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Teleoperation in the Presence of Varying Time Delays and Sandwich Linearity in Actuators

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Abstract

In this paper, a novel control scheme is proposed to guarantee global asymptotic stability of bilateral teleoperation systems that are subjected to time-varying time delays in their communication channel and *sandwich linearity* in their actuators. This extends prior art concerning control of nonlinear bilateral teleoperation systems under time-varying time delays to the case where the local and the remote robots' control signals pass through saturation or similar nonlinearities that belong to a class of systems we name *sandwich linear systems*. Our proposed controller is similar to the proportional plus damping (P+D) controller with the difference that it takes into account the actuator saturation at the outset of control design and alters the proportional term by passing it through a nonlinear function; thus, we call the proposed method as nonlinear proportional plus damping (nP+D).The asymptotic stability of the closed-loop system is established using a Lyapunov-Krasovskii functional under conditions on the controller parameters, the actuator saturation characteristics, and the maximum values of the time-varying time delays. To show the effectiveness of the proposed method, it is simulated on a variable-delay teleoperation system comprising a pair of planar 2-DOF robots subjected to actuator saturation. Furthermore, the controller is experimentally validated on a pair of 3-DOF PHANToM Premium 1.5A robots, which have limited actuation capacity, that form a teleoperation system with a varying-delay communication channel.

Keywords: Nonlinear teleoperation, varying time delay, sandwich linearity, actuator saturation, asymptotic stability.

1. Introduction

In telerobotic applications with a distance between the local and the remote robots (e.g., telesurgery and space exploration), there will be a time delay in the communication channel of the system, which can destabilize the telerobotic (Sheridan (1993)). In practice, the communication delay can be time-varying and asymmetric in the forward and backward paths between the operator and the remote environment (Gao (2007) and Gao (2008)). There are a number of control schemes for time-varying delay compensation in the literature, e.g., Nuño (2009b), Polushin (2008) and Hua (2010). On the other hand, in almost all applications of control systems including teleoperation systems, the actuator output (i.e., control signal) has a limited amplitude, i.e., is subject to saturation. Controllers that ignore actuator saturation may cause undesirable responses and even closed-loop instability (Kothare (1994)).Although it may be possible to avoid actuator saturation by using sufficiently high-torque actuators in robots, the large size of the actuators will cause further problems in robot design and control. Therefore, it is highly desirable to develop control methods that take any actuator saturation into account at the design outset and, therefore, allow for efficient and stable control with small-size actuators that inevitably possess a limited output capacity.

In order to address the stability of the position control loop for a single robotic manipulator subjected to bounded actuator output, several approaches have been proposed in the literature. An anti-windup approach is presented to guarantee global asymptotic stability of Euler-Lagrange systems in Morabito (2004). In Loria (1997), a controller is proposed involving a gravity compensation term plus a saturating function through which the position errors pass. A velocity and position feedback method with adaptive gravity compensation is reported in Zergeroglu (2000) in which the velocity and position errors separately pass through two nonlinear saturating functions and the outputs are then added to an adaptive gravity compensation term. In Zavala (2006), a brief review of PD plus gravity compensation controllers is provided. None of the above research has been done in the context of teleoperation systems.

Going beyond anti-saturation control for a single robot, there has been some attention paid recently to actuator saturation in bilateral teleoperation systems (Ahn (2001)). Combining wave variable with a nonlinear proportional controller, an architecture to handle actuator saturation is discussed in Lee (2010) for the case where the delay in the communication channel is constant. In Lee (2011), an anti-windup approach combined with wave variables is used for constant-delay teleoperation subjected to bounded control signals.

In this paper, a control scheme is introduced to cope with actuator saturation in nonlinear bilateral teleoperation systems that are subjected to time-varying delays. Asymptotic stability of the position error in the teleoperation system is studied, resulting in conditions on the controller parameters, the actuator saturation characteristics and the maximum values of varying delays. The main advantage of the proposed nP+D controller is in unifying the study of actuator saturation and varying time delay in the same framework and in guaranteeing the stability of system in the presence of both time varying delays and actuator saturation.

This paper is organized as follows. Section II states the preliminaries while the proposed control and main results are presented in Section III and IV respectively. In Section V, simulation and experimental results are provided followed by the conclusions in Section VI.

Notation. We denote the set of real numbers by $R=(-\infty,\infty)$, the set of positive real numbers by $R_{>0}=(0,\infty)$, and the set of nonnegative real numbers by $R_{\geq 0}=[0,\infty)$. Also, $\|X\|_\infty$ and $\|X\|_2$ stand for the Euclidian ∞-norm and 2-norm of a vector $X\epsilon R^{n\times 1}$, and $|X|$denotes element-wise absolute value of the vector X. The $\mathcal{L}_\infty$ and $\mathcal{L}_2$ norms of a time function

$f: R_{\geq 0} \rightarrow R^{n\times 1}$ are shown as $\|f\|_{\mathcal{L}\infty} = sup_{t\epsilon[0,\infty)} \|f(t)\|_\infty$ and $\|f\|_{\mathcal{L}2} = \left(\int_0^\infty \|f(t)\|_2{}^2 dt\right)^{0.5}$, respectively. The $\mathcal{L}_\infty$ and $\mathcal{L}_2$ spaces are defined as the sets $\{f: R_{\geq 0} \rightarrow R^{n\times 1}, \|f\|_{\mathcal{L}\infty} < +\infty\}$ and $\{f: R_{\geq 0} \rightarrow R^{n\times 1}, \|f\|_{\mathcal{L}2} < +\infty\}$, respectively. For simplicity, we refer to $\|f\|_{\mathcal{L}\infty}$ as $\|f\|_\infty$ and to $\|f\|_{\mathcal{L}2}$ as $\|f\|_2$. We also simplify the notation $lim_{t\rightarrow\infty} f(t) = 0$ to $f(t) \rightarrow 0$.

2. Preliminaries

2. A. Teleoperation System Dynamical Model

Consider the master (local) and slave (remote) robots with saturated inputs as follows:

$$M_m\big(q_m(t)\big)\ddot{q}_m(t) + C_m\big(q_m(t), \dot{q}_m(t)\big)\dot{q}_m + G_m\big(q_m(t)\big) = \tau_h(t) - S\big(\tau_m(t)\big) \quad (1)$$

$$M_s\big(q_s(t)\big)\ddot{q}_s(t) + C_s\big(q_s(t), \dot{q}_s(t)\big)\dot{q}_s(t) + G_s\big(q_s(t)\big) = S\big(\tau_s(t)\big) - \tau_e(t) \quad (2)$$

Here, q_i, $\dot{q}_i$ and $\ddot{q}_i \epsilon R^{n\times 1}$ for $i\epsilon\{m, s\}$ are the joint positions, velocities and accelerations of the master and slave robots, respectively. Also, $M_i(q_i(t)) \,\epsilon\, R^{n\times n}$, $C_i(q_i(t), \dot{q}_i(t)) \,\epsilon\, R^{n\times n}$, and $G_i(q_i(t)) \epsilon\, R^{n\times 1}$ are the inertia matrix, the Coriolis/centrifugal matrix, and the gravitational vector, respectively. Moreover, τ_h and $\tau_e \epsilon\, R^{n\times 1}$ are the torques applied by the human operator and the environment, respectively. Lastly, τ_m and $\tau_s \epsilon\, R^{n\times 1}$ are the control signals (torques) for the master and the slave robots skewed by the vector function $S: R^{n\times 1} \rightarrow R^{n\times 1}$, which can be nonlinear.

Important properties of the nonlinear dynamic models (1) and (2), which will be used in this paper, are (Kelly (2005) and Spong (2005))

P-1. For a manipulator with revolute joints, the inertia matrix $M(q)$ is symmetric positive-definite and has the following upper and lower bounds:

$$0 < \lambda_{min}\big(M(q(t))\big)I \leq M(q(t)) \leq \lambda_{Max}\big(M(q(t))\big)I \leq \infty$$

where $I\epsilon R^{n\times n}$ is the identity matrix.

P-2. For a manipulator, the relation between the Coriolis/centrifugal and the inertia matrices is as follows:

$$\dot{M}(q(t)) = C(q(t), \dot{q}(t)) + C^T(q(t), q(t))$$

P-3. For a manipulator with revolute joints, there exists a positive η bounding the Coriolis/centrifugal term as follows:

$$\|C(q(t), x(t))y(t)\|_2 \leq \eta\|x(t)\|_2\|y(t)\|_2$$

P-4. The time derivative of $C(q(t), \dot{q}(t))$ is bounded if $\dot{q}(t)$ and $\ddot{q}(t)$ are bounded.

P-5. For a manipulators with revolute joints, the gravity vector $G\big(q(t)\big)$ is bounded. (there exist positive constants γ_j such that every elements of the gravity vector, $g_j\big(q(t)\big)$, $j = 1, \cdots, n$, satisfies $\big|g_j\big(q(t)\big)\big| \leq \gamma_j$).

2. B. Actuator Model with Sandwich Linearity

In the following, sandwich linearity of the actuator as a vector function $S(\cdot)$ is introduced. It is assumed that *n* is the number of joints in the master and slave robots and the elements of $S(X)$, where $X \triangleq [x_1 \cdots x_n]^T$, are $s_j(x_j): R \rightarrow R, j = 1, \cdots, n$, defined by

$$s_j(x_j)\begin{cases} > M_j, & \text{if} \quad x_j > M_j \\ = x_j, & \text{if} \quad -M_j \leq x_j \leq M_j \; M_j > 0 \\ < -M_j, & \text{if} \quad x_j < -M_j \end{cases} \quad (3)$$

With this definition of $S(X)$, it is possible to define different sandwich linearity characterizations for different joints of the manipulator. Note that the function $s_j(x_j)$ is only required to be linear for $-M_j \leq x_j \leq M_j$, and can be nonlinear (unbounded or bounded) for $|x_j| > M_j$. An example of such a function is saturation.

It is imperative to have $\gamma_j < M_j, j = 1, \cdots, n$, where γ_j is the upper bound of $\big|g_j\big(q(t)\big)\big|$. This condition implies that the actuators of each of the master and the slave manipulators have the capacity to overcome the corresponding robot's gravity within their workspaces.

3. Proposed Control Law

In this paper, a nonlinear Proportional plus Damping (nP+D) controller that incorporates gravity compensation is proposed for the master and the slave robots as

$$\tau_m(t) = -G_m\big(q_m(t)\big) + P\Big(q_m(t) - q_s\big(t - T_2(t)\big)\Big) + K_m\dot{q}_m(t) \quad (4)$$

$$\tau_s(t) = G_s\big(q_s(t)\big) - P\big(q_s(t) - q_m(t - T_1(t))\big) - K_s\dot{q}_s(t) \quad (5)$$

Where $P(X): R^{n\times 1} \rightarrow R^{n\times 1}$ is a vector on linear function with elements $p_j(x_j): R \rightarrow R$. The function $p_j(x_j)$ is required to be strictly increasing, bounded, continuous, passing through the origin, concave for positive x and convex for negative x, with continuous first derivative around the origin, such that $|p_j(x_j)| \leq |x_j|$ and $p_j(-x_j) = -p_j(x_j)$.

Under the above assumptions, we will have the following properties for $p_j(x_j)$:

P-I. For any $x, y\epsilon R$, $|p_j(x) - p_j(y)| \leq 2p_j(|x - y|)$

P-II. For any $x, y\epsilon R$, if $x < y$ then $p_j(x) < p_j(y)$

P-III. For any $x, y\epsilon R_{\geq 0}$, $p_j(x + y) \leq p_j(x) + p_j(y)$

P-IV. For any $x\epsilon R$, $lim_{\varepsilon\rightarrow 0}\, p_j(\varepsilon x) = lim_{\varepsilon\rightarrow 0}\, \varepsilon p_j(x)$

P-V. For any $x, y\epsilon R$, $xp_j(y) \leq xp_j(x) + yp_j(y)$

P-VI. For any $x\epsilon R$, $|p_j(x)| \leq min\{|x|, N_j\}$ where $N_j \triangleq sup_{x\epsilon R}\, p_j(x)$

P-VII. For any $x(t)\epsilon R$, time derivative of $p_j(x(t))$ is bounded.

P-VIII. For any $x\epsilon R$, $xp_j(x) \geq 0$

4. Main Results

Let us start by a few preliminary lemmas that will be needed in the proof of our first main result in Theorem I. For simplicity, we use $G_m(\cdot)$, $G_s(\cdot)$, $P_m(\cdot)$ and $P_s(\cdot)$ instead of $G_m\big(q_m(t)\big)$, $G_s\big(q_s(t)\big)$, $P\big(q_m(t) - q_s(t - T_2(t))\big)$ and $P\big(q_s(t) - q_m(t - T_1(t))\big)$, respectively. Similarly, we denote by $g_{m_j}(\cdot)$, $g_{s_j}(\cdot)$, $p_{m_j}(\cdot)$ and $p_{s_j}(\cdot)$ the j^{th} element of $G_m(\cdot)$, $G_s(\cdot)$, $P_m(\cdot)$ and $P_s(\cdot)$, respectively.

Lemma I. Given $g_{m_j}(\cdot) \leq \gamma_j < M_j$, $g_{s_j}(\cdot) \leq \gamma_j < M_j$, $p_{m_j}(\cdot) \leq N_j \leq M_j - \gamma_i$ and $p_{s_j}(\cdot) \leq N_j \leq M_j - \gamma_i$, for positive-definite diagonal matrices K_m and K_s the following inequalities hold:

$$\begin{aligned} &\dot{q}_m^T\big(S(G_m(\cdot) - P_m(\cdot) - K_m\dot{q}_m) - (G_m(\cdot) - P_m(\cdot))\big) \leq 0 \\ &\dot{q}_s^T\big(S(G_s(\cdot) - P_s(\cdot) - K_s\dot{q}_s) - (G_s(\cdot) - P_s(\cdot))\big) \leq 0 \end{aligned} \quad (6)$$

Lemma II. For any $T(t)\epsilon R_{\geq 0}$, $x(t)\epsilon R_{\geq 0}$, we have

$$p_i\left(\int_{t-T(t)}^{t} x(\tau)d\tau\right) \leq \int_{t-T(t)}^{t} p_i\big(x(\tau)\big)d\tau \quad (7)$$

Lemma III. For any vector functions $q_m(t) \triangleq \big[q_{m_1}(t) \cdots q_{m_n}(t)\big]^T$ and $q_s(t) \triangleq \big[q_{s_1}(t) \cdots q_{s_n}(t)\big]^T$ and for any positive time-varying scalars $T_1(t)$ and $T_2(t)$, the following inequality holds:

$$\dot{q}_m^T(t)\Big(P\big(q_m(t) - q_s(t)\big) - P\Big(q_m(t) - q_s\big(t - T_2(t)\big)\Big)\Big) \leq 2|\dot{q}_m(t)|^T \int_{t-T_2(t)}^{t} P(|\dot{q}_s(\tau)|)d\tau$$

$$\dot{q}_s^T(t)\left(P\big(q_s(t)-q_m(t)\big)-P\big(q_s(t)-q_m(t-T_1(t))\big)\right)\leq 2|\dot{q}_s(t)|^T\int_{t-T_1(t)}^{t}P(|\dot{q}_m(\tau)|)d\tau \quad (8)$$

where $\dot{q}_m(t)$ and $\dot{q}_s(t)$ are the time derivatives of $q_m(t)$ and $q_s(t)$, respectively.

Lemma IV. For any vectors $A(t)\triangleq[a_1(t)\cdots a_n(t)]^T$ and $B(t)\triangleq[b_1(t)\cdots b_n(t)]^T$ and for any bounded time-varying scalar $0\leq T(t)\leq T_m$, the following inequality holds:

$$A^T(t)\int_{t-T(t)}^{t}P\big(B(\tau)\big)d\tau-\int_{t-T(t)}^{t}B^T(\tau)P\big(B(\tau)\big)d\tau \leq T_mA^T(t)P\big(A(t)\big) \quad (9)$$

Proofs of the above four lemmas are provided in Appendix.

Theorem I. Assuming the human operator and the environment are passive, in the bilateral teleoperation system (1)-(2) with controllers (4)-(5), the velocities $\dot{q}_m$ and $\dot{q}_s$ and the position error q_m-q_s are bounded for any bounded time-varying time delays $T_1(t)$ and $T_2(t)$ provided that

1) $T_{1max}+T_{2max}\leq\frac{-d+M_{min}}{2N_{max}}$
2) $2(T_{1max}+T_{2max})<K_s$
3) $2(T_{1max}+T_{2max})<K_m$
4) $\gamma_j+N_j\leq M_j$

In the above $M_{min}\triangleq min\{M_j\}$ and $N_{max}\triangleq max\{N_j\}$ for $j=1\cdots n$. Also, $d_m\triangleq sup_{(x,y)\epsilon R^{n\times1}\times R^{n\times1}}\|G_m(x)-P(y)\|_\infty$, $d_s\triangleq sup_{(x,y)\epsilon R^{n\times1}\times R^{n\times1}}\|G_s(x)-P(y)\|_\infty$,and $d\triangleq max\{d_m,d_s\}$. Lastly, $T_{1max}\triangleq sup_{t\epsilon R}T_1(t)$ and $T_{2max}\triangleq sup_{t\epsilon R}T_2(t)$.

Proof of Theorem I:

Applying controller (4)-(5) to the system (1)-(2), we have following closed loop dynamics

$$M_m\big(q_m(t)\big)\ddot{q}_m(t)+C_m\big(q_m(t),\dot{q}_m(t)\big)\dot{q}_m(t)+G_m\big(q_m(t)\big)= -S\Big(-G_m\big(q_m(t)\big)+P\big(q_m(t)-q_s(t-T_2(t))\big)+K_m\dot{q}_m(t)\Big) +\tau_h(t) \quad (10)$$

$$M_s\big(q_s(t)\big)\ddot{q}_s(t)+C_s\big(q_s(t),\dot{q}_s(t)\big)\dot{q}_s(t)+G_s\big(q_s(t)\big)= +S\Big(G_s\big(q_s(t)\big)-P\big(q_s(t)-q_m\big(t-T_1(t)\big)\big)-K_s\dot{q}_s(t)\Big) -\tau_e(t) \quad (11)$$

To show the stability of the system (10)-(11), define $x_t=x(t+\psi)$, as the state of the system where $x(t)\triangleq[q_m(t)\quad \dot{q}_m(t)\quad q_s(t)\quad \dot{q}_s(t)]$, $-T_{max}\leq\psi\leq0$ and $T_{max}=max(T_{1max},T_{2max})$, (Chopra (2006) and Hale (1993)). So Lyapunov-Krasovskii functional $V(x_t)$ can be defined as

$$V(x_t)=V_1(x_t)+V_2(x_t)+V_3(x_t)+V_4(x_t) \quad (12)$$

$$V_1(x_t)=\frac{1}{2}\dot{q}_m^T(t)M_m\big(q_m(t)\big)\dot{q}_m(t) +\frac{1}{2}\dot{q}_s^T(t)M_s\big(q_s(t)\big)\dot{q}_s(t) \quad (13)$$

$$V_2(x_t)=\int_0^t\big(-\dot{q}_m(\tau)\tau_h(\tau)+\dot{q}_s(\tau)\tau_e(\tau)\big)\,d\tau \quad (14)$$

$$V_3(x_t)=\sum_{j=1}^{n}\int_0^{q_{m_j}(t)-q_{s_j}(t)}p_j(\gamma_j)\,d\gamma_j \quad (15)$$

$$V_4(x_t)=2\int_{-T_{1max}}^{0}\int_{t+\gamma}^{t}\dot{q}_m^T(\eta)P\big(\dot{q}_m(\eta)\big)d\eta\,d\gamma +2\int_{-T_{2max}}^{0}\int_{t+\gamma}^{t}\dot{q}_s^T(\eta)P\big(\dot{q}_s(\eta)\big)d\eta\,d\gamma \quad (16)$$

Note that based on the assumption of passivity of the operator and the environment, $V_2(x_t)$ is a lower-bonded function. In other words, there exist positive constants κ_m and κ_s such that

$\int_0^t\big(-\dot{q}_m(\tau)\tau_h(\tau)\big)\,d\tau+\kappa_m\geq0$ and $\int_0^t\big(\dot{q}_s(\tau)\tau_e(\tau)\big)\,d\tau+\kappa_s\geq0$

Considering property P-2, the time derivative of $V_1(t)$ is simplified to

$$\dot{V}_1(x_t)=\dot{q}_m^T(t)\bigg(-S\Big(P\Big(q_m(t)-q_s\big(t-T_2(t)\big)\Big)- G_m\big(q_m(t)\big)+K_m\dot{q}_m(t)\Big)\bigg)+\dot{q}_m^T(t)\Big(\tau_h(t)-G_m\big(q_m(t)\big)\Big) +\dot{q}_s^T(t)\Big(S\Big(G_s\big(q_s(t)\big)-P\big(q_s(t)-q_m(t-T_1(t))\big)- K_s\dot{q}_s(t)\Big)\Big)+\dot{q}_s^T(t)\Big(-\tau_e(t)-G_s\big(q_s(t)\big)\Big) \quad (17)$$

The time derivatives of $V_2(t)$ and $V_3(t)$ are

$$\dot{V}_2(x_t)=\dot{q}_m(t)\tau_h(t)-\dot{q}_s(t)\tau_e(t) \quad (18)$$

$$\dot{V}_3(x_t)=\big(\dot{q}_m(t)-\dot{q}_s(t)\big)P\big(q_m(t)-q_s(t)\big) \quad (19)$$

By adding and subtracting each of $\dot{q}_m(t)P\big(q_m(t)-q_s(t-T_2(t))\big)$ and $\dot{q}_s(t)P\big(q_s(t)-q_m(t-T_1(t))\big)$ to and from $\dot{V}_1$ and noting that $P(-X)=-P(X)$, it is possible to see that

$$\dot{V}_1(x_t)+\dot{V}_2(x_t)+\dot{V}_3(x_t)= \dot{q}_m^T(t)\Big\{S\Big(G_m\big(q_m(t)\big)-P\Big(q_m(t)-q_s\big(t-T_2(t)\big)\Big)-K_m\dot{q}_m(t)\Big) -\Big(G_m\big(q_m(t)\big)-P\Big(q_m(t)-q_s\big(t-T_2(t)\big)\Big)\Big)\Big\} +\dot{q}_m^T(t)\Big\{P\big(q_m(t)-q_s(t)\big)-P\Big(q_m(t)-q_s\big(t-T_2(t)\big)\Big)\Big\} +\dot{q}_s^T(t)\Big\{S\Big(G_s\big(q_s(t)\big)-P\big(q_s(t)-q_m(t-T_1(t))\big)-K_s\dot{q}_s(t)\Big) -\Big(G_s\big(q_s(t)\big)-P\big(q_s(t)-q_m(t-T_1(t))\big)\Big)\Big\} +\dot{q}_s^T(t)\{P\big(q_s(t)-q_m(t)\big)-P\big(q_s(t)-q_m(t-T_1(t))\big)\} \quad (20)$$

Considering Lemma I, it is easy to see that there exist positive δ_m and δ_s such that

$$\dot{V}_1(x_t)+\dot{V}_2(x_t)+\dot{V}_3(x_t)\leq-\delta_mK_m\dot{q}_m^T(t)\dot{q}_m(t) +\dot{q}_m^T(t)\Big\{P\big(q_m(t)-q_s(t)\big)-P\Big(q_m(t)-q_s\big(t-T_2(t)\big)\Big)\Big\} -\delta_sK_s\dot{q}_s^T(t)\dot{q}_s(t)+\dot{q}_s^T(t)\{P\big(q_s(t)-q_m(t)\big) -P\big(q_s(t)-q_m(t-T_1(t))\big)\} \quad (21)$$

Where δ_m and δ_s are defined as

$$\delta_m\triangleq\frac{-1}{K_m\|\dot{q}_m(t)\|_2^2}\dot{q}_m^T(t)\{S\big(G_m\big(q_m(t)\big) -P\Big(q_m(t)-q_s\big(t-T_2(t)\big)\Big)-K_m\dot{q}_m(t)\Big) -\Big(G_m\big(q_m(t)\big)-P\Big(q_m(t)-q_s\big(t-T_2(t)\big)\Big)\Big)\Big\},\quad \|\dot{q}_m(t)\|_2\neq0$$

$$\delta_s\triangleq\frac{-1}{K_s\|\dot{q}_s(t)\|_2^2}\dot{q}_s^T(t)\{S\big(G_s\big(q_s(t)\big) -P\big(q_s(t)-q_m(t-T_1(t))\big)-K_s\dot{q}_s(t)\Big) -\Big(G_s\big(q_s(t)\big)-P\Big(q_s(t)-q_m\big(t-T_1(t)\big)\Big)\Big)\Big\},\quad \|\dot{q}_s(t)\|_2\neq0 \quad (22)$$

Applying the result of Lemma III to the last two terms in the right hand side of (21), we get

$$\dot{V}_1(x_t)+\dot{V}_2(x_t)+\dot{V}_3(x_t)\leq-\delta_mK_m\dot{q}_m^T(t)\dot{q}_m(t) -\delta_sK_s\dot{q}_s^T(t)\dot{q}_s(t)+2|\dot{q}_m(t)|^T\int_{t-T_2(t)}^{t}P(|\dot{q}_s(\tau)|)d\tau +2|\dot{q}_s(t)|^T\int_{t-T_1(t)}^{t}P(|\dot{q}_m(\tau)|)d\tau \quad (23)$$

On the other hand, the time derivative of $V_4(x_t)$ is

$$\dot{V}_4(x_t) = 2T_{1max}\dot{q}_m^T(t)P\big(\dot{q}_m(t)\big) - 2\int_{t-T_1(t)}^{t} \dot{q}_m^T(\tau)P\big(\dot{q}_m(\tau)\big)d\tau$$
$$+2T_{2max}\dot{q}_s^T(t)P\big(\dot{q}_s(t)\big) - 2\int_{t-T_2(t)}^{t} \dot{q}_s^T(\tau)P\big(\dot{q}_s(\tau)\big)d\tau \qquad (24)$$

Considering (23) and (24) together, we have

$$\dot{V}(x_t) \le -\delta_m K_m \dot{q}_m^T(t)\dot{q}_m(t) - \delta_s K_s \dot{q}_s^T(t)\dot{q}_s(t)$$
$$+2|\dot{q}_m(t)|^T \int_{t-T_2(t)}^{t} P(|\dot{q}_s(\tau)|)d\tau$$
$$+2|\dot{q}_s(t)|^T \int_{t-T_1(t)}^{t} P(|\dot{q}_m(\tau)|)d\tau + 2T_{1max}\dot{q}_m^T(t)P\big(\dot{q}_m(t)\big)$$
$$-2\int_{t-T_1(t)}^{t} \dot{q}_m(\tau)P\big(\dot{q}_m(\tau)\big)d\tau + 2T_{2max}\dot{q}_s^T(t)P\big(\dot{q}_s(t)\big)$$
$$-2\int_{t-T_2(t)}^{t} \dot{q}_s^T(\tau)P\big(\dot{q}_s(t)\big)d\tau \qquad (25)$$

Applying Lemma IV to (25), we get

$$\dot{V}(x_t) \le -\delta_m K_m \dot{q}_m^T(t)\dot{q}_m(t) - \delta_s K_s \dot{q}_s^T(t)\dot{q}_s(t) + 2(T_{1max} + T_{2max})\dot{q}_m^T(t)P\big(\dot{q}_m(t)\big) + 2(T_{1max} + T_{2max})\dot{q}_s^T(t)P\big(\dot{q}_s(t)\big) \qquad (26)$$

Defining η_m and η_s as

$$\eta_m = \frac{\dot{q}_m^T(t)P\big(\dot{q}_m(t)\big)}{\|\dot{q}_m(t)\|_2^2}, \qquad \|\dot{q}_m(t)\|_2 \ne 0 \qquad (27)$$
$$\eta_s = \frac{\dot{q}_s^T(t)P\big(\dot{q}_s(t)\big)}{\|\dot{q}_s(t)\|_2^2}, \qquad \|\dot{q}_s(t)\|_2 \ne 0 \qquad (28)$$

we have

$$\begin{aligned}\dot{V}(x_t) \le &-(\delta_m K_m - 2(T_{1max} + T_{2max})\eta_m)\dot{q}_m^T(t)\dot{q}_m(t)\\ &-(\delta_s K_s - 2(T_{1max} + T_{2max})\eta_s)\dot{q}_s^T(t)\dot{q}_s(t)\end{aligned} \qquad (29)$$

Now, let us find conditions on δ_m and δ_s such that $\dot{V}(x_t) \le 0$. It is possible to see from (29) that a sufficient condition for $\dot{V}(x_t) \le 0$ is

$$\delta_m K_m \ge 2(T_{1max} + T_{2max})\eta_m \qquad (30)$$
$$\delta_s K_s \ge 2(T_{1max} + T_{2max})\eta_s \qquad (31)$$

Next, we will investigate conditions under which the inequalities (30) and (31) are satisfied. Given the definitions of η_m and η_s in (27) and (28) and using property P-VI of function $P(.)$, we have

$$\begin{aligned}\eta_m &\le min\left\{1, \frac{\sum_{j=1}^{n}\left|\dot{q}_{m_j}(t)\right|N_j}{\|\dot{q}_m(t)\|_2^2}\right\}\\ \eta_s &\le min\left\{1, \frac{\sum_{j=1}^{n}\left|\dot{q}_{s_j}(t)\right|N_j}{\|\dot{q}_s(t)\|_2^2}\right\}\end{aligned} \qquad (32)$$

To study the lower bounds of δ_m and δ_s for replacement in (30)-(31), let us consider two regions $\mathcal{X}_1$ and $\mathcal{X}_2$ as

$$\mathcal{X}_1 \triangleq \left\{\dot{q}_m(t)\colon \left|\dot{q}_{m_j}(t)\right| \le \frac{M_j - d_j}{K_m} \quad j = 1 \cdots n\right\} \qquad (33)$$
$$\mathcal{X}_2 \triangleq \left\{\dot{q}_m(t)\colon \left|\dot{q}_{m_j}(t)\right| > \frac{M_j - d_j}{K_m} \quad j = 1 \cdots n\right\} \qquad (34)$$

where $d_j \triangleq sup_{(x,y)\epsilon R\times R}\left|g_j(x) - p_j(y)\right|$. We distinguish the following two cases:

- Case 1: $\dot{q}_m(t)\epsilon\mathcal{X}_1$

Based on $g_{m_j}(.) - p_j(.) - K_m\dot{q}_{m_j}(t) \le g_{m_j}(.) - p_j(.) + K_m\left|\dot{q}_{m_j}(t)\right| \le g_{m_j}(.) - p_j(.) + M_j - d_j \le M_j$ and the definition of δ_m in (22), we have $\delta_m = 1$. Also, from (32), we know that $\eta_m \le 1$.

Applying $\delta_m = 1$ and $\eta_m \le 1$ to (30), the following inequality is found as a sufficient condition for (30):

$$K_m > 2(T_{1max} + T_{2max}) \qquad (35)$$

Therefore (35) is a sufficient condition to have $-(\delta_m K_m - 2(T_{1max} + T_{2max})\eta_m)\dot{q}_m^T(t)\dot{q}_m(t) \le 0$.

- Case 2: $\dot{q}_m(t)\epsilon\mathcal{X}_2$

Then, $\left|g_{m_j}(.) - p_j(.) - K_m\dot{q}_{m_j}(t)\right|$ could be greater than M_j or less than M_j. So,

1. If $\left|g_{m_j}(.) - p_j(.) - K_m\dot{q}_{m_j}(t)\right| \le M_j$, based on the definition of δ_m in (22), $\delta_m = 1$.
2. If $\left|g_{m_j}(.) - p_j(.) - K_m\dot{q}_{m_j}(t)\right| > M_j$, then $s_i\left(g_{m_j}(.) - p_j(.) - K_m\dot{q}_{m_j}(t)\right) > M_j$ and using reverse triangle inequality, we will have the following inequality:

$$\left|\left(g_{m_j}(.) - p_j(.)\right) - \left(s_i\left(g_{m_j}(.) - p_j(.) - K_m\dot{q}_{m_j}(t)\right)\right)\right| > \left|M_j - \left|g_{m_j}(.) - p_j(.)\right|\right| > M_j - d_j$$

Using Lemma I, we have

$$\dot{q}_{m_j}(t)\left(\left(g_{m_j}(.) - p_j(.)\right) - \left(s_j\left(g_{m_j}(.) - p_j(.) - K_m\dot{q}_{m_j}(t)\right)\right)\right) = \left|\dot{q}_{m_j}(t)\right|\left|\left(g_{m_j}(.) - p_j(.)\right) - \left(s_j\left(g_{m_j}(.) - p_j(.) - K_m\dot{q}_{m_j}(t)\right)\right)\right| \qquad (36)$$

Given that $M_j - d_j \ge M_{min} - d$ and based on the definition of δ_m in (22), we get

$$\delta_m > \frac{\sum_{i=1}^{n}\left|\dot{q}_{m_j}(t)\right|(M_{min} - d)}{K_m\|\dot{q}_m(t)\|_2^2} \qquad (37)$$

Knowing from (32) that $\eta_m \le \frac{\sum_{j=1}^{n}\left|\dot{q}_{m_j}(t)\right|N_j}{\|\dot{q}_m(t)\|_2^2}$ and using (37), it is possible to see that $\delta_m K_m N_{max} \ge (M_{min} - d)\eta_m$.

Using this, we can find following condition to satisfy the inequality (30).

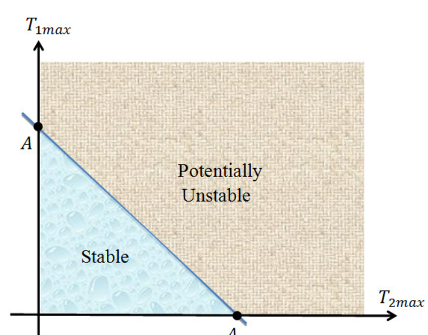
$$\frac{M_{min} - d}{N_{max}} \ge 2(T_{1max} + T_{2max}) \qquad (38)$$

Therefore if (35) and (38) are satisfied, then $-(\delta_m K_m - 2(T_{1max} + T_{2max})\eta_m)\dot{q}_m^T(t)\dot{q}_m(t) \le 0$. Finally, conducting a similar analysis to find a condition for (31) to hold will result in (38) and following inequality

$$K_s > 2(T_{1max} + T_{2max}) \qquad (39)$$

Using the above analysis, it is possible to see that if M_{max} satisfies (38) and K_m and K_s fulfill inequalities (35) and (39), then $\dot{V}(x_t) \le 0$ meaning that all terms in $V(x_t)$ are bounded. Therefore, $\dot{q}_m$, $\dot{q}_s$ and $q_m - q_s\epsilon\mathcal{L}_\infty$ and the poof is complete.

□



$$A \triangleq \frac{1}{2}min\left\{K_m, K_s, \frac{M_{min} - \gamma_{max}}{N_{max}} - 1\right\}$$

Fig. 1. The stability region based on inequalities (35), (38) and (39).

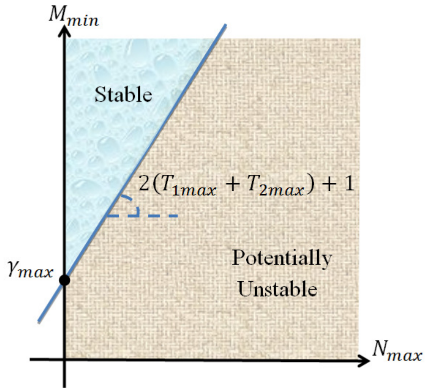


Fig. 2. the stability region provided by (38) in M_{min} versus N_{max} plane given T_{1max} and T_{2max}.

Note that the parameter d defined in Theorem I is equal to $N_{max} + \gamma_{max}$ where $\gamma_{max} \triangleq max\{\gamma_j\}$. Using the inequalities (35), (38) and (39), a schematic representation of the stability condition in terms of T_{1max} and T_{2max} is shown in Fig. 1.

The stability condition (38) in terms of M_{min} and N_{max} is shown in Fig. 2.

Theorem II. In the bilateral teleoperation system (1)-(2) with the controller (4)-(5), the absolute values of the velocities $|\dot{q}_m(t)|$ and $|\dot{q}_s(t)|$ and the position error $|q_m(t) - q_s(t)|$ tend to zero asymptotically in free motion (i.e., $\tau_h(t), \tau_e(t) \to 0$) if all conditions in Theorem I are satisfied and both $\dot{T}_1(t)$ and $\dot{T}_2(t)$ are bounded.

Proof of Theorem II:

Integrating both sides of (29), it is possible to see that, $\dot{q}_m(t)$ and $\dot{q}_s(t)\epsilon\mathcal{L}_2$. Based on the result of theorem I, $V(t)$ is a lower bounded decreasing function. Therefore, $\dot{q}_m(t)$, $\dot{q}_s(t)$ and $q_m(t) - q_s(t)\, \epsilon\mathcal{L}_\infty$. Using the fact that $q_m(t) - q_s\big(t - T_2(t)\big)$ $= q_m(t) - q_s(t) + \int_{t-T_2(t)}^{t} \dot{q}_s(t)dt$ and $\int_{t-T_2(t)}^{t} \dot{q}_s(t)dt\, \epsilon\mathcal{L}_\infty$, we have $q_m(t) - q_s\big(t - T_2(t)\big)\epsilon\mathcal{L}_\infty$. Since the gravity terms g_m and g_s are bounded, using property P-1 and P-2 of system dynamics and given the boundedness of $S(\tau_m(t))$ and $S(\tau_s(t))$, it can be seen that $\ddot{q}_m(t)$ and $\ddot{q}_s(t)\epsilon\mathcal{L}_\infty$. Because $\dot{q}_m(t)\epsilon\mathcal{L}_2$ and $\ddot{q}_m(t)\epsilon\mathcal{L}_\infty$, using Barbalat's lemma we have that $\dot{q}_m(t) \to 0$. Similarly, it can be reasoned that $\dot{q}_s(t) \to 0$.

Now, if $\dddot{q}_m$ and $\dddot{q}_s$ are continuous in time, or equivalently $\dddot{q}_m(t)$ and $\dddot{q}_s(t)\epsilon\mathcal{L}_\infty$, then $\dot{q}_m(t)$ and $\dot{q}_s(t) \to 0$ ensures that $\ddot{q}_m(t)$ and $\ddot{q}_s(t) \to 0$. Let us investigate the boundedness of $\dddot{q}_i(t)$ for $i\epsilon\{m, s\}$. The closed-loop dynamics found from combining the open-loop system (1) and (2) with the controllers (4) and (5) is

$$\begin{aligned}
\ddot{q}_m(t) &= \Big(M_m\big(q_m(t)\big)\Big)^{-1}\{-C_m\big(q_m(t), \dot{q}_m(t)\big)\dot{q}_m(t) \\
&-G_m\big(q_m(t)\big) - S\Big(-G_m\big(q_m(t)\big) + P\Big(q_m(t) - q_s\big(t - T_2(t)\big)\Big) \\
&+K_m\dot{q}_m(t)\Big)\} \\
\ddot{q}_s(t) &= \Big(M_s\big(q_s(t)\big)\Big)^{-1}\{-C_s\big(q_s(t), \dot{q}_s(t)\big)\dot{q}_s(t) - G_s\big(q_s(t)\big) \\
&-S\Big(-G_s\big(q_s(t)\big) + P\Big(q_s(t) - q_m\big(t - T_1(t)\big)\Big) \\
&+K_s\dot{q}_s(t)\Big)\}
\end{aligned} \tag{40}$$

Differentiating both sides with respect to time and given

$$\begin{aligned}
\frac{d}{dt}\Big(M_i\big(q_i(t)\big)\Big)^{-1} &= -\Big(M_i\big(q_i(t)\big)\Big)^{-1}\{C_i\big(q_i(t), \dot{q}_i(t)\big) \\
&+C_i^T\big(q_i(t), \dot{q}_i(t)\big)\}M_i\big(q_i(t)\big) \qquad i\epsilon\{m, s\}
\end{aligned} \tag{41}$$

and based on properties P-1 and P-3 and given the boundedness of $\dot{q}_s$ and $\dot{q}_m$, it is easy to see that $\frac{d}{dt}\Big(M_m\big(q_m(t)\big)\Big)^{-1}$ and $\frac{d}{dt}\Big(M_s\big(q_s(t)\big)\Big)^{-1}$ are bounded. Given that $\dot{P}(\cdot)$ is bounded and using properties P-1, P-3 and P-4 of system dynamics and the boundedness of $q_s(t) - q_m\big(t - T_1(t)\big)$, $q_m(t) - q_s\big(t - T_2(t)\big)$,$\dot{q}_m, \dot{q}_s, \ddot{q}_m, \ddot{q}_s$, $\dot{T}_1$ and $\dot{T}_2$, it can be seen that $\dddot{q}_m$ and $\dddot{q}_s$ are bounded. Given that $\dot{q}_m(t)$ and $\dot{q}_s(t) \to 0$ and $\dddot{q}_m(t)$ and $\dddot{q}_s(t)\epsilon\mathcal{L}_\infty$, using Barbalat's lemma we have that $\ddot{q}_m(t)$ and $\ddot{q}_s(t) \to 0$.

Considering the dynamic equation of the master and slave robots in (10) and (11), having shown that $\ddot{q}_i(t) \to 0$ and $\dot{q}_i(t) \to 0$, $i\epsilon\{m, s\}$, it is easy to see that

$$G_m\big(q_m(t)\big) \to S\Big(G_m\big(q_m(t)\big) - P\big(q_m(t) - q_s(t - T_2(t))\big)\Big) \tag{42}$$

$$G_s\big(q_s(t)\big) \to S\Big(G_s\big(q_s(t)\big) - P\big(q_s(t) - q_m(t - T_1(t))\big)\Big) \tag{43}$$

Given $S\Big(G_i\big(q_i(t)\big)\Big) = G_i\big(q_i(t)\big)$, we find that

$$P\big(q_m(t) - q_s(t - T_2(t))\big) \to 0 \tag{44}$$

$$P\big(q_s(t) - q_m(t - T_1(t))\big) \to 0 \tag{45}$$

and using assumptions of $P(\cdot)$,

$$|q_m(t) - q_s(t - T_2(t))| \to 0 \tag{46}$$

$$|q_s(t) - q_m(t - T_1(t))| \to 0 \tag{47}$$

It is possible to see the asymptotic zero convergence of velocities and the position tracking error and proof is completed. □

Remark I. Based on the assumption $T_{1max} + T_{2max} \le \frac{M_{min} - (N_{max} + \gamma_{max})}{2N_{max}}$ in Theorem I, it is possible to see a trade-off between the robustness to the maximum values of time delays and the tracking performance in controller design. For instance, if N_{max} is lowered, then the position difference between the master and the slave robots contributes less to the control signal, resulting in an increase in the settling time for the position tracking response. At the same time, as N_{max} is lowered, the maximum admissible values of time delays increase; i.e., the robustness of the system stability to larger time delays improves. The above trends in performance and stability are understandable once one pays attention to the control laws (4)-(5). When N_{max} has a small value, the nonlinear proportional terms in (4)-(5) are suppressed, leaving more "room" for the derivative signals $\dot{q}(t)$ to contribute to the control signal, i.e., the velocity gains K_m and K_s are allowed to be larger. It is clear from the stability conditions (35) and (39) that larger K_m and K_s allow for larger values for the maximum time delays. At the same time, with small N_{max}, settling time of position tracking increases, degrading the performance of the teleoperation system. Therefore, it can be concluded that, for a fixed M_{min} , there is a trade-off between stability and performance of the system and this trade-off can be tuned by changing N_{max}.

Remark III. The discussion in this paper has focused on joint-space dynamics of the master and the slave robots and joint-level position tracking. It is noteworthy that the dynamics of a robot in the task space are similar to the joint-space dynamics (1)-(2) and the inertia, Coriolis/centrifuge, and gravity matrices/vectors in task-space dynamics also have the same properties (Lee (2011)). Therefore, the proposed controller and stability results in this paper can be used in the task space in the same way that they are used in the joint space. This is advantageous because, in different applications, one may be facing with different saturation effects. For instance, for safety reasons, a low-level controller may be enforcing preset limits on the (Cartesian) forces applied by the end-effector of a robot; this would be a case of actuator saturation in the task space.

5. Simulation and Experiment Results

In this section, simulation and experimental results for the proposed teleoperation controller are provided.

A) Simulation on a teleoperated pair of 2-DOF planar robots

To verify the theoretical results of this paper, the master and the slave manipulators are considered to be a pair of 2-DOF planar robots with revolute joints. In this simulation, it is assumed that the control signals for the both the master and the slave robots are subjected to actuator saturation at levels +1 and -1. Also the single-way forward and backward time delays in the communication channel are chosen to be random variables with uniform distributions over $[0 \quad 0.3]$ second. The random nature of time delays makes it possible to show the effectiveness of the proposed method for fast-varying time delays. Note that the single-way time delays used in this experiment, which vary between 0 and 300 ms, are reasonable in Internet-based teleoperation applications because the round-trip delay has been shown to have an average of 350 ms if the distance between the local and the remote robots is up 10,000 km (Oboe (2003)). The input human force, F_{hX} and F_{hY}, applied to the master robot are zero at the beginning and starts to increase after 0.5 and 1.5 s, reaching 10 N at 1 and 2 s and staying at that level until 2 and 3 s, respectively. After that, they decrease until 2.5 and 3.5 s, respectively, when they reach zero. The application of this human force results in the joint positions profiles for the master and the slave robots shown in Fig. 3.

To show the efficiency of the proposed method, the propose nP+D controller has been applied to the 2-DOF planar robots assuming saturation on joint actuators as well as the previously-discussed time varying delays in the communication channel. The nonlinear function $P(\cdot)$ in the controllers (4) and (5) is chosen as $0.2\,tan^{-1}(\cdot)$. Note that all conditions listed for Theorem I are satisfied with the function $tan^{-1}(\cdot)$; see the required properties of $P(\cdot)$ in section III. Also, K_m and K_s are set to 3. This means that, in this simulation, the controllers (4) and (5) become

$$\tau_m(t) = -G_m\big(q_m(t)\big) + 0.2\,tan^{-1}\Big(q_m(t) - q_s\big(t - T_2(t)\big)\Big) + 3\dot{q}_m(t)$$
$$\tau_s(t) = G_s\big(q_s(t)\big) - 0.2\,tan^{-1}\Big(q_s(t) - q_m\big(t - T_1(t)\big)\Big) - 3\dot{q}_s(t) \quad (48)$$

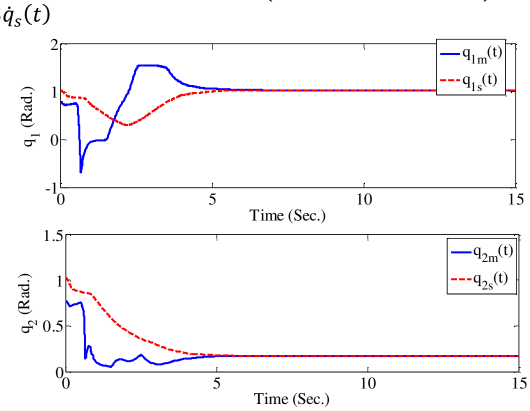


Fig. 3. First joint (up) and second joint (down) positions of the 2-DOF planar robot teleoperation with actuator saturation and variable time delay controlled by the proposed nP+D controller.

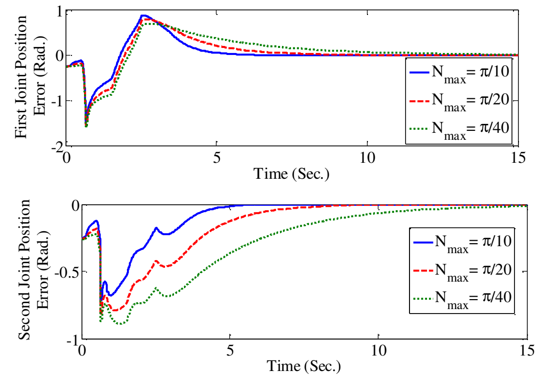


Fig. 4. First joint (up) and second joint (down) position errors between master and slave robots for different values of N_{max}.

Note that in this simulation, $M_{min} = 1$, $N_{max} = \pi/10$, $\gamma_{max} = 0.05$ and $T_{1max} = T_{2max} = 0.3$, for which it is easy to see that the sufficient condition (38) is satisfied.

As mentioned in Remark I, the maximum value of the nonlinear function $P(\cdot)$ in the control laws (4)-(5), N_{max}, can affect the performance of the closed-loop system. To show this, in Fig. 4, the position tracking errors between the master and the slave robots are shown for three different values of N_{max}. Evidently, smaller values of N_{max} make the closed-loop response slower and the settling time larger.

B) Experiment on a teleoperated pair of 3-DOF PHANToM robots

In this section, experimental results for the proposed control method are reported. In the experimental setup shown in Fig. 5, two 3-DOF PHANToM Premium 1.5A robots are connected via a communication channel with varying time delays with uniform distributions between 81 and 100 ms. *Please note that, in addition to the experimental results reported below, a video showing experiments on this teleoperation test-bed accompanies this paper*[1].

In the proposed nP+D method, the following controllers are used:

$$\tau_m(t) = -G_m\big(q_m(t)\big) + 0.1\,p\left(5\left(q_m(t) - q_s\big(t - T_2(t)\big)\right)\right) + \dot{q}_m(t)$$
$$\tau_s(t) = G_s\big(q_s(t)\big) - 0.1\,p\left(5\left(q_s(t) - q_m\big(t - T_1(t)\big)\right)\right) - \dot{q}_s(t) \quad (49)$$

Fig. 5. The experimental teleoperation setup consisting of two PHANToM Premium 1.5A robots and the schematic of the PHANToM robot with its corresponding joint angles.

In the above, we choose the nonlinear function in the controllers to be $p(x) \triangleq sgn(x)min\{|x|, 1\}$ with $sgn(\cdot)$ being the signum function. As a result, when the position errors $q_m(t) - q_s\big(t - T_2(t)\big)$ (the same holds for the position error $q_s(t) - q_m\big(t - T_1(t)\big)$) is between -0.2 and 0.2 radian, it appears linearly through a gain of 0.5 in the control signal. Otherwise, its contribution to the control signal is maxed out at -0.1 or 0.1. Note that the function $0.1p(5x)$ meets all the required properties listed in Section III. Also note that the

[1] http://www.ece.ualberta.ca/~mtavakol/Automatica_submission/

approximate levels of the actuator saturation for the first, second and third joint of the PHANToM robot in generalized coordinate are $M_1 = 0.29$, $M_2 = 0.29$ and $M_3 = 0.23$ Nm, respectively. Also $\gamma_{max} = 0.07$, $M_{min} = 0.28$, $T_{1max} = T_{2max} = 0.1$, $N_{max} = 0.1$ and it is easy to see that for these values the sufficient condition (38) for stability is satisfied. Experimental results of the proposed nP+D controller in terms of joint position tracking between the master and the slave robots are shown in Fig. 6.

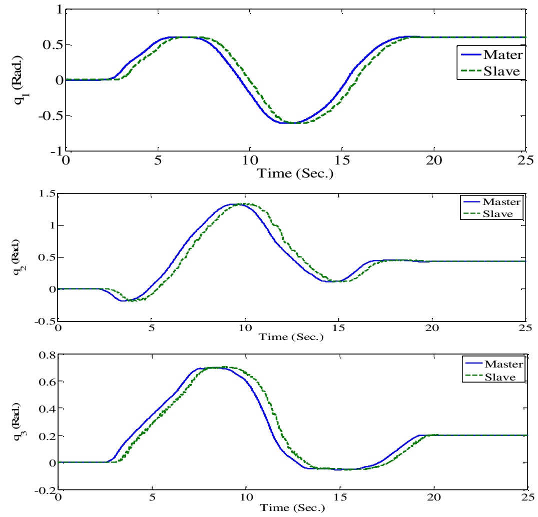


Fig. 6. Joint position tracking between the master and the slave using the proposed nP+D control method.

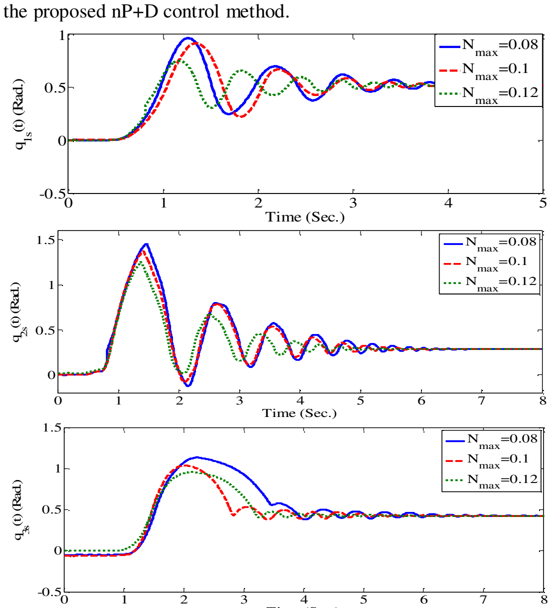


Fig. 7. Step response of slave robot's joint positions for different values of N_{max}.

Let us study the effect of N_{max}, which is the maximum value of the nonlinear function $P(\cdot)$ in (4)-(5), on the performance of the teleoperation system in experiments. To show the slave's joint position trajectory in response to a step set-point (corresponding to a fixed position for the master), the control signal (50) is applied to the slave robot for different values of N_{max}.

$$\tau_s(t) = G_s(q_s(t)) - N_{max}\, p\left(5\left(q_s(t) - q_m(t - T_1(t))\right)\right) \quad (50)$$

where $p(.)$ is the same as that used in (49), i.e., $p(x) \triangleq sgn(x)min\{|x|, 1\}$. In Fig. 7, the step responses of the slave robot's joint positions are shown for three different values of N_{max}. Evidently, a smaller N_{max}, which corresponds to a reduced contribution of the tracking error $q_s(t) - q_m(t - T_1(t))$ to the slave robot's control signal, leads to a slower step response (i.e., larger settling time). This is a result that is consistent with Remark I.

6. Conclusion and future work

In this paper, we developed a novel method to cope with actuator saturation in bilateral teleoperation systems that are subjected to time-varying time delays in their communication channels. The proposed controller, which we call nP+D method, is similar to the conventional P+D controller except for the fact that we have replaced the proportional term by a nonlinear function through which the position errors pass. This makes the proposed nP+D method capable of handling actuator saturation and guaranteeing position tracking even in the presence of time-varying time delays in the communication channel. We analyzed the stability of the system using a Lyapunov Krasovskii functional and showed asymptotic position tracking between the master and the slave robots. The derived stability conditions involve relationships between the nP+D controller parameters, the actuator saturation characteristics, and the maximum values of the time-varying delays. We simulated the proposed controller on a teleoperated pair of 2-DOF planar robots subjected to actuator saturation and time-varying delays. We also experimentally tested the proposed controller on a teleoperated pair of 3-DOF PHANToM Premium robots, which are naturally subject to actuator saturation. Simulation and experimental results of the proposed nP+D control method have demonstrated its efficiency.

In the future, it is possible to account for the simultaneous presence of time-varying time delay, actuator sandwich linearity, and a third source of problem in teleoperation control such as model uncertainties, unmodeled dynamics, external disturbances, etc.

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Appendix

Here, proofs of Lemmas I, II, III and IV are provided.

Proof of Lemma I: The assumption in the lemma can be summed up as $\left|g_{m_j}(\cdot)-p_{m_j}(\cdot)\right|<M_j$ and $\left|g_{s_j}(\cdot)-p_{s_j}(\cdot)\right|<M_j$. Regarding increasing monotone property of function $s_j(\cdot)$, if $\dot q_{m_j}\ge 0$ then $s_j\left(g_{m_j}(\cdot)-p_{m_j}(\cdot)-k_j\dot q_{m_j}\right)\le s_j\left(g_{m_j}(\cdot)-p_{m_j}(\cdot)\right)$ and if $\dot q_{m_j}<0$ then $s_j\left(g_{m_j}(\cdot)-p_{m_j}(\cdot)-k_j\dot q_{m_j}\right)>s_j\left(g_{m_j}(\cdot)-p_{m_j}(\cdot)\right)$. Based on the fact that $g_{m_j}(\cdot)-p_{m_j}(\cdot)$ belongs to the linear part of s_j, $s_j\left(g_{m_j}(\cdot)-p_{m_j}(\cdot)\right)=g_{m_j}(\cdot)-p_{m_j}(\cdot)$, therefore for all $\dot q_{m_j}$

$$\dot q_{m_j}\left(s_j\left(g_{m_j}(\cdot)-p_{m_j}(\cdot)-k_j\dot q_{m_j}\right)-\left(g_{m_j}(\cdot)-p_{m_j}(\cdot)\right)\right)\le 0 \quad \text{(A1)}$$

$$\dot q_m^T\left(S(G_m(\cdot)-P_m(\cdot)-K_m\dot q_m)-\left(G_m(\cdot)-P_m(\cdot)\right)\right)$$
$$=\sum_{j=1}^{n}\dot q_{m_j}\left(s_j\left(g_{m_j}(.)-p_{m_j}(.)-k_j\dot q_{m_j}\right)-\left(g_{m_j}(.)-p_{m_j}(.)\right)\right)\le 0$$
(A2)

Using similar study,

$$\dot q_s^T\left(S(G_s(\cdot)-P_s(\cdot)-K_s\dot q_s)-(G_s(\cdot)-P_s(\cdot))\right)\le 0 \quad \text{(A3)}$$

and proof completed. □

Proof of Lemma II: Based on the definition of integral, we know that

$$\int_{t-T(t)}^{t}x(\tau)d\tau=\lim_{n\to\infty}\sum_{k=0}^{n-1}\frac{T(t)}{n}x\left(t-T(t)+k\frac{T(t)}{n}\right) \quad \text{(A4)}$$

$$\int_{t-T(t)}^{t}p_i\big(x(\tau)\big)d\tau=\lim_{n\to\infty}\sum_{k=0}^{n-1}\frac{T(t)}{n}p_i\left(x\left(t-T(t)+k\frac{T(t)}{n}\right)\right) \quad \text{(A5)}$$

Using the properties P-III and P-IV of $p_i(.)$ and knowing that $x(\tau)$ and $T(t)$ are positive,

$$p_i\left(\int_{t-T(t)}^{t}x(\tau)d\tau\right)=p_i\left(\lim_{n\to\infty}\sum_{k=0}^{n-1}\frac{T(t)}{n}x\left(t-T(t)+k\frac{T(t)}{n}\right)\right)$$
$$\le\lim_{n\to\infty}\sum_{k=0}^{n-1}p_i\left(\frac{T(t)}{n}x\left(t-T(t)+k\frac{T(t)}{n}\right)\right) \quad \text{(A6)}$$
$$=\lim_{n\to\infty}\sum_{k=0}^{n-1}\frac{T(t)}{n}p_i\left(x\left(t-T(t)+k\frac{T(t)}{n}\right)\right)=\int_{t-T(t)}^{t}p_i\big(x(\tau)\big)d\tau$$

and proof completed. □

Proof of lemma III: Considering property P-I of $p_j(.)$,

$$\left|p_j\left(q_{m_j}(t)-q_{s_j}(t)\right)-p_j\left(q_{m_j}(t)-q_{s_j}\big(t-T_2(t)\big)\right)\right|$$
$$\le 2p_j\left(\left|q_{s_j}(t)-q_{s_j}\big(t-T_2(t)\big)\right|\right)=2p_j\left(\left|\int_{t-T_2(t)}^{t}\dot q_{s_j}(\tau)d\tau\right|\right) \quad \text{(A7)}$$

and similarly

$$\left|p_j\left(q_{s_j}(t)-q_{m_j}(t)\right)-p_j\left(q_{s_j}(t)-q_{m_j}\big(t-T_1(t)\big)\right)\right|$$
$$\le 2p_j\left(\left|\int_{t-T_1(t)}^{t}\dot q_{m_j}(\tau)d\tau\right|\right) \quad \text{(A8)}$$

Using property P-II of $p_j(.)$ and knowing that for any $\dot q\epsilon R$ and for any positive$T(t)$,$\left|\int_{t-T(t)}^{t}\dot q(\tau)d\tau\right|\le\int_{t-T(t)}^{t}|\dot q(\tau)|d\tau$, then

$$p_j\left(\left|\int_{t-T(t)}^{t}\dot q(\tau)d\tau\right|\right)\le p_j\left(\int_{t-T(t)}^{t}|\dot q(\tau)|d\tau\right) \quad \text{(A9)}$$

Considering lemma II,

$$p_j\left(\int_{t-T(t)}^{t}|\dot q(\tau)|d\tau\right)\le\int_{t-T(t)}^{t}p_j(|\dot q(\tau)|)d\tau \quad \text{(A10)}$$

Considering (A7), (A9) and (A10),

$$\left|p_j\left(q_{m_j}(t)-q_{s_j}(t)\right)-p_j\left(q_{m_j}(t)-q_{s_j}\big(t-T_2(t)\big)\right)\right|$$
$$\le 2\int_{t-T(t)}^{t}p_j(|\dot q(\tau)|)d\tau$$

and therefore

$$\dot q_m^T(t)\left(P\big(q_m(t)-q_s(t)\big)-P\left(q_m(t)-q_s\big(t-T_2(t)\big)\right)\right)$$
$$=\sum_{j=1}^{n}\left(\dot q_{m_j}(t)\left(p_j\left(q_{m_j}(t)-q_{s_j}(t)\right)-p_j\left(q_{m_j}(t)-q_{s_j}\big(t-T_2(t)\big)\right)\right)\right)$$
$$\le\sum_{j=1}^{n}\left(\left|\dot q_{m_j}(t)\right|\left|p_j\left(q_{m_j}(t)-q_{s_j}(t)\right)-p_j\left(q_{m_j}(t)-q_{s_j}\big(t-T_2(t)\big)\right)\right|\right)$$
$$=2\sum_{j=1}^{n}\left(\left|\dot q_{m_j}(t)\right|\left(\int_{t-T(t)}^{t}p_j\left(\left|\dot q_{s_j}(\tau)\right|\right)d\tau\right)\right)$$
$$=2|\dot q_m(t)|^T\int_{t-T_2(t)}^{t}P(|\dot q_s(\tau)|)d\tau \quad \text{(A11)}$$

Similarly

$$\dot q_s^T(t)\left(P\big(q_s(t)-q_m(t)\big)-P\left(q_s(t)-q_m\big(t-T_1(t)\big)\right)\right)$$
$$\le 2|\dot q_s(t)|^T\int_{t-T_1(t)}^{t}P(|\dot q_m(\tau)|)d\tau \quad \text{(A12)}$$

and proof completed. □

Proof of Lemma IV: given property P-V of function$p_j(.)$,

$$\sum_{j=1}^{n}\left(a_j(t)p_j\left(b_j(\tau)\right)-b_j(\tau)p_j\left(b_j(\tau)\right)\right)\le\sum_{j=1}^{n}\left(a_j(t)p_j\left(a_j(t)\right)\right) \quad \text{(A13)}$$

Therefore

$$A^T(t)P\big(B(\tau)\big)-B^T(\tau)P\big(B(\tau)\big)\le A^T(t)P\big(A(t)\big) \quad \text{(A14)}$$

Integrating both sides based on $d\tau$ from $t-T(t)$ to t,

$$\int_{t-T(t)}^{t}A^T(t)P\big(B(\tau)\big)d\tau-\int_{t-T(t)}^{t}B^T(\tau)P\big(B(\tau)\big)d\tau$$
$$\le\int_{t-T(t)}^{t}A^T(t)P\big(A(t)\big)d\tau \quad \text{(A15)}$$

That can be simplified to

$$A^T(t)\int_{t-T(t)}^{t}P\big(B(\tau)\big)d\tau-\int_{t-T(t)}^{t}B^T(\tau)P\big(B(\tau)\big)d\tau\le T(t)A^T(t)P\big(A(t)\big)$$
(A16)

Given property P-VIII of function $p_j(\cdot)$,$A^T(t)P\big(A(t)\big)>0$ and so

$$A^T(t)\int_{t-T(t)}^{t}P\big(B(\tau)\big)d\tau-\int_{t-T(t)}^{t}B^T(\tau)P\big(B(\tau)\big)d\tau\le T_mA^T(t)P\big(A(t)\big)$$
(A17)

and proof is completed. □