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A Method for Passivity Analysis of Multilateral Haptic Systems

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Abstract

This paper presents a novel criterion to study the stability of multilateral teleoperation systems based on passivity. Such systems (modelled as n-port networks) have recently found interesting applications in cooperative haptic teleoperation and haptic-assisted training. The criterion provides researchers with an analytical, closed-form, necessary and sufficient condition useful for both analysis and design of multilateral haptic teleoperation systems. The paper shows that when $n = 2$ the proposed conditions reduce to the well-known Raisbeck's passivity criterion for 2-port networks. The proposed conditions are used to study the passivity (and consequently the stability) of a dual-user haptic system for control of a single teleoperated robot. Simulations and experiments are performed to further test the validity of the proposed criterion.

Keywords: Passivity, absolute stability, n-port networks, multilateral haptic systems, teleoperator, teleoperation, trilateral system.

1. Introduction

1.1 Motivation

The sense of touch allows us to explore and manipulate an object by feeling its roughness, size, stiffness, etc. When an object we intend to manipulate is not physically reachable, we can use tools as extensions to our arms. Sometimes, the extension tool is capable of recreating for us the sense of touch. In that case, we are able to manipulate remote objects and "feel" as if we are in direct contact with them. The described scenario is realized by haptic teleoperation systems. These systems are made up of one or more human operator(s) coupled to one or more master robot(s) in order to control the movement of a remote slave to perform a task on a remote environment.

The key motivation for this research is to establish a criterion for investigating the stability of multilateral haptic teleoperation systems, which can be modeled as n-port networks. The realization of a *teleoperator* involve one or more *master* robots (i.e., user interfaces), one or more *slave* robots (i.e., remote robots), control units, and communication channels between the masters and the slaves. A multilateral *teleoperation system* is formed once the above teleoperator is coupled to human operators in one end and to external environments in the other end; naturally, human operators are coupled to the masters while the environments interact with the slaves. The teleoperation system is said to provide haptic feedback if all of the slave/environment interaction forces are reflected back to the human operators via the masters.

Figure 1 shows a multilateral *teleoperation system* made up of n robots. One potential application scenario for Figure 1 is that $n-1$ master's robots are sharing the execution of a task in a remote environment by collaboratively controlling the movement of a slave robot [4][5][6][7][8]. In Figure 1, each human operator/master interaction is denoted by F_{hi}, $i = 1, \ldots, n-1$ and the slave/environment interaction is denoted by F_e. Also, V_{hi}, V_e, F_{cmi}, and F_{cs} are the masters' and the slave's velocities and control signals, respectively. Impedances Z_{hi} and Z_e denote the dynamic characteristics of the human operators and the remote environment, respectively. Z_{mi} and Z_s denote the linear impedances of the masters and the slave, respectively. Moreover, F_{hi}^* and F_e^* are the operators' and the environment's exogenous input forces.

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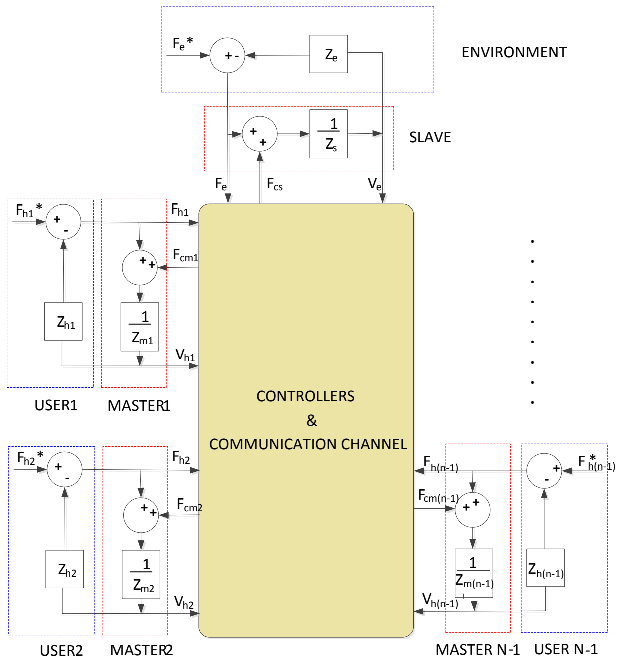

Figure 1. A multilateral haptic teleoperation system consisting of $n-1$ master robots and one slave robot.

Stability criteria can give formal and accurate information necessary for obtaining the best teleoperation transparency once one keeps in mind the trade-offs between performance and stability of any teleoperation system [20]. Consider a teleoperation task involving flipping the three-way switch shown in Figure 2. Assume that the human operator has to move the switch from state 1 to state 2 but not to state 3. The system should exhibit a sufficiently satisfactory performance so that the human operator can flip the switch by teleoperation of the slave robot through the master robot; for this, the slave robot's overshoot should be no more than the position difference between states 2 and 3. It is evident that master-slave position error, which is a measure of teleoperation system performance, directly affects the performance of the task. To achieve a small enough master-slave position error, the slave's position controller gains have to be selected large. However, selecting too large a controller gain risks making the system non-passive or even unstable [16][17][18]. *The upper limit on the controller gains before stability is lost is what can be determined using the passivity criterion developed in this paper.* The passivity criterion developed in this paper is, therefore, a valuable result that allow for obtaining maximum performance in the stable region. In practice, the upper limit imposed on the control gain for ensuring stability may restrict the performance to the extent that task performance is severely undermined. For example, in the same 3-way switch task, one may find that the highest slave's controller gain for which the system remains stable is still not high enough to complete the task successfully (especially if the switch is sticky and the position difference between states 2 and 3 is small) even though the same task is done readily under direct touch. Therefore, it is also informative to study if successful completion of this task is possible at all and this study can be facilitated using the passivity criterion proposed in this paper.

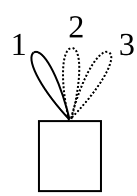

Figure 2. A three-way switch.

Human operators and environments are part of the closed-loop teleoperation system and thus their models are necessary for stability analysis. In practice, such models are next to impossible to acquire. For instance, the dynamics of a human operator changes according to the task at hand [2][3] and identification of the human arm dynamics requires a meticulous off-line process of data collection and analysis [19].

For the simplest case, the bilateral teleoperation system (Figure 3a), there exist well-known methods to investigate the stability. Such a study of stability is valid when the 2-port network (Figure 3b) is connected to *unknown* terminations (human operator and environment) that are passive. These methods are known as Llewellyn's absolute stability criterion and Raisbeck's passivity criterion [9]. A compact method to study the stability of multilateral teleoperation systems beyond the bilateral case, which is the subject of this paper, is still in demand.

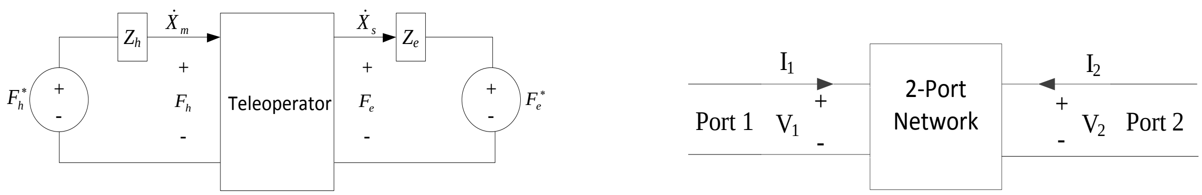

Figure 3a (left): A bilateral teleoperation system comprising a human operator, a teleoperator (a master, a slave, controllers, and a communication channel), and an environment. Figure 3b (right): A 2-port network.

1.2 Emerging Applications for Multilateral Teleoperation Systems

Multilateral teleoperation systems beyond the bilateral one can offer greater advantages: They can be used to haptically train people in performing remote tasks, they can increase task efficiency where it helps to use two hands instead of one, they can help to perform a task in cooperation among several human operators, etc.

1.3 Literature Survey

The stability of a multilateral haptic teleoperation can be determined by using passivity criteria. The following are the existing criteria for passivity of n-port networks, to the best of the author's knowledge.

In 1954, Raisbeck proposed a general definition of passivity of n-port networks [12]. From the definition, Raisbeck presented a passivity criterion for a 2-port network, however, he did not extend the criterion for the general case of n-port networks where n can be an integer greater than 2. In 1959, Youla et al. published the first formal justification of the passivity definition for n-port networks based on Raisbeck's general passivity definition (with minor differences) [13]. The paper presented a rigorous theory of passive LTI n-port networks but stopped short of proposing a passivity criterion. Wyatt et al. presented another rigorous definition for passivity of n-ports [14], however, like the previous case, this paper stopped short of proposing a passivity criterion for n-port networks.

In [15], Anderson and Spong introduced a tool for checking the passivity of an n-port network based on the singular value of the scattering matrix of the network. They showed that a network is passive if and only if the norm of its scattering operator is less than or equal to one. The scattering operator S is defined as

$$F - v = S(F + v) \tag{1}$$

In (1), F is the effort measured across the network's ports and v is the flow entering the network's ports. In relation to haptic teleoperation systems, the effort variable is equivalent to force and the flow variable is equivalent to velocity. In relation to electrical networks, effort is equivalent to voltage and flow is equivalent to current. In the Laplace domain, (1) becomes

$$F(s) - V(s) = S(s)(F(s) + V(s)) \tag{2}$$

According to [15], the n-port network is passive if and only if

$$\|S\|_\infty \leq 1 \tag{3}$$

This is equivalent to

$$\sup_\omega \lambda^{1/2}(S^*(j\omega)S(j\omega)) \leq 1 \tag{4}$$

where λ denotes the eigenvalue of a square matrix, * denotes the complex conjugate transpose, and ω is the frequency. Condition (4) is difficult to verify, especially without knowledge of the model parameters for the robots and the controllers, making it not suitable for control synthesis.

This paper presents a closed-form and practically-useful criterion for passivity of n-port networks (n equal or greater than 2), which can be used to investigate the stability of multilateral haptic teleoperation systems. Section 2 presents an overview of passivity of 2-port networks. In Section 3, a novel method to investigate the passivity of n-port networks, based on immittance parameters of the network, is presented. The method is given as a closed-form criterion for passivity of n-port networks and can be used to investigate the stability of multilateral haptic teleoperation systems. In Section 4, the passivity of a dual-user haptic system for control of a single teleoperated robot is investigated through simulations and experiments in order to verify the findings in Section 3. Section 5 presents the conclusions as well as directions for future research.

2. Passivity of Bilateral Teleoperators with Unknown Terminations

Closed-loop stability is crucial for safe teleoperation. For the analysis of closed-loop stability of a teleoperation system, according to Figure 3a, the knowledge of the human operator and the environment dynamics are needed in addition to that of the teleoperation system's immittance parameters ($z, y, h,$ or g). In practice, however, the models of the human operator and the environment are usually unknown, uncertain, and/or time-varying. This makes it impossible to use conventional techniques to investigate the closed-loop stability of a teleoperation system.

Assuming that $Z_h(s)$ and $Z_e(s)$ in Figure 3a are passive, we can draw stability conditions that are independent of the human operator and the environment by using Raisbeck's passivity criterion. The following definitions are needed before presenting this criterion.

Definition: Passivity *[9]*

A 2-port network is passive if, for all excitations, the total energy delivered to the network at its input and output ports is non-negative. Hence, passivity is a property of the 2-port network that establishes that it cannot deliver more energy than what is delivered to it. Assuming that the 2-port network has zero energy stored at time $t = 0$, the 2-port network is said to be passive if it satisfies

$$E(t) = \int_0^t (i_1(\tau)v_1(\tau) + i_2(\tau)v_2(\tau))\, d\tau \geq 0 \tag{5}$$

where $i_i(t)$ and $v_i(t)$ are the instantaneous values of the current and voltages at port i with $i = 1, 2,$ and $E(t)$ represents the total energy exchange for the 2-port network.

Definition: Activity *[9]*

If a network is not passive, then it is active.

Definition: Positive realness *[9]*

A rational function $F(s)$ is positive real if and only if, in addition to being real for real s, it meets the following conditions:

- **a.** $F(s)$ has no poles neither zeros in the right half plane (RHP),
- **b.** Any poles of $F(s)$ on the imaginary axis are simple with real and non-negative residues, and
- **c.** $\Re e\{F(j\omega)\} \geq 0, \quad \forall\, \omega.$

where $\Re e\{*\}$ denotes the real part of a complex number.

Theorem: Equivalence between positive realness and passivity for LTI systems *[11]*

Consider a linear time invariant system H defined by $Hx = h * x$, where h has a Laplace transform that has no poles in the RHP. System H is passive if and only if $\Re e\{\widehat{H}(j\omega)\} \geq 0,$ for all real frequencies ω, where $\widehat{H}(j\omega)$ is the Fourier transform of $h(t)$.

This theorem establishes that an LTI system is passive if and only if its transfer function is a positive real function. This theorem is stated for a 1-port network.

Raisbeck's Passivity Criterion *[9]*

The necessary and sufficient conditions for passivity of a 2-port network with the immittance parameter p are

1. The p-parameters have no RHP poles.
2. Any poles of the p-parameters on the imaginary axis are simple, and the residues of the p-parameters at these poles satisfy the following conditions:
   If k_{ij} denotes the residue of p_{ij} and k_{ij}^* is the complex conjugate of k_{ji}, then

$$k_{11} \geq 0$$
$$k_{22} \geq 0$$
$$k_{11}k_{22} - k_{12}k_{21} \geq 0 \;\; with \;\; k_{21} = k_{12}^* \tag{6}$$

3. The real and imaginary part of the p-parameters satisfy the following conditions for all real frequencies ω

$$\Re e(p_{11}) \geq 0$$
$$\Re e(p_{22}) \geq 0$$
$$4\Re e(p_{11})\Re e(p_{22}) - \left(\Re e(p_{12}) + \Re e(p_{21})\right)^2 - \left(\mathcal{I}m(p_{12}) - \mathcal{I}m(p_{21})\right)^2 \geq 0 \tag{7}$$

where $\mathcal{I}m(\;)$ denotes the imaginary part of a complex expression.

3. Passivity of Multilateral Teleoperators (n-port Networks) with Unknown Terminations

An n-port network contain n pairs of terminals for external connections (Figure 4). Each pair of terminals represents a port. The external behavior of the n-port network can be determined if all the I_i currents and V_i voltages are known. If for any given port the product of current and voltage is positive, then power is entering that port. As a natural extension from 2-ports, passivity of an n-port network is a sufficient condition for the stability of the network when coupled to passive termination. In this section, the necessary and sufficient conditions for passivity of an n-port network are presented.

3.1 Passivity Conditions for Linear n-port Networks

By analogy with the case of 2-port networks, an n-port network is passive if, for all excitations, the total energy exchange at the network's input and output ports is non-negative. Assuming that the 2-port network has zero energy stored at time $t = 0$, this passivity definition is expressed as

$$E(t) = \int_0^t (i_1(\tau)v_1(\tau) + i_2(\tau)v_2(\tau) + \ldots + i_n(\tau)v_n(\tau))\, d\tau \geq 0 \tag{8}$$

where $E(t)$ is the total energy delivered to the n-port network.

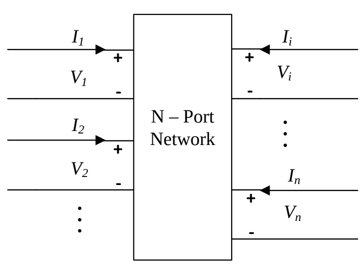

Figure 4. A general n-port network.

The n-port network passivity theorem that we propose later holds for **any of the four immittance parameters**, yet for brevity it is written only in terms of impedance parameters. Using the impedance parameters of the n-port network, the relation in the s-domain between voltages and currents is given by

$$\begin{bmatrix} V_1(s) \\ V_2(s) \\ \vdots \\ V_n(s) \end{bmatrix} = \begin{bmatrix} z_{11}(s) & z_{12}(s) & \cdots & z_{1n}(s) \\ z_{21}(s) & z_{22}(s) & \cdots & z_{2n}(s) \\ \vdots & \vdots & \ddots & \vdots \\ z_{n1}(s) & z_{n2}(s) & \cdots & z_{nn}(s) \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \\ \vdots \\ I_n(s) \end{bmatrix} \tag{9}$$

which can be compactly described as $\boldsymbol{V} = \boldsymbol{ZI}$ (note that $\boldsymbol{I}$ is the vector of current and not the identity matrix). In the proof of the theorem that will follow, we will need the following definitions.

Definition: Hermitian matrix

A Hermitian matrix $\boldsymbol{H}$ is a square matrix with complex elements h_{ij} for which the following property holds: $h_{ij} = h_{ji}^*$. Consequently, a Hermitian matrix $\boldsymbol{H}$ is equivalent to its own conjugate transpose.

The eigenvalues of a Hermitian matrix are always real-valued. Another important attribute of a Hermitian matrix $\boldsymbol{H}$ is that it is always possible to find a square unitary matrix $\boldsymbol{U}$ (i.e., $\boldsymbol{U}^*\boldsymbol{U}$ is the identity matrix) such that $\boldsymbol{U}^*\boldsymbol{HU}$ is a diagonal matrix with the eigenvalues of $\boldsymbol{H}$ on its diagonal. Hence, it is always possible to diagonalize a Hermitian matrix.

Definition: Hermitian form

A Hermitian form is an expression of the form $\sum h_{ij} a_j\, a_i^*$ in which the coefficients h_{ij} are the complex elements of a Hermitian matrix $\boldsymbol{H}$.

The following theorem and proof constitute the main result of this paper.

Theorem 1: Passivity of an $\boldsymbol{n}$-**port** ***network***

The necessary and sufficient conditions for passivity of an n-port network are

A. The z-parameters have no RHP poles.

B. Any poles of the z-parameters on the imaginary axis are simple, and the residues k_{ij} of z-parameters at these poles satisfy the following conditions:

1. $k_{ii} \geq 0 \qquad\qquad i = 1, 2, \dots, n$

2. $\dfrac{k_{11}k_{22} - k_{12}k_{21}}{k_{11}} \geq 0$

3. $\dfrac{k_{11}k_{33} - k_{12}k_{21}}{k_{11}} - \dfrac{(k_{11}k_{23} - k_{21}k_{13})(k_{11}k_{32} - k_{31}k_{12})}{k_{11}(k_{11}k_{22} - k_{12}k_{21})} \geq 0$

$\vdots$

$\vdots$

$$n.\ \ k_{nn} - \sum_{i=1}^{n-1} |u_{in}|^2 k_{ii}' \geq 0 \qquad \forall\, u_{ij} \text{ with } i \leq j \tag{10}$$

where

- k_{ij} denotes the residue of z_{ij} .
- The terms u_{ij} are the elements of an upper triangular matrix $\boldsymbol{U}$ used to diagonalize the residues matrix $\boldsymbol{K}$ according to $\boldsymbol{U}^*\boldsymbol{K}'\boldsymbol{U} = \boldsymbol{K}$, with $\boldsymbol{U}^*$being equal to the transpose complex conjugate of $\boldsymbol{U}$.
- The coefficients k_{ii}' are the elements of the diagonal matrix $\boldsymbol{K}'$.

C. The complex z'-parameters satisfy the following conditions for all real frequencies ω

1. $z_{ii}' \geq 0 \qquad\qquad i = 1, 2, \dots, n$

2. $\dfrac{z_{11}'z_{22}' - z_{12}'z_{21}'}{z_{11}'} \geq 0$

3. $\dfrac{z_{11}'z_{33}' - z_{13}'z_{31}'}{z_{11}'} - \dfrac{(z_{11}'z_{23}' - z_{21}'z_{13}')(z_{11}'z_{32}' - z_{31}'z_{12}')}{z_{11}'(z_{11}'z_{22}' - z_{12}'z_{21}')} \geq 0$

$\vdots$

$\vdots$

$$n.\ \ z'_{nn} - \sum_{i=1}^{n-1} |w_{in}|^2 z''_{ii} \geq 0 \qquad \forall\, w_{ij} \text{ with } i \leq j \tag{11}$$

where

- $z'_{ij} = \frac{1}{2}\left(z_{ij} + z^*_{ji}\right)$ are the elements of the matrix $\boldsymbol{Z}'$.
- The terms w_{ij} are the elements of an upper triangular matrix $\boldsymbol{W}$ used to diagonalize the matrix $\boldsymbol{Z}'$ according to $\boldsymbol{W}^*\boldsymbol{Z}''\boldsymbol{W} = \boldsymbol{Z}'$, with $\boldsymbol{W}^*$equal to the transpose complex conjugate of $\boldsymbol{W}$.
- The elements z''_{ii} are the entries of the main diagonal matrix $\boldsymbol{Z}''$. ■

3.2 Proof of Theorem 1

In Section 2 it was shown that for LTI systems, passivity and positive realness of the network's transfer function are equivalent. Hence, for the simple case of a 1-port network ($n = 1$) the energy requirement in (8) in the *s*-domain is equivalent to

$$\Re e\{Z(s)\} \geq 0 \quad \text{for} \quad \Re e\{s\} \geq 0 \tag{12}$$

where $Z(s)$ represents the input impedance of the 1-port network. $Z(s)$ can be expressed as

$$Z(s) = \frac{V(s)}{I(s)} \tag{13}$$

where $V(s)$ is the voltage across the 1-port and $I(s)$ is the current flowing through the port. By manipulating (13) as

$$Z(s) = \frac{V(s)}{I(s)} = \frac{V(s)\, I^*(s)}{I(s)\, I^*(s)} = \frac{V(s)\, I^*(s)}{|I(s)|^2} \tag{14}$$

Equation (12) is equivalent to

$$\Re e\{V(s)\, I^*(s)\} \geq 0 \quad \text{for} \quad \Re e\{s\} \geq 0 \tag{15}$$

where $I^*(s)$ is the complex conjugate of $I(s)$. Notice that $|I(s)|^2$ in the denominator of (14) is always positive.

By analogy with (15), (8) is equivalent to the following condition

$$\Re e\{V_1(s)I_1^* + V_2(s)I_2^* \cdots V_n(s)I_n^*\} \geq 0 \qquad \text{for} \quad \Re e\{s\} \geq 0 \tag{16}$$

Eliminating the voltages in (16) by using (9), we find that the n-port network passivity is equivalent to

$$\Re e\{F(s)\} \geq 0 \qquad \text{for} \quad \Re e\{s\} \geq 0 \tag{17}$$

where

$$\begin{aligned}\Re e\{F(s)\} = \Re e\{&z_{11}(s)I_1(s)I_1^*(s) + \cdots + z_{1n}(s)I_n(s)I_1^*(s) + z_{21}(s)I_1(s)I_2^*(s) + z_{2n}(s)I_n(s)I_2^*(s) + \cdots \\ &+ z_{n1}(s)I_1(s)I_n^*(s) + \cdots + z_{nn}(s)I_n(s)I_n^*(s)\}\end{aligned} \tag{18}$$

On the other hand, we know that the rational function $F(s)$ is positive real (i.e., (17) holds) if and only if, in addition to being real for real s, $F(s)$ meets the following conditions:

1. $F(s)$ has no poles in the right half plane (RHP)
2. Any poles of $F(s)$ on the imaginary axis are simple with real and non-negative residues
3. $\Re e\{F(j\omega)\} \geq 0 \quad \forall\, \omega$

For condition **1**, we require that none of the z-parameters of the n-port network have any poles in the RHP. To investigate condition **2**, assume that $F(s)$ has a simple pole at $s = j\omega_0$ with a residue k_0. Let $k_{11}, k_{12}, \cdots, k_{21} \cdots k_{nn}$ denote the residues of $z_{11}, z_{12}, \cdots, z_{21} \cdots z_{nn}$, respectively, at this pole. Expanding $F(s)$ in a Laurent series about $s = j\omega_0$ and keeping only the dominant terms in the immediate neighborhood of the pole, we get

$$\frac{k_0}{s-j\omega_0} = \frac{k_{11}(j\omega_0)\, I_1(j\omega_0)\, I_1^*(j\omega_0)}{s-j\omega_0} + \cdots + \frac{k_{1n}(j\omega_0)\, I_n(j\omega_0)\, I_1^*(j\omega_0)}{s-j\omega_0} + \cdots + \frac{k_{n1}(j\omega_0)\, I_1(j\omega_0)\, I_n^*(j\omega_0)}{s-j\omega_0} + \cdots + \frac{k_{nn}(j\omega_0)\, I_n(j\omega_0)\, I_n^*(j\omega_0)}{s-j\omega_0} \tag{19}$$

which is equivalent to

$$k_0 = k_{11}(j\omega_0)\, I_1(j\omega_0)\, I_1^*(j\omega_0) + \cdots + k_{1n}(j\omega_0)\, I_n(j\omega_0)\, I_1^*(j\omega_0) + \cdots + k_{n1}(j\omega_0)\, I_1(j\omega_0)\, I_n^*(j\omega_0) + \cdots + k_{nn}(j\omega_0)\, I_n(j\omega_0)\, I_n^*(j\omega_0) \tag{20}$$

In (20), k_0 must be a real and non-negative number to satisfy condition **2**. Terms k_{ii} for $i = 1, 2, \ldots, n$ are real and positive since the impedances z_{ii} are positive real functions. Also, $I_i(j\omega_0)\, I_i^*(j\omega_0)$ is real and positive. Note that in the pairs $k_{ij}(j\omega_0)\, I_j(j\omega_0)\, I_i^*(j\omega_0) + k_{ji}(j\omega_0)\, I_i(j\omega_0)\, I_j^*(j\omega_0)$, since $I_j(j\omega_0)\, I_i^*(j\omega_0)$ and $I_i(j\omega_0)\, I_j^*(j\omega_0)$ are complex conjugates, k_{ij} and k_{ji} are also complex conjugates.

Since the right side of (20) is a Hermitian form (with $h_{ij} = k_{ij}$), it can be diagonalized with respect to the Hermitian matrix with coefficients k_{ij}. To do so, (20) can be written in matrix form as

$$k_0 = [I_1^* \quad I_2^* \quad \cdots \quad I_n^*] \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & \cdots & k_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ k_{n1} & k_{n2} & \cdots & k_{nn} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = \boldsymbol{I}^* \boldsymbol{K} \boldsymbol{I} \tag{21}$$

The $\boldsymbol{K}$-matrix is diagonalizable and we want to find a linear transformation $\boldsymbol{U}^*\boldsymbol{K}'\boldsymbol{U} = \boldsymbol{K}$ where $\boldsymbol{K}'$ is a diagonal matrix, $\boldsymbol{U}$ is an upper triangular matrix, and $\boldsymbol{U}^*$ (the transpose complex conjugate of $\boldsymbol{U}$) is a lower triangular matrix.

$$\begin{bmatrix} u_{11}^* & 0 & 0 & \cdots & 0 \\ u_{12}^* & u_{22}^* & 0 & \cdots & 0 \\ u_{13}^* & u_{23}^* & u_{33}^* & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ u_{1n}^* & u_{2n}^* & u_{3n}^* & \cdots & u_{nn}^* \end{bmatrix} \begin{bmatrix} k_{11}' & 0 & 0 & \cdots & 0 \\ 0 & k_{22}' & 0 & \cdots & 0 \\ 0 & 0 & k_{33}' & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & k_{nn}' \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} & \cdots & u_{1n} \\ 0 & u_{22} & u_{23} & \cdots & u_{2n} \\ 0 & 0 & u_{33} & \ddots & u_{3n} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & u_{nn} \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & k_{23} & \cdots & k_{2n} \\ k_{31} & k_{32} & k_{33} & \ddots & k_{3n} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ k_{n1} & k_{n2} & k_{n3} & \cdots & k_{nn} \end{bmatrix} \tag{22}$$

which represents the system $\boldsymbol{U}^*\boldsymbol{K}'\boldsymbol{U} = \boldsymbol{K}$**.** Solving for $\boldsymbol{K}'$ and $\boldsymbol{U}$ will lead us to expressions for each k_{ij}' as a function of k_{ij} elements. The solution will follow.

The left hand side of system (22) can be written as

$$\begin{bmatrix} |u_{11}|^2 k_{11}' & u_{11}^* k_{11}' u_{12} & u_{11}^* k_{11}' u_{13} & \cdots & u_{11}^* k_{11}' u_{1n} \\ u_{12}^* k_{11}' u_{11} & |u_{12}|^2 k_{11}' + |u_{22}|^2 k_{22}' & u_{12}^* k_{11}' u_{13} + u_{22}^* k_{22}' u_{23} & \cdots & \sum_{i=1}^{n} u_{i2}^* k_{ii}' u_{in} \\ u_{13}^* k_{11}' u_{11} & u_{13}^* k_{11}' u_{12} + u_{23}^* k_{22}' u_{22} & |u_{13}|^2 k_{11}' + |u_{23}|^2 k_{22}' + |u_{33}|^2 k_{33}' & \cdots & \sum_{i=1}^{n} u_{i3}^* k_{ii}' u_{in} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ u_{1n}^* k_{11}' u_{11} & \sum_{i=1}^{n} u_{in}^* k_{ii}' u_{i2} & \sum_{i=1}^{n} u_{in}^* k_{ii}' u_{i3} & \cdots & \sum_{i=1}^{n} |u_{in}|^2 k_{ii}' \end{bmatrix} \tag{23}$$

for all u_{ij} and u_{ij}^* with $i \le j$. In (23), we used $u_{ij}^* u_{ij} = |u_{ij}|^2$.

Equation (22) is equivalent to the following system of equations:

$$
\begin{aligned}
&|u_{11}|^2 k'_{11} = k_{11} \\
&u_{11}^* u_{12} k'_{11} = k_{12} \\
&u_{11}^* u_{13} k'_{11} = k_{13} \\
&u_{12}^* u_{11} k'_{11} = k_{21} \\
&|u_{12}|^2 k'_{11} + u_{22}^2 k'_{22} = k_{22} \\
&\vdots \\
&|u_{13}|^2 k'_{11} + |u_{23}|^2 k'_{22} + u_{33}^2 k'_{33} = k_{33} \\
&\vdots \\
&\sum_{i=1}^{n} |u_{in}|^2 k'_{ii} = k_{nn} \qquad \forall\, u_{ij} \text{ and } u_{ij}^* \text{ with } i \le j
\end{aligned} \tag{24}
$$

Solution to the system of equation (24) is straightforward:

$$
\begin{aligned}
&k'_{11} = \frac{k_{11}}{|u_{11}|^2} \\
&k'_{22} = \frac{k_{11}k_{22} - k_{12}k_{21}}{|u_{22}|^2 k_{11}} \\
&k'_{33} = \frac{k_{11}k_{33} - k_{12}k_{21}}{|u_{33}|^2 k_{11}} - \frac{(k_{11}k_{23} - k_{21}k_{13})(k_{11}k_{32} - k_{31}k_{12})}{|u_{33}|^2 k_{11}(k_{11}k_{22} - k_{12}k_{21})} \\
&\vdots \\
&k'_{nn} = k_{nn} - \textstyle\sum_{i=1}^{n-1} |u_{in}|^2 k'_{ii} \qquad \forall\, u_{ij} \text{ with } i \le j
\end{aligned} \tag{25}
$$

Now, (21) can be rewritten as

$$k_0 = \boldsymbol{I}^*\boldsymbol{K}\boldsymbol{I} = \boldsymbol{I}^*\boldsymbol{U}^*\boldsymbol{K}'\boldsymbol{U}\boldsymbol{I} = (\boldsymbol{U}\boldsymbol{I})^*\boldsymbol{K}'(\boldsymbol{U}\boldsymbol{I}) \tag{26}$$

implying that k_0 will be non-negative and equivalently condition **B** in the theorem holds iff k'_{ii} in (25) are all non-negative. The expressions on the right hand side of (25) are all divided by coefficients of the form $|u_{ii}|^2$. Those coefficients are clearly positive and, hence, conditions $k'_{ii} \ge 0$ become

$$
\begin{aligned}
&1.\ \ k_{ii} \ge 0 \qquad\qquad i = 1, 2, \dots, n \\
&2.\ \ \frac{k_{11}k_{22} - k_{12}k_{21}}{k_{11}} \ge 0 \\
&3.\ \ \frac{k_{11}k_{33} - k_{12}k_{21}}{k_{11}} - \frac{(k_{11}k_{23} - k_{21}k_{13})(k_{11}k_{32} - k_{31}k_{12})}{k_{11}(k_{11}k_{22} - k_{12}k_{21})} \ge 0 \\
&\vdots \\
&\vdots \\
&n.\ \ k_{nn} - \textstyle\sum_{i=1}^{n-1} |u_{in}|^2 k'_{ii} \ge 0 \qquad \forall\, u_{ij} \text{ with } i \le j
\end{aligned} \tag{27}
$$

Therefore, it is established that condition **2** holds iff (10) holds.

Regarding condition **3**, the real part of $F(j\omega)$ can be obtained from

$$\Re e\{F(j\omega)\} = \frac{1}{2}[F(j\omega) + F^*(j\omega)] \tag{28}$$

where $F(j\omega)$ is given as

$$\begin{aligned} F(j\omega) = {} & z_{11}(j\omega)I_1(j\omega)I_1^*(j\omega) + \cdots + z_{1n}(j\omega)I_n(j\omega)I_1^*(j\omega) \\ & + z_{21}(j\omega)I_1(j\omega)I_2^*(j\omega) + \cdots + z_{2n}(j\omega)I_n(j\omega)I_2^*(j\omega) \\ & + \cdots + \\ & \vdots \\ & + z_{n1}(j\omega)I_1(j\omega)I_n^*(j\omega) + \cdots + z_{nn}(j\omega)I_n(s)I_n^*(j\omega) \end{aligned} \tag{29}$$

and $F^*(j\omega)$ is given as

$$\begin{aligned} F^*(j\omega) = {} & z_{11}^*(j\omega)I_1^*(j\omega)I_1(j\omega) + \cdots + z_{1n}^*(j\omega)I_n^*(j\omega)I_1(j\omega) \\ & + z_{21}^*(j\omega)I_1^*(j\omega)I_2(j\omega) + \cdots + z_{2n}^*(j\omega)I_n^*(j\omega)I_2(j\omega) \\ & + \cdots + \\ & \vdots \\ & + z_{n1}^*(j\omega)I_1^*(j\omega)I_n(j\omega) + \cdots + z_{nn}^*(j\omega)I_n^*(j\omega)I_n(j\omega) \end{aligned} \tag{30}$$

Substituting (29) and (30) in (28) we have

$$\begin{aligned} \Re e\{F(j\omega)\} = \frac{1}{2}[& z_{11}(j\omega)I_1(j\omega)I_1^*(j\omega) + \cdots + z_{1n}(j\omega)I_n(j\omega)I_1^*(j\omega) \\ & + z_{21}(j\omega)I_1(j\omega)I_2^*(j\omega) + \cdots + z_{2n}(j\omega)I_n(j\omega)I_2^*(j\omega) \\ & + \cdots + \\ & \vdots \\ & + z_{n1}(j\omega)I_1(j\omega)I_n^*(j\omega) + \cdots + z_{nn}(j\omega)I_n(s)I_n^*(j\omega) \\ & + z_{11}^*(j\omega)I_1^*(j\omega)I_1(j\omega) + \cdots + z_{1n}^*(j\omega)I_n^*(j\omega)I_1(j\omega) \\ & + z_{21}^*(j\omega)I_1^*(j\omega)I_2(j\omega) + \cdots + z_{2n}^*(j\omega)I_n^*(j\omega)I_2(j\omega) \\ & + \cdots + \\ & \vdots \\ & + z_{n1}^*(j\omega)I_1^*(j\omega)I_n(j\omega) + \cdots + z_{nn}^*(j\omega)I_n^*(j\omega)I_n(j\omega)] \end{aligned} \tag{31}$$

By using $z'_{ij} = \frac{1}{2}\left[z_{ij} + z_{ji}^*\right]$, $\Re e\{F(j\omega)\}$ can be written as

$$\begin{aligned} \Re e\{F(j\omega)\} = {} & z'_{11}(j\omega)I_1(j\omega)I_1^*(j\omega) + \cdots + z'_{1n}(j\omega)I_n(j\omega)I_1^*(j\omega) \\ & + z'_{21}(j\omega)I_1(j\omega)I_2^*(j\omega) + \cdots + z'_{2n}(j\omega)I_n(j\omega)I_2^*(j\omega) \\ & + \cdots + \\ & \vdots \\ & z'_{n1}(j\omega)I_1(j\omega)I_n^*(j\omega) + \cdots + z'_{nn}(j\omega)I_n(s)I_n^*(j\omega) \end{aligned} \tag{32}$$

or equivalently as

$$\Re e\{F(j\omega)\} = \boldsymbol{I}^* \, \boldsymbol{Z}' \, \boldsymbol{I} \tag{33}$$

where

$$\boldsymbol{Z}' = \begin{bmatrix} z'_{11}(j\omega) & z'_{12}(j\omega) & \cdots & z'_{1n}(j\omega) \\ z'_{21}(j\omega) & z'_{22}(j\omega) & \cdots & z'_{2n}(j\omega) \\ \vdots & \vdots & \ddots & \vdots \\ z'_{n1}(j\omega) & z'_{n2}(j\omega) & \cdots & z'_{nn}(j\omega) \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} z_{11}(j\omega)+z^*_{11}(j\omega) & z_{12}(j\omega)+z^*_{21}(j\omega) & \cdots & z_{1n}(j\omega)+z^*_{n1}(j\omega) \\ z_{21}(j\omega)+z^*_{12}(j\omega) & z_{22}(j\omega)+z^*_{22}(j\omega) & \cdots & z_{2n}(j\omega)+z^*_{n2}(j\omega) \\ \vdots & \vdots & \ddots & \vdots \\ z_{n1}(j\omega)+z^*_{1n}(j\omega) & z_{n2}(j\omega)+z^*_{2n}(j\omega) & \cdots & z_{nn}(j\omega)+z^*_{nn}(j\omega) \end{bmatrix} \quad (34)$$

In general, the z-parameters have complex values, i.e., $z_{ij} = r_{ij} + jx_{ij}$ where r_{ij} is the real part and x_{ij} is the imaginary part of z_{ij} .

(32) is a Hermitian form. Using a procedure similar to (21)-(26), which was for the residue matrix, the matrix $\boldsymbol{Z}'$ can be expressed as $\boldsymbol{Z}' = \boldsymbol{W}^* \boldsymbol{Z}'' \boldsymbol{W}$ where $\boldsymbol{Z}''$ is a diagonal matrix and $\boldsymbol{W}$ is an upper triangular matrix. By reducing the $\boldsymbol{W}$ matrix in the reduced row-echelon form ($w_{ij} = 1, \forall\, i = j$) the calculations of conditions **C** of Theorem 1 can be greatly simplified.

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ w^*_{12} & 1 & 0 & \cdots & 0 \\ w^*_{13} & w^*_{23} & 1 & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ w^*_{1n} & w^*_{2n} & w^*_{3n} & \cdots & 1 \end{bmatrix}\begin{bmatrix} z''_{11} & 0 & 0 & \cdots & 0 \\ 0 & z''_{22} & 0 & \cdots & 0 \\ 0 & 0 & z''_{33} & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & z''_{nn} \end{bmatrix}\begin{bmatrix} 1 & w_{12} & w_{13} & \cdots & w_{1n} \\ 0 & 1 & w_{23} & \cdots & w_{2n} \\ 0 & 0 & 1 & \ddots & w_{3n} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$= \begin{bmatrix} z'_{11} & z'_{12} & z'_{13} & \cdots & z'_{1n} \\ z'_{21} & z'_{22} & z'_{23} & \cdots & z'_{2n} \\ z'_{31} & z'_{32} & z'_{33} & \ddots & z'_{3n} \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ z'_{n1} & z'_{n2} & z'_{n3} & \cdots & z'_{nn} \end{bmatrix} \quad (35)$$

The solution to (35) is

$$z''_{11} = z'_{11}$$

$$z''_{22} = \frac{z'_{11}z'_{22} - z'_{12}z'_{21}}{z'_{11}}$$

$$z''_{33} = \frac{z'_{11}z'_{33} - z'_{13}z'_{31}}{z'_{11}} - \frac{(z'_{11}z'_{23} - z'_{21}z'_{13})(z'_{11}z'_{32} - z'_{31}z'_{12})}{z'_{11}(z'_{11}z'_{22} - z'_{12}z'_{21})}$$

$$\vdots$$

$$\vdots$$

$$z''_{nn} = z'_{nn} - \sum_{i=1}^{n-1} |w_{in}|^2 z''_{ii} \qquad w_{ij} \text{ with } i \le j \quad (36)$$

Now, (33) can be rewritten as

$$\Re e\{F(j\omega)\} = \boldsymbol{I}^* \boldsymbol{Z}' \boldsymbol{I} = \boldsymbol{I}^* \boldsymbol{W}^* \boldsymbol{Z}'' \boldsymbol{W}\boldsymbol{I} = (\boldsymbol{W}\boldsymbol{I})^* \boldsymbol{Z}''(\boldsymbol{W}\boldsymbol{I}) \quad (37)$$

Therefore, $\Re e\{F(j\omega)\} \ge 0, \forall\, \omega$ (i.e., condition 3) holds iff the z''-parameters in (36) are non-negative (this also implies $z'_{22} \ge 0, z'_{33} \ge 0, \cdots, z'_{nn} \ge 0$). Therefore, condition 3 holds iff (11) holds.

In summary, conditions **A**, **B** and **C** (Theorem 1) are necessary and sufficient for (17) or equivalently (16), which defines the n-port network passivity. This concludes the proof. ■

3.3 Case Study: Passivity Conditions of a 2-port Network

In this section, the special case of passivity of 2-port networks is considered. We will proceed in the same way as we did when we arrived at the passivity Theorem 1. The result will be compared with the well-known Raisbeck' Criterion. The network has been modeled using the impedance parameters as shown in (38).

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \tag{38}$$

Assuming that the z-parameters have no RHP poles, we move into the analysis of the residues for which Equation (22) has to be solved for the case of $n = 2$. The system is represented below:

$$\begin{bmatrix} 1 & 0 \\ u_{12}^* & 1 \end{bmatrix} \begin{bmatrix} k'_{11} & 0 \\ 0 & k'_{22} \end{bmatrix} \begin{bmatrix} 1 & u_{12} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \tag{39}$$

It is easy to see that solving (39) results in

$u_{12} = k_{12}/k_{11}$ and $u_{12}^* = k_{21}/k_{11}$

and

$$\begin{aligned} k'_{11} &= k_{11} \geq 0 \\ k'_{22} &= \frac{k_{11}k_{22} - k_{12}k_{21}}{k_{11}} \geq 0 \end{aligned} \tag{40}$$

Therefore, it is straightforward that condition **B** in Theorem 1 is same as condition 2 in Raisbeck's criterion. Also, in the following we show that solving (35) for $n = 2$ results in condition 3 in the Raisbeck's criterion. Writing z_{ij} as $r_{ij} + jx_{ij}$ where r_{ij} is the real part and x_{ij} is the imaginary part of z_{ij}, we have

$$\begin{bmatrix} z'_{11}(j\omega) & z'_{12}(j\omega) \\ z'_{21}(j\omega) & z'_{22}(j\omega) \end{bmatrix} = \begin{bmatrix} r_{11} & \frac{1}{2}(r_{12} + r_{21}) + \frac{j}{2}(x_{12} - x_{21}) \\ \frac{1}{2}(r_{12} + r_{21}) - \frac{j}{2}(x_{12} - x_{21}) & r_{22} \end{bmatrix} \tag{41}$$

Using (35) for the case of $n = 2$ the following system is formed:

$$\begin{bmatrix} 1 & 0 \\ w_{12}^* & 1 \end{bmatrix} \begin{bmatrix} z''_{11} & 0 \\ 0 & z''_{22} \end{bmatrix} \begin{bmatrix} 1 & w_{12} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} z'_{11} & z'_{12} \\ z'_{21} & z'_{22} \end{bmatrix} \tag{42}$$

Solving (42) results in:

$w_{12} = z'_{12}/z'_{11}$ and $w_{12}^* = z'_{21}/z'_{11}$.

and

$$\begin{aligned} z''_{11} &= z'_{11} \geq 0 \\ z''_{22} &= \frac{z'_{11}z'_{22} - z'_{12}z'_{21}}{z'_{11}} \geq 0 \end{aligned} \tag{43}$$

By using $z'_{ij} = \frac{1}{2}\left(z_{ij} + z_{ji}^*\right)$, the second condition in (43) reduces to:

By using $z'_{ij} = \frac{1}{2}\left(z_{ij} + z_{ji}^*\right)$, the second condition in (43) reduces to:

$$4r_{11}r_{22} - (r_{12} + r_{21})^2 - (x_{12} - x_{21})^2 \geq 0 \tag{44}$$

with $r_{ij=} \Re e(z_{ij})$ and $x_{ij} = \Im m(z_{ij})$ with $i, j = 1, 2$

Inequality (44) is the same as the last of condition 3 in Raisbeck's criterion with $r_{ij=} \Re e(p_{ij})$ and $x_{ij} = \Im m(p_{ij})$.

We conclude that for 2-ports, by using similar procedure as the one used for finding conditions of passivity of n-port networks, the final result is the same as Raisbeck's criterion. In the future, one does not have to go through all these calculations; on the contrary, we have presented Theorem 1 which allows for direct investigation of passivity of n-port networks where n can be any positive integer number equal or larger than 2.

4. Application of Passivity to a Dual-User Haptic Teleoperation System

In this section, the criterion proposed in Section 3 will be used in order to find passivity conditions of a trilateral haptic teleoperation system. The 3-port network is represented by its impedance matrix. The network is a dual-user haptic teleoperation system, in which two master robots for two operators share the control of one slave robot to perform a task in a remote environment. This configuration has many real-world applications such as training a trainee to do a task under haptic guidance from a mentor. In Section 4.1, the impedance matrix of the dual-user haptic teleoperation system is found by using the four-channel multilateral shared control architecture proposed in [10]. Section 4.2 is devoted to finding passivity conditions of such a system. Sections 4.3 and 4.4 are concerned with simulations and experiments (respectively) of the dual-user haptic teleoperation system.

4.1 A Dual-User Shared Haptic Control Teleoperation System

In a dual-user haptic teleoperation system, the goal is that two users coupled to two master robots (one user per one master robot) collaboratively control a slave robot to perform a task in a remote environment. The desired position and force for each robot are weighted sums of positions and forces of the other two robots, with the weights being determined by a parameter α whose value ranges from 0 to 1 [10] (see Figure 5). For instance, if α = 1, the slave robot will be fully controlled by User 1 and User 2 only receives large force feedback urging him/her to follow User 1's motions. The same parameter α can be given a value of 0, in which case the slave robot is fully controlled by User 2, allowing User 1 to assess the skill level of User 2 by feeling the reflected forces. If $0 < \alpha < 1$, then the two users collaborate and each contributes to the position command while receiving some force feedback. This provides "hand-over-hand" training using haptic assistance.

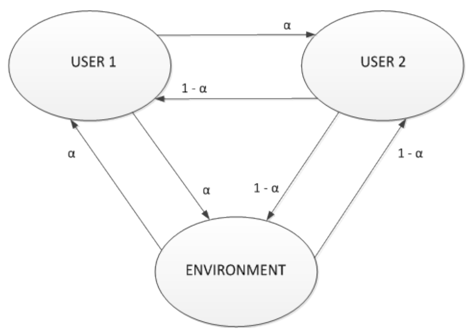

Figure 5. A dual-user haptic teleoperation system.

Consider the four-channel multilateral shared control architecture given in [10] and depicted in Figure 6. Under the assumption that each user is interfaced with his/her master robot and the slave is in contact with the environment, the dynamics of the two masters and slave can be model in frequency domain as

$$\begin{aligned} Z_{m1}V_{h1} &= F_{h1} + F_{cm1} \\ Z_{m2}V_{h2} &= F_{h2} + F_{cm2} \\ Z_s V_e &= F_e + F_{cs} \end{aligned} \tag{45}$$

In (45), $Z_{m1} = M_{m1}s$, $Z_{m2} = M_{m2}s$ and $Z_s = M_s s$ are the models of the two masters and the single slave, respectively. Also, F_{h1}, F_{h2} and F_e are the contact forces between each master and its human operator, and between the slave and its environment, respectively. Lastly, V_{h1}, V_{h2}, and V_e are the velocities of the two users and the environment respectively. In Figure 6, ${F_{h1}}^*$, ${F_{h2}}^*$, and ${F_e}^*$ are the two operator's and environment's exogenous input forces, which are independent of the teleoperation system behavior [1].

The controller outputs in the 4-channel architecture are

$$F_{cm1} = -C_{m1}V_{h1} - C_{4m1}V_{h1d} + C_{6m1}F_{h1} - C_{2m1}F_{h1d}$$

$$F_{cm2} = -C_{m2}V_{h2} - C_{4m2}V_{h2d} + C_{6m2}F_{h2} - C_{2m2}F_{h2d}$$
$$F_{cs} = -C_sV_e + C_1V_{ed} + C_5F_e + C_3F_{ed} \tag{46}$$

for $i = 1, 2$. C_{mi} and C_s are local position controllers, and C_{6mi} and C_5 are local force controllers for the two masters and the slave, respectively. Also, the controllers C_1, C_{4i} are position compensators similar to C_s and C_{mi}, respectively. C_{2mi} and C_3 are feedforward force terms for the two masters and the slave, respectively. Lastly, V_{hid} and V_{ed} are the desired positions, and F_{hid} and F_{ed} are the desired forces for the two masters and the slave, respectively.

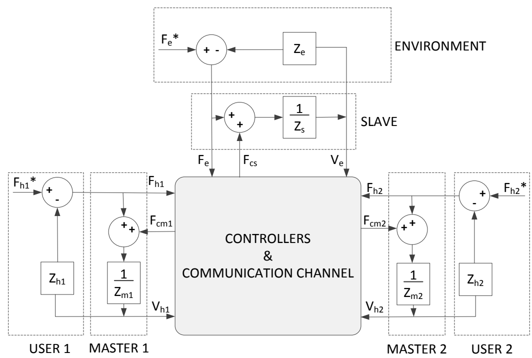

Figure 6. A dual-user haptic teleoperation system under four-channel control.

In this 3-robot shared control architecture, the desired velocity and force of each robot is a function of the velocities and forces of the other two robots, as the following set of equations state:

$$V_{h1d} = \alpha V_e + (1-\alpha)\, V_{h2}$$
$$V_{h2d} = (1-\alpha)V_e + \alpha\, V_{h1}$$
$$V_{ed} = \alpha V_{h1} + (1-\alpha)\, V_{h2}$$
$$F_{h1d} = \alpha F_e + (1-\alpha)F_{h2}$$
$$F_{h2d} = (1-\alpha)F_e + \alpha\, F_{h1}$$
$$F_{ed} = \alpha F_{h1} + (1-\alpha)\, F_{h2} \tag{47}$$

where $\alpha \epsilon [0, 1]$ is the weight parameter specifying the relative authority that each operator has over the slave and the corresponding share of force feedback he/she receives.

Position-error based (PEB) control is a special case of dual-user shared control architecture, which does not need any force sensor measurements. The PEB controller works by minimizing the difference between the weighted master and slave positions, thus reflecting a force related to this difference to each user once the slave makes contact with an object. In the PEB control architecture the following choices are made: $C_3 = C_5 = C_{2m1} = C_{2m2} = C_{6m1} = C_{6m2} = 0$. Also, for good position tracking the common choice is $C_1 = C_s$, $C_{4m1} = -C_{m1}$ and $C_{4m2} = -C_{m2}$. Here, we have

$$C_{m1} = \frac{K_{pm1} + K_{vm1}s}{s}$$
$$C_{m2} = \frac{K_{pm2} + K_{vm2}s}{s}$$
$$C_s = \frac{K_{ps} + K_{vs}s}{s} \tag{48}$$

In (48), K_{pm1}, K_{pm2}, K_{ps} represent master 1, master 2, and slave position controllers' gains. Similarly, K_{vm1}, K_{vm2}, K_{vs} represent master 1, master 2, and slave velocity controllers' gains.

By using (45), (46), (47), and (48), the impedance matrix of the closed-loop multilateral system in

$$\begin{bmatrix} F_{h1} \\ F_{h2} \\ F_e \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & z_{13} \\ z_{21} & z_{22} & z_{23} \\ z_{31} & z_{32} & z_{33} \end{bmatrix} \begin{bmatrix} V_{h1} \\ V_{h1} \\ V_e \end{bmatrix} \tag{49}$$

is found as

$$\begin{aligned}
z_{11} &= \left(M_{m1}s^2 + K_{vm1}s + K_{pm1}\right)/s \\
z_{12} &= \left(-K_{vm1}s + K_{pm1}\alpha + K_{vm1}s\alpha - K_{pm1}\right)/s \\
z_{13} &= \left(-K_{vm1}s\alpha - K_{pm1}\alpha\right)/s \\
z_{21} &= \left(-K_{vm2}s\alpha - K_{pm2}\alpha\right)/s \\
z_{22} &= \left(M_{m2}s^2 + K_{vm2}s + K_{pm2}\right)/s \\
z_{23} &= \left(-K_{vm2}s + K_{pm2}\alpha + K_{vm2}s\alpha - K_{pm2}\right)/s \\
z_{31} &= -\left(K_{vs}s\alpha + K_{ps}\alpha\right)/s \\
z_{32} &= \left(-K_{vs}s + K_{ps}\alpha + K_{vs}s\alpha - K_{ps}\right)/s \\
z_{33} &= \left(M_s s^2 + K_{vs}s + K_{ps}\right)/s
\end{aligned} \tag{50}$$

4.2 Applying Passivity Criterion to the Dual-User Shared Haptic Control Teleoperation System

The passivity criterion of n-port networks formulated in Chapter 3 reduces to the following conditions for the case of a 3-port network:

A. The z-parameters have no RHP poles.

B. Any poles of the z-parameters on the imaginary axis are simple, and the residues k_{ij} of the z-parameters at these poles satisfy the following conditions:

$$\begin{aligned}
&1.\ k_{ii} \geq 0 \qquad\qquad i = 1, 2, 3 \\
&2.\ \frac{k_{11}k_{22} - k_{12}k_{21}}{k_{11}} \geq 0 \\
&3.\ \frac{k_{11}k_{33} - k_{12}k_{21}}{k_{11}} - \frac{(k_{11}k_{23} - k_{21}k_{13})(k_{11}k_{32} - k_{31}k_{12})}{k_{11}(k_{11}k_{22} - k_{12}k_{21})} \geq 0
\end{aligned} \tag{51}$$

C. The complex z'-parameters satisfy the following conditions for all real frequencies ω

$$\begin{aligned}
&1.\ z'_{ii} \geq 0 \qquad\qquad i = 1, 2, 3 \\
&2.\ \frac{z'_{11}z'_{22} - z'_{12}z'_{21}}{z'_{11}} \geq 0 \\
&3.\ \frac{z'_{11}z'_{33} - z'_{13}z'_{31}}{z'_{11}} - \frac{(z'_{11}z'_{23} - z'_{21}z'_{13})(z'_{11}z'_{32} - z'_{31}z'_{12})}{z'_{11}(z'_{11}z'_{22} - z'_{12}z'_{21})} \geq 0
\end{aligned} \tag{52}$$

where $z'_{ij} = \frac{1}{2}\left(z_{ij} + z^*_{ji}\right)$.

Analysis of (50) shows that all the elements of the 3-port network impedance matrix have only a simple pole on the imaginary axis, thus fulfilling condition **A**. Analysis of the residues (condition **B**) leads to the following conditions:

$$k_{11} = K_{pm1} \geq 0 \tag{53}$$
$$k_{22} = K_{pm2} \geq 0 \tag{54}$$
$$k_{33} = K_{ps} \geq 0 \tag{55}$$

$$\frac{k_{11}k_{22} - k_{12}k_{21}}{k_{11}} = (1 - \alpha + \alpha^2)K_{pm1}K_{pm2} \geq 0 \tag{56}$$
$$\frac{k_{11}k_{33} - k_{12}k_{21}}{k_{11}} - \frac{(k_{11}k_{23} - k_{21}k_{13})(k_{11}k_{32} - k_{31}k_{12})}{k_{11}(k_{11}k_{22} - k_{12}k_{21})} = 0 \tag{57}$$

The inequality (56) always holds as $(1 - \alpha + \alpha^2) > 0$ for all $\alpha \in [0, 1]$.

Analysis of the impedance matrix according to Condition **C** leads to the following conditions on the controllers' gains:

$$K_{vm1} \geq 0 \tag{58}$$
$$K_{vm2} \geq 0 \tag{59}$$
$$K_{vs} \geq 0 \tag{60}$$
$$4K_{vm1}K_{vm2} - (K_{vm1} - \alpha K_{vm1} + \alpha K_{vm2})^2 - \frac{\left(K_{pm1} - \alpha K_{pm1} + \alpha K_{pm2}\right)^2}{\omega^2} \geq 0 \tag{61}$$

Condition (61) will be fulfilled for all real frequencies ω if the gains of the PD controllers satisfy:

$$\frac{K_{pm1}}{K_{pm2}} = \frac{\alpha}{1 - \alpha} \tag{62}$$

$$4K_{vm1}K_{vm2} - (K_{vm1} - \alpha K_{vm1} + \alpha K_{vm2})^2 \geq 0 \tag{63}$$

Using (62) in the last condition of **C** (condition 3 of (52)), we get the following inequality:

$$\begin{aligned}
&\frac{-1}{\omega^2}\left\{\frac{\left(K_{pm1} - K_{ps}\right)^2[K_{vm1}(1-\alpha)^2(2-\alpha) + K_{vm2}\alpha^2(1+\alpha)]}{2\alpha} + \frac{(1-2\alpha)^2 K_{vm1}}{\alpha^2}\right.\\
&\left.+\frac{\left({K_{pm1}}^2 - {K_{ps}}^2\right)(1-2\alpha)(1-\alpha)[\alpha^2 K_{vm2} + (\alpha+2)K_{vm1}]}{2\alpha}\right\}\\
&+\{(1+\alpha)(2-\alpha)K_{vm1}K_{vm2}K_{vs} - \alpha^2(2-\alpha)K_{vm2}K_{vs}(K_{vm2} + K_{vs})\\
&-(1-\alpha+\alpha^2)K_{vm1}K_{vm2}[(1-\alpha)K_{vm1} + \alpha K_{vm2}]\\
&-(1-\alpha)^2(1+\alpha)K_{vm1}K_{vs}(K_{vm1} + K_{vs})\} \geq 0
\end{aligned} \tag{64}$$

Equation (64) will be fulfilled for all real frequencies ω if the controller's gains and parameter α satisfy the following conditions:

$$\begin{aligned}
K_{pm1} &= K_{pm2} = K_{ps}\\
K_{vm1} &= K_{vm2} = K_{vs}\\
\alpha &= {}^{1}\!/_{2}
\end{aligned} \tag{65}$$

The reader may well think that α = 0.5 is a limitation. However, this is not caused by our passivity criterion; rather, it is a limitation due to using the four-channel multilateral shared control laws for authority sharing between the two operators in conjunction with position-position laws for teleoperation control. The four-channel multilateral shared control laws for authority sharing, which are given by the six equations (47) in our paper, were proposed in [10]. Choosing a different authority sharing law or a different teleoperation control law can eliminate the limitation on α when using our passivity criterion.

As a conclusion, the dual-user haptic teleoperation system is passive if the set of equations (65) holds. Notice that (65) is a sufficient, frequency-independent, and compact condition for passivity of the PEB dual-user haptic teleoperation system described in Section 4.2.

4.3 Simulation Study: The Dual-User Shared Haptic Control Teleoperation System

In this section, the passivity conditions for the PEB dual-user haptic teleoperation system found in Section 4.2 will be verified via MATLAB/Simulink simulations. The simulation assumes no time delay in the communication channels between the three robots. According to Equation (8) and assuming that the energy stored in the system for $t < 0$ is zero, the 3-port network is passive if and only if

$$E(t) = \int_0^t (i_1(\tau)v_1(\tau) + i_2(\tau)v_2(\tau) + i_3(\tau)v_3(\tau))\, d\tau \geq 0 \quad (66)$$

A *passivity observer* is incorporated in the simulations in order to evaluate (66). In the simulations, all ports of the 3-port network are connected to the passive terminations with a transfer function $\frac{1}{1+s}$. An input F_{h1}^* in the form of a sine wave is applied by the Master 1's operator. The three robots are modeled by masses $M_{m1} = 0.7$, $M_{m2} = 0.9$, and $M_s = 0.5$.

According to the previous section, the dual-user haptic teleoperation system is passive if the set of equations (65) holds. Table 1 shows two sets of controllers' gains used for these simulations, one set is in agreement with conditions given in (65), thus representing a passive trilateral system. The other set violates (65), representing a non-passive system. For all simulations $\alpha = {}^{1}/_{2}$.

Table 1 Controllers' gains for (A) passive and (B) non-passive PEB trilateral system.

System	**Master 1's controller**	**Master 2's controller**	**Slave's controller**
(A) Passive	$K_{pm1} = 5$ $K_{vm1} = 10$	$K_{pm2} = 5$ $K_{vm2} = 10$	$K_{ps} = 5$ $K_{vs} = 10$
(B) Non-Passive	$K_{pm1} = 5$ $K_{vm1} = 10$	$K_{pm2} = 100$ $K_{vm2} = 10$	$K_{ps} = 5$ $K_{vs} = 10$

Figure 7 shows that choosing the controllers' gains according to the conditions found in previous section results in a passive system (to which positive energy is delivered at all times). Figure 8 clearly shows that a violation of such conditions may result in a non-passive system (the energy delivered to the network is not always positive).

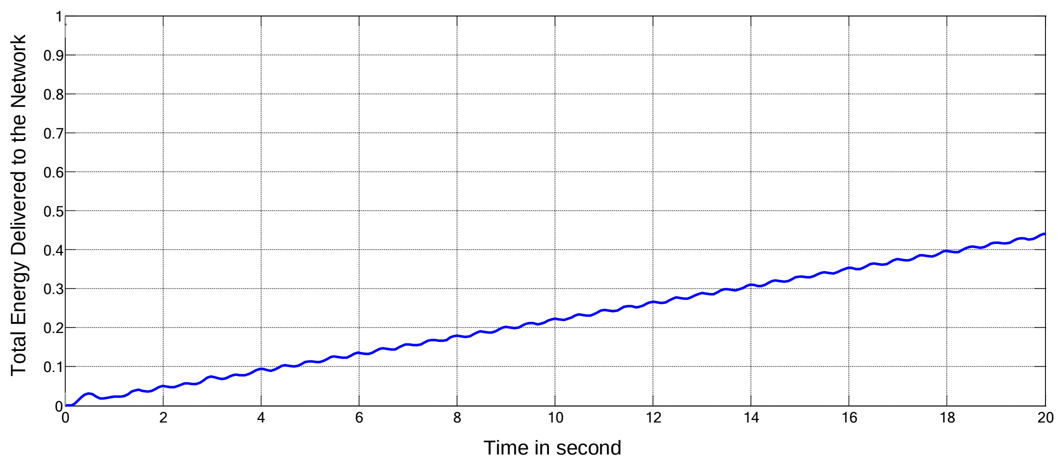

Figure 7. Passivity observer: Total energy delivered to a passive trilateral system.

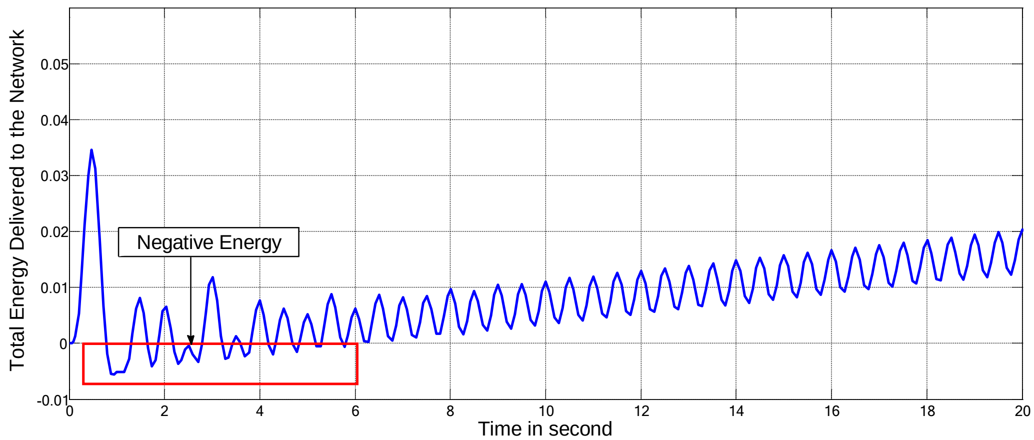

Figure 8. Passivity observer: Total energy delivered to a non-passive trilateral system.

4.4 Experiment Study: The Dual-User Shared Haptic Control Teleoperation System

For experiments with a dual-user shared haptic control teleoperation system, we use two Phantom Premium 1.5A robots as the master #1 and master #2, and a Rehab robot (Quanser, Inc. Markham, ON, Canada) as the slave. Out of the actuated joints of each robot, the first joint, which rotates about the vertical, is considered in the experiments while others joints are locked using high-gain position controllers. The Phantom Premium robots for master #1 and master #2 are equipped with JR3 6-DOF force/torque sensors (JR3, Inc., Woodland, CA, USA), and Rehab robot is equipped with ATI force/torque sensor (ATI Industrial Automation, Apex, NC, USA) for measuring the external contact forces.

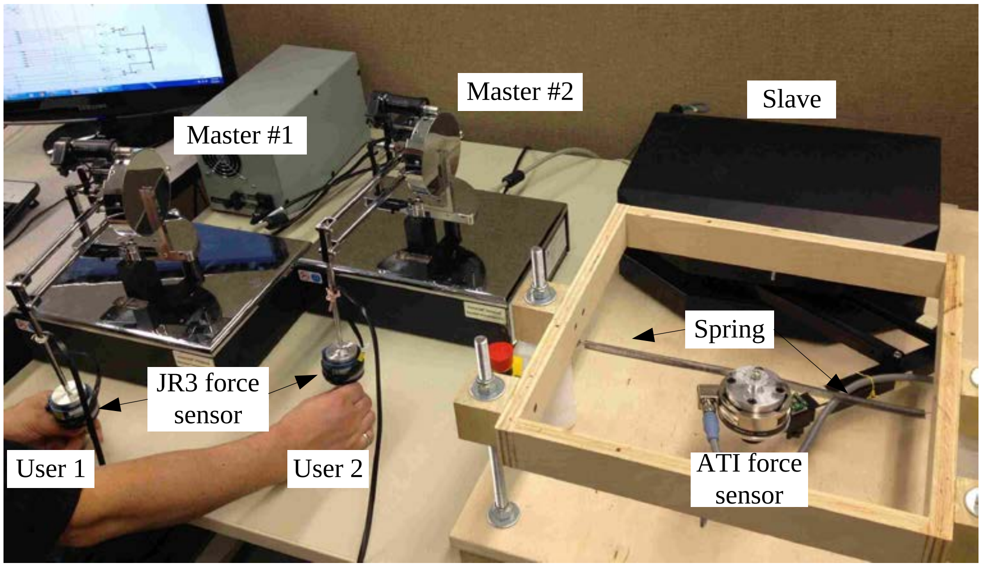

Figure 9. Experimental setup where the master #1 and the master #2 are controlled by human users and the slave is connected via passive spring to a stiff wall.

Table 2. Controllers' gains of the PEB trilateral system for experiments.

System	Master 1's controller	Master 2's controller	Slave's controller
Passive	$K_{pm1} = 6$ $K_{vm1} = 8$	$K_{pm2} = 6$ $K_{vm2} = 8$	$K_{ps} = 6$ $K_{vs} = 8$

The experimental setup is shown in Figure 9, the master #1 and master #2 are controlled by human users, the slave is connected via a pair of passive springs to a stiff wall. The controller's gains shown in Table 2 satisfy set of equations (65). The dissipated energy (66) profiles $E(t)$ are plotted in Figure 10. As it can be seen, if the controllers gains are selected according to (65), e.g., as listed in Table 2, then the passivity observer output $E(t)$ is non-negative at all times. Figure 11 depicts the average positions of the two masters versus the slave position. These profiles of positions further corroborate the passivity (stability) of the system. These experimental results agree with the passive case (65).

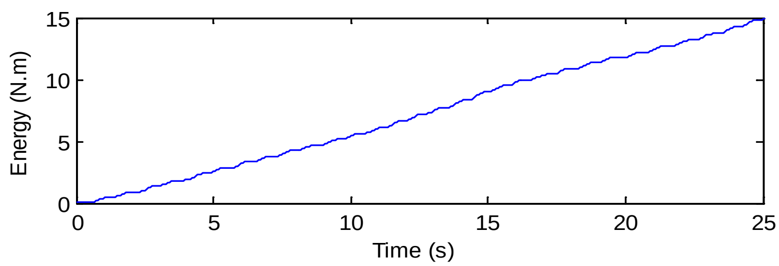

Figure 10. Experimental results for the dual-user teleoperation system. Passivity observer's output is used for passivity analysis.

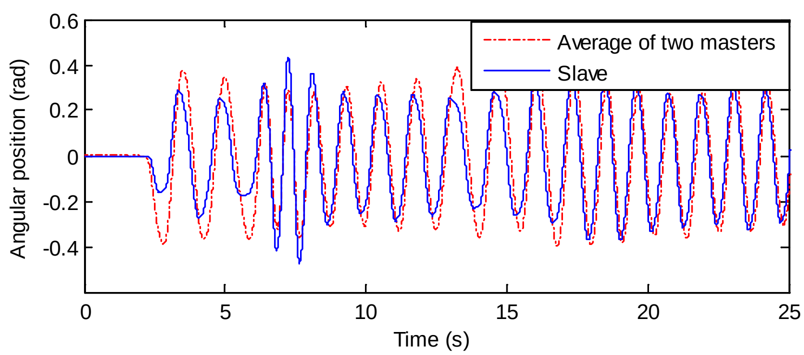

Figure 11. Experiment results for the dual-user teleoperation system. The desired and actual positions for the passive system are shown.

5. Conclusions and Future Directions

This paper presents a novel method for stability analysis of n-port networks with passive terminations. The proposed method can be used for analysis and design of multilateral systems involving haptic information sharing between a number of users. The major contributions of the paper are summarized below:

- A new passivity theorem for investigation of passivity of n-port networks is proposed. The theorem gives the necessary and sufficient conditions for passivity of the n-port network based on the immittance parameters of the network. The use of immittance parameters is preferable compared to more complex techniques found in the literature, which are based on scattering parameters and reflection coefficients. Moreover, the literature has tried to investigate the passivity of 3-port networks by assuming one known/fixed termination, thus reducing the 3-port into a 2-port network, this is a cumbersome process that involves an infinite number of applications of Raisbeck's criterion, also, a degree of freedom is lost when the third port is coupled to a known termination. In contrast, the closed-form conditions given in this paper make it possible to investigate the passivity of n-port networks (thus not necessarily limited to $n = 3$) directly and without resorting to using any known/fixed terminations, assuring a complete general solution to the problem. As for future work, **t**he passivity theorem condition given in this paper has been developed in the frame of 1 degree of freedom (DOF) systems. A step forward would be its extension to 2- and 3-DOF systems.

Acknowledgements

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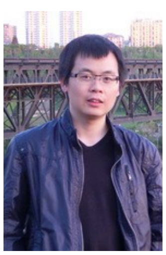

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