

# A passivity criterion for sampled-data bilateral teleoperation systems

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## Abstract

In a bilateral teleoperation system, conditions involving open-loop model parameters and controller parameters for ensuring teleoperator passivity are useful as control design guidelines to attain maximum teleoperation transparency (due to passivity/transparency tradeoffs). By teleoperator, we mean the teleoperation system excluding the human operator and the remote environment. The rationale behind considering teleoperator passivity instead of teleoperation system stability is that, unlike the former, the latter is influenced by the dynamics of the human operator and the remote environment, which are typically uncertain, time-varying, and/or nonlinear. In this paper, a condition for the passivity of a teleoperator is found when the teleoperation controllers are implemented in the discrete-time domain. Such as new passivity analysis is necessary because discretization causes energy leaks and does not necessarily preserve passivity. We show that the passivity criterion for the sampled-data teleoperator imposes a lower bound on the robot damping and upper bounds on the control gains and the sampling time. The criterion has been verified through computer simulations as well as experimental tests involving a bilateral teleoperation system consisting of a pair of Phantom Omni robots.

## 1 Introduction

A teleoperation system consists of a human operator, a remote environment, and a teleoperator, which itself includes a master robot, a slave robot, a communication channel and master/slave controllers. The slave robot is in contact with an environment and the master robot is controlled by a human operator. In bilateral teleoperation, the operator can feel the forces occurring between the slave robot and the environment. Literature surveys on bilateral teleoperation systems are given in [1, 2, 3].

The controller of a teleoperation system is designed to achieve two objectives. First, the system should have stability, which is defined as the boundedness of the signals in the system. Second, the teleoperation system should be transparent meaning that the positions of the robots and the contact forces in the master and slave sides should be similar [4].

Due to the unknown, time-varying and nonlinear dynamics of the environment and/or the operator, it is easier to analyze the passivity of the teleoperator in lieu of the stability of the closed-loop teleoperation system. Indeed, the combination of a passive teleoperator and passive environment and operator terminations will be passive and consequently the overall teleoperation system will be passive [5]. For a teleoperation system, the teleoperator is modelled as a two-port network and the teleoperator's passivity condition is expressed as a bound on the maximum singular value of the scattering matrix of the two-port network [6].

Similar to bilateral teleoperation, in a force-reflective virtual reality simulation system the operator feels virtual contact forces while applying position commands through the haptic user interface. Colgate and Schenkel have found a passivity condition for such a system considering the discrete-time components of the system [7]. The passivity condition for the discrete-time counterpart model  $K + sB$  of a virtual wall is found to be  $b > KT/2 + B$ , where  $b$  is the haptic interface damping and  $T$  is the sampling time. The stability of the virtual wall system has been investigated using the Routh-Hurwitz method [8]. The condition for the stability of a similar system is  $b > KT/2 - B$ , which is clearly less conservative than the passivity condition. Previous research has also considered the impact of other non-idealities such as quantization and friction on the stability of the virtual wall system [9, 10].

Passivity of a teleoperation system can be jeopardized if any of the components in the system are discretized. Discretization of the teleoperation controller does not necessarily preserve the passivity of the teleoperator. In other words, the passivity of a teleoperation system is not guaranteed if the continuous-time controllers are substituted with their discrete-time counterparts because of energy leaks caused by the Zero Order Hold [11]. To consider the effect of discretization on the passivity of the teleoperation system, the sampled-data system including the continuous-time models of the master/slave robots, the discrete-time models of the master/slave controllers, and also the Zero Order Hold (ZOH) and sampler blocks must be considered in the analysis.

For a discrete-time teleoperation system, stability can be achieved using six low-pass filters [12]. In some approaches, the whole teleoperation system is converted to the digital domain [11, 13] or the continuous-time domain [14], which simplifies the stability analysis for known models of the environment or the operator. Also, Ryu et al proposed a passivity controller / passivity observer for monitoring and controlling the energy in the communication channel of a discrete-time teleoperation system [15, 16]. In addition, Secchi et al proposed a geometric method to investigate the problem of having both continuous-time and discrete-time signals in a single system where the system is represented by a continuous-time port-Hamiltonian system [17].

In this paper, the passivity analysis starts with considering the dynamics of the master and the slave controllers as well as the dynamics of the master and the slave robots. A condition will be found for passivity of the teleoperator based on parameters such as the sampling time, the controller gains and the robots damping terms. It is important to determine the lower bound on the damping term of the robots because most of the newly designed haptic devices have low damping terms to deliver sensitivity and fidelity to the operator. The upper bounds on the controllers gains give a useful guideline for control design because the transparency of the system is degraded when the controller gains decrease. In other words, this paper gives the conditions that can be used as design guidelines for achieving higher transparency.

The rest of this paper is organized as follows. The teleoperation system is modelled in Section 2 and this model is used in Section 3 to find a condition for the passivity of the teleoperator

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system. This condition has been tested via computer simulations in Section 4, which allows the damping terms of the robots to vary while they are fixed in the experiments. The experimental results using two Phantom Omni robots are reported in Section 5 and the concluding remarks are given in Section 6.

## 2 Teleoperation System Modelling

The block diagram of a position-error based (PEB) sampled-data teleoperation system is shown in Fig. 1. The master and slave robots are modelled as

$$\begin{aligned} f_h - f_m &= m_m \ddot{x}_m + b_m \dot{x}_m \\ f_e - f_s &= m_s \ddot{x}_s + b_s \dot{x}_s \end{aligned} \quad (1)$$

where  $f_h$  and  $f_e$  are the operator's hand and environment's forces, respectively. The subscripts  $m$  and  $s$  indicate the master and the slave sides, respectively, and  $f_i$ 's are the controller output forces, and  $x_i$ 's are the robots position signals. Also,  $m$  and  $b$  denote the mass and the damping of each robot.

As depicted in Fig. 1, the positions of the master and slave robots are discretized using sampler blocks. The superscript \* denotes sampled signals such as samplers' and controllers' output signals. The sampled signals are converted back to the continuous-time domain using zero-order-hold blocks.

The environment and the operator are modelled as impedances  $Z_e$  and  $Z_h$ , that are assumed to be passive but otherwise arbitrary. In Fig. 1,  $\tilde{f}_h$  is the exogenous input force from the operator's hand and  $\tilde{f}_e$  is the exogenous input force from the environment. The environment and the operator models are

$$\begin{aligned} \tilde{F}_h - F_h &= Z_h s X_m \\ \tilde{F}_e - F_e &= Z_e s X_s \end{aligned} \quad (2)$$

The robots' dynamic (1) can be rewritten in the Laplace domain as

$$\begin{aligned} s X_m &= \frac{1}{m_m s + b_m} (F_h - F_m) \\ s X_s &= \frac{1}{m_s s + b_s} (F_e - F_s) \end{aligned} \quad (3)$$

The sampler output is a Dirac comb weighted by the sampled signal:

$$x^*(t) = \sum_{k=0}^{\infty} x(kT) \delta(t - kT) \quad (4)$$

In the s-domain, the sampler (4) can be rewritten as

$$X^*(s) = \mathcal{L}\{x^*(t)\} = \sum_{k=0}^{\infty} x(kT) e^{-skT} \quad (5)$$

The z-domain equivalent of (5) is

$$X(z) = \mathcal{Z}\{x^*(t)\} = X^*(s)|_{s=1/T \ln z} \quad (6)$$

## 3 Passivity condition for the discrete-time controller teleoperator

The teleoperator passivity condition in the time domain is based on the dissipated energy in the equivalent 2-port network, which can be measured by the input-output energy integral at the system ports:

$$\int_0^t f_h(\tau) \dot{x}_m(\tau) d\tau + \int_0^t f_e(\tau) \dot{x}_s(\tau) d\tau > 0 \quad (7)$$

The system is passive, if and only if for all time  $t > 0$ , condition (7) holds. Condition (7) is satisfied if

$$\int_0^t f_h(\tau) \dot{x}_m(\tau) d\tau + \int_0^t f_e(\tau) \dot{x}_s(\tau) d\tau > \frac{1}{2} m_m \dot{x}_m^2 + \frac{1}{2} m_s \dot{x}_s^2 \quad (8)$$

Based on the energy balance in the master and the slave robot dynamics, (8) can be simplified to

$$\begin{aligned} \int_0^t f_m(\tau) \dot{x}_m(\tau) d\tau + \int_0^t f_s(\tau) \dot{x}_s(\tau) d\tau \\ + \int_0^t b_m \dot{x}_m^2(\tau) d\tau + \int_0^t b_s \dot{x}_s^2(\tau) d\tau > 0 \end{aligned} \quad (9)$$

Parseval's theorem is then used to convert (9) into the frequency domain counterpart

$$\begin{aligned} \int_{-\infty}^{\infty} F_m(j\omega) V_m^c(j\omega) d\omega + \int_{-\infty}^{\infty} F_s(j\omega) V_s^c(j\omega) d\omega \\ + \int_{-\infty}^{\infty} b_m V_m(j\omega) V_m^c(j\omega) d\omega + \int_{-\infty}^{\infty} b_s V_s(j\omega) V_s^c(j\omega) d\omega > 0 \end{aligned} \quad (10)$$

where superscript  $c$  denotes the complex conjugate operator. In (10),  $F_i(j\omega)$  is the Fourier transforms of  $f_i(t)$  and  $V_i(j\omega)$  is the Fourier transforms of  $\dot{x}_i(t)$  (for  $i = m, s$ ).

The zero-order-hold block and the controllers in the master side determine the control signal. The master controller output in Fig. 1 is

$$F_m^*(s) = C_m(z)|_{z=e^{sT}} [\beta X_s^*(s) - \alpha X_m^*(s)] \quad (11)$$

On the other hand, the ZOH's transfer function in s-domain is

$$G_{ZOH}(s) = \frac{1 - e^{-sT}}{sT} \quad (12)$$

Combining (11) and (12), the master control signal can be expressed as

$$\begin{aligned} F_m(j\omega) &= \beta \frac{1 - e^{-j\omega T}}{j\omega T} C_m(e^{j\omega T}) X_s^*(j\omega) \\ &\quad - \alpha \frac{1 - e^{-j\omega T}}{j\omega T} C_m(e^{j\omega T}) X_m^*(j\omega) \end{aligned} \quad (13)$$

Similarly, for the slave side, the controller output is

$$\begin{aligned} F_s(j\omega) &= \alpha \frac{1 - e^{-j\omega T}}{j\omega T} C_s(e^{j\omega T}) X_m^*(j\omega) \\ &\quad - \beta \frac{1 - e^{-j\omega T}}{j\omega T} C_s(e^{j\omega T}) X_s^*(j\omega) \end{aligned} \quad (14)$$

On the other hand, the position signals can be defined as

$$\begin{aligned} X_m^* &= \frac{1}{T} \sum_{n=-\infty}^{\infty} \frac{V_m(j\omega + jn\omega_s)}{j\omega + jn\omega_s} \\ X_s^* &= \frac{1}{T} \sum_{n=-\infty}^{\infty} \frac{V_s(j\omega + jn\omega_s)}{j\omega + jn\omega_s} \end{aligned} \quad (15)$$

where  $\omega_s = 2\pi/T$ . Substituting (13), (14) and (15) into (10) will result in the following condition for passivity of the 2-port teleoperator:

$$\begin{aligned} \int_{-\infty}^{\infty} \bar{C}_m(\omega) S_m(j\omega) \left[ \frac{V_m(j\omega)}{j\omega} \right]^c d\omega \\ + \int_{-\infty}^{\infty} \bar{C}_s(\omega) S_s(j\omega) \left[ \frac{V_s(j\omega)}{j\omega} \right]^c d\omega \\ + \int_{-\infty}^{\infty} b_m V_m(j\omega) V_m^c(j\omega) d\omega \\ + \int_{-\infty}^{\infty} b_s V_s(j\omega) V_s^c(j\omega) d\omega > 0 \end{aligned} \quad (16)$$

where  $\bar{C}_m$  and  $\bar{C}_s$  are functions of the master and slave controller transfer functions:

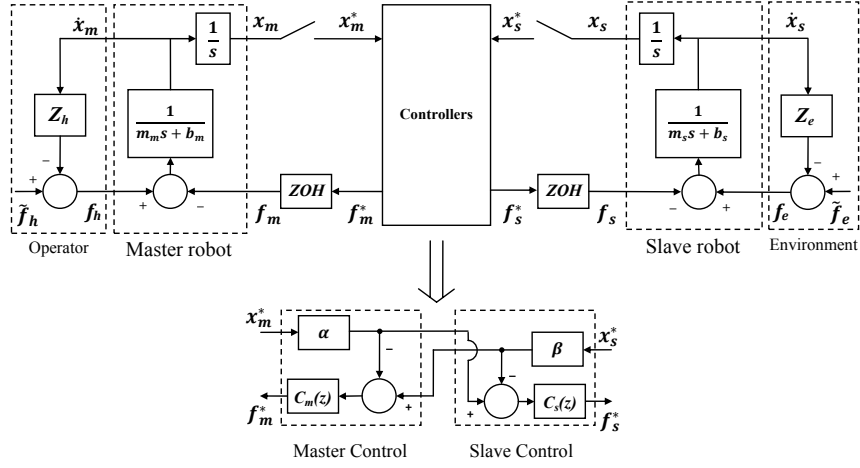


Figure 1: The structure of a teleoperation system, which includes discretized controller models.

$$\begin{aligned}\bar{C}_m(\omega) &= -\frac{1 - e^{-j\omega T}}{T} C_m(e^{j\omega T}) \\ \bar{C}_s(\omega) &= -\frac{1 - e^{-j\omega T}}{T} C_s(e^{j\omega T})\end{aligned}\quad (17)$$

and  $S_m$  and  $S_s$  are the following summations:

$$\begin{aligned}S_m &= \sum_{n=-\infty}^{\infty} \frac{\alpha V_m(j\omega + jn\omega_s) - \beta V_s(j\omega + jn\omega_s)}{j\omega + jn\omega_s} \\ S_s &= \sum_{n=-\infty}^{\infty} \frac{\beta V_s(j\omega + jn\omega_s) - \alpha V_m(j\omega + jn\omega_s)}{j\omega + jn\omega_s}\end{aligned}\quad (18)$$

To simplify (16), the master and slave controllers are selected to be proportional to each other (same proportion as the position scaling):

$$C(j\omega) = \frac{C_m(j\omega)}{\alpha} = \frac{C_s(j\omega)}{\beta}\quad (19)$$

where  $\alpha$  and  $\beta$  are the same as the position scales in (11). It can be shown that for (16) to be valid, it is sufficient to have

$$\begin{aligned}& \int_{-\infty}^{\infty} \left[ b_m + (\alpha^2 + \alpha\beta)\bar{C}^-(\omega) \sum_{n=-\infty}^{\infty} \frac{1}{(\omega + n\omega_s)^2} \right] \\ & \times V_m(j\omega)V_m^c(j\omega)dw \\ & + \int_{-\infty}^{\infty} \left[ b_s + (\beta^2 + \alpha\beta)\bar{C}^-(\omega) \sum_{n=-\infty}^{\infty} \frac{1}{(\omega + n\omega_s)^2} \right] \\ & \times V_s(j\omega)V_s^c(j\omega)dw > 0\end{aligned}\quad (20)$$

where

$$\bar{C}^-(\omega) = \begin{cases} \Re\{\bar{C}(\omega)\} & , \text{ if } \Re\{\bar{C}(\omega)\} < 0 \\ 0 & , \text{ otherwise} \end{cases}\quad (21)$$

where  $\Re$  is the real operator. The summation inside (20) is equal to

$$\sum_{n=-\infty}^{\infty} \frac{1}{(\omega + n\omega_s)^2} = \frac{T^2}{2} \frac{1}{1 - \cos \omega T}\quad (22)$$

Substituting (22) into (20), the conditions that guarantee the teleoperator passivity will be

$$\begin{aligned}b_m + (\alpha^2 + \alpha\beta)\bar{C}^-(\omega) \frac{T^2}{2} \frac{1}{1 - \cos \omega T} &> 0 \\ b_s + (\beta^2 + \alpha\beta)\bar{C}^-(\omega) \frac{T^2}{2} \frac{1}{1 - \cos \omega T} &> 0\end{aligned}\quad (23)$$

Combining (17), (19), (21) and (23), the passivity condition is simplified to

$$\begin{aligned}b_m &> \frac{T}{2} \frac{\alpha + \beta}{1 - \cos \omega T} \Re\{(1 - e^{-j\omega T})C_m(e^{j\omega T})\} \\ b_s &> \frac{T}{2} \frac{\alpha + \beta}{1 - \cos \omega T} \Re\{(1 - e^{-j\omega T})C_s(e^{j\omega T})\}\end{aligned}\quad (24)$$

If the master and the slave controllers are the same, the scaling factors are  $\alpha = \beta = 1$  and the passivity condition will be

$$\begin{aligned}b_m &> \frac{T}{1 - \cos \omega T} \Re\{(1 - e^{-j\omega T})C_m(e^{j\omega T})\} \\ b_s &> \frac{T}{1 - \cos \omega T} \Re\{(1 - e^{-j\omega T})C_s(e^{j\omega T})\}\end{aligned}\quad (25)$$

The passivity condition (25) is valid for all controller  $C_m = C_s$ . If the controller's structure is known, the condition can be further simplified. For instance, for a PD controller

$$C_s(z) = C_m(z) = K + B \frac{z-1}{Tz}\quad (26)$$

the passivity condition is simplified to

$$b > KT - 2B \cos \omega T\quad (27)$$

Condition (27) is dependent on the frequency  $\omega$ . The term  $\cos \omega T$  can vary from -1 to 1 and, thus, the sufficient condition for teleoperator passivity over all frequencies will be

$$b > KT + 2B\quad (28)$$

which has to hold for both the master and the slave robots. Condition (28) implies that increasing the sampling time and controller gains drive the system closer to non-passivity. The master or slave robot damping term  $b$  is a physical characteristic of the robot and cannot be easily changed whereas the other parameters in (28) can be appropriately set in the control design process. The sampling time is practically limited by an upper bound because of the discrete-time processing of control laws. In a teleoperation system, the controller gains need to large for teleoperation transparency. This represents a trade-off between the passivity requirement and the transparency requirement.

## 4 Simulation study

The teleoperation system of Fig. 1 has been simulated in MATLAB/Simulink and the passivity condition (28) has been tested for a pair of 1-DOF robots modelled by mass and damping terms. The simulation allows for variation of the parameters that are constant in an experimental teleoperation system such as the robot damping term  $b$ .

To determine the passivity of the teleoperator, a passivity observer has been incorporated into the simulations. The passivity observer calculates the dissipated energy in the system. The dissipated energy is represented as the input-output energy integral in (7). The system is passive if the energy integral is non-negative at all times.

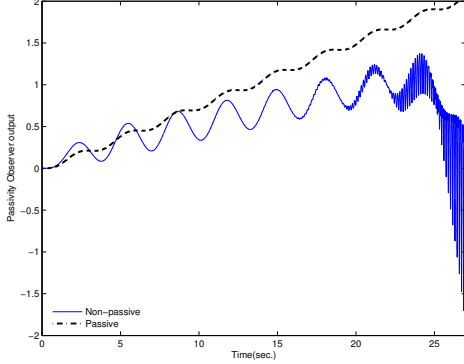


Figure 2: Example of the passivity observer output for passive and non-passive sampled-data teleoperator. Simulation parameters are  $T = 1\text{ms}$ ,  $K=1000$ ,  $B=0$ ,  $m=1$  and  $b=2$ , for the passive cases and  $b=0.1$  for the non-passive case.

Examples of the passivity observer output for sample passive and non-passive teleoperator are shown in Fig. 2. As it can be seen, in the non-passive teleoperator the energy term may be positive for a period of time. However, eventually the energy goes below zero.

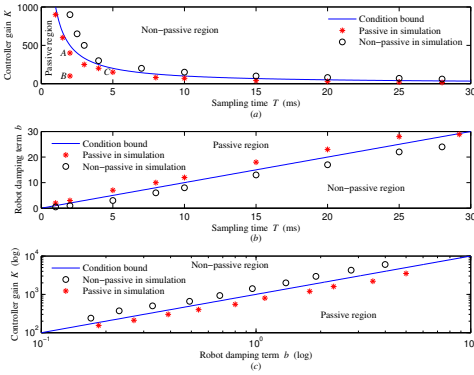


Figure 3: Passive and non-passive regions (a) in K-T plane for  $B = 0$ ,  $b = 1$ , (b) in b-T plane for  $K = 1000$  and  $B = 0$ , and (c) in K-b plane for  $B = 0$  and  $T = 1\text{ms}$ .

The simulation has been repeated for various model parameters of the system, for which passivity can be determined by condition (28). In the simulation results of Fig. 3, the solid line indicates the border between passivity and non-passivity as explained by condition (28). Each star or circle in Fig. 3

shows a simulation result for certain parameters of the system. If the simulation is passive, the point is marked with a star and if the system is non-passive, the point is marked with a circle. The passivity observer was used to determine passivity of the system. To have consistent results with the theory, it is expected to have circles only in non-passive region.

As shown in Fig. 3, the regions found by the passivity condition (28) match the simulation results. In one case, however, the passivity condition of (28) turns out to be conservative: In Fig. 3-b, for the case of  $T = 28\text{ms}$  and  $b = 28$ , the passivity condition predicts that the system is non-passive while the simulation results indicates that the system is passive. The conservatism of condition (28) is due to the fact that it was found as a sufficient condition for passivity.

In addition to the passivity in the teleoperator, it is desirable to have similar master and slave position profiles (for transparency). Fig. 4 depicts samples of master/slave position profiles for three passive teleoperation systems corresponding to points A, B and C in Fig. 3a. By comparing the simulation results of Fig. 4, it is observed that increasing the controller gain will increase the degree of telepresence. For instance, the controller gain in Fig. 4a is greater than that in Fig. 4b and Fig. 4c. In the continuous-time counterpart, it can be proven that increasing the controller gain makes the system more transparent.

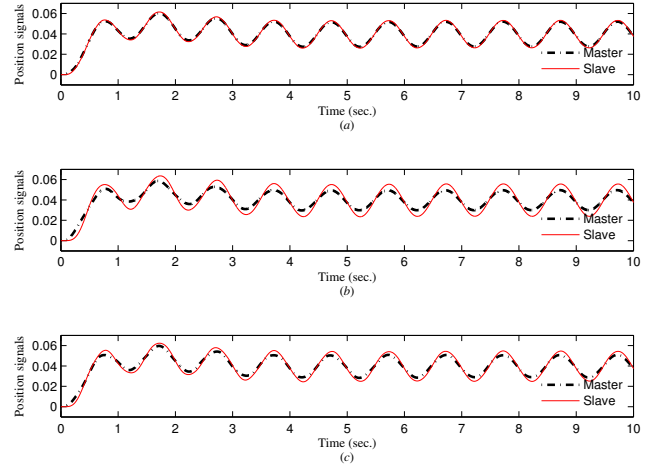


Figure 4: The master robot is shaken intentionally and the slave robot follows the motion of the master arm. The parameters are (a)  $T = 2\text{ms}$ ,  $b = 1$ ,  $B = 0$  and  $K = 400$  (corresponding to point A in Fig. 3a), (b)  $T = 2\text{ms}$ ,  $b = 1$ ,  $B = 0$  and  $K = 100$  (corresponding to point B in Fig. 3a) and (c)  $T = 5\text{ms}$ ,  $b = 1$ ,  $B = 0$  and  $K = 150$  (corresponding to point C in Fig. 3a).

Condition (28) determines an upper bound on the controller gain for sampled-data teleoperation. Moreover, as the sampling time  $T$  increases, the upper bound on the passivifying controller gain decreases (Fig. 3a). Consequently, any increase in the sampling time degrades the degree of telepresence of the teleoperation system.

## 5 Experimental results

To verify the passivity condition, an experimental setup has been designed with two identical Sensable Phantom Omni robots, which are 6-DOF haptic devices with 3-DOF actuated and 3-DOF free-running joints. The operator interacts with the master robot while the slave robot is attached to a nonlinear spring connected to a stiff wall (Fig. 5). Out of the three actuated joints of the robot, the first one is used

in the experiment while the second and the third joints, which form a parallel mechanism, are locked.

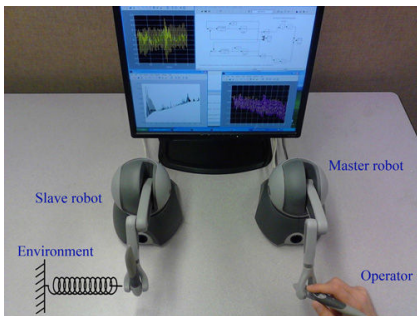


Figure 5: Experimental setup – The master arm is controlled by the human operator and the slave arm interacts with the environment.

To verify the passivity condition, the damping terms of the robots must be known. The damping and mass terms of the robot were found through grey-box system identification in a separate experiment. The values were found to be  $m = 1.503 \times 10^{-2} \pm 1.7 \times 10^{-4}$  and  $b = 4.624 \times 10^{-2} \pm 1.1 \times 10^{-3}$ .

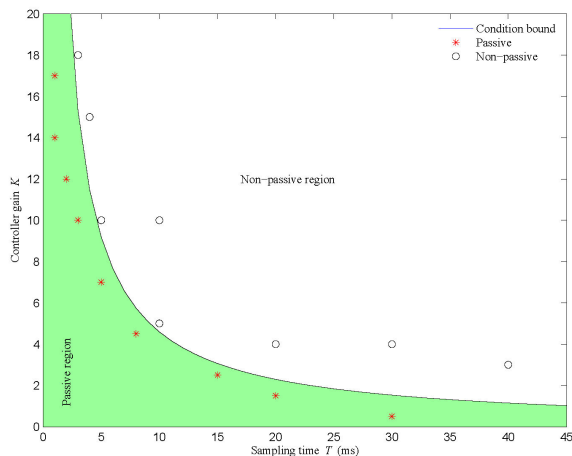


Figure 6: Passivity and non-passivity regions in  $K - T$  plane for a teleoperator consisting of two Phantom Omni robots. The passive region is where condition (28) is satisfied and the non-passive region is where condition (28) is violated.

In Section 4, the passivity of the simulated teleoperator was found using a passivity observer. The passivity observer uses force and position signals. In our experimental setup, however, the hand and the environment force measurements are not available. Two approaches have been described in the following subsections to determine whether the system is passive or non-passive. The passivity bound given by (28) is then compared to the experimental points (shown as \* and o) in Fig. 6. The robot model has been identified and has been used to estimate the forces. The model requires the first and the second derivatives of the position signals. To minimize the impact of measurement noise a 5th-order Butterworth filter is applied after taking the derivative of the position signals.

### 5.1 Passivity monitoring with a non-passive operator

To investigate the passivity of the experimental system, the operator injects energy into the teleoperation system by

exciting the master robot. When the system starts to display underdamped oscillations, the stabilizing influence of the operator is removed. If from this point on the oscillations of the system increase *without* bound the system is clearly non-passive. If the system remains unstable and the oscillation amplitude tends to increase over time – which due to physical constraints drives the system to a high-amplitude limit cycle with saturation on the force signals – it means that the teleoperation system did not damp the excess energy and hence the teleoperation system is non-passive (Fig. 7). In principle, the non-passivity of the teleoperation might have resulted from the non-passivity of the operator, the environment, or the teleoperator. The operator has released the master robot and is therefore passive. The environment can also be chosen to be a passive system. Consequently, it is the teleoperator itself that is non-passive. In other words, the only block that can cause non-passivity of the teleoperation system is the teleoperator block. In short, if the teleoperation system stays unstable after intentional destabilization followed by releasing the master by the operator, it can be concluded that the teleoperator is non-passive. It is worthy to note that stabilization in this scenario cannot establish passivity – only non-passivity can be shown.

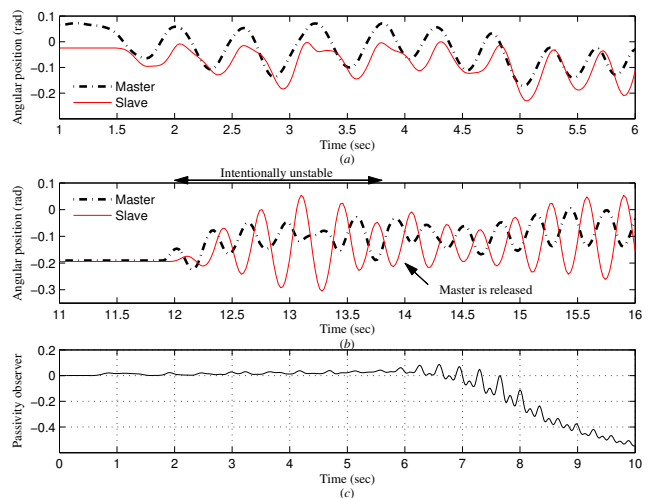


Figure 7: An example of a non-passive teleoperation system. (a) The position profiles of the master and the slave robots. (b) Position profiles when the operator injects energy to the system and deliberately destabilizes the system. After releasing the master arm at  $t = 14^s$ , the teleoperation system remains unstable. The parameters are  $T = 20^{ms}$ ,  $K = 4$ ,  $B = 0.01$ , (c) The energy observer output, which goes negative at  $t = 7.8^s$ .

□

### 5.2 Passivity monitoring using a force-estimating passivity observer

Another approach to determine the passivity of the experimental teleoperation system is to design a force estimator, where the energy integral (7) can be calculated using (1). In (1), the controller output forces  $f_m$  and  $f_s$  are known and the positions are measured. The forces of the environment and the operator are estimated. If the energy integral remains non-negative for all times, the system is passive. The resulting energy integrals are shown in Fig. 7c and Fig. 8c for a non-passive and a passive teleoperation system, respectively. The energy integral becomes negative at some time for the non-passive system whereas for the passive system the energy integral is never negative.

□



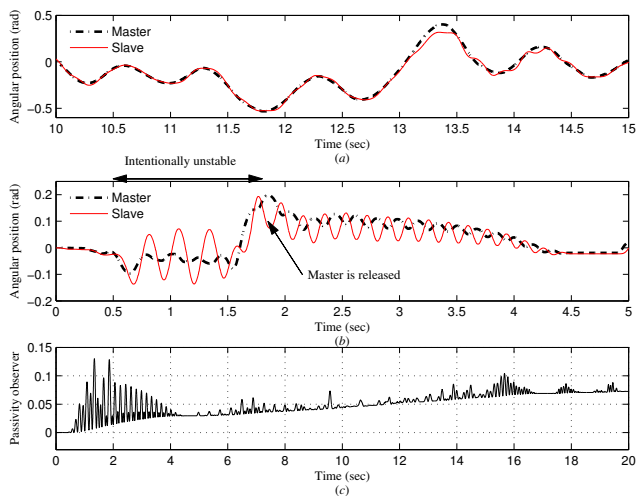


Figure 8: An example of a passive teleoperation system. (a) The master robot is moved by the operator and the slave robot follows the master position (b) The operator deliberately destabilizes the system by shaking the master robot violently which injects energy to the system. After releasing the master arm at  $t = 1.8^s$  the teleoperation system returns to the stability. The parameters are  $T = 1^{\text{ms}}$ ,  $K = 15$ ,  $B = 0.01$ , (c) The energy observer output, which is non-negative at all times.

The experimental results of Fig. 7 and 8 correspond to non-passive and passive teleoperation systems, respectively, where passivity/non-passivity has been judged based on meeting or violating condition (28). Fig. 7a and Fig. 8a show the master/slave position tracking in the teleoperation system – when the operator moves the master robot, the slave robot follows the motions of it. As it can be seen in these figures, the position tracking is better in the passive case than the non-passive case. Fig. 7b shows an experiment involving intentional destabilization of the system by the operator. The fact that the oscillation are not damped out highlight the non-passivity of the teleoperator. Also, as shown in Fig. 7c, the corresponding energy integral becomes negative, further confirming that the teleoperator non-passive. On the other hand, in Fig. 8b, the operator injects energy to the system in a similar manner but the oscillation is damped out by the passive teleoperator. Also, in Fig. 8c, the energy integral remains positive at all times, corroborating the passivity of the teleoperator.

## 6 Conclusion

In this paper, a passivity condition has been found for a bilateral teleoperation system in which the controllers are implemented in discrete-time and the rest of the system works in continuous-time. To find the condition, the models of the zero-order-hold and the sampler are considered in an appropriate frequency-domain analysis. The condition imposes a lower bound on the robot damping and upper bounds on the controller gains and the sampling time. The importance of the bound is that as it allows the controller gains to increase, the degree of telepresence is allowed to improve in the teleoperation system. The conditions have been tested by simulating the teleoperation system and monitoring the energy integral. The condition is found to be an accurate criterion for passivity for both the simulated system and the experimental system. To implement the passivity observer in a situation where a force sensor is not available, a method based on intentionally destabilizing the system by the operator has been introduced, which can be used to detect non-passivity. Experimental tests

on teleoperation system has confirmed the passivity condition. Both the destabilization method and the force-estimation-based method for passivity observation led to results consistent with the theoretical passivity condition found in this paper.

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## References

- [1] P. F. Hokayem and M. W. Spong, “Bilateral teleoperation: An historical survey,” *Automatica*, vol. 42, no. 12, pp. 2035 – 2057, 2006.
- [2] S. Salcudean, “Control for teleoperation and haptic interfaces,” in *Control Problems in Robotics and Automation*, ser. Lecture Notes in Control and Information Sciences, B. Siciliano and K. Valavanis, Eds. Springer Berlin / Heidelberg, 1998, vol. 230, pp. 51–66.
- [3] J. Sheng and P. Liu, “A review of bilateral sampled-data control of teleoperators,” Aug 2004, pp. 385 – 390.
- [4] D. A. Lawrence, “Stability and transparency in bilateral teleoperation,” *IEEE Transactions on Robotics & Automation*, vol. 9, pp. 624–637, October 1993.
- [5] A. van der Schaft, *L2-Gain and Passivity in Nonlinear Control*, 2nd ed. Secaucus, NJ, USA: Springer-Verlag New York, Inc., 1999.
- [6] R. J. Anderson and M. W. Spong, “Asymptotic Stability for Force Reflecting Teleoperators with Time Delay,” *The International Journal of Robotics Research*, vol. 11, no. 2, pp. 135–149, 1992.
- [7] J. Colgate and G. Schenkel, “Passivity of a class of sampled-data systems: Application to haptic interfaces,” *Journal of Robotic Systems*, vol. 14, no. 1, pp. 37–47, 1997.
- [8] J. Gil, A. Avello, A. Rubio, and J. Florez, “Stability analysis of a 1 dof haptic interface using the routh-hurwitz criterion,” *IEEE Transactions on Control Systems Technology*, vol. 12, no. 4, pp. 583–588, July 2004.
- [9] N. Diolaiti, G. Niemeyer, F. Barbagli, and J. J.K. Salisbury, “Stability of haptic rendering: Discretization, quantization, time delay, and coulomb effects,” *IEEE Transactions on Robotics*, vol. 22, no. 2, pp. 256– 268, April 2006.
- [10] J. Abbott and A. Okamura, “Effects of position quantization and sampling rate on virtual wall passivity,” *IEEE Transactions on Robotics*, vol. 21, no. 5, pp. 952–964, October 2005.
- [11] M. Tavakoli, A. Aziminejad, R. Patel, and M. Moallem, “Discrete-time bilateral teleoperation: modelling and stability analysis,” *Control Theory and Applications, IET*, vol. 2, no. 6, pp. 496–512, June 2008.
- [12] G. Leung and B. Francis, “Bilateral control of teleoperators with time delay through a digital communication channel,” in *Proceedings of the Thirtieth Annual Allerton Conference on Communication, Control and Computing*, 1992, pp. 692–701.
- [13] P. Berestesky, N. Chopra, and M. Spong, “Discrete time passivity in bilateral teleoperation over the internet,” in *Proceedings of International Conference on Robotics and Automation*, New Orleans, LA, April 2004, pp. 4557–4564.
- [14] K. Kosuge and H. Murayama, “Bilateral feedback control of tele-manipulator via computer network in discrete time domain,” in *Robotics and Automation, 1997. Proceedings., 1997 IEEE International Conference on*, vol. 3, Apr. 1997, pp. 2219 –2224 vol.3.
- [15] J.-H. Ryu, D.-S. Kwon, and B. Hannaford, “Stable teleoperation with time-domain passivity control,” *Robotics and Automation, IEEE Transactions on*, vol. 20, no. 2, pp. 365 – 373, 2004.
- [16] J.-H. Ryu, J. Artigas, and C. Preusche, “A passive bilateral control scheme for a teleoperator with time-varying communication delay,” *Mechatronics*, vol. 20, no. 7, pp. 812 – 823, 2010, special Issue on Design and Control Methodologies in Telerobotics.
- [17] C. Secchi, S. Stramigioli, and C. Fantuzzi, *Control of Interactive Robotic Interfaces: A Port-Hamiltonian Approach (Springer Tracts in Advanced Robotics)*. Secaucus, NJ, USA: Springer-Verlag New York, Inc., 2007.