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Stability of Cooperative Teleoperation Using Haptic Devices with Complementary Degrees of Freedom

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Abstract

In bilateral teleoperation of a dexterous task, to take full advantage of the human's intelligence, experience, and sensory inputs, a possibility is to engage multiple human arms through multiple masters (haptic devices) in controlling a single slave robot with high degrees-of-freedom (DOF); the total DOFs of the masters will be equal to the DOFs of the slave. A multi-master/single-slave cooperative haptic teleoperation system with w DOFs can be modeled as a two-port network where each port (terminal) connects to a termination defined by w inputs and w outputs. The stability analysis of such a system is not trivial due to dynamic coupling across the different DOFs of the robots, the human operators, and the physical or virtual environments. The unknown dynamics of the users and the environments exacerbate the problem. We present a novel, straightforward and convenient frequency-domain method for stability analysis of this system. As a case study, two 1-DOF and 2-DOF master haptic devices are considered to teleoperate a 3-DOF slave robot. It is qualitatively discussed how such a trilateral haptic teleoperation system may result in better task performance by splitting the various DOFs of a dexterous task between two arms of a human or two humans. Simulation and experimental results demonstrate the validity of the stability analysis framework.

Index Terms

Cooperative teleoperation, absolute stability.

I. Introduction

Robotic manipulators with multiple degrees of freedom (DOF) have recently found many applications such as in robotic-assisted surgery and therapy, space exploration, and navigation systems [1], [2], [3]. For successful teleoperation of a multi-DOF task, it is generally assumed that the human is always in the loop in the sense that every move made to the master by the human has been informed by continuous visual and haptic updates received from the slave, and that the teleoperator comprised

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of the master, the slave, the controllers, and the communication channel is transparent. Then, the human would feel as if he/she is performing the task via direct touch with the environment while actually doing it via the teleoperation system. In other words, given a multi-DOF slave robot in a conventional teleoperation setting, if the human controls it and receives haptic feedback via a master robot with the same DOFs, it is expected that the natural sensorimotor capabilities of the human in terms of execution of multi-DOF tasks is transferred to teleoperation. Even more, it is expected that due to the super-human capabilities of machines such as tremor filtering, high-accuracy positioning, and motion or force scaling, the task performance in teleoperation is better than that in direct touch.

In practice, despite the vast amount of research aimed at teleoperation transparency enhancement in recent years [4], there are still many electromechanical transparency-limiting imperfections in teleoperators including communication delays [5], [6], model uncertainties, model nonlinearities [7], control sampling [8], sensor quantization [9], and actuator switching [10]. Also, while the teleoperator amplifies certain skills of the human, it may attenuate some other skills. The bottom line is that successful completion of multi-DOF tasks via haptic teleoperation system is still harder than that in the case of direct touch.

To mitigate the above problem, haptic virtual fixtures as software-generated forces have been used to guide the human through a task with a specified path. A virtual fixture effectively creates motion constraints in a subset of the haptic device's DOFs, allowing the human to focus on the remaining DOFs, which can result in improved task performance. An informative examples is given in [11] where for retinal vein cannulation, a needle of about 20-50 μm in diameter must be inserted into the lumen of a retinal vein, which is about 100 μm in diameter. Given the very small scale at which the task is to be executed, there is a need to drastically enhance the precision of human motion. A virtual fixture can do so by constraining the needle motion in the lateral directions to stabilize the human hand while allowing the needle motion in the axial direction.

As explained above, virtual fixtures deliberately but temporarily eliminate a subset of the haptic device's DOFs in order to improve the performance of a multi-DOF task. An alternative strategy conjectured in this paper is to allow multiple human arms (e.g., two hands of a human) to manipulate multiple haptic devices, each of which provides a subset of the DOFs required in the task. This can offer advantages over single-handed, virtual-fixture-based assistance. First, for a multi-DOF slave robot such as the Kinova arm for performing dexterous manipulation tasks by the disabled [12], two master haptic devices help make most of the limited but possibly complementary motions of each of the patient's two hands. Second, using two (or more) haptic devices allows to separate the required motions in a multi-DOF task into gross vs. fine, position-controlled vs. force-controlled, translational vs. rotational, fast vs. slow, etc. This separation has the potential to improve dexterous task performance. Third, building a highly-dexterous master for every new application can be very costly whereas combining the capabilities of two (or more) less-dexterous, off-the-shelf masters can be a more affordable solution. These facts provide the motivation for considering cooperative teleoperation systems that involve two or more haptic devices with complementary degrees of freedom for performing dexterous tasks [13]. In this paper, we are interested in the stability analysis of such cooperative haptic teleoperation systems.

Closed-loop system stability is critical for safe and effective teleoperation. However, investigation of teleoperation system stability using common closed-loop stability analysis tools in the control systems literature is not possible because the models of the human and the environment are usually unknown, uncertain, and/or time-varying. Research has proved it possible to draw stability conditions for a haptic teleoperation system under unknown human and environment as long as they are passive [14], [15], [16], [17], [18]. While this is referred to as absolute or unconditional stability in the literature, for brevity we call it stability in this paper. In the literature, control schemes have been proposed for conventional multi-DOF teleoperation systems [19], [20], [21], [22]. Lee and Li in [19] presented a passive bilateral feedforward control scheme for linear dynamically similar teleoperated manipulators with kinematic and power scaling. In [20], Lee and Li studied a passive bilateral teleoperation control law for a pair of multi-DOF nonlinear robotic systems. In [21], Speich and Goldfarb proposed a teleoperation control architecture for a 3-DOF scaled masterslave system. Also, Kim et al. in [22] proposed a control framework that decoupled a multi-DOF bilateral teleoperation system such that 1-DOF bilateral teleoperation system stability criteria can be applied independently to each DOF.

In the above studies, only single-master/single-slave, multi-DOF teleoperation systems were considered. When the slave is controlled by two or more master devices, the stability analysis cannot be done using the above techniques. In [23], two masters in interaction with different DOFs of a slave were considered. The authors proposed a control structure using which, the slave is *decoupled* to two parts, each of which shares the same DOFs with one of the masters. It is desirable to have stability criteria that do not necessarily require the aforementioned decoupling to take place. In general, a multi-master/single-slave w-DOF teleoperation system can be modeled as a $2w$-port network. This means that the class of systems we consider is that of n-port network represented by a frequency-domain, linear time-invariant model called the impedance matrix. An "n-port network" is a network (e.g., mechanical device or electrical circuit) with n pairs of terminals (ports), each of which is connected to an external network. Thus, each terminal constitutes an input/output interface where the network connects to another network or "termination". The n-port network together with its n terminations constitute a "coupled system".

In this paper, we present a novel approach for stability analysis of multi-user cooperative teleoperation systems with multiple haptic devices with complementary DOFs. As a case study, we consider a 1-DOF robot and a 2-DOF robot as the two masters and a 3-DOF robot as the slave in a bilateral teleoperation system, and invoke the proposed stability criterion to design stabilizing teleoperation controllers for the system.

This paper is organized as follows. The next section discusses the example of dual-master/single-slave, 3-DOF teleoperation system for performing a peg-in-the-hole task and further justifies the utility of cooperative teleoperation as opposed to conventional teleoperation. Section III gives mathematical definitions and lemmas for analysis of stability. Section IV models a teleoperation system with multiple haptic devices with complementary motions. Next, in Section V, the proposed frequency-domain stability analysis method for multi-master/single-slave, w-DOF haptic systems is derived. Then, as a case study to show how the proposed stability criterion can be utilized, in Section VI, a dual-master/single-slave, 3-DOF teleoperation system with position-position control is considered. The stability conditions in terms of system parameters including controller gains are found, and simulations and experiments to verify the validity of the calculated stability conditions are presented. Section VII contains concluding remarks.

II. A Motivating Example

In the following, for a dual-master/single-slave 3-DOF teleoperation system, we discuss how splitting the degrees of freedom of a dexterous task between two user interfaces (1-DOF and 2-DOF) can result in better task performance. For such a system, planar peg-in-the-hole insertion is an interesting manipulation task. As shown in Figure 1, this is a 3-DOF task that involves two translations (y and z) and one rotation (ϕ). We will consider two distinct cases: (i) A 1-DOF master 1 is manipulated by a hand of the human to control ϕ while a 2-DOF master 2 is manipulated by the other hand of the human (or by another human) to control y and z; (ii) A 3-DOF master is manipulated by a single hand of the human to control the y, z and ϕ. In both cases, a 3-DOF slave robot holds the peg and performs the y, z and ϕ maneuvers based on the position commands from the corresponding degrees of freedom of the master(s).

In case (i), as shown in Figure 1, the procedure is divided into three steps. In the first step, the peg is moved at an angle toward the hole; this step is completed when the peg makes contact with the edge of the hole. In the second step, an insertion force is applied in the z direction and a force is applied in the y direction to maintain a contact between the peg and the hole's edge. At the same time, the peg is turned along the ϕ direction to become co-axial to the hole axis. This step is completed once the peg is aligned with and slightly inside the hole. In the third step, an insertion force in the z direction is applied while the lateral force in the y direction and the moment in the ϕ direction are kept to zero. Clearly, master 2 controls the first and the third steps while master 1 controls the second step (the role of master 2 during the second step is only to maintain contact and does not involve considerable motions). This provides a tangible separation between the translational and rotational degrees of freedom in order to simplify the performance of this dexterous task. On the other hand, in case (ii), the procedure may take longer because the peg can more easily be jammed inside the hole as the master can inadvertently move in an unintended degree of freedom. For instance, in the third step, the human may cause ϕ motions when he/she tries to apply z motions simply because both motions are controlled from the same user interface [24]. Obviously, two-handed teleoperation is more effective in this task. As to be the best of authors' knowledge no work has been done on direct absolute stability analysis of multi-master/single slave coupled teleoperators with complementary motions, we propose a novel absolute stability criterion for such systems.

III. Mathematical Preliminaries

Notation 1. *a is a scalar, A is a vector, $\mathbf{A}$ is a matrix, and $\mathcal{A}$ is a block matrix (i.e., with matrix elements) or a block vector (i.e., with vector elements).*

Definition 1. *A n-port network is stable (weakly stable) if the coupled system remains bounded-input bounded-output stable under all possible passive (strictly passive) terminations.*

Definition 2. *[17] A n-port network is passive (strictly passive) if the total energy delivered to the network at its ports is non-negative (positive).*

Property 1. *A Hermitian matrix, i.e., a square matrix equal to its conjugate transpose, is positive definite (positive semidefinite) if its principal minors are all positive (non-negative).*

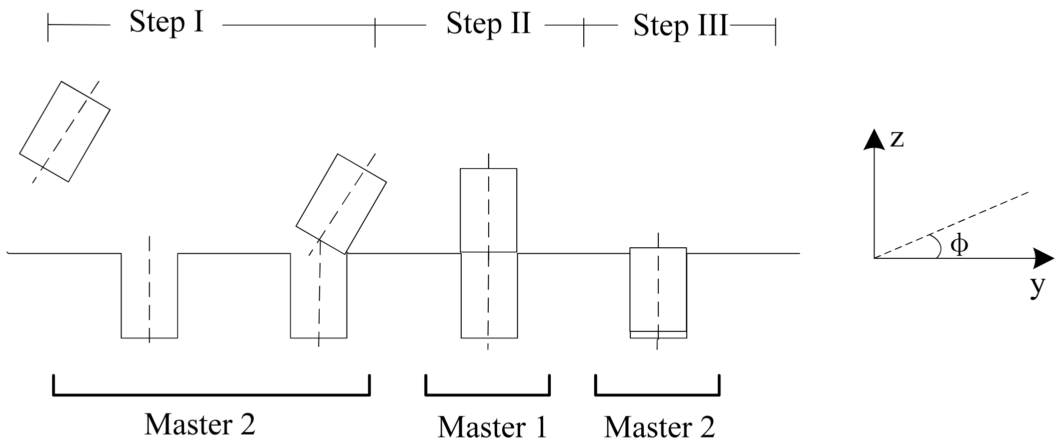

Figure 1. The insertion procedure.

Definition 3. *[25] A $n \times n$ proper rational transfer matrix $\mathbf{G(s)}$ is positive real if*

i) *Poles of all elements of $\mathbf{G(s)}$ are in $\mathrm{Re}[s] \leq 0$,*

ii) *Any pure imaginary pole $j\omega$ of any element of $\mathbf{G(s)}$ is a simple pole and the residue matrix $\lim_{s \to j\omega}(s - j\omega)\mathbf{G(s)}$ is positive semidefinite Hermitian,*

iii) *For all real ω for which $j\omega$ is not a pole of any element of $\mathbf{G(s)}$, the matrix $\mathbf{G}(j\omega) + \mathbf{G}^T(-j\omega)$ is positive semidefinite.*

Lemma 1. *[25] A linear time-invariant minimal realization model with transfer matrix $\mathbf{G(s)}$ is passive (strictly passive) if $\mathbf{G(s)}$ is non-negative real (positive real).*

Lemma 2. *[26] Let $\mathbf{Z} = \mathbf{Z}^{\mathrm{T}}$ be the impedance matrix of a reciprocal n-port network [27]. Then, the network is passive (strictly passive) if and only if it is weakly stable (stable).*

Lemma 3. *[28] Let $\mathbf{Z_1}$ and $\mathbf{Z_2}$ be the impedance matrices of two n-port networks. Then, if $\mathbf{Z_1}$ and $\mathbf{Z_2}$ possess identical principal minors of all orders, then $\mathbf{Z_1}$ is stable (weakly stable) if and only if $\mathbf{Z_2}$ is stable (weakly stable).*

Another way to look at Definition 1 is that an n-port network is stable (weakly stable) if the port currents $I_1, I_2, \cdots, I_n$ are zero under all passive (strictly passive) terminations $z_1, z_2, \cdots, z_n$ for ports [26]. In other words, an n-port network with an impedance matrix $\mathbf{Z_{n\times n}}$ is stable (weakly stable) if and only if the equation $(\mathbf{Z} + \mathbf{Z_0})\mathbf{I} = \mathbf{0}$, where $I = [I_1, I_2, \cdots, I_n]^T$ and $Z_0 = \mathrm{diag}[z_1, z_2, \cdots, z_n]$, has only the trivial solution $I = 0$ for every passive (strictly passive) choice of $\mathbf{Z_0}$; this happens if and only if $\det(\mathbf{Z} + \mathbf{Z_0}) \neq \mathbf{0}$. Now, according to [28], if two $n \times n$ matrices $\mathbf{Z_1}$ and $\mathbf{Z_2}$ have identical principal minors of all orders, then

$$\det(\mathbf{Z_1} + \mathbf{Z_0}) = \det(\mathbf{Z_2} + \mathbf{Z_0}) \tag{1}$$

for any $\mathbf{Z_0} = \mathrm{diag}[\mathbf{z_1}, \mathbf{z_2}, \cdots, \mathbf{z_n}]$. This implies that the stability (weak stability) of two n-port networks with impedance matrices $\mathbf{Z_1}$ and $\mathbf{Z_2}$ will happen at the same time (Lemma 3).

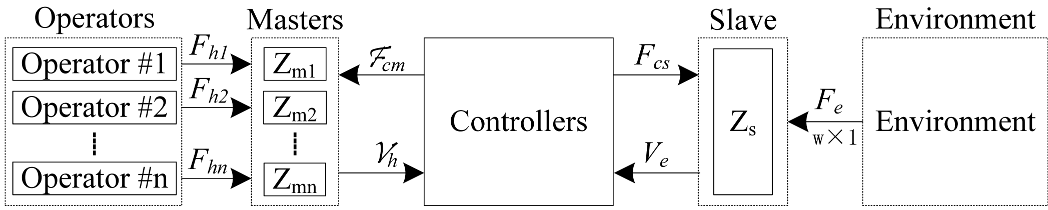

Figure 2. A multi-master/w-DOF single-slave teleoperation system

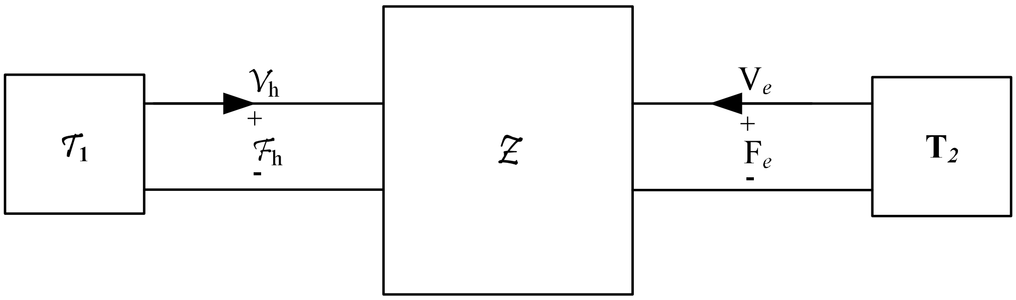

Figure 3. An two-port network where each port (terminal) connects to an w-DOF termination.

IV. Modeling a Teleoperation System With Multiple Haptic Devices With Complementary Motions

The proposed multi-master/single-slave w-DOF teleoperation system is shown in Figure 2. To analyze the stability (weak stability) of such a teleoperation system, first the possibly nonlinear dynamics of the multi-DOF masters and the w-DOF slave need to be modeled around their operating points by linear-time-invariant (LTI) impedances $\mathbf{Z}_{mi}$ and $\mathbf{Z}_s$, respectively, where $i = 1, 2, \cdots, n$ denotes each of the n masters. These impedances either relate joint torques to joint angular velocities or end-effector forces to end-effector Cartesian velocities; we will assume the latter is the case in the rest of this paper. Then, the teleoperation system is modeled as

$$\mathcal{Z}_m \mathcal{V}_h = \mathcal{F}_h + \mathcal{F}_{cm} \tag{2a}$$

$$\mathbf{Z}_s V_e = F_e + F_{cs} \tag{2b}$$

where $\mathcal{Z}_m = \text{diag}[\mathbf{Z}_{m1}, \mathbf{Z}_{m2}, \cdots, \mathbf{Z}_{mn}]$. Also, $\mathcal{F}_h = \left[\begin{array}{c:c:c:c} F_{h1} & F_{h2} & \cdots & F_{hn} \end{array}\right]^T$ denotes the interaction force vector between the n humans (or the n hands of several humans) and the n masters while $F_e = \left[\begin{array}{cccc} f_{e1} & f_{e2} & \cdots & f_{ew} \end{array}\right]^T$ denotes the interaction force vector between the slave and the environment. Furthermore, $\mathcal{V}_h = \left[\begin{array}{c:c:c:c} V_{h1} & V_{h2} & \cdots & V_{hn} \end{array}\right]^T$ and $V_e = \left[\begin{array}{cccc} v_{e1} & v_{e2} & \cdots & v_{ew} \end{array}\right]^T$ are the n masters and the slave velocity vectors while $\mathcal{F}_{cm} = \left[\begin{array}{c:c:c:c} F_{cm1} & F_{cm2} & \cdots & F_{cmn} \end{array}\right]^T$ and $F_{cs} = \left[\begin{array}{cccc} f_{cs1} & f_{cs2} & \cdots & f_{csw} \end{array}\right]^T$ denote the control signals sent to the n masters and the slave, respectively.

Thus, the system (2) can be modeled as a two-port network in which each port (terminal) connects to a w-DOF termination as shown in Figure 3. Once the control laws $\mathcal{F}_{cm}$ F_{cs} are substituted in (2), the network impedance model resemble

$$\mathcal{F} = \mathcal{Z}\mathcal{V} \tag{3}$$

where

$$\mathcal{F} = \begin{bmatrix} \mathcal{F}_h & F_e \end{bmatrix}^T, \quad \mathcal{V} = \begin{bmatrix} \mathcal{V}_h & V_e \end{bmatrix}^T \tag{4}$$

and $\mathcal{F}$ and $\mathcal{V}$ represent the $w \times 1$ vectors of force and velocity at each port of the network. The impedance matrix which is a transfer matrix of the teleoperator network will be of the form

$$\mathcal{Z} = \begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} \end{bmatrix} = \begin{bmatrix} z_{1,1} & \cdots & z_{1,2w} \\ \vdots & \ddots & \vdots \\ z_{2w,1} & \cdots & z_{2w,2w} \end{bmatrix} \tag{5}$$

where $\mathbf{Z}_{ij}$, $i,j = 1,2$, are $w \times w$ matrices given in (6).

$$\mathbf{Z}_{ij} = \begin{bmatrix} z_{(i-1)w+1,(j-1)w+1} & z_{(i-1)w+1,(j-1)w+2} & \cdots & z_{(i-1)w+1,jw} \\ z_{(i-1)w+2,(j-1)w+1} & z_{(i-1)w+2,(j-1)w+2} & \cdots & z_{(i-1)w+2,jw} \\ \vdots & \vdots & \cdots & \vdots \\ z_{iw,(j-1)w+1} & z_{iw,(j-1)w+2} & \cdots & z_{iw,jw} \end{bmatrix}_{w \times w} \tag{6}$$

The pair of w-dimensional terminations are represented by

$$\mathcal{T} = \operatorname{diag}[\mathbf{T}_1, \mathbf{T}_2] \tag{7}$$

where $\mathbf{T}_i$, $i = 1,2$, represents the $w \times w$ impedance matrix of each port termination. For future purposes, let us define the matrices $\mathcal{Z}'$ as in (8).

$$\begin{aligned} \mathcal{Z}' &= \begin{bmatrix} \mathbf{Z}'_{11} & \mathbf{Z}'_{12} \\ \mathbf{Z}'_{21} & \mathbf{Z}'_{22} \end{bmatrix} \\ &= \begin{bmatrix} z_{1,1} & \gamma_{1,2}\sqrt{z_{1,2}z_{2,1}} & \cdots & \gamma_{1,2w-1}\sqrt{z_{1,2w-1}z_{2w-1,1}} & \gamma_{1,2w}\sqrt{z_{1,2w}z_{2w,1}} \\ \gamma_{2,1}\sqrt{z_{1,2}z_{2,1}} & z_{2,2} & \cdots & \gamma_{2,2w-1}\sqrt{z_{2,2w-1}z_{2w-1,2}} & \gamma_{2,2w}\sqrt{z_{2,2w}z_{2w,2}} \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ \gamma_{2w,1}\sqrt{z_{1,2w}z_{2w,1}} & \gamma_{2w,2}\sqrt{z_{2,2w}z_{2w,2}} & \cdots & \gamma_{2w,2w-1}\sqrt{z_{2w-1,2w}z_{2w,2w-1}} & z_{2w,2w} \end{bmatrix} \\ &\text{where,} \quad \gamma_{i,i} = 1, \quad \gamma_{i,j} = \gamma_{j,i} = \pm 1, \quad i \neq j, \quad \text{and} \quad i,j = 1,2,\cdots,2w \end{aligned} \tag{8}$$

V. Main Result: An Absolute Stability Criterion for Multi-master/Single-Slave w-DOF Teleoperation Systems

Theorem 1. *A multi-master/single-slave w-DOF teleoperation system with impedance matrix $\mathcal{Z}$ in (5) satisfying the symmetrization conditions*

A) $z_{i,j}z_{j,k}z_{k,i} = z_{j,i}z_{k,j}z_{i,k}$*, where* $i,j,k = 1,2,\cdots,2w$*, and* $i \neq j \neq k$*, and* $i \neq k$*.*

B) $\mathbf{Z}_{\ell\ell}$ *is symmetric, where* $\ell = 1,2$*,*

is stable (weakly stable) if and only if

C) The elements of $\mathcal{Z}$ in (5) have no poles in the right-half plane (RHP).

D) Any poles of the elements of the $\mathcal{Z}$ matrix in (5) on the imaginary axis are simple, and the principal minors of the residue matrix of the $\mathcal{Z}$ matrix at these poles are greater than zero.

E) For all real values of frequencies ω, the principal minors of the real part of the $\mathcal{Z}'$ matrix in (8) are greater than zero (greater than or equal to zero), or equivalently the following $2w$ equations are satisfied:

$$\mathrm{Re}(z_{i,i}) > 0 \quad (\geq 0), \quad i = 1, 2, \cdots 2w \tag{9a}$$

$$\mathrm{Re}(z_{1,1})\mathrm{Re}(z_{2,2}) - \frac{|z_{1,2}z_{2,1}| + \mathrm{Re}(z_{1,2}z_{2,1})}{2} > 0 \quad (\geq 0) \tag{9b}$$

$$\vdots$$

$$\det(\mathrm{Re}(\mathcal{Z}')) > 0 \quad (\geq 0) \tag{9c}$$

□

Proof. Consider a linear time-invariant system with impulse response $h(t)$. The system's transfer function is the Laplace transform of $h(t)$ defined as

$$H(s) = \int_0^\infty h(t)e^{-st}dt \tag{10}$$

where $s = \sigma + j\omega$. $H(s)$ is stable if every bounded input produces a bounded output and this happens if the poles of $H(s)$ have negative real parts. This stability definition is equivalent to the *absolute convergence* (defined below) of $H(s)$ in the region $\mathrm{Re}(s) \geq 0$. If h is locally integrable, then $H(s)$ is said to converge if the limit $H(s) = \lim_{r\to\infty} \int_0^r h(t)e^{-st}dt$ exists. Also, $H(s)$ is said to converge absolutely if the integral $\int_0^\infty |h(t)e^{-st}|dt$ exists. The set of values of s for which $H(s)$ converges is known as the region of convergence (ROC) and is of the form $\mathrm{Re}(s) \geq a$, where a is a real constant. Importantly, if $H(s)$ converges at $s = s_0$, then it automatically converges for all s with $\mathrm{Re}(s) > Re(s_0)$. The above means that for stability analysis it suffices to focus on the convergence of $H(s)$ when $\mathrm{Re}(s) = 0$, i.e., on the $j\omega$ axis. This is sometimes referred to as real-frequency stability. Similarly, a MIMO linear system with an impulse response matrix comprised of $h_{ij}(t)$ elements is BIBO stable if and only if $h_{ij}(t)$ is absolutely integrable for all i, j [29]. Thus, as a linear time-invariant system, the stability (weak stability) of a multi-master/single-slave w-DOF teleoperation system needs to be analyzed only for $s = j\omega$.

According to Lemma 3, if there exists a reciprocal n-port network with impedance matrix $\mathcal{Z}'$ that has the same stability (weak stability) characteristics as the original nonreciprocal n-port network with impedance matrix $\mathcal{Z}$, then

$$\det(\mathcal{Z}' + \mathcal{T}) = \det(\mathcal{Z} + \mathcal{T}) \tag{11}$$

for any passive (strictly passive) $\mathcal{T}$ in (7). Thus,

$$\det \begin{bmatrix} \mathbf{Z}'_{11} + \mathbf{T}_1 & \mathbf{Z}'_{12} \\ \mathbf{Z}'_{21} & \mathbf{Z}'_{22} + \mathbf{T}_2 \end{bmatrix} = \det \begin{bmatrix} \mathbf{Z}_{11} + \mathbf{T}_1 & \mathbf{Z}_{12} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} + \mathbf{T}_2 \end{bmatrix}$$

The above is to hold for *any* passive (strictly passive) $\mathcal{T}$. It is easy to show that calculating the two determinants and equating the coefficients of $\mathbf{T}_1, \mathbf{T}_2$ gives the matrix $\mathcal{Z}'$ in (8) as well as the symmetrization conditions A and B.

According to Lemma 2, the reciprocal n-port network with impedance matrix $\mathcal{Z}'$ is stable (weakly stable) if and only if it is passive (strictly passive). In turn, according to Lemma 1, $\mathcal{Z}'$ is passive (strictly passive) if and only if it is non-negative (positive) real, which can be verified through Definition 3.

From the above, we conclude that the original nonreciprocal n-port network with impedance matrix $\mathcal{Z}$ is stable (weakly stable) if and only if the equivalent reciprocal n-port network's impedance matrix $\mathcal{Z}'$ is non-negative (positive) real. In this context, it is straightforward to show that Conditions C and D in Theorem 1 are the same as Conditions i) and ii) in Definition 3. Also, according to Condition iii) of Definition 3, the Hermitian matrix

$$\mathcal{Z}'(j\omega) + \mathcal{Z}'^T(-j\omega) = 2\mathrm{Re}(\mathcal{Z}'(j\omega)) \tag{12}$$

needs to be positive definite (positive semidefinite) for the n-port network with impedance matrix $\mathcal{Z}$ to be stable (weakly stable). Using Property 1, and simplifying the conditions by

$$(\mathrm{Re}(\sqrt{z_{i,j}z_{j,i}})) = \sqrt{\frac{|z_{i,j}z_{j,i}| + \mathrm{Re}(z_{i,j}z_{j,i})}{2}} \tag{13}$$

where $i, j = 1, 2, \cdots, 2w$, we arrive at conditions (9a)-(9c). This concludes the proof. $\square$

VI. Case Study: Stability of a Dual-master/Single-slave 3-DOF Teleoperation System

In this section, the aim is to apply the proposed stability (weak stability) criterion to a 1-DOF + 2-DOF dual-master/3-DOF single-slave teleoperation system. For brevity, we only think about weak stability case. Then, simulations and experiments will be conducted for verifying the theoretical weak stability conditions. The dynamics of the two masters and the slave in contact with the users and the environment, respectively, were considered as (2) in Section IV. In the dual-master/single-slave 3-DOF teleoperation system, assume that the 1-DOF master moves along the x direction while the 2-DOF master moves along the y and z directions. Modeling each robot by a mass, we have $\mathcal{Z}_m = \mathcal{M}_m s$ and $\mathbf{Z}_s = \mathbf{M}_s s$ as the impedance matrices of the masters and the slave, respectively, where

$$\mathcal{M}_m = \left[\begin{array}{c:cc} m_{mxx} & 0 & 0 \\ \hdashline 0 & m_{myy} & m_{myz} \\ 0 & m_{myz} & m_{mzz} \end{array}\right], \quad \mathbf{M}_s = \begin{bmatrix} m_{sxx} & m_{sxy} & m_{sxz} \\ m_{sxy} & m_{syy} & m_{syz} \\ m_{sxz} & m_{syz} & m_{szz} \end{bmatrix} \tag{14}$$

Also, $\mathcal{F}_h = \left[\begin{array}{c:cc} f_{hx} & f_{hy} & f_{hz} \end{array}\right]^T$ denotes the interaction force vector between the users and the masters and $F_e = \begin{bmatrix} f_{ex} & f_{ey} & f_{ez} \end{bmatrix}^T$ denotes the interaction force vector between the slave and the environment. Lastly, $\mathcal{V}_h = \left[\begin{array}{c:cc} v_{hx} & v_{hy} & v_{hz} \end{array}\right]^T$ and $V_e = \begin{bmatrix} v_{ex} & v_{ey} & v_{ez} \end{bmatrix}^T$ are the masters and the slave velocities.

For simplicity, let us consider the position-position teleoperation control laws [30]:

$$\mathcal{F}_{cm} = -\mathcal{C}_m \mathcal{V}_h + \mathcal{C}_4 V_e \tag{15a}$$

$$F_{cs} = -\mathbf{C}_s V_e + \mathbf{C}_1 \mathcal{V}_h \tag{15b}$$

where, since impedances relate forces and velocities, the normally PD position controllers show up as PI velocity controllers:

$$\mathbf{C}_m = \begin{bmatrix} \frac{k_{pmxx}+k_{vmxx}s}{s} & 0 & 0 \\ 0 & \frac{k_{pmyy}+k_{vmyy}s}{s} & \frac{k_{pmyz}+k_{vmyz}s}{s} \\ 0 & \frac{k_{pmyz}+k_{vmyz}s}{s} & \frac{k_{pmzz}+k_{vmzz}s}{s} \end{bmatrix}$$

$$\mathbf{C}_s = \begin{bmatrix} \frac{k_{psxx}+k_{vsxx}s}{s} & \frac{k_{psxy}+k_{vsxy}s}{s} & \frac{k_{psxz}+k_{vsxz}s}{s} \\ \frac{k_{psxy}+k_{vsxy}s}{s} & \frac{k_{psyy}+k_{vsyy}s}{s} & \frac{k_{psyz}+k_{vsyz}s}{s} \\ \frac{k_{psxz}+k_{vsxz}s}{s} & \frac{k_{psyz}+k_{vsyz}s}{s} & \frac{k_{pszz}+k_{vszz}s}{s} \end{bmatrix}$$

$$\mathbf{C}_4 = \begin{bmatrix} \frac{k_{p4xx}+k_{v4xx}s}{s} & 0 & 0 \\ 0 & \frac{k_{p4yy}+k_{v4yy}s}{s} & \frac{k_{p4yz}+k_{v4yz}s}{s} \\ 0 & \frac{k_{p4yz}+k_{v4yz}s}{s} & \frac{k_{p4zz}+k_{v4zz}s}{s} \end{bmatrix}$$

$$\mathbf{C}_1 = \begin{bmatrix} \frac{k_{p1xx}+k_{v1xx}s}{s} & \frac{k_{p1xy}+k_{v1xy}s}{s} & \frac{k_{p1xz}+k_{v1xz}s}{s} \\ \frac{k_{p1xy}+k_{v1xy}s}{s} & \frac{k_{p1yy}+k_{v1yy}s}{s} & \frac{k_{p1yz}+k_{v1yz}s}{s} \\ \frac{k_{p1xz}+k_{v1xz}s}{s} & \frac{k_{p1yz}+k_{v1yz}s}{s} & \frac{k_{p1zz}+k_{v1zz}s}{s} \end{bmatrix} \tag{16}$$

By substituting (15) and (16) in (2), the impedance matrix representation of this dual-master/single-slave teleoperator is found as

$$\begin{bmatrix} \mathcal{F}_h \\ F_e \end{bmatrix} = \begin{bmatrix} \mathcal{C}_m + \mathcal{Z}_m & -\mathcal{C}_4 \\ -\mathbf{C}_1 & \mathbf{C}_s + \mathbf{Z}_s \end{bmatrix} \begin{bmatrix} \mathcal{V}_h \\ V_e \end{bmatrix} \tag{17}$$

Now, let us investigate the weak stability of the teleoperator via Theorem 1. With $s = j\omega$, the symmetrization conditions A and B boil down to the following conditions involving the control gains and the frequency ω:

$$k_{v1xy} = k_{p1xy} = k_{v1xz} = k_{p1xz} = 0 \tag{18}$$

$$\omega^2(k_{v1yz}k_{v4yy} - k_{v4yz}k_{v1yy}) + k_{p4yz}k_{p1yy} - k_{p1yz}k_{p4yy}$$
$$+ j\omega(k_{v4yz}k_{p1yy} + k_{p4yz}k_{v1yy} - k_{v1yz}k_{p4yy} - k_{p1yz}k_{v4yy}) = 0 \tag{19}$$

$$\omega^2(k_{v1zz}k_{v4yz} - k_{v4zz}k_{v1yz}) + k_{p4zz}k_{p1yz} - k_{p1zz}k_{p4yz}$$
$$+ j\omega(k_{v4zz}k_{p1yz} + k_{p4zz}k_{v1yz} - k_{v1zz}k_{p4yz} - k_{p1zz}k_{v4yz}) = 0 \tag{20}$$

Conditions (18)-(20) will be fulfilled for all frequencies ω if the gains of the PD controllers (16) satisfy

$$k_{v1xy} = k_{p1xy} = k_{v1xz} = k_{p1xz} = 0 \tag{21a}$$

$$\frac{k_{v4yy}}{k_{v1yy}} = \frac{k_{v4yz}}{k_{v1yz}} = \frac{k_{v4zz}}{k_{v1zz}} = \frac{k_{p4yy}}{k_{p1yy}} = \frac{k_{p4yz}}{k_{p1yz}} = \frac{k_{p4zz}}{k_{p1zz}} \tag{21b}$$

It is easy to see that, under (21), all the elements of the impedance matrix (17) have only a simple pole on the imaginary axis, thus satisfying Condition C. Analysis of the residues according to Condition D leads to the following constraints:

$$k_{pmxx} \geq 0, \quad k_{pmyy} \geq 0, \quad k_{pmzz} \geq 0, \quad k_{psxx} \geq 0, \quad k_{psyy} \geq 0, \quad k_{pszz} \geq 0 \tag{22a}$$

$$k_{pmyy}k_{pmzz} - k_{pmyz}^2 \geq 0 \tag{22b}$$

$$(k_{p1xx}k_{p4xx} - k_{pmxx}k_{psxx}) \geq 0 \tag{22c}$$

$$k_{p1yz}k_{pmyy} = k_{pmyz}k_{p1yy}, \quad k_{psxy} = 0 \tag{22d}$$

$$k_{p1xx} = k_{psxx}, \quad k_{p1yy} = k_{psyy}, \quad k_{v1xx} = k_{vsxx} \tag{22e}$$

$$k_{v1yy} = k_{vsyy}, \quad k_{p4xx} = k_{pmxx}, \quad k_{v4xx} = k_{vmxx} \tag{22f}$$

Now, let us deal with Condition E of Theorem 1, which itself consists of $2w = 6$ inequality conditions. Condition (9a) turns out to state

$$k_{vmxx} \geq 0, \quad k_{vmyy} \geq 0, \quad k_{vmzz} \geq 0 \tag{23a}$$

$$k_{vsxx} \geq 0, \quad k_{vsyy} \geq 0, \quad k_{vszz} \geq 0 \tag{23b}$$

Under (21) and (22), the second principal minor condition, i.e., (9b), gives

$$k_{vmxx}k_{vmyy} \geq 0 \tag{24}$$

Similarly, the third principal minor condition requires

$$k_{vmxx}(k_{vmyy}k_{vmzz} - k_{vmyz}^2) \geq 0 \tag{25}$$

The fourth principal minor condition mandates

$$-\left(k_{pmxx}k_{vsxx} - k_{vmxx}k_{psxx}\right)^2 \geq 0 \tag{26}$$

Condition (26) will be fulfilled if the PD control gain satisfy

$$\frac{k_{pmxx}}{k_{vmxx}} = \frac{k_{psxx}}{k_{vsxx}} \tag{27}$$

The fifth principal minor condition mandates

$$k_{vsxy} = 0 \tag{28}$$

Finally, under the above weak stability conditions, the fifth principal minor condition and the sixth principal minor condition are met, i.e., (9c) is always positive.

All in all, a *sufficient, frequency-independent*, and compact condition set for weak stability of the above-described teleoperator

Table I
THE CONTROLLERS GAINS OF THE DUAL-MASTER/SIGNAL-SLAVE 3-DOF TELEOPERATION SYSTEM USED IN SIMULATIONS.

Master	k_{pmxx}	400	k_{pmyy}	300	k_{pmzz}	250
	k_{vmxx}	80	k_{vmyy}	60	k_{vmzz}	50
	k_{p4xx}	400	k_{p4yy}	300	k_{p4yz}	30
	k_{v4xx}	80	k_{v4yy}	60	k_{v4yz}	10
	k_{p4zz}	250	k_{pmyz}	**30 or 300**		
	k_{v4zz}	50	k_{vmyz}	10		
Slave	k_{psxx}	200	k_{psyy}	150	k_{pszz}	125
	k_{vsxx}	40	k_{vsyy}	30	k_{vszz}	25
	k_{psxy}	0	k_{psxz}	15	k_{psyz}	15
	k_{vsxy}	0	k_{vsxz}	20	k_{vsyz}	5
	k_{p1xx}	200	k_{p1yy}	150	k_{p1zz}	125
	k_{v1xx}	40	k_{v1yy}	30	k_{v1zz}	25
	k_{p1xy}	0	k_{p1xz}	0	k_{p1yz}	15
	k_{v1xy}	0	k_{v1xz}	0	k_{v1yz}	5

is

$$k_{v1xy} = k_{p1xy} = k_{v1xz} = k_{p1xz} = k_{psxy} = k_{vsxy} = 0 \tag{29a}$$

$$\frac{k_{v4yy}}{k_{v1yy}} = \frac{k_{v4yz}}{k_{v1yz}} = \frac{k_{v4zz}}{k_{v1zz}} = \frac{k_{p4yy}}{k_{p1yy}} = \frac{k_{p4yz}}{k_{p1yz}} = \frac{k_{p4zz}}{k_{p1zz}} \tag{29b}$$

$$k_{pmyy}k_{pmzz} - k_{pmyz}^2 \geq 0, \quad k_{vmyy}k_{vmzz} - k_{vmyz}^2 \geq 0 \tag{29c}$$

$$k_{p1xx} = k_{psxx}, \quad k_{p1yy} = k_{psyy}, \quad k_{v1xx} = k_{vsxx} \tag{29d}$$

$$k_{v1yy} = k_{vsyy}, \quad k_{p4xx} = k_{pmxx}, \quad k_{v4xx} = k_{vmxx} \tag{29e}$$

$$\frac{k_{pmxx}}{k_{vmxx}} = \frac{k_{psxx}}{k_{vsxx}}, \quad k_{p1yz}k_{pmyy} = k_{pmyz}k_{p1yy} \tag{29f}$$

where all control gains are nonnegative. The ratios in (29) are merely artifacts of our presentation of the weak stability conditions meaning that division by zero can be avoided.

A. Simulations

The position-position dual-master/single-slave teleoperation system has been simulated in MATLAB/Simulink. There is no time delay in the communication channel between the masters and the slave. In (14), the 1-DOF and 2-DOF master robots posses $M_{mxx} = 1.7$, $M_{myy} = 1.9$, $M_{mzz} = 1.3$, and $M_{myz} = 0.3$ while the slave possess $M_{sxx} = 23$, $M_{sxy} = 5$, $M_{syy} = 5.6$, $M_{szz} = 15$ $M_{sxz} = 0.5$, and $M_{syz} = 1.3$.

According to (29), the weak stability of the position-position teleoperation system should depend on the controllers gains. In the simulations, the controllers gains were chosen according to Table I.

For checking the weak stability of a two-port network such as a bilateral teleoperator, port #2 (environment port) can be connected to a passive termination while the input energy at port #1 (operator port) is measured. The bilateral teleoperator is weakly stable if and only if, at all times $t > 0$, we have [31]:

$$E_s(t) = \int_0^t F_h^T(\tau)V_h(\tau)\,\mathrm{d}\tau \geq 0. \tag{30}$$

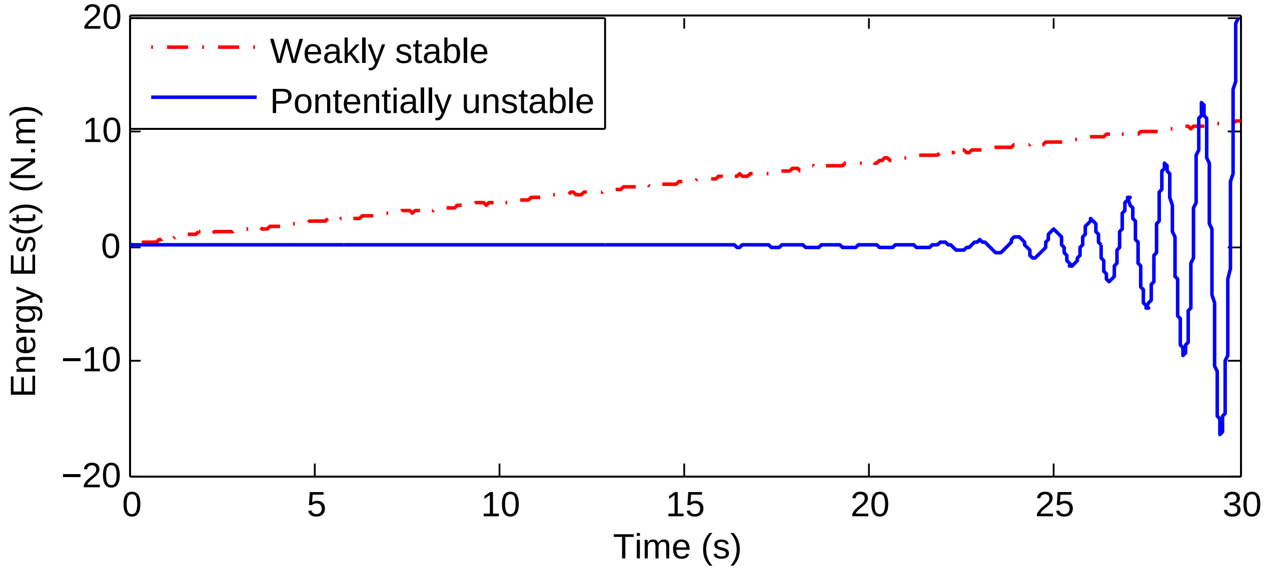

Figure 4. Simulation results for the dual-master/signal-slave 3-DOF teleoperation system. Input energy at the masters' port is shown while the slave is connected to LTI passive terminations. The control gains are listed in Table I for the **weakly stable** case with $Kpmyz = 30$ and for the potentially unstable case with $Kpmyz = 300$.

Thus, in our simulations, to check the weak stability of the 3-DOF two-port network, the slave port is connected to the LTI termination

$$\mathbf{T}_2 = \begin{bmatrix} \frac{9}{s+1} & -\frac{2}{s+2} & -\frac{1}{s+3} \\ -\frac{2}{s+2} & \frac{5}{s+1} & \frac{2}{s+1} \\ -\frac{1}{s+3} & \frac{2}{s+1} & \frac{4}{s+1} \end{bmatrix}$$

which are passive. Port 1 is open and three sine-wave inputs f_{hx}, f_{hy}, f_{hz} are applied to the two masters. The input energy $E_s(t)$ in (30) is plotted in Figure 4. As it can be seen, if the control gains are selected according to (29), e.g., as listed in Table I, with $K_{pmyz} = 30$, then the input energy at port 1 is non-negative at all times, indicating the weak stability of the bilateral teleoperator. However, when we change K_{pmyz} to 300, which violates (29), the input energy $E_s(t)$ will become negative at least for a period of time, indicating potential instability of the bilateral teleoperator. The above show that there is an agreement between the theoretical weak stability condition (29) and the simulations.

B. Experiments

For experiments with the dual-master/single-slave teleoperation system, we use a three-joint Phantom Premium 1.5A (Geomagic, Wilmington, MA, USA) as master #1 in which the first joint, which rotates about the vertical, is free to move while the second and the third joints, which form a parallel mechanism in a vertical plane, are locked using high-gain controllers. We also use a 2-DOF planar robot (Quanser Inc., Markham, ON, Canada) as master #2, and a three-joints Phantom Premium 1.5A as the slave. The first joint of the slave, which rotates about the vertical, is controlled by master #1 while the second and the third joints, which form a parallel mechanism, are controlled by master #2. The experimental setup is shown in Figure 5, where two human arms interact with the two masters while the slave is physically connected via a 2D passive spring array to a stiff wall, the same as Figure 1 in [32]. Even though we will only implement position-position teleoperation control, master #1

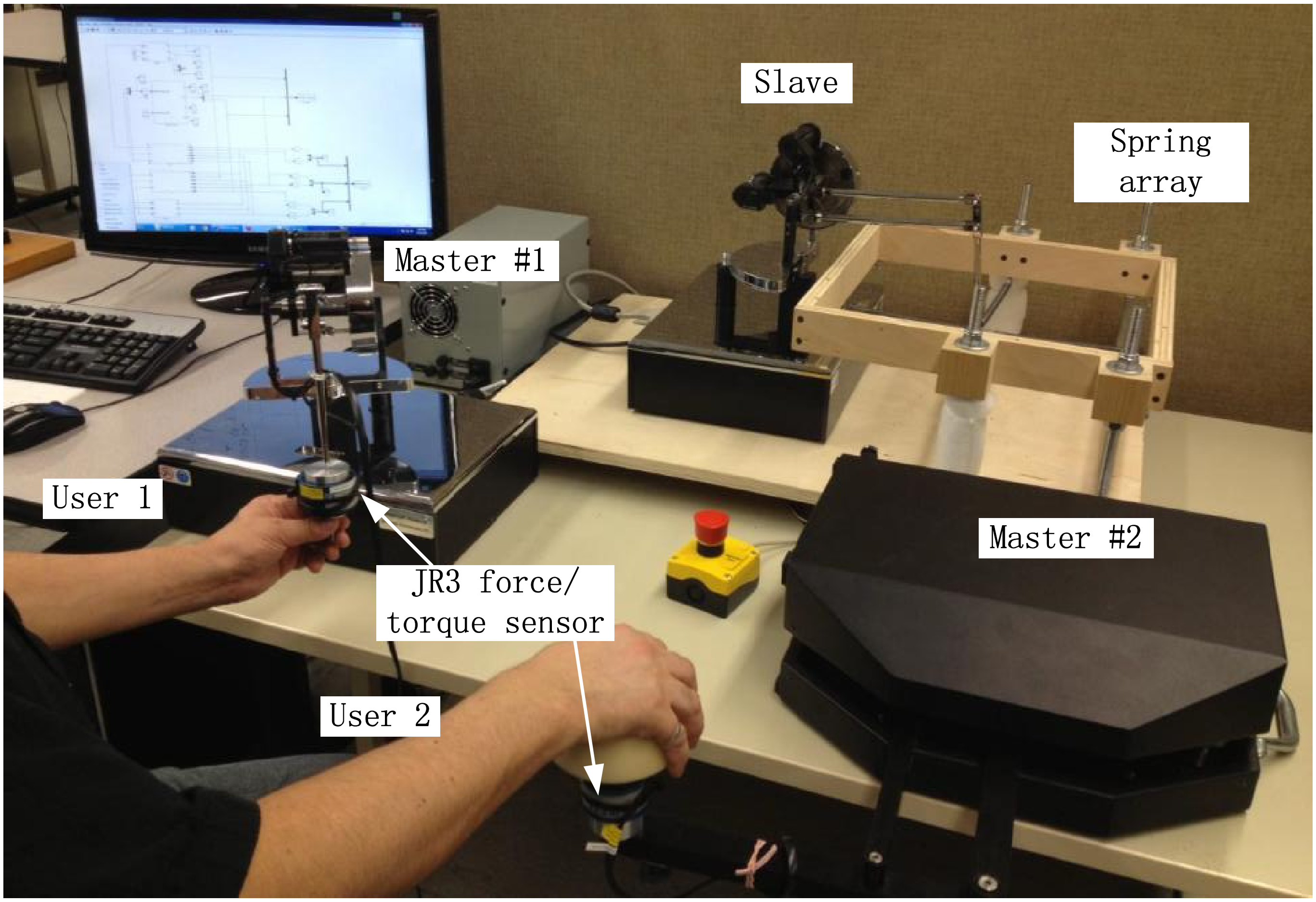

Figure 5. Experimental setup where the master #1 and the master #2 are controlled by two human arms and the slave is physically connected via a passive spring array to a stiff wall.

and master #2 are equipped with two JR3 6-DOF force/torque sensors (JR3 Inc., Woodland, CA, USA) for measuring the external contact forces to be used in (30).

According to the condition set (29), the weak stability of the dual-master/single-slave teleoperator should depend on the control gains. In the experiments, the control gains were chosen according to Table II, meeting the theoretical weak stability condition (29). The input energy $E_s(t)$ in (30) is plotted in Figure 6. As it can be seen, the input energy at the masters port is non-negative at all times, indicating the weak stability of this teleoperator. Figure 7 depicts the master position versus the slave position for each of the three joints for the parameters listed in Table II, further showing the weak stability. The above show that there is an agreement between the theoretical weak stability condition (29) and the experiments.

VII. Conclusions

Humans are usually better than autonomous robots in operating in complex unstructured environments. To take full advantage of the intelligence, experience, and sensory inputs of the human, it is proposed in this paper that the multiple human arms manipulate multiple master haptic devices in order to control a multi-DOF slave robot for performing a dexterous task. The total DOFs of all the masters is equal to the DOFs of the slave. The paper presented a closed-form, compact and easy-to-use stability weak stability criterion for such a multi-master/signal-slave w-DOF teleoperation systems. Through a case study, we

Table II
THE CONTROLLERS GAINS OF THE DUAL-MASTER/SIGNAL-SLAVE 3-DOF TELEOPERATION SYSTEM USED IN EXPERIMENTS.

Master	k_{pmxx}	7.5	k_{pmyy}	7.5	k_{pmzz}	6.25
	k_{vmxx}	6	k_{vmyy}	6	k_{vmzz}	5
	k_{p4xx}	7.5	k_{p4yy}	7.5	k_{p4yz}	0.05
	k_{v4xx}	6	k_{v4yy}	6	k_{v4yz}	0.04
	k_{p4zz}	6.25	k_{pmyz}	0.05		
	k_{v4zz}	5	k_{vmyz}	0.04		
Slave	k_{psxx}	6	k_{psyy}	6	k_{pszz}	5
	k_{vsxx}	4.8	k_{vsyy}	4.8	k_{vszz}	4
	k_{psxy}	0	k_{psxz}	0.04	k_{psyz}	0.04
	k_{vsxy}	0	k_{vsxz}	0.036	k_{vsyz}	0.036
	k_{p1xx}	6	k_{p1yy}	6	k_{p1zz}	5
	k_{v1xx}	4.8	k_{v1yy}	4.8	k_{v1zz}	4
	k_{p1xy}	0	k_{p1xz}	0.04	k_{p1yz}	0.04
	k_{v1xy}	0	k_{v1xz}	0.036	k_{v1yz}	0.036

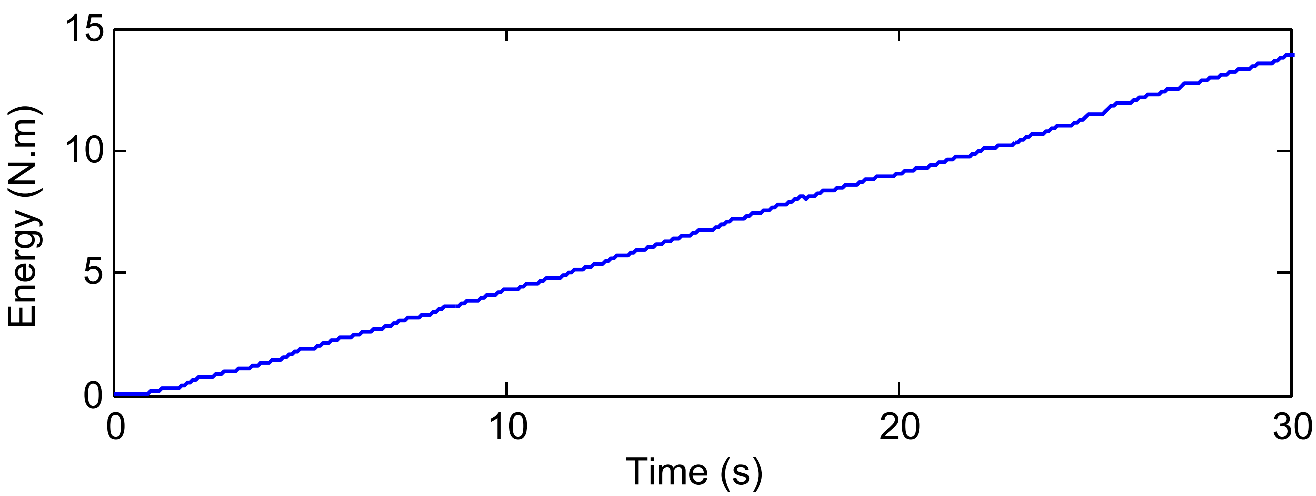

Figure 6. Experiment results for the dual-master/signal-slave teleoperation system. Input energy at the masters' port is shown while the slave is physically connected via a passive spring array to a stiff wall. The control gains are listed in Table II.

elaborated on its application in weak stability analysis of a 1-DOF + 2-DOF dual-master/signal-slave 3-DOF teleoperation system. Through simulations and experiments, the proposed stability weak stability criterion was validated.

VIII. ACKNOWLEDGMENT

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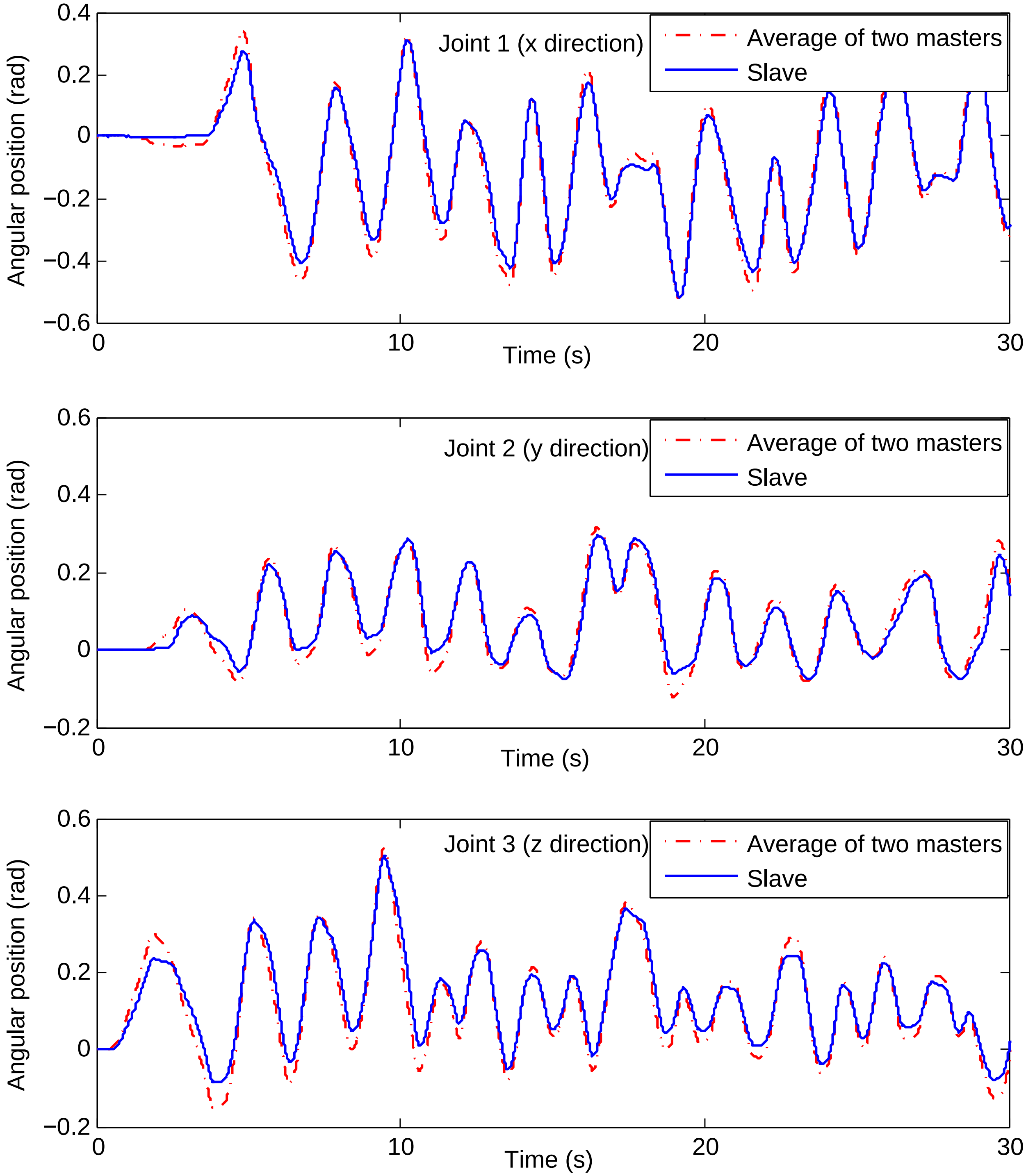

Figure 7. Experimental results for the dual-master/signal-slave teleoperation system. The master and slave positions in terms of their first, the second and the third joints are shown when using the control gains listed in Table II, which amount to **weak stability** of the teleoperator.

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