DISTURBANCE OBSERVER-BASED TRAJECTORY FOLLOWING CONTROL OF NONLINEAR ROBOTIC MANIPULATORS

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ABSTRACT

Robotic manipulators are highly nonlinear and coupled dynamic systems, which may be subject to different types of unknown disturbances such as joint frictions and end-effector external payloads. Such disturbances, when unaccounted for, cause poor tracking performance of the robot and may even destabilize the robot control system. In this paper we propose a novel nonlinear control scheme for robotic manipualtors subject to disturbances using the concept of *disturbance observer-based control* by modifying the disturbance observers proposed in [1] and [2]. The proposed control scheme and disturbance observer guarantee global asymptotic position and disturbance tracking and remove the previous restrictions on the number of degrees of freedom (DOFs), joint types, or manipulator configuration. Computer simulations are presented for a 4-DOF SCARA manipulator to show the effectiveness of the proposed disturbance observer-based control scheme.

Keywords: Robotic manipulator, trajectory following, disturbance observer, SCARA.

INTRODUCTION

Robotic manipulators are often subject to different types of unknown disturbances such as joint frictions and end-effector external payloads. Such disturbances, when unaccounted for, tend to degrade the tracking performance of the robot and may even cause the instability of control system. An approach to suppress these disturbances is to use disturbance observers. The idea behind such an approach is to lump all the internal and external disturbances acting on the manipulator into a single disturbance term, estimate the lumped term using a disturbance observer, and then introduce feedforward compensation to cancel it. Because of the feedforward nature of this compensation, disturbance observers can lead to fast, good trajectory tracking performance and smooth control actions without the use of large feedback gains [3]. Because of their disturbance attenuation capability, disturbance observers have found applications in a variety of areas such as independent joint control [4], friction estimation and compensation [5].

A considerable part of the existing literature on disturbance observer design uses linearized models or linear system techniques. In order to overcome the limitations and shortcomings of the linear disturbance observers given the highly nonlinear and coupled dynamics of robotic manipulators, Chen et al. proposed a nonlinear disturbance observer for a certain class of nonlinear robotic manipulators and designed it such that no acceleration measurement was needed [1]. The design problem was only solved for a 2-link planar manipulator with revolute joints. Later, Nikoobin et al. solved the disturbance observer design problem for n-link planar manipulators with revolute joints [2]. Although these disturbance observers show promising results in terms of disturbance estimation, their application domain is limited to planar, serial manipulators with revolute joints. Industrial robots including 6-DOF articulated robotic arms such as EPSON C3 and PUMA 560 are, however, non-planar. Moreover, some of the industrial arms such as SCARA manipulators have prismatic joints in addition to revolute joints. Moreover, the disturbance observers proposed in [1] and [2] need the knowledge of the maximum joint velocities of the robot. In addition to these limitations in terms of the manipulator configuration, the closed-loop stability of the overall system including the disturbance observer and the controller has not been investigated yet. This serves as the motivation for this paper to look for a general design method which also guarantees disturbance and trajectory tracking of the closed-loop system.

In this paper, we propose a nonlinear disturbance observer-based control scheme for general robotic manipulators by modifying the disturbance observers proposed in [1] and [2]. While our proposed control law and disturbance observer guarantee asymptotic trajectory and disturbance tracking, the previous limitations on the number of DOFs, joint types, or manipulator configuration are removed. In order to illustrate the effectiveness of the proposed disturbance observer-based control scheme, simulations are done using a SCARA robotic manipulator subject to disturbances.

PROBLEM FORMULATION

The following equation gives the dynamics of an n-DOF rigid manipulator [6]:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \boldsymbol{\tau} + \boldsymbol{\tau}_d \tag{1}$$

where $\mathbf{q}$,$\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ are the $n \times 1$ vectors of joint positions, velocities and accelerations, respectively. Here, $\mathbf{M}(\mathbf{q})$ is the $n \times n$ inertia matrix, $\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$ is the $n \times n$ Coriolis/centrifugal matrix, $\mathbf{G}(\mathbf{q})$ is the $n \times 1$ vector of gravitational forces, $\boldsymbol{\tau}$ is the $n \times 1$ vector of input control torques, and $\boldsymbol{\tau}_d$ is the $n \times 1$ vector of lumped disturbances. The disturbance $\boldsymbol{\tau}_d$ lumps the effect of friction torques, external disturbance torques such as unknown end-effector payloads, unmodeled dynamics, etc. Note that the inertia matrix $\mathbf{M}(\mathbf{q})$ of the robot is a symmetric and positive definite matrix. Therefore, it is invertible and its inverse is also a symmetric and positive definite matrix [7].

Assuming that robot joint acceleration measurements are available, the following nonlinear disturbance observer for the robot (1) was proposed in [1]:

$$\dot{\hat{\boldsymbol{\tau}}}_d = -\mathbf{L}\hat{\boldsymbol{\tau}}_d + \mathbf{L}\{\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) - \boldsymbol{\tau}\} \tag{2}$$

where $\mathbf{L}$ is the observer gain matrix. Defining $\Delta\boldsymbol{\tau}_d = \boldsymbol{\tau}_d - \hat{\boldsymbol{\tau}}_d$ as the disturbance tracking error and using (1), we have

$$\dot{\hat{\boldsymbol{\tau}}}_d = \mathbf{L}\Delta\boldsymbol{\tau}_d \tag{3}$$

Here, we assume that the rate of change of the lumped disturbance is neglibgible in comparsion with the estimation error dynamics, i.e. $\dot{\boldsymbol{\tau}}_d \approx \mathbf{0}$. This assumption is not overly restrictive and is commonly made in the robotics literature (see for example [1]). Under this assumption, (3) becomes

$$\Delta\dot{\boldsymbol{\tau}}_d = -\mathbf{L}\Delta\boldsymbol{\tau}_d \tag{4}$$

Note that despite the above assumption, the simulations will show that the proposed nonlinear disturbance observer is also able to track fairly fast time-varying disturbances.

A shortcoming of the disturbance observer (2) is the need for acceleration measurement. Accurate accelerometers are not available in many robotic applications. It is possible to modify the disturbance observer, as in [1], in a way that no acceleration measurement is needed. For this purpose, let us define the auxiliary variable

$$\mathbf{z} = \hat{\boldsymbol{\tau}}_d - \mathbf{p}(\mathbf{q},\dot{\mathbf{q}}) \tag{5}$$

where the vector $\mathbf{p}(\mathbf{q},\dot{\mathbf{q}})$ can be determined from the observer gain matrix $\mathbf{L}(\mathbf{q},\dot{\mathbf{q}})$:

$$\frac{d}{dt}\mathbf{p}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{L}(\mathbf{q},\dot{\mathbf{q}})\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} \tag{6}$$

Therefore, the modified disturbance observer, which does not need acceleration measurement, takes the following form [1]:

$$\begin{aligned}&\dot{\mathbf{z}} = -\mathbf{L}(\mathbf{q},\dot{\mathbf{q}})\mathbf{z} + \\ &\mathbf{L}(\mathbf{q},\dot{\mathbf{q}})\{\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) - \boldsymbol{\tau} - \mathbf{p}(\mathbf{q},\dot{\mathbf{q}})\} \\ &\hat{\boldsymbol{\tau}}_d = \mathbf{z} + \mathbf{p}(\mathbf{q},\dot{\mathbf{q}})\end{aligned} \tag{7}$$

Again, assuming slow-varying disturbances with respect to the observer's dynamics, the error dynamics becomes similar to (4):

$$\Delta\dot{\boldsymbol{\tau}}_d = -\mathbf{L}(\mathbf{q},\dot{\mathbf{q}})\Delta\boldsymbol{\tau}_d \tag{8}$$

NONLINEAR DISTURBANCE OBSERVER-BASED CONTROL LAW

We propose the following nonlinear control law for the robotic manipualtor described by (1):

$$\begin{aligned}\boldsymbol{\tau} \;=\;& \mathbf{M}(\mathbf{q})[\ddot{\mathbf{q}}_d + \mathbf{K}_v\Delta\dot{\mathbf{q}} + \mathbf{K}_p\Delta\mathbf{q}] + \\ & \mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) - \hat{\boldsymbol{\tau}}_d\end{aligned} \tag{9}$$

where $\mathbf{q}_d$, $\dot{\mathbf{q}}_d$, and $\ddot{\mathbf{q}}_d$ are the desired position, velocity and acceleration of the joints of the robot, $\Delta\mathbf{q} = \mathbf{q}_d - \mathbf{q}$ and $\Delta\dot{\mathbf{q}} = \dot{\mathbf{q}}_d - \dot{\mathbf{q}}$ are the position and velocity tracking errors, and $\mathbf{K}_v$ and $\mathbf{K}_p$ are constant, symmetric and positive definite matrices. Note the use of disturbance estimate $\hat{\boldsymbol{\tau}}_d$ in the proposed control law (9). Applying the nonlinear disturbance observer-based control law (9) to the robot described by (1) results in the following closed-loop equation:

$$\Delta\ddot{\mathbf{q}} + \mathbf{K}_v\Delta\dot{\mathbf{q}} + \mathbf{K}_p\Delta\mathbf{q} = \mathbf{M}^{-1}(\mathbf{q})\Delta\boldsymbol{\tau}_d \tag{10}$$

where $\Delta\boldsymbol{\tau}_d = \boldsymbol{\tau}_d - \hat{\boldsymbol{\tau}}_d$.

Now, we modify the disturbance observer in the previous section such that global asymptotic trajectory and disturbance tracking of the closed-loop system encompassing the robot, the controller, and the observer are achieved. We modify the disturbance observer in (7) by incorporating the following term in it:

$$\mathbf{M}^{-1}(\mathbf{q})(\Delta\dot{\mathbf{q}} + \gamma\Delta\mathbf{q}), \gamma > 0$$

We will have

$$\begin{aligned}&\dot{\mathbf{z}} = -\mathbf{L}(\mathbf{q},\dot{\mathbf{q}})\mathbf{z} + \\ &\mathbf{L}(\mathbf{q},\dot{\mathbf{q}})\{\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) - \boldsymbol{\tau} - \mathbf{p}(\mathbf{q},\dot{\mathbf{q}})\} \\ &+\mathbf{M}^{-1}(\mathbf{q})(\Delta\dot{\mathbf{q}} + \gamma\Delta\mathbf{q}) \\ &\hat{\boldsymbol{\tau}}_d = \mathbf{z} + \mathbf{p}(\mathbf{q},\dot{\mathbf{q}})\end{aligned} \tag{11}$$

Assuming slow-varying disturbances with respect to the observer's dynamics, the error dynamics of the modified disturbance observer in (11) becomes

$$\Delta\dot{\boldsymbol{\tau}}_d = -\mathbf{L}(\mathbf{q},\dot{\mathbf{q}})\Delta\boldsymbol{\tau}_d - \mathbf{M}^{-1}(\mathbf{q})(\Delta\dot{\mathbf{q}} + \gamma\Delta\mathbf{q}) \tag{12}$$

Now, we propose the following disturbance observer gain matrix:

$$\mathbf{L}(\mathbf{q}) = \alpha \mathbf{M}^{-1}(\mathbf{q}) \tag{13}$$

where α is an arbitrary positive constant. According to (6), we have

$$\mathbf{p}(\dot{\mathbf{q}}) = \alpha \dot{\mathbf{q}} \tag{14}$$

The following theorem gives the sufficient conditions for global asymptotic trajectory and disturbance tracking of the robot (1) using the control law (9) and the disturbance observer (11).

Theorem 1. *Consider the robotic manipulator subject to disturbances described by the dynamic equation* (1)*. The disturbance observer has the dynamics given in* (11)*, where the disturbance observer gain matrix* $\mathbf{L}(\mathbf{q})$ *is given in* (13) *and the disturbance observer auxiliary vector* $\mathbf{p}(\dot{\mathbf{q}})$ *is given in* (14)*. The control law* (9) *guarantees global asymptotic disturbance and trajectory tracking in the overall closed-loop system if the following conditions hold:*

1. $\mathbf{K}_v$ *is a constant symmetric and positive definite matrix satisfying* $\mathbf{K}_v > \gamma \mathbf{I}$*,*
2. $\mathbf{K}_p$ *is a constant symmetric and positive definite matrix,*
3. $\dot{\tau}_d \cong \mathbf{0}$*, i.e., the rate of change of the disturbance acting on the manipulator is negligible in comparison with the estimation error dynamics* (12)*.*

Proof. Under the control law (9) and according to condition 3 of the Theorem, the position tracking error dynamics and the disturbance tracking error dynamics are given by (10) and (12), respectively. Let us consider the following candidate Lyapunov function:

$$V(\Delta\dot{\mathbf{q}}, \Delta\mathbf{q}, \Delta\boldsymbol{\tau}_d) = \frac{1}{2}(\Delta\dot{\mathbf{q}} + \gamma\Delta\mathbf{q})^T(\Delta\dot{\mathbf{q}} + \gamma\Delta\mathbf{q}) + \frac{1}{2}\Delta\mathbf{q}^T(\mathbf{K}_p + \gamma\mathbf{K}_v - \gamma^2\mathbf{I})\Delta\mathbf{q} + \frac{1}{2}\Delta\boldsymbol{\tau}_d^T\Delta\boldsymbol{\tau}_d$$

Taking the time derivative of the above function and using (10) and (12) we get

$$\dot{V} = -\Delta\dot{\mathbf{q}}^T(\mathbf{K}_v - \gamma\mathbf{I})\Delta\dot{\mathbf{q}} - \gamma\Delta\mathbf{q}^T\mathbf{K}_p\Delta\mathbf{q} - \alpha\Delta\boldsymbol{\tau}_d^T\mathbf{M}^{-1}(\mathbf{q})\Delta\boldsymbol{\tau}_d \tag{15}$$

Note that $\mathbf{M}^{-1}(\mathbf{q})$ is positive definite according to the Property 1 in the previous section. According to (15) and condition 2 of the theorem, the Lyapunov function V is positive definite in the entire state space $[\Delta\dot{\mathbf{q}}^T, \Delta\mathbf{q}^T, \Delta\boldsymbol{\tau}^T]^T$ and is radially unbounded. Conditions 1–3 of the theorem and positive definiteness of $\mathbf{M}^{-1}(\mathbf{q})$ guarantee that $\dot{V}$ is negative definite in the entire state space. Therefore, the velocity, position and disturbance tracking errors converge to zero. □

SIMULATION STUDY

SCARA (Selective Compliance Assembly Robot Arm) is an industrial 4-DOF robotic arm, which is widely used in the assembly of electronic circuits and devices. Figure 1 depicts a schematic digram of this manipulator. The dynamics of the SCARA manipulator is given by (1) where [7]:

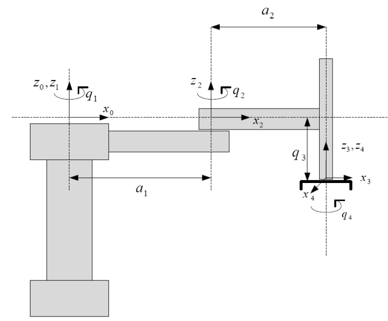

Fig.1: Schematic diagram of the SCARA robotic arm.

$$\mathbf{M}(\mathbf{q}) = \begin{bmatrix} p_1 + p_2c_2 & p_3 + 0.5p_2c_2 & 0 & -p_5 \\ p_3 + 0.5p_2c_2 & p_3 & 0 & -p_5 \\ 0 & 0 & p_4 & 0 \\ -p_5 & -p_5 & 0 & p_5 \end{bmatrix}$$

$$\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} -p_2s_2\dot{q}_2 & -0.5p_2s_2\dot{q}_2 & 0 & 0 \\ 0.5p_2s_2\dot{q}_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{G}(\mathbf{q}) = \begin{bmatrix} 0 & 0 & -p_4g & 0 \end{bmatrix}^T \tag{16}$$

where P_i, $i = 1, \dots, 5$ are constant terms. The Jacobian of the SCARA manipulator, with respect to the robot base frame, is [7]:

$$\mathbf{J}(\mathbf{q}) = \begin{bmatrix} -a_1s_1 - a_2s_{12} & -a_2s_{12} & 0 & 0 \\ a_1c_1 + a_2c_{12} & a_2c_{12} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \tag{17}$$

In the above, $s_2 = \sin(q_2)$, $c_2 = \cos(q_2)$, $s_{12} = \sin(q_1 + q_2)$, $c_{12} = \cos(q_1 + q_2)$

In the simulations, two types of disturbances are exerted to the robot, namely joint-level friction and external task-level payload. The external end-effector payload is modeled by a force exerted to the robot end-effector in the z direction (vertical direction). This payload is equal to 10^N from $t = 0^{sec}$ to $t = 7^{sec}$ and is then reduced to 5^N at $t = 7^{sec}$. The friction torques acting on the joints of the robots are generated based on the model in [8]. For the $i - th$ joint of the robot, $i = 1, 2, 3, 4$, we have the frictions

$$\tau_{i_{friction}} = F_{ci}\mathrm{sgn}(\dot{q}_i)[1 - \exp(\frac{-\dot{q}_i^2}{v_{si}^2})] + F_{si}\mathrm{sgn}(\dot{q}_i)\exp(\frac{-\dot{q}_i^2}{v_{si}^2}) + F_{vi}\dot{q}_i \tag{18}$$

where F_{ci}, F_{si}, F_{vi} are the Coulomb, static, and viscous friction coefficients, respectively. The parameter v_{si} is the Stribeck parameter. The SCARA parameters are chosen as in [7]. Other simulation parameters are

$$\begin{aligned} &F_{c2} = 2.8,\ F_{c3} = 0.7,\ F_{c4} = 0.7,\ F_{v1} = 0.15 \\ &F_{v2} = 0.12,\ F_{v3} = 0.03,\ F_{v4} = 0.03,\ v_{s1} = 0.19 \\ &v_{s2} = 0.15,\ v_{s3} = 0.03,\ v_{s4} = 0.03 \end{aligned} \tag{19}$$

Square-wave reference position trajectories are supplied for all joints of the robot. In the first case, no disturbance observer is used with the control law (9), i.e., $\hat{\boldsymbol{\tau}}_d = 0$. Note that when $\hat{\boldsymbol{\tau}}_d = 0$, the control scheme (9) is a regular computed-torque control law. In the second case, the proposed disturbance observer is used (to estimate the joint frictions and the external payload) together with the control law (9) with $\mathbf{K}_v = 6\mathbf{I}$ and $\mathbf{K}_p = 16\mathbf{I}$ where $\mathbf{I}$ is the identity matrix. The disturbance observer is given by (11), (13) and (14) with $\alpha = 10$ and $\gamma = 10$. Figure 2 illustrates the time profiles of the positions of the joints of the robot. As it can be observed, the computed-torque control law fails to track the position commands accurately when no disturbance observer is used. Note that the third joint, i.e., q_3, is worst affected since a weight was attached to the vertical link. On the other hand, when the disturbance observer is used, the robot performs the position commands accurately. Figure 3 depicts the actual and estimated disturbances. Despite the relatively fast time-varying disturbance, the estimated disturbance is able to track it.

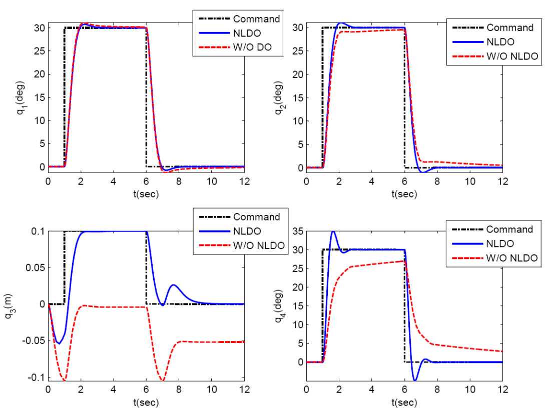

Fig.2: Position of the joints of the SCARA robot.

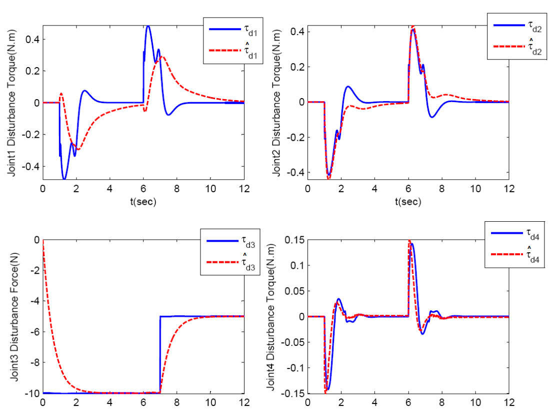

Fig.3: Disturbance tracking of the proposed disturbance observer.

CONCLUSIONS

A nonlinear disturbance observer-based control scheme for general robotic manipulators has been proposed in this paper. While past methods merely dealt with planar serial manipulators with revolute joints, the proposed disturbance observer removes the previous restrictions on the number of degrees-of-freedom, the types of joints, and the manipulator configuration. Moreover, the asymptotic position and disturbance tracking of the closed-loop system including the robotic manipulator, the controller and the observer are guaranteed. Simulations using an industrial manipulator were presented to verify the effectiveness of the proposed approach.

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