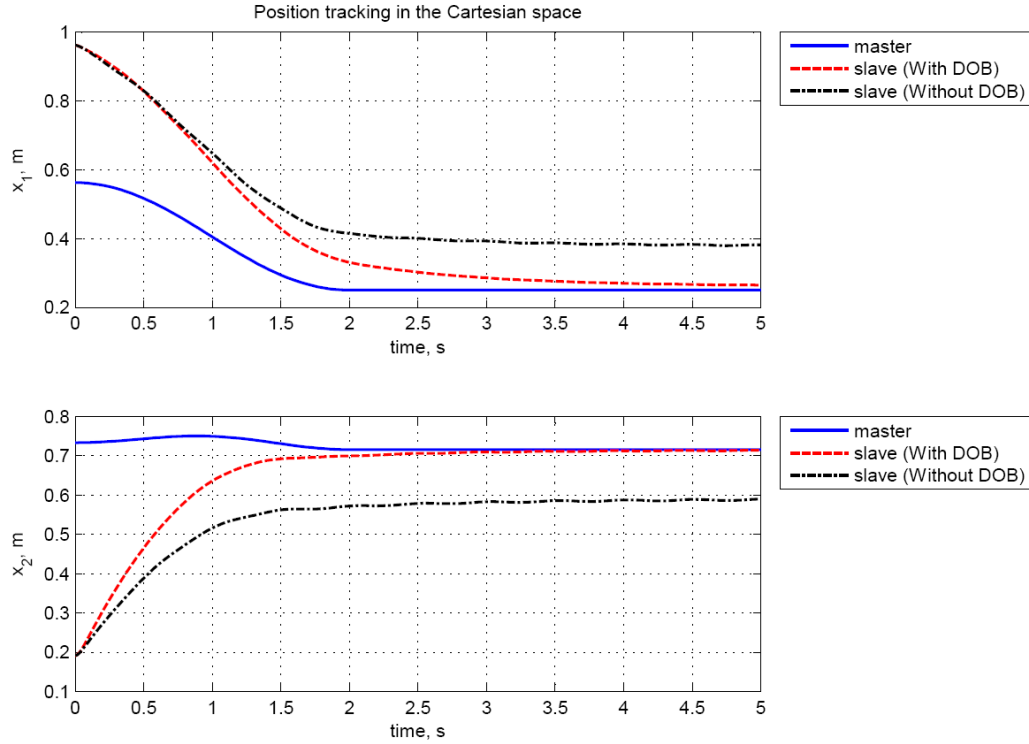


Fig. 3. Position tracking of the teleoperation system with and without disturbance observer.

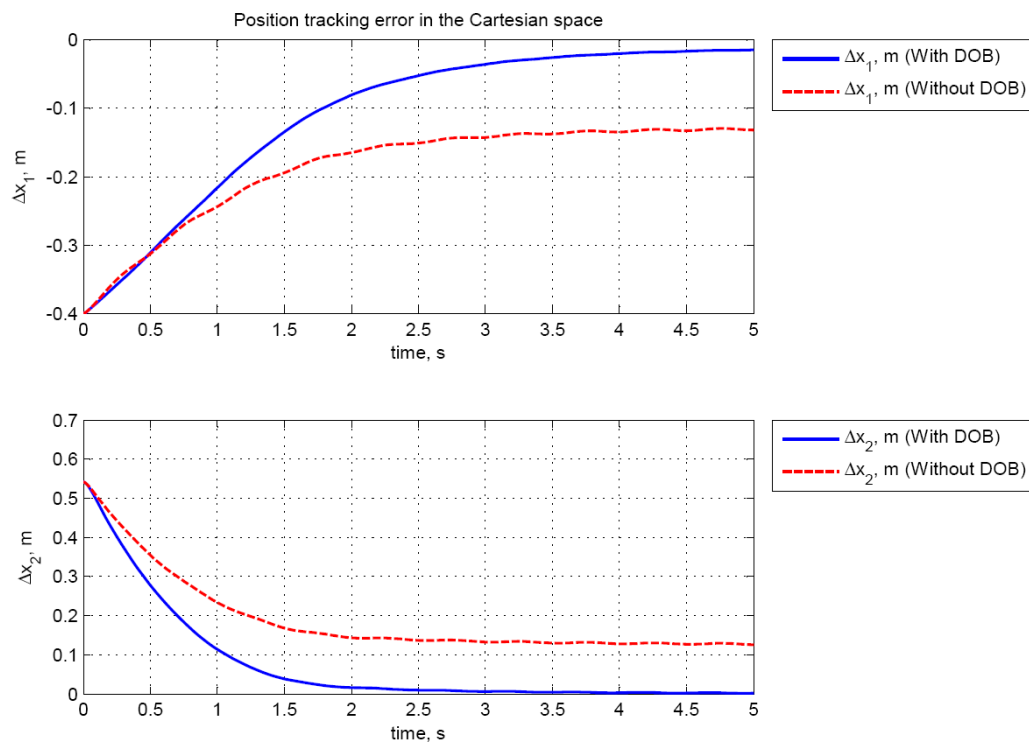


Fig. 4. Position tracking error of the teleoperation system with and without disturbance observer.

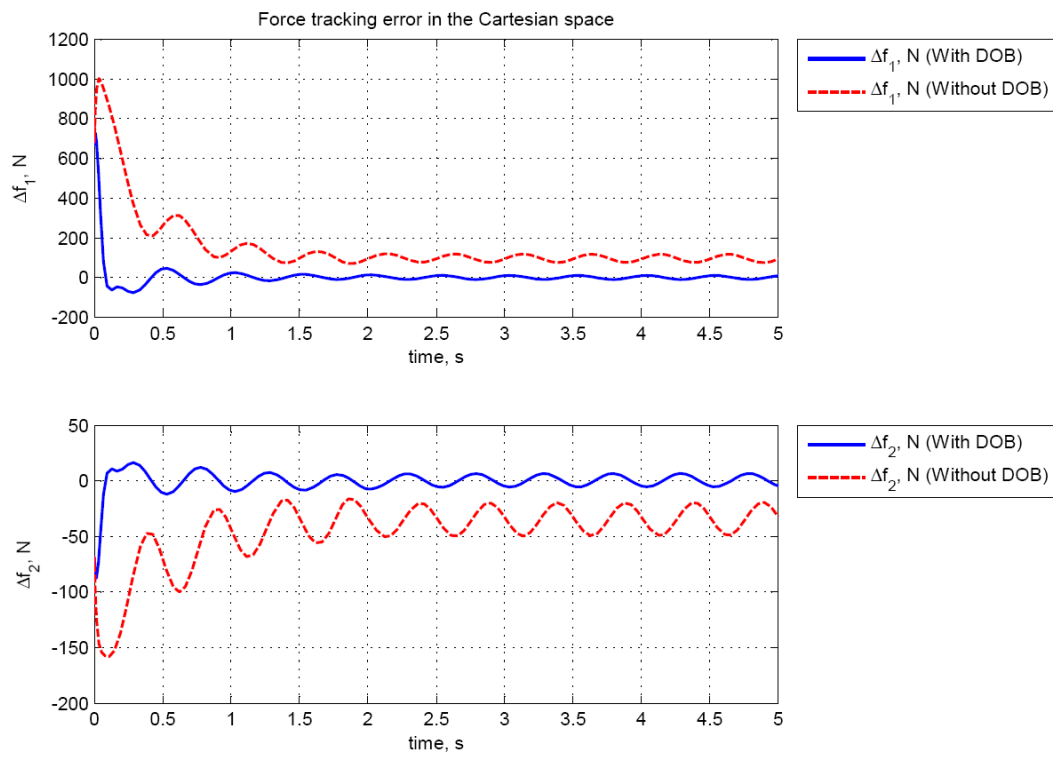


Fig. 5. Force tracking of the teleoperation system with and without disturbance observer.

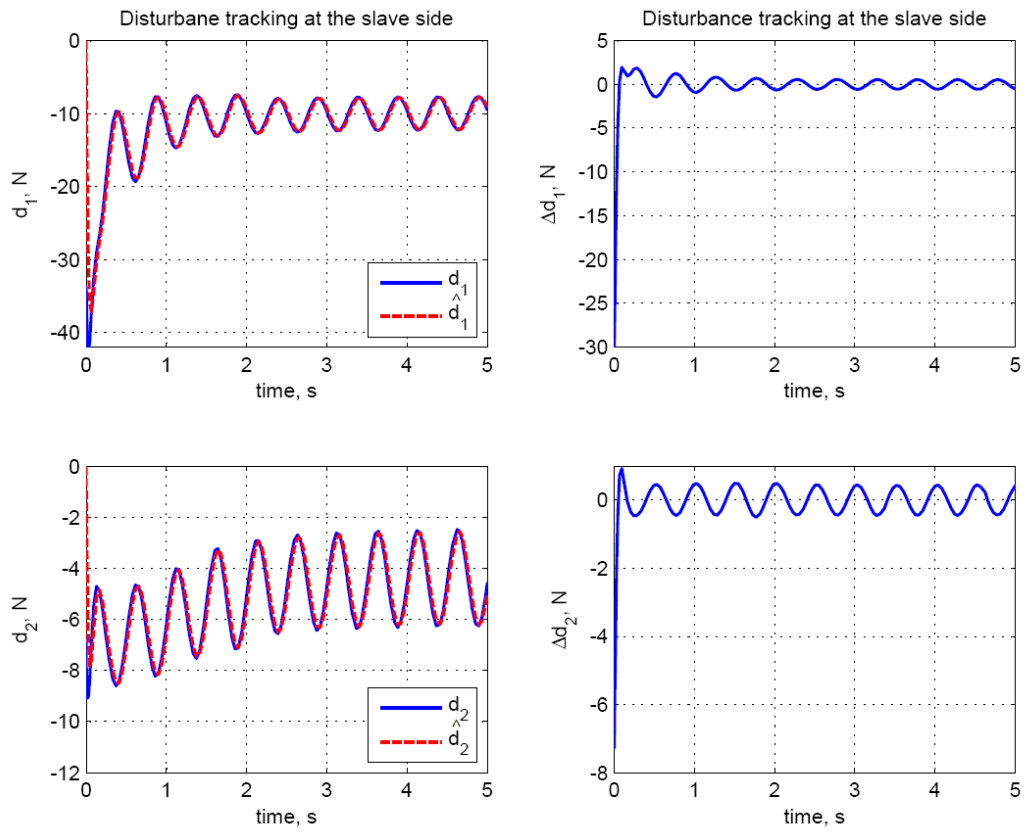


Fig. 6. Disturbance tracking at the slave side.

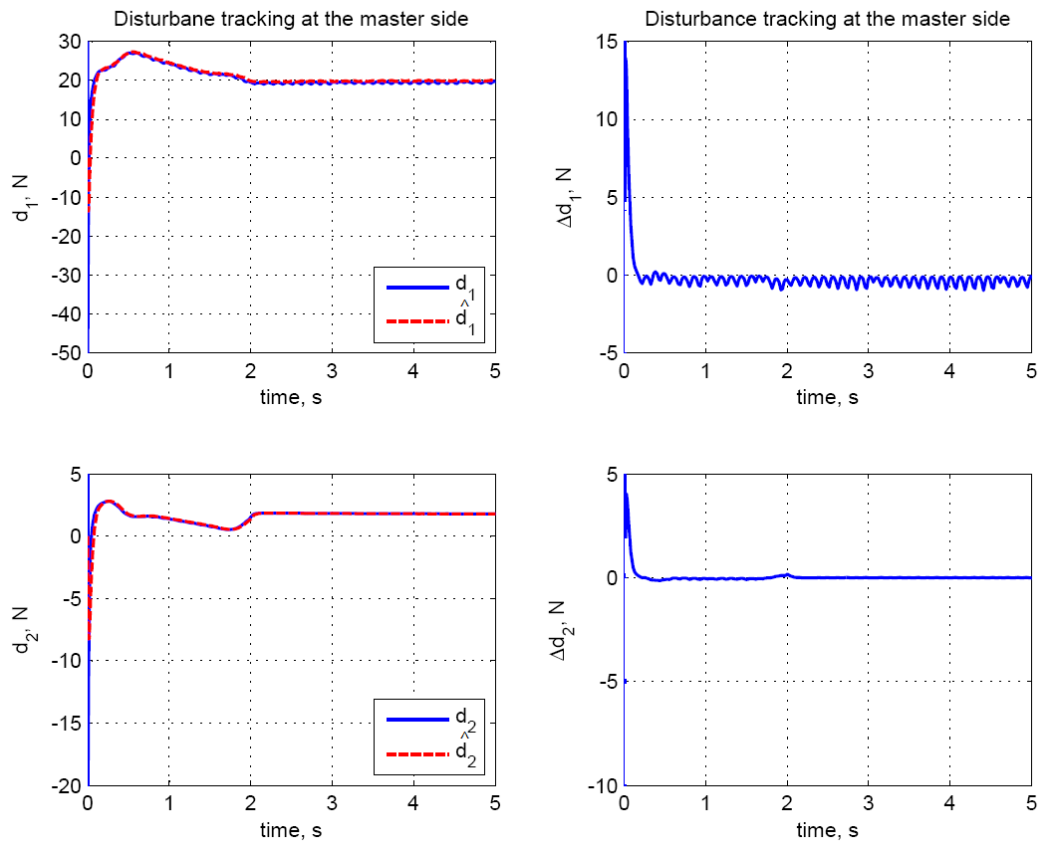


Fig. 7. Disturbance tracking at the master side.