Multi-lateral Nonlinear Time-Delayed Teleoperation in a Multi-agent Systems Framework

I. Sharifi[1], H. A. Talebi[2], and M. Tavakoli[3]

Abstract—**In this paper, the issue of non-linear multi-lateral teleoperation has been considered. The multifaceted nature of controller designation for multi-lateral teleoperation frameworks increment by increasing the quantity of operators. The multi-agent System (MAS) based structure is presented as a solution to this issue. The MAS-based framework focuses in light of structures that many intelligent and smart agent interact with each other. A sort of self-intelligence exists inside every agents; implying that each of them knows how different agents are working in the system. Along these lines, the strategy in this paper has some preferences over multi-lateral structure in conventional teleoperation frameworks. In light of the MAS structure, a brought together structure will deal with the overall system. Thusly, there is no compelling reason to upgrade the controller while exchanging the topography of multi-lateral framework. Besides, there is no constraint in the quantity of operators. Moreover, the structure of the proposed controller is to such an extent that some scenarios about the behavior of the operators can be characterized and used in practical manners. At long last, this paper presents a structure for concurrent training and therapy in multi-lateral rehabilitation frameworks as a case study. In this framework, a specialist therapist, various understudy learners and a patient interact with each other to perform both patient recovery and student training. Experiments are done to affirm the performance in presence of stability of the proposed system.**

Index Terms—**Multi-lateral Teleoperation, Multi-agent Systems (MAS), Cooperative Teleoperation, Nonlinear Control, Tele-Rehabilitation.**

I. Introduction

A noteworthy issue in multi-lateral teleoperation frameworks occurs with expanding the quantities of robots which are in cooperation. In this circumstance, the control plan and solidness examination issues turn out to be all the more difficult. An answer can be looked for in the self-intelligence between numerous agents connecting with each other in a multi-agent system (MAS).

A multi-agent system is worked of agents that are able to communicate with their neighbors for decision making. The shared information between the agents can help them reach a desired goal together. The goal might be synchronization, coverage, or consensus [1], [2]. One of the essential objectives in MAS is synchronization [3] which implies the agreement of agents over a goal notwithstanding constraints in the system. The application for the proposed strategy is tele-rehabilitation for patient and training the understudies of recovery process caused by neurological wounds. Neurological wounds are caused by two reasons, in particular stroke and spinal cord wounds.

In the US, stroke cost the economy 34.3 billion dollars in 2008 and this is surveyed to raise up to 69.1 billion dollars in 2017 [4], [5].

Task-oriented movement practices have direct effect on improvement of muscle quality and development in patients with neurological clutters [6]. Physiotherapy is a labor-intensive process and its sessions are commonly not sufficiently long to acquire appropriate remedial outcomes [7]. Additionally, traditional preparing through interaction between a therapist (as a mentor), and a trainee (rehabilitation student) can be extremely tedious [8].

Along these lines, another technique called "simultaneous training and therapy" is presented in this paper. In this technique, the numbers of trainees are trained at the same time with the patient who is rehabilitating. The technique is relied upon to simultaneously decrease the period of treatment and the period of preparing. To actualize the proposed strategy the idea of multi-lateral teleoperation is used.

The idea of multi-lateral tele-rehabilitation in view of synchronization was beforehand presented in [9] by the authors. It was demonstrated that the bilateral tele-operation issue can be considered as a synchronization problem, in which the operators synchronize their powers and positions with each other. In this paper, new controllers are produced for some rehabilitation scenarios, that can manage non-linear uncertain manipulators. Also, the controllers has flexibility to design a desired hand force for every operator, which is so useful to manage training and therapy, simultaneously.

The rest of this paper is organized as follows. In Section 2, some mathematical preliminaries about the teleoperation and MAS are presented. In Section 3, the concept of centralized controller for multi-lateral teleoperation systems based on MAS is introduced. Afterwards, in Section 4, the controller is improved in the presences of uncertainty in the environment and the operator using a passivity-based adaptive control scheme. Afterwards, novel schemes for multi-lateral tele-rehabilitation systems are introduced and investigated in Section 5. Several experimentation are done in this Section. Finally, conclusion and future works are discussed in Section 6.

The rest of this paper is sorted out as follows. In Section 2,

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some preliminaries about the teleoperation and MAS are depicted. In Section 3, the idea of centralized controller for multi-lateral teleoperation frameworks in view of MAS is presented. Then, in Section 4, the controller is enhanced in the existences of uncertainty in the earth and the environment utilizing a passivity-based adaptive control. Thereafter, novel plans for multi-lateral tele-rehabilitation frameworks are presented and researched in Section 5. Some experimentation are done in this Section. At last, conclusion and future works are examined in Section 6.

II. Mathematical Preliminaries

In this section, a brief introductory about the expressions and terms used in the proposed structure is investigated.

A. Multi-Agent Systems

Graph theory is a useful tool to study MAS and their behavior. An undirected graph $\mathcal{G}$ on vertex set $\mathcal{V} = 1, 2, ..., N$ contains $\mathcal{V}$ and a set of unordered pairs $\mathcal{E} = \{(i, j) : i, j \in \mathcal{V}\}$, which are called $\mathcal{G}$'s edges. Two vertices are called adjacent, if there is an edge between them. Here, a system consisting of N agents is considered. The state of the i^{th} agent is denoted by x_i for $i = 1, ..., N$. Considering the N agents as the vertices in $\mathcal{V}$, the relationships between the N agents can be explained by a simple and undirected graph $\mathcal{G}$.

Moreover, $\mathcal{N}_i(t)$ indicates the set of labels of those agents that are neighbors of agent i at time t. The weighted adjacency matrix of $\mathcal{G}$ is denoted by $A = [a_{ij}] \in \mathcal{R}^{n\times n}$ where $a_{ij} = 0$ if there exist no input from the j^{th} agent to i^{th} agent; otherwise, $a_{ij} = 1$. The degree matrix $\mathcal{D} = diag\{d_1, d_2, ..., d_N\} \in \mathcal{R}^{N\times N}$ is a diagonal matrix, where diagonal elements are $d_i = \sum_{j=1}^{N} a_{ij}$ for $i = 1, ..., N$. Then, the Laplacian matrix of the weighted graph is defined as $L \triangleq \mathcal{D} - A$. A directed graph is connected if there is if there exists a path between any two vertices.

Remark 1. *[10] In a multi-agent system with connected graph and positive weights, there exists a vector γ (with positive elements) satisfying $\gamma^T L = 0$, where the vector γ is defined as $\gamma = [\gamma_1, ..., \gamma_N]^T$, $\gamma_i > 0$, $i = 1, \ ..., \ N$ for the case with N agents. Moreover, the the agents will be synchronized provided that the graph of the system is connected.*

Remark 2. *[11] For graph G, the Laplacian L has real eigenvalues that can be ordered sequentially as $0 = \lambda_1(L) \leq \lambda_2(L) \leq ... \leq \lambda_n(L) \leq 2d_{max}$. The second smallest eigenvalue $\lambda_2(L)$ is termed the algebraic connectivity of the graph.*

B. Teleoperation Systems

In teleoperation systems, the robot interacting with master(s) and slave(s) are considered as n-DOF serial links with revolute joints. Their associated nonlinear dynamics are described by

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = -\tau_{ext_i} + \tau_{c_i} \tag{1}$$

in which $q_i, \dot{q}_i$ and $\ddot{q}_i \in R_{n\times 1}$ for $i = 1, \ 2, \ ..., N$ are the joint angle, angular velocities and angular accelerations of the i^{th} robot [12]. Additionally, $M_i(q_i(t)) \in R_{n\times n}$, $C_i(q_i(t), \dot{q}_i(t)) \in R_{n\times n}$, and $g(q_i(t)) \in R_{n\times 1}$ are inertia matrix, Coriolis/centrifugal matrix, and gravitational vector, respectively. If i^{th} robot is the one that is interacting with the human, then $\tau_{ext_i} = -\tau_{h i}$ (torque applied by the operator to i^{th} robot). If it is interacting with the environment, then $\tau_{ext_i} = \tau_{e i}$ (torque applied by the i^{th} environment). Lastly, $\tau_{c_i} \in R_{n\times 1}$ is control torques for master and slave robots.

The following are some essential properties of robotics manipulators that are utilized in this paper and can be found in [13]:

Property 1. *For a serial manipulator, the relation between the Coriolis/centrifugal and the inertia matrix is that $\dot{M}_i(q_i)\ddot{q}_i - 2C_i(q_i, \dot{q}_i)$ is skew symmetric, or equivalently*

$$x^T\left(\dot{M}(q) - 2C(q, \dot{q})\right)x = 0, \qquad \forall x \in R^{N.n\times 1}$$

Property 2. *For revolute joint manipulators, the Coriolis/centrifugal terms are bounded as follows*

$$\|C_i(q_i, x)y\|_2 \leq \|x\|_2\|y\|_2$$

This fact can be generalized easily to the augmented equation (3), *which is*

$$\|\mathcal{C}(\mathcal{Q}, \mathcal{X}).\mathcal{Y}\|_2 \leq \|\mathcal{X}\|_2\|\mathcal{Y}\|_2$$

Property 3. *For a manipulator with revolute joints, the inertia matrix $M(q)$ is symmetric positive-definite and has the following upper and lower bounds:*

$$0 < \lambda_{min}(M(q(t)))I \leq M(q(t)) \leq \lambda_{Max}(M(q(t)))I < \infty$$

or equivalently,

$$0 < \tfrac{1}{\lambda_{Max}}(M^{-1}(q(t)) \ I \leq M^{-1}(q(t)) \leq \tfrac{1}{\lambda_{Min}}(M^{-1}(q))I < \infty$$

where $I \in R^{n\times n}$ is the identity matrix and λ_i denotes the eigenvalue of a matrix. Moreover, for determining derivative of inverse of a matrix we have:

$$\frac{d}{dt}\left\{M(q)^{-1}\right\} = -M(q)^{-1}\frac{d}{dt}\left\{M(q)\right\}M(q)^{-1}$$

Property 4. *The manipulator dynamics, written in equation* (1) *can be linearly parametrized as*

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + k_i q_i = \theta_i(q_i, \dot{q}_i)\mathcal{Y}_i + \tau_{c_i} - \tau_{h_i}$$

where $\theta_i(q_i, \dot{q}_i)$ is the vector of unknown parameters of the robot and the regressor matrix $\mathcal{Y}_i$ contains known functions [5].

III. Multi-Agent Based Approach for Multi-lateral Teleoperation

In order to develop a correspondence between multi-lateral teleoperation systems and MAS, the following assumptions are considered. All of the master robots in the teleoperation system are considered as leaders and all of the slave robots in the teleoperation system are considered as the followers in the multi-agent system. Therefore, the cooperative teleoperation structure can be considered as a leader-follower in MAS.

Besides, the positions of the masters and slaves have to track each other. This objective is similar to the convergence of the agents' positions in the MAS. Furthermore, any latency in communication channels of the teleoperation system is considered as delay between the agents in MAS. A property of MAS is the possibility of synchronization in spite of the limited connectivity between neighbors [14]. Hence, in this paper the tracking of positions in a multi-lateral teleoperation system is shown to be possible even if some network connections are broken, provided that the graph of the system is connected (see Remark 1).

Assume a graph of multi-agent system with topology network $\mathcal{G}$ and adjacency matrix $\mathcal{A}$ as defined in Section II-A. If agent i gets position information from Agent j, then α_{ij} is not zero. The indices are defined such that the first letter shows the destination and the second one specifies the source of position command. Theses coefficients can be adjusted based on the performance/interference index [15].

In this paper, the position error for the i^{th} agents is defined as $e_i(t) = \sum\limits_{j\in\mathcal{N}_i} \alpha_{ij}\left(q_i(t) - q_j(t)\right)$. Moreover, the torque effort for i^{th} manipulator should contain the term

$$\overline{\tau}_{ci}(t) = -\sum_{j\in\mathcal{N}_j} \alpha_{ji}\bar{p}_i e_j(t) \tag{2}$$

where $\bar{p}_i \geq 0$ is a weight scaler. It will be shown in the Section IV that using (2) as a part of the control effort helps to make the multi-lateral teleoperation system have transparency.

Assumption 1. *The term $T_{ji}(t)$ is the variable communication delay for transmitted signals from the j^{th} agent to the i^{th} agent. It is assumed that*

$$\dot{T}_{ji}(t) \leq \psi.$$

IV. Designing Centralized Controller for Multi-lateral Teleoperation System

Here, another centralized controller in view of centralized MAS is presented for a multi-lateral teleoperation framework. Consider the nonlinear dynamic equation of n-DOF manipulator as (1). The condition of the N robots (agents) can be augmented with the accompanying definitions

$$\begin{array}{l}\mathcal{M}_{n.N\times n.N} = diag\{M_1, M_2, ..., M_N\},\ \mathcal{C}_{n.N\times n.N} = diag\{C_1, C_2, ..., C_N\}\\ \mathcal{G}_{n.N\times 1} = \left[{G_1}^T, {G_2}^T, ..., {G_N}^T\right]^T,\ \mathcal{T}_{Ext\ n.N\times 1} = \left[\begin{array}{ccc}\tau_{ext_1}^T & \cdots & \tau_{ext_N}^T\end{array}\right]^T\\ \mathcal{T}_{Cn.N\times 1} = \left[\begin{array}{ccc}\tau_{c_1}^T & \cdots & \tau_{c_N}^T\end{array}\right]^T,\ \mathcal{Q}_{n.N\times 1} = \left[\begin{array}{ccc}q_1^T & \cdots & q_N^T\end{array}\right]^T\end{array}$$

as

$$\begin{array}{l}\mathcal{M}(\mathcal{Q}(t))\ddot{\mathcal{Q}}(t) + \mathcal{C}(\mathcal{Q}(t), \dot{Q}(t))\dot{\mathcal{Q}}(t) + \mathcal{G}(\mathcal{Q}(t))\\ = -\mathcal{T}_{Ext}(t) + \mathcal{T}_C(t)\end{array} \tag{3}$$

Property 5. *It is easy to demonstrate that Property 1 can be summed up to the augmented dynamic of the operators in* (3). *The augmented version of Property 1 is*

$$X^T\left(\dot{\mathcal{M}}(\mathcal{Q}) - 2\mathcal{C}(\mathcal{Q}, \dot{\mathcal{Q}})\right)X = 0 \qquad \forall X \in R^{N.n\times 1}$$

Remark 3. *Consider matrix $\bar{P} = diag\{\bar{p}_1, ..., \bar{p}_N\}$ and the following equation:*

$$P_{N.n\times N.n} = \bar{P}_{N\times N} \otimes I_{n\times n}$$

Based on the Kronecker product properties, the following equation can be shown directly:

$$\left(L\otimes I_{n\times n}\right)^T P\left(L\otimes I_{n\times n}\right) = \left(L^T\bar{P}L\right)\otimes I_{n\times n}$$

Moreover, it is obvious that if P is chosen to be positive definite, then $\bar{P}$ will be positive definite, too.

Moreover, we have

$$\mathcal{E}(t) = [\varepsilon_1, ..., \varepsilon_N]^T = \left(L_{N\times N}\otimes I_{n\times n}\right)_{N.n\times N.n} .\ \mathcal{Q}(t)_{N.n\times 1} \tag{4}$$

where

$$\varepsilon_i(t) = (L\otimes I)q_i(t) \tag{5}$$

which is the position errors of the i^{th} agent.

The controller is considered as

$$\tau_{c_i}(t) = g_i(q(t)) - \Gamma_i\dot{q}_i(t) + \overline{\tau}_{c_i}(t) \tag{6}$$

in which $\overline{\tau}_{c_i}(t)$ is defined as (2). To use (2) inside (6), it is required to augment $\tau_{c_i}(t)$ values. The augmented form of $\overline{\tau}_{c_i}(t)$ and $\tau_{c_i}(t)$ are as

$$\mathcal{T}_{\mathcal{C}}(t) = \mathcal{G}(\mathcal{Q}) - \Gamma_{N.n\times N.n}\ .\ \dot{\mathcal{Q}}(t) + \bar{\mathcal{T}}_{\mathcal{C}}(t) \tag{7}$$

$$\overline{\mathcal{T}}_C(t) = \left(\left(L^T\bar{P}L\right)\otimes I_{n\times n}\right)_{N.n\times N.n} .\ \mathcal{Q}_{N.n\times 1}(t) \tag{8}$$

where Γ is the damping factor of the system and is a positive definite matrix which can be chosen as $diag\{\Gamma_1, ..., \Gamma_N\}$. Consequently, the closed-loop system becomes

$$\begin{array}{l}\mathcal{M}(\mathcal{Q}(t))\ddot{\mathcal{Q}}(t) + \mathcal{C}(\mathcal{Q}(t), \dot{\mathcal{Q}}(t))\dot{\mathcal{Q}}(t) = -\mathcal{T}_H(t)\\ +\left(\left(L^T\bar{P}L\right)\otimes I_{n\times n}\right) .\ \mathcal{Q}_{N.n\times 1}(t) - \Gamma_{N.n\times N.n}\ .\ \dot{\mathcal{Q}}(t)\end{array} \tag{9}$$

Assumption 2. *Human operators and the environment are considered to be passive iff there exist positive constants κ_i such that for the i^{th} operator we have [16]*

$$\int_0^t \dot{q}(s)_i{}^T\tau_{h_i}(s)ds + \kappa_i \geq 0$$

Theorem 1. *By assumption 3 in the multi-lateral teleoperation system with the augmented dynamics* (3), *the controller* (7), *and* (8), *with positive-definite Γ as damping coefficient, the augmented joint velocity and acceleration $\dot{\mathcal{Q}}(t)$, $\ddot{\mathcal{Q}}(t)$, and the augmented joint position error $\mathcal{E}(t)$ are bounded for $\alpha_{ij} \geq 0$.*

Proof. Consider the following positive scalar functionals:

$$\begin{array}{l}V_1(t) = \frac{1}{2}\sum\limits_{i=1}^{N}\dot{q}^T M_i(q(t))\dot{q} = \frac{1}{2}\dot{\mathcal{Q}}^T\mathcal{M}(\mathcal{Q}(t))\dot{\mathcal{Q}}\\ V_2(t) = \frac{1}{2}\sum\limits_{i=1}^{N} e_i(t)^T .\ p_i .\ e_i(t) = \frac{1}{2}\mathcal{E}(t)^T{}_{1\times N.n}\ \ P_{N.n\times N.n}\\ \qquad\qquad\qquad\qquad\qquad\qquad\times\mathcal{E}(t)_{N.n\times 1}\\ \qquad = \mathcal{Q}^T\left(\left(L\otimes I_{n\times n}\right)^T P\left(L\otimes I_{n\times n}\right)\right)\mathcal{Q}(t)\\ \qquad = \mathcal{Q}^T\left(\left(L^T\bar{P}L\right)\otimes I_{n\times n}\right)\mathcal{Q}(t)\\ V_3(t) = \int_o^t \dot{\mathcal{Q}}(s)^T\mathcal{T}_H(s)dt + \Upsilon\end{array}$$

So, by summing up V_is and using (7) and (8), it can be concluded that

$$\begin{aligned}\dot{V}(t) &= \dot{\mathcal{Q}}^T(-\mathcal{T}_H + \mathcal{T}_C - \mathcal{C}\dot{\mathcal{Q}} - \mathcal{G} \\ &\quad + \tfrac{1}{2}\dot{\mathcal{M}}(Q(t))\dot{\mathcal{Q}}) + \dot{\mathcal{Q}}^T((L^T\bar{P}L)\otimes I_{n\times n})\mathcal{Q} + \dot{\mathcal{Q}}^T\mathcal{T}_H \\ &= \dot{\mathcal{Q}}^T(-\mathcal{T}_H + \mathcal{T}_C - \mathcal{C}\,\dot{\mathcal{Q}} - \mathcal{G}) \\ &= \dot{\mathcal{Q}}^T(\overline{\mathcal{T}}_C - ((L^T\bar{P}L)\otimes I_{n\times n})\mathcal{Q}) - \dot{\mathcal{Q}}^T(t)\Gamma\dot{\mathcal{Q}}(t) \\ &= -\dot{\mathcal{Q}}^T(t)\Gamma\dot{\mathcal{Q}}(t) \leq 0\end{aligned} \tag{10}$$

Since the proposed positive function $V(t)$ is non-increasing for any $\alpha_{ij} \geq 0$, the boundedness of $\dot{\mathcal{Q}}(t)$, and $\mathcal{E}(t)$ is satisfied, which completes the proof. □

Corollary 1. *Consider the system and controllers exactly as in Theorem 1 but working in free motion ($\mathcal{T}_{Ext}(t) = 0$), then the joint velocities ($\dot{q}_i(t)$) and the position errors $\varepsilon_i(t)$ converge to zero asymptotically.*

Proof. Integrating (10), while knowing that $V(\mathcal{Q}(t)) \geq 0$, results in

$$0 \geq \int_0^t \left(-\dot{\mathcal{Q}}^T(s)\Gamma\,\dot{\mathcal{Q}}(s)\right)ds = V(t) - V(0) \geq -V(0)$$

Therefore, $0 \leq \lambda_{\min}(\Gamma)\left\|\dot{Q}(t)\right\|_2^2 \leq V(0)$. So, $\dot{Q} \in \mathbf{L}_2$, which yields in $\dot{q}_i(t) \in \mathbf{L}_2,\ \forall i \in \{1, ..., N\}$. Moreover, $V(\mathcal{Q}(t))$ is a lower-bounded decreasing function, we conclude that

$$\mathcal{E}(t)_{N.n\times 1} = (L_{N\times N}\otimes I_{n\times n})_{N.n\times N.n}\,.\,\mathcal{Q}(t)_{N.n\times 1} \in \mathbf{L}_\infty$$

On the other hand,

$$\begin{aligned}\ddot{\mathcal{Q}}(t) = \ &\mathcal{M}^{-1}(\mathcal{Q}(t))\left(-\Gamma_{N.n\times N.n}\,.\,\dot{Q}(t)\right. \\ &\left.+((L^T\bar{P}L)_{N\times N}\otimes I_{n\times n})_{N.n\times N.n}Q(t) + \mathcal{C}(\mathcal{Q},\dot{Q})\,\dot{\mathcal{Q}}\right)\end{aligned} \tag{11}$$

So, from (11), it can be seen that, $\ddot{\mathcal{Q}}(t) \in \mathbf{L}_\infty$, which yields in $\ddot{q}_i(t) \in \mathbf{L}_\infty,\ \forall i \in \{1, ..., N\}$. Up to now, it is shown that $\dot{q}_i(t) \in \mathbf{L}_\infty \cap \mathbf{L}_2$, and $\ddot{q}(t) \in L_\infty$ for all $i \in \{1, ..., N\}$. Consequently, using Barbalet's lemma, $\dot{q}_i(\infty) \to 0$. Therefore, from (5), $\varepsilon_i(t)$ converge to zero, asymptotically.

Accordingly, from (9), $\mathcal{T}_H(\infty) = (L\bar{P}\otimes I)^T\mathcal{E}(\infty) \to 0$. Consequently, the sensed force from the operators, asymptotically converges to zero which means that the operators feel the free motion. □

Remark 4. *From* (9) *it can be seen that, in the steady state, the sensed force by the operator is as follows:*

$$\mathcal{T}_H(\infty) = \left[(L^T\bar{P}L)\otimes I_{n\times n}\right]\ \mathcal{Q}(\infty) \tag{12}$$

This fact will be used in Section VI.

V. Case of Uncertainty in Manipulators and Environment

In this section, uncertainty in the dynamics of manipulators are investigated. Recall the augmented dynamics of the manipulators.

$$\begin{aligned}&\mathcal{M}(\mathcal{Q}(t))\ddot{\mathcal{Q}}(t) + \mathcal{C}(\mathcal{Q}(t),\dot{Q}(t))\dot{\mathcal{Q}}(t) + \mathcal{G}(\mathcal{Q}(t)) \\ &= -\mathcal{T}_H(t) + \mathcal{T}_C(t)\end{aligned} \tag{13}$$

The controller $\mathcal{T}_\mathcal{C}(t)$ is now defined as

$$\begin{aligned}\mathcal{T}_\mathcal{C}(t) &= \hat{\mathcal{M}}(\mathcal{Q}(t))\dot{\mathcal{V}}(t) + \hat{\mathcal{C}}(\mathcal{Q}(t),\dot{\mathcal{Q}}(t))\mathcal{V}(t) \\ &\quad + \hat{\mathcal{G}}(t)(\mathcal{Q}(t)) - K\mathcal{R}(t) + \bar{\mathcal{T}}_C(t) \\ &= \mathcal{Y}(t)\hat{\Theta}(t) - K\mathcal{R}(t) + \overline{\mathcal{T}}_C(t)\end{aligned} \tag{14}$$

when $\bar{\mathcal{T}}_C(t)$ is defined as

$$\bar{\mathcal{T}}_C(t) = -((L^T\bar{P})\otimes I_{n\times n})\mathcal{E}(t) \tag{15}$$

The adaptation law is considered as

$$\dot{\hat{\Theta}} = \Omega^{-T}\mathcal{Y}^T(t)\mathcal{R}(t) \tag{16}$$

The controller (14) can be re-written as

$$\begin{aligned}&\mathcal{T}_\mathcal{C}(t) = \hat{\mathcal{M}}(\mathcal{Q})\dot{\mathcal{V}}(t) + \hat{\mathcal{C}}(t)(\mathcal{Q}(t),\dot{\mathcal{Q}}(t))V(t) + \mathcal{G}\hat{(t)}(\mathcal{Q}(t)) \\ &\pm(\mathcal{M}(\mathcal{Q})\dot{\mathcal{V}}(t) + \mathcal{C}(\mathcal{Q},\dot{\mathcal{Q}})\mathcal{V}(t) + \mathcal{G}(\mathcal{Q})) - K\mathcal{R}(t) + \bar{\mathcal{T}}_C(t) \\ &\ = \mathcal{Y}\hat{\Theta} \pm \mathcal{Y}\Theta - K\mathcal{R}(t) + \overline{\mathcal{T}}_C(t)\end{aligned}$$

in which the symbol $\pm$ means that the parameters in the parenthesis are added to and subtracted from the equation. So, the closed-loop dynamics of the system (13) using the controller (14) can be re-arranged as

$$\begin{aligned}&\mathcal{M}(\mathcal{Q})(\ddot{\mathcal{Q}} - \dot{\mathcal{V}}) + \mathcal{C}(\mathcal{Q},\dot{\mathcal{Q}})(\dot{\mathcal{Q}} - \mathcal{V}) + K.\mathcal{R}(t) \\ &= (\hat{\mathcal{M}}(\mathcal{Q}) - \mathcal{M}(\mathcal{Q}))\dot{\mathcal{V}}(t) + (\hat{\mathcal{C}}(\mathcal{Q},\dot{\mathcal{Q}}) - \mathcal{C}(\mathcal{Q},\dot{\mathcal{Q}}))\mathcal{V}(t) \\ &\quad + (\hat{\mathcal{G}} - \mathcal{G}) + \bar{\mathcal{T}}_C(t) - \mathcal{T}_H(t)\end{aligned}$$

which yields

$$\begin{aligned}\mathcal{M}(\mathcal{Q})\dot{\mathcal{R}}(t) = \ &\mathcal{Y}(t)\tilde{\Theta}(t) + \bar{\mathcal{T}}_C(t) - \mathcal{C}(\mathcal{Q}(t),\dot{\mathcal{Q}})\mathcal{R}(t) \\ &-K.\mathcal{R}(t) - \mathcal{T}_H(t)\end{aligned} \tag{17}$$

Now, based on (17), we choose the parameter $\mathcal{R}(t)$ as

$$\mathcal{R}(t) = \dot{\mathcal{Q}}(t) - \mathcal{V}(t) \tag{18}$$

$R(t)$ can be considered as a low-pass filter. Let us define this filter as follow

$$\mathcal{R}(t) = \dot{\mathcal{Q}}(t) + \lambda(L\otimes I_{n\times n})\mathcal{E}(t) \tag{19}$$

which means

$$\mathcal{V}(t) = -\lambda(L\otimes I_{n\times n})\mathcal{Q}(t) \tag{20}$$

Assumption 3. *Human operators and the environment are said to be* ***pre-filtered passive*** *iff there exist positive constants κ_i such that for the i^{th} operator we have*

$$\int_0^t \dot{r}(s)_i^T\tau_{h_i}(s)ds + \kappa_i \geq 0$$

where r_i is defined as in (19)*.*

Theorem 2. *By Assumption 3 in the multi-lateral teleoperation system with the uncertain augmented dynamics* (13)*, and the controllers* (14), (16), (18) *and* (20) *with positive-definite damping coefficient Γ, the augmented joint position error $\mathcal{E}(t)$ goes to zero asymptotically for $\alpha_{ij} \geq 0$.*

Proof. Take the following Lyapunov functional

$$\begin{aligned} V_1(t) &= \tfrac{1}{2}\mathcal{R}^T(t)\mathcal{M}\mathcal{R}(t) \\ V_2(t) &= \tfrac{1}{2}\tilde{\Theta}^T(t)\Omega\tilde{\Theta}(t) \\ V_3(t) &= \left(\int_0^t \mathcal{R}^T(t)\mathcal{T}_H(t)dt + \Upsilon\right) \\ V_4(t) &= \tfrac{1}{2}\mathcal{E}^T(t)P\mathcal{E}(t) \end{aligned}$$

and

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t)$$

we have

$$\begin{aligned} \dot{V}_1(t) &= \mathcal{R}^T(t)\left(Y\tilde{\Theta}(t) + \overline{\mathcal{T}}_C(t) - \mathcal{C}\mathcal{R}(t) - K\mathcal{R}(t)\right) \\ \dot{V}_2(t) &= \dot{\tilde{\Theta}}^T(t)\Omega\tilde{\Theta}(t) \\ \dot{V}_3(t) &= \mathcal{R}^T(t)\mathcal{T}_H(t) \\ \dot{V}_4(t) &= \dot{\mathcal{E}}^T(t)P\mathcal{E}(t) \end{aligned} \tag{21}$$

Using (19) inside $V_4(t)$, we have

$$\begin{aligned} \dot{V}_4(t) &= (\mathcal{R}^T(t) - \lambda\mathcal{Q}^T(t)(L\otimes I_{n\times n}))(L\otimes I_{n\times n})^T P\mathcal{E}(t) \\ &= \mathcal{R}^T(t)(L\otimes I_{n\times n})^T P\mathcal{E}(t) \\ &\quad -\lambda\mathcal{Q}^T(t)\left((L\otimes I_{n\times n})(L\otimes I_{n\times n})^T P\right)\mathcal{E}(t) \\ &= \mathcal{R}^T(t)(L\otimes I_{n\times n})^T P\mathcal{E}(t) \\ &\quad -\lambda\mathcal{Q}^T(t)\left(\left(LL^T\otimes I_{n\times n}\right)P\right)\mathcal{E}(t) \end{aligned}$$

So,

$$\begin{aligned} \dot{V}(t) = &\ -\mathcal{R}^T(t)K\mathcal{R}(t) \\ &-\lambda\mathcal{E}^T(t)\left(\underbrace{\left(L^T\otimes I_{n\times n}\right)P}_{\text{Positive Definite}}\right)\mathcal{E}(t) \leq 0 \end{aligned} \tag{22}$$

Therefore, $\tilde{\Theta}(t)$, $\mathcal{E}(t)$, and $\mathcal{R}(t) \in \mathbf{L}_\infty$. So, from (18), $\dot{\mathcal{Q}}(t) \in \mathbf{L}_\infty$. Considering $\left(LL^T\otimes I_{n\times n}\right)P = \Upsilon$ and integrating (22), we have

$$\begin{aligned} \int_0^t\left(-\mathcal{R}^T(s)K\mathcal{R}(s)\right)ds &+ \int_0^t\left(-\mathcal{E}^T(s)\Upsilon\mathcal{E}(s)\right)ds \\ &= V(t) - V(0) \geq -V(0) \end{aligned}$$

Therefore, given $0 \leq \lambda_{min}(K)I \leq K$ and $0 \leq \lambda_{min}(\Upsilon)I \leq \Upsilon$, it is concluded that

$$0 \leq \lambda_{\min}(K)\left\|\mathcal{R}(t)\right\|_2^2 + \lambda_{\min}(\Upsilon)\left\|\mathcal{E}(t)\right\|_2^2 \leq V(0)$$

Thus, $\mathcal{R}(t)$ and $\mathcal{E}(t) \in \mathbf{L}_2$. Therefore, based on Barbalet's Lemma, the parameter $\mathcal{E}(t)$ converges to zero asymptotically. □

Corollary 2. *Under the assumptions of Theorem 2 and in free motion, i.e,* $\tau_{ext_i} = 0$ *(or equivalently* $\mathcal{T}_{Ext} = 0$*), joint velocity errors (*$\dot{e}_i(t)$*) and position errors* $e_i(t)$ *converge to zero asymptotically.*

Proof. Integrating (21) yields

$$\int_0^t \dot{V}dt = \int_0^t -\mathcal{R}^T(t)K\mathcal{R}(t)dt = V(t) - V(0) \geq -V(0)$$

Therefore, $\int_0^t \mathcal{R}^T(t)K\mathcal{R}(t)dt \leq V(0),\ \forall t > 0$, which yields $\mathcal{R}(t) \in \mathcal{L}_2$. Also,

$$\begin{aligned} \dot{\mathcal{R}}(t) = &\ \mathcal{M}^{-1}(\mathcal{Q})\left(\mathcal{Y}(t)\tilde{\Theta}(t) + \overline{\mathcal{T}}_C(t) - \mathcal{C}(\mathcal{Q}(t),\dot{\mathcal{Q}})\mathcal{R}(t)\right. \\ &\left. -K.\mathcal{R}(t) - \mathcal{T}_H(t)\right) \end{aligned}$$

which results in $\dot{\mathcal{R}}(t) \in L_\infty$. So, using Barbalet's Lemma, $\mathcal{R}(\infty) \to 0$. Since $\mathcal{R}(t)$ is the input of the passive low pass filter, as $\mathcal{R}(t)$ converges to zero asymptotically at the steady state, $\mathcal{E}$ diminishes asymptotically at steady state, too. Equivalently, the position errors $e_i(t)$ converge to zero asymptotically.

$$\begin{aligned} \ddot{\mathcal{R}}(t) = &\ \tfrac{d}{dt}\left\{\mathcal{M}^{-1}(\mathcal{Q})\right\}\left(\mathcal{Y}(t)\tilde{\Theta}(t) + \overline{\mathcal{T}}_C(t)\right. \\ &\left. -\mathcal{C}(\mathcal{Q}(t),\dot{\mathcal{Q}})\mathcal{R}(t) - K.\mathcal{R}(t)\right) \\ &+\mathcal{M}^{-1}(\mathcal{Q})\tfrac{d}{dt}\left(\mathcal{Y}(t)\tilde{\Theta}(t) + \bar{\mathcal{T}}_C(t)\right. \\ &\left. -\mathcal{C}(\mathcal{Q}(t),\dot{\mathcal{Q}})R(t) - K.\mathcal{R}(t)\right) \end{aligned}$$

Again, using *properties 1-3*, it can be concluded that $\ddot{R}(t) \in \mathbf{L}_\infty$. Up to know it was shown that $\mathcal{R}(t) \in \mathbf{L}_2 \cap \mathbf{L}_\infty$, and $\mathcal{R}(t)$, $\dot{\mathcal{R}}(t)$, $\ddot{\mathcal{R}}(t) \in \mathbf{L}_\infty$. Therefore, using Barbalet's Lemma it can be concluded that $\mathcal{R}(t)$, $\dot{\mathcal{R}}(t)$ converges to zero, asymptotically. Therefore, $\mathcal{E}(t)$, $\dot{\mathcal{E}}(t)$ converge asymptotically to zero, which completes the proof. □

VI. Multi-lateral Design for Tele-rehabilitation Tasks

In this section, novel designs are categorized for tasks which are used in the therapy of patients and training of trainees in tele-rehabilitation systems. Two examples of useful scenarios in tele-rehabilitation systems are brought. The main item in this structure is the matrix P (positive definite) which can be used to design the structure of the tele-rehabilitation system. It was shown in Theorems of this paper that the controller guarantees position synchronization for any positive definite P. So, by choosing a proper positive definite matrix P, tracking the desired force for the patient's hand can be achieved. As described in *Remark 4*, by choosing a proper P, we can design the desired forces which are sensed by operator's hand at the steady state. Hence, by choosing $F_{des} = DQ$, the equation $D = L^TPL$ should be solved. However, we know from *Remark 2* that the Laplacian matrix is inherently singular. To find a solution to this equation, a small positive scalar is added to the zero eigenvalue(s) of the Jordan matrix corresponding to the Laplacian matrix (L) of the system, which will be called L_{new}. Note that all uses of L should be replaced with L_{new}.

Remark 5. *As mentioned in the theorems of this article, matrix* P *should be positive semi-definite to guarantee the stability. Moreover, as Remark 2 said, all eigenvalues of L are positive or zero. So, the matrix L is positive semi-definite [17]. The addition of small positive value to zero eigen value(s) of L preserves the positive-definiteness of Laplacian matrix. Furthermore, the desired force matrix* D *is chosen to be positive semi-definite. Consequently,* $P = L_{new}^{-T}DL_{new}^{-1}$ *is positive semi-definite.*

Remark 6. *It is to be noted that the Laplacian matrix in the system specifies how the operators are haptically connected to each other. The final positions of the operators are not functions of the initial values of positions of the operators. The final values will synchronize, if the graph of the system*

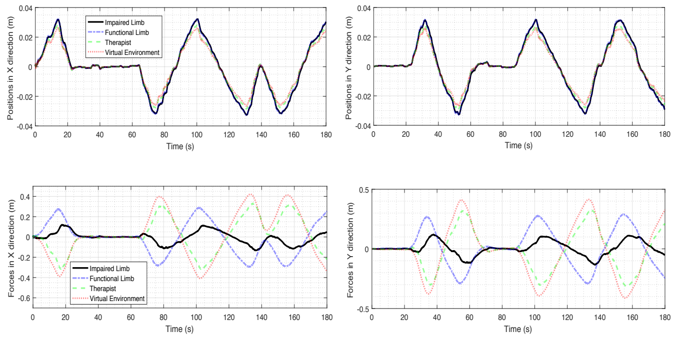

Fig. 1: Positions and forces of manipulators in mirror therapy.

is connected. Consequently, by utilizing L_{new} in the proposed controllers, new strategies can be defined.

Therapists must complete training sessions to receive a license to practice. At minimum, therapists have completed a substantial amount of supervised clinical hours before independently seeing a patient [18].

In next subsections, the proposed structure will be examined on the scenarios of rehabilitation. The results shows the efficiency of the proposed methods.

A. Supervised Mirror Therapy

In this scenario, the patient attempts bi-manual symmetric movements while the (remote) therapist helps her/him to move in a desired path. The limbs are maintained in symmetry by the robots, which assists the affected limb to rehabilitate. The desired hand force matrix D and the Laplacian matrix L for position synchronization are chosen as follows

$$L = \begin{bmatrix} 1 & -0.5 & -0.5 & 0 \\ -0.5 & 1 & -0.5 & 0 \\ -0.5 & -0.5 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 1 & -0.6 & 0 & -0.4 \\ 0 & 1 & -1 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} \tag{23}$$

The results of the experiments are depicted in Fig. 1.

They show that the positions of the patient's hands, and the positions of the therapist's hand are synchronized. The forces of the operators are such that the summation values of the forces will be zero in the steady states.

$$\tau_{\text{Therapist}} + \tau_{\text{ImpairedLimb}} + \tau_{\text{Therapist}} + \tau_{\text{SVE}} = 0$$

The above equation can be easily verified by (12) and (23). The fact is depicted in Fig. 1.

VII. Conclusion

In this paper, the issue of nonlinear multi-lateral tele-rehabilitation was investigated. A novel structure in light of MAS was utilized to solve the problems of complexity of an expansion in the quantity of operators, in designing multi-lateral teleoperation controllers. The self-intelligence between the agents connecting with each other was considered as the principle key to collaboration of the controllers. Besides, uncertainties in the agent dynamics and time-varying delays in the correspondence channels were investigated.

Also, this paper presented a structure for simultaneous training and therapy in multi-lateral tele-rehabilitation frameworks. The technique can lessen the time of treatment and the time of training. Various Experiments were done to affirm the unwavering reliability and performance of the proposed structure.

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