On the Optimality of Multiuser Zero-Forcing Precoding in MIMO Broadcast Channels

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Abstract—In this paper, we consider a multiuser Gaussian broadcast channel where the base station and the users both are equipped with multiple antennas. The problem of optimal zero-forcing (ZF) precoding design subject to sum power constraint is discussed. Block diagonalization (BD) is applied as the ZF precoding method. We consider two common optimization criteria: maximum throughput and maximal fairness, where by maximizing fairness we mean maximizing the minimum rate among all users [1]. Under these criteria, the structure of the optimal BD precoders is presented and analyzed. It is shown that the conventional BD precoding design with sum power constraint is indeed optimal under the throughput and fairness criteria.

Index Terms—Block diagonalization (BD), linear precoder design, multiple-input multiple-output (MIMO) broadcast channel (BC), zero-forcing (ZF) precoding

I. INTRODUCTION

Due to the increased demand on data throughput and quality of service (QoS), multiple-input multiple-output (MIMO) wireless systems are usually considered to provide such benefits. The capacity region of a multiuser MIMO Gaussian broadcast channel (BC) was recently characterized [2]–[5]. It is well-known that the optimal dirty paper coding (DPC) technique achieves the capacity region of the multiuser MIMO BC [2]. However, the DPC technique is based on noncausal knowledge of each user’s interfering signal at the base station. Furthermore, successive encoding to suppress interference is very complex and makes DPC extremely challenging to implement. Therefore, practical sub-optimal linear precoding techniques are of great interest [6]–[8].

The most common linear precoding schemes involve zero-forcing (ZF). This simple approach isolates the users’ transmissions and hence decouples the multiuser channel into multiple independent single-user sub-channels. Therefore, ZF schemes have been studied extensively in multiuser systems. While for multiuser multiple-input single-output (MISO) downlink (each user equipped with a single antenna) the channel inversion is known as ZF precoding, this has been generalized to downlink MIMO multiuser with multiple-antenna users in [9] and is often referred to as block diagonalization (BD).

Traditionally, the ZF precoder is assumed to implement a pseudo-inverse of the channel. The optimal ZF forms in multiuser MISO systems have been discussed in [1] under different power constraints. In particular, for multiuser MISO downlink under individual per-antenna power constraints [10], [11] the pseudo-inverse based precoder is no longer optimal ZF precoder and the optimal precoding is obtained through convex optimizations over other generalized inverses [1]. In [1], Wiesel et. al. have also shown that under a sum power constraint, the pseudo-inverse based precoder is the optimal form of ZF precoder for any performance criterion. However, in [1], this is only proved for multiuser MISO downlink (each user is equipped with only single antenna). In this paper, the problem of optimal ZF precoding in multiuser MIMO systems (multiple antennas at each user) is discussed. We prove that for these systems the BD scheme proposed in [9] is the optimal form of ZF precoder.

The BD scheme in [9] is based on the transmission to each user in the null space of other user channels. However, in [9] it is assumed that the ZF precoder employs only a specific linear combination of the null space basis vectors of other users’ channels (i.e., power allocation) and therefore the optimization of ZF precoder has not been attempted over the entire null space. The main contribution of this paper is optimizing the ZF precoder over the entire null space of other users’ transmissions, which is done through convex optimization [12]. The paper proves the optimality of the traditionally used BD in [9], [10]. The structure of ZF precoders is also applied to obtain maximal fairness among users (equal-rate transmission among the scheduled users). An algorithm to find the optimal ZF precoders is proposed. Similar work has been done under per-antenna power constraints [1], [13], where the optimum ZF precoding for a multiuser MISO downlink system has been studied. Hence, the results in this paper are the extension of the work done in [1], [13] to MIMO systems (i.e., more than one receive antenna per user) under sum power constraint.

II. SYSTEM MODEL

We consider a single-cell multiuser MIMO downlink system with $K$ users and a base station. Each user is equipped with $N_r$, $(N_r > 1)$ receive antennas and the base station is equipped with $N_t$ transmit antennas. The downlink channel between the base station and user $k$ is described by the matrix $\mathbf{H}_k \in \mathbb{C}^{N_r\times N_t}$. Therefore, the aggregate downlink channel is described by $\mathbf{H} = [\mathbf{H}_1 \mathbf{H}_2 \cdots \mathbf{H}_K]^{T}$, where $(\cdot)^T$ denotes the matrix transpose. In the MIMO BC, let $\mathbf{x} \in \mathbb{C}^{N_t\times 1}$ denote the transmitted signal vector from the base station’s antennas , $\mathbf{y}_k \in \mathbb{C}^{N_r\times 1}$ be the received signal vector at the mobile receiver $k$. The noise at receiver $k$ is represented by $\mathbf{z}_k \in \mathbb{C}^{N_r\times 1}$ and is assumed to be circularly symmetric zero-mean complex Gaussian noise ($\mathbf{z}_k \sim \mathcal{CN}(0, \sigma^2 \mathbf{I})$). The
received signal for user $k$ can be expressed as

$$y_k = H_k x + z_k, \quad k = 1, \ldots, K \quad (1)$$

The covariance matrix of the transmitted signal is $S_x = \mathbb{E}[xx^H]$. Under an average total power constraint $P$, we require that $\text{tr}(S_x) \leq P$.

The transmitted symbol of user $k$ is an $N_r$-dimensional vector $u_k$, which is multiplied by a $N_t \times N_r$ precoding matrix $W_k$ to produce the transmitted signal intended for user $k$, $x_k$. Thus, the complex transmit antenna output vector $x$ is composed of signals for all $K$ users given by

$$x = \sum_{k=1}^{K} x_k = \sum_{k=1}^{K} W_k u_k \quad (2)$$

where $\mathbb{E}[u_k u_k^H] = I_{N_r}$. The received signal $y_k$ for user $k$ can be represented as

$$y_k = H_k W_k u_k + \sum_{j=1, j \neq k}^{K} H_k W_j u_j + z_k \quad (3)$$

where $z_k$ denotes the additive white Gaussian noise (AWGN) vector for user $k$ with variance $\mathbb{E}[z_k z_k^H] = \sigma^2 I$. Each entry of $H_k$ is a zero mean i.i.d. complex Gaussian variable with variance $0.5$ per dimension. i.i.d. Gaussian distributed channel gains ensure $\text{rank}(H_k) = \min(N_r, N_t)$ for all $k$ with probability one.

### III. BLOCK DIAGONALIZATION

The key idea of multiuser MIMO ZF precoding is block diagonalization (BD) [9]. The transmitted symbol vector of user $k$, $u_k$, is precoded with the matrix $W_k$ such that

$$H_k W_j = 0 \quad \text{for all} \quad k \neq j \quad \text{and} \quad 1 \leq k, j \leq K. \quad (4)$$

Hence, the received signal for user $k$ simplifies to

$$y_k = H_k W_k u_k + z_k. \quad (5)$$

Let $\tilde{H}_k = [H_1 \cdots H_k \cdots H_{k+1}^T \cdots H_K^T]^T$. Zero-interference constraint in (4) forces $\tilde{W}_k$ to lie in the null space of $\tilde{H}_k$ which requires a dimension condition $N_t \geq N_r K$ to be satisfied. For simplicity of our setup, we assume that $N_t = N_r K$ and we focus on $K$ users which are scheduled. To simplify further analysis, we normalize the vectors in (5) and divide each vector by the standard deviation of the additive noise component, $\sigma$. Then, the components of $z_k$ have unit variance. Assuming that $H_k$ is a full rank matrix, $\text{rank}(H_k) = (K-1)N_r$, let us define the singular value decomposition (SVD)

$$\tilde{H}_k = \tilde{U}_k \tilde{\Sigma}_k \tilde{V}_k^T \quad (6)$$

where $\tilde{Y}_k$ holds the first $(K-1)N_r$ right singular vectors corresponding to non-zero eigenvalues, and $V_k \in \mathbb{C}^{N_r \times N_r}$ contains the last $N_r$ right singular vectors corresponding to zero eigenvalues of $H_k$. It can be observed that $V_k^H V_k = I_{N_r}$.

The columns of $V_k$ form a basis set in the null space of $\tilde{H}_k$, and hence $W_k$ can be any linear combination of $V_k$, i.e.

$$W_k = V_k T_k, \quad k = 1, \ldots, K \quad (7)$$

where $T_k \in \mathbb{C}^{N_r \times N_r}$ can be any arbitrary matrix subject to the sum power constraints. Hence, the received signal for user $k$ can be rewritten as

$$y_k = H_k V_k T_k u_k + z_k. \quad (8)$$

Denote $S_k = T_k T_k^H \in \mathbb{C}^{N_r \times N_r}$, $k = 1, \ldots, K$ which are positive semidefinite matrices. The user $k$’s rate is then given by

$$R_k = \log|I + H_k V_k S_k V_k^H H_k^H|. \quad (9)$$

Due to the application, there are two different objective functions considered: throughput ($\sum_k R_k$), and fairness ($\min_k R_k$). In the following sections, the problem of optimal ZF precoding design is addressed to maximize these objective functions.

#### A. Throughput

The problem of ZF precoding design for throughput maximization subject to the total sum power constraint can be formulated as

$$\begin{align*}
\text{maximize} & \quad \sum_{k=1}^{K} \log|I + H_k V_k S_k V_k^H H_k^H| \\
\text{subject to} & \quad \sum_{k=1}^{K} \text{tr}\{S_k\} \leq P \quad (10) \\
\text{s.t.} & \quad S_k \succeq 0, \quad k = 1, \ldots, K
\end{align*}$$

where $\succeq$ denotes matrix inequality defined on the cone of positive semidefinite matrices. Therefore, the maximization in (10) is over the set of positive semidefinite matrices ($S_k \succeq 0$, $k = 1, \ldots, K$). Notice that the objective function in (10) is concave because $\log|\cdot|$ is concave (see [14, p. 466], [12, p. 48]). The constraints are also affine functions [12]. Thus, the problem is categorized as a convex optimization problem which can be solved either through the Karush-Kuhn-Tucker (KKT) optimality conditions or via efficient numerical optimization. To simplify the formulations, let $G_k = H_k V_k$ be the effective channel for user $k$. The Lagrangian function of this convex problem is given by

$$\begin{align*}
\mathcal{L}(\{S_k\}, \{Q_k\}, \mu) = & \quad \sum_{k=1}^{K} \log|I + G_k S_k G_k^H| \\
- & \quad \mu (\sum_{k=1}^{K} \text{tr}\{S_k\} - P) + \sum_{k=1}^{K} \text{tr}\{Q_k S_k\} \\
\mu \geq 0, \quad Q_k \succeq 0, \quad k = 1, \ldots, K
\end{align*} \quad (11)$$

where positive semidefinite matrices $Q_k$ are slack variables [12] to guarantee that $S_k$ are positive semidefinite. Real non-negative value $\mu$ is a slack variable to satisfy the total sum power constraint. Since the above problem is convex, the KKT conditions [12] are necessary and sufficient for the optimality of certain solutions $S_k$, $k = 1, \ldots, K$. Taking into account the
complementary slackness conditions [12], the KKT conditions can be expressed as

\[
G_k^H(\mathbf{I} + G_k \mathbf{S}_k G_k^H)^{-1} G_k = \mu \mathbf{I} + \mathbf{Q}_k,
\]

\[
\text{tr} \{ \mathbf{S}_k \mathbf{Q}_k \} = 0,
\]

\[
\mu \left( \sum_{k=1}^{K} \text{tr} \{ \mathbf{S}_k \} - P \right) = 0,
\]

\[
\mu \geq 0, \mathbf{Q}_k \succeq 0, \quad k = 1, \ldots, K
\]

(12)

where the first KKT condition is similar to the water-filling condition for a Gaussian vector channel [15]. The KKT conditions in (12) require that \( \mu \) be a positive value, thus the power inequality constraint is equivalent to the equality for the optimal solutions. Thus, to obtain the optimal ZF precoders for the convex optimization problem (11) the KKT conditions need to be solved. From (12) we get

\[
\mathbf{S}_k = (\mu \mathbf{I} + \mathbf{Q}_k)^{-1} - G_k^H G_k^{-1}.
\]

(13)

The key to solve the KKT conditions is SVD of the matrices. Let the SVD, \( G_k^H G_k = \mathbf{U}_k \Lambda_k \mathbf{U}_k^H \) where \( \Lambda_k = \text{diag} \{ \lambda_{k1}, \ldots, \lambda_{kN_r} \} \) is a diagonal matrix of the eigenvalues. We assume that \( \mathbf{Q}_k \) can also be decomposed as \( \mathbf{Q}_k = \mathbf{U}_k \Gamma_k \mathbf{U}_k^H \), where \( \Gamma_k = \text{diag} \{ \gamma_{k1}, \ldots, \gamma_{kN_r} \} \) are diagonal matrices of non-negative singular values of \( \mathbf{Q}_k \). Further, when the solutions satisfy the KKT conditions, this assumption leads to the global optimality. Hence, the transmit precoding matrix \( \mathbf{S}_k \) can be expressed as

\[
\mathbf{S}_k = \mathbf{U}_k \left[ (\mu \mathbf{I} + \Gamma_k)^{-1} - \Lambda_k^{-1} \right] \mathbf{U}_k^H
\]

for \( k = 1, \ldots, K \). The positive semidefiniteness of \( \mathbf{S}_k \) implies

\[
\lambda_{ki} \geq \mu + \gamma_{ki}, \quad k = 1, \ldots, K, \quad i = 1, \ldots, N_r.
\]

(15)

Substituting this into KKT conditions requires that

\[
\text{tr} \{ \mathbf{S}_k \mathbf{Q}_k \} = \text{tr} \left\{ \left[ (\mu \mathbf{I} + \Gamma_k)^{-1} - \Lambda_k^{-1} \right] \Gamma_k \right\} = 0
\]

(16)

Since each of the diagonal matrices in the trace contains non-negative elements on its diagonal, therefore it can be concluded that

\[
\gamma_{ki} (\gamma_{ki} + \mu - \lambda_{ki}) = 0, \quad k = 1, \ldots, K, \quad i = 1, \ldots, N_r
\]

(17)

which means that \( \gamma_{ki} \) is either zero or \( \mu - \lambda_{ki} \). On the other hand, \( \gamma_{ki} \) is non-negative. Therefore, \( \mathbf{S}_k \) can be described as

\[
\mathbf{S}_k = \mathbf{U}_k \text{diag} \left[ \left( \frac{1}{\mu} - \frac{1}{\lambda_{k1}} \right)_+, \ldots, \left( \frac{1}{\mu} - \frac{1}{\lambda_{kN_r}} \right)_+ \right] \mathbf{U}_k^H
\]

(18)

where \((\cdot)_+ = \max(0, \cdot)\). The average total power constraint requires that

\[
\sum_{k=1}^{K} \sum_{i=1}^{N_r} \left( \frac{1}{\mu} - \frac{1}{\lambda_{ki}} \right)_+ = P
\]

(19)

which is an eigenvalue water-filling used in [9]. Thus the procedure is eigenvalue water-fill up to a fixed level. The above discussion demonstrates that this eigenvalue waterfilling procedure is indeed the optimal multiuser ZF precoding. This is concluded from the fact that the problem involves convex optimization and the optimal solutions must satisfy the KKT conditions in (12). The result in this section is indeed an extension of results in [1, Section IV] to the MIMO BC with multiple receive antennas (i.e. \( N_r > 1 \)).

B. Fairness

QoS requirements of users specify a fairness metric (minimum user rate) which has to be maximized under a power constraint. The fairness criterion yields the following optimization problem:

\[
\begin{align*}
\text{maximize} & \quad r \\
\text{subject to} & \quad r \leq \log |\mathbf{I} + \mathbf{G}_k \mathbf{S}_k \mathbf{G}_k^H|, \quad k = 1, \ldots, K \\
& \quad \sum_{k=1}^{K} \text{tr} \{ \mathbf{S}_k \} \leq P \\
& \quad \mathbf{S}_k \succeq 0, \quad k = 1, \ldots, K.
\end{align*}
\]

(20)

where the maximization of the fairness metric \( r \) is over a set of positive semidefinite matrices \( \mathbf{S}_1, \ldots, \mathbf{S}_K \). Thus, in this problem the objective is to design a ZF precoder so as to maximize the fairness metric in the network subject to the average total power constraint. By maximizing fairness metric, we mean maximizing the minimum rate over all users. The objective function to be maximized is concave and the constraints are affine. Thus, the problem is a convex optimization problem [12]. Therefore, finding the Lagrangian function and consequently solving the KKT conditions gives the optimal precoder structure.

The main idea to solve the convex optimization problem (20) is dual decomposition technique discussed in [16]. In this technique the problem is solved iteratively by dividing it into a number of problems for each of the users. However, the dual decomposition technique in [16], [17] is used to maximize the sum capacity of MIMO Gaussian vector channel while in this section the similar technique is employed to find the optimal ZF precoders. We begin by introducing new scalar variables \( p_k, k = 1, \ldots, K \). Therefore, the optimization problem (20) can be rewritten as

\[
\begin{align*}
\text{maximize} & \quad r \\
\text{subject to} & \quad r \leq \log |\mathbf{I} + \mathbf{G}_k \mathbf{S}_k \mathbf{G}_k^H|, \quad k = 1, \ldots, K \\
& \quad \text{tr} \{ \mathbf{S}_k \} \leq p_k, \quad k = 1, \ldots, K \\
& \quad \mathbf{S}_k \succeq 0, \quad k = 1, \ldots, K \\
& \quad \sum_{k=1}^{K} p_k \leq P.
\end{align*}
\]

(21)

Therefore, in this optimization problem the objective functions and power constraints are separated and the only coupled constraint is \( \sum_{k=1}^{K} p_k \leq P \). Considering this constraint the Lagrangian of the problem can be expressed as

\[
\mathcal{L} \left\{ \{ \mathbf{S}_k \}, \{ p_k \}, \{ \mu_k \}, v \right\} = \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad
\]

\[
\quad r - \sum_{k=1}^{K} \mu_k (r - \log |\mathbf{I} + \mathbf{G}_k \mathbf{S}_k \mathbf{G}_k^H|) - v \left( \sum_{k=1}^{K} p_k - P \right).
\]

Thus, the dual objective function can be described as

\[
\mathcal{G} \left( v, \{ \mu_k \} \right) = \max_{\{ \mathbf{S}_k \} \in \{ \mathbf{S} \}, \{ p_k \} \in \{ p \}} \mathcal{L} \left( \{ \mathbf{S}_k \}, \{ p_k \}, \{ \mu_k \}, v \right).
\]

(22)
The dual problem can be established as [16]

\[
\begin{align*}
\text{maximize} & \quad r - v \left( \sum_{k=1}^{K} p_k - P \right) \\
\text{subject to} & \quad r \leq \log \left| I + G_k S_k G_k^H \right|, \quad k = 1, \ldots, K \\
& \quad \text{tr} \left( S_k \right) \leq p_k, \quad k = 1, \ldots, K \\
& \quad S_k \succeq 0, \quad k = 1, \ldots, K
\end{align*}
\] (23)

where the maximization is over \(v, \) \(v \geq 0\). Notice that the above optimization problem is indeed a decomposition of the primal problem to a number of subproblems for each of the users. Therefore the problem can be solved iteratively by solving for each user. However, these subproblems are linked together by the sum power constraint \(\sum_{k=1}^{K} p_k \leq P\). At each iteration a pair of \((S_k, p_k)\) is optimized. The iterative algorithm must converge because the objective function is nondecreasing with each iteration. The problem is convex and optimal solution satisfies the KKT conditions. Thus, the fixed values of optimization are the global optimal values.

At each iteration the optimal pair \((S_k, p_k)\) must meet the KKT requirements and thus it is necessary to derive the Lagrangian function for each pair \((S_k, p_k)\). The Lagrangian function can be expressed as

\[
\mathcal{L}(S_k, p_k, u_k, \mu_k, Q_k) = r - \mu_k \left( r - \log \left| I + G_k S_k G_k^H \right| \right) - v \left( \sum_{k=1}^{K} p_k - P \right) + \text{tr} \left\{ Q_k S_k \right\} - u_k \left( \text{tr} \left\{ S_k \right\} - p_k \right)
\]

\[
\mu_k \geq 0, \quad Q_k \geq 0, \quad k = 1, \ldots, K
\] (24)

where positive semidefinite matrices \(Q_k\) are slack variables associated with the positive semidefinite constraint of \(S_k\), \(\mu_k\) are real non-negative values and dual variables \(u_k\) are considered for the power constraint \(\text{tr} \left\{ S_k \right\} \leq p_k\). The KKT conditions can be obtained by differentiating this Lagrangian function and complementary slackness conditions [12]. Since the pair \((S_k, p_k)\) must be optimized in each iteration, the differentiation of the Lagrangian function \(\partial \mathcal{L}_k / \partial S_k = 0\) and \(\partial \mathcal{L}_k / \partial p_k = 0\) gives

\[
G_k^H \left( G_k S_k G_k^H + I \right)^{-1} G_k = \frac{u_k}{\mu_k} I + \frac{1}{\mu_k} Q_k
\]

\[
u = \frac{u_k}{\mu_k}, \quad \text{tr} \left\{ S_k \right\} = p_k.
\] (25)

The first KKT condition is similar to the water-filling condition for a Gaussian vector channel [15]. Similarly to the previous section, the optimal \(S_k\) is given by

\[
S_k = U_k \text{diag} \left[ \left( \frac{\mu_k}{v} - \frac{1}{\lambda_{k1}} \right)_+, \ldots, \left( \frac{\mu_k}{v} - \frac{1}{\lambda_{kN_r}} \right)_+ \right] U_k^H
\] (26)

The optimal \(p_k\) is then

\[
p_k = \sum_{i=1}^{N_r} \left( \frac{\mu_k}{v} - \frac{1}{\lambda_{ki}} \right)_+.
\] (27)

Introducing new variable \(\eta_k = \mu_k/v\), the optimal \(S_k\) can be determined by \(\eta_k\). The rate of user \(k\) is given by [2]

\[
R_k = \sum_{i=1}^{N_r} \left[ \log \left( \eta_k \lambda_{ki} \right) \right]_+.
\] (28)

The algorithm works as follows. We start with a very low, easily achievable target rate \(r\), and then initialize values of \(\eta_k\) to zero. Next, the values of \(\eta_k\) will be updated to achieve the target fairness, i.e. \(R_k = r\). All the pairs \((S_k, p_k)\) will be updated as defined in (26) and (27). In the second iteration, the value of the target fairness metric \(r\) will be increased. Again, we increase the values of \(\eta_k\) to meet the fairness target requirement. The same procedure will be followed for the higher target fairness and with greater accuracy, until the sum power constraint is violated \(\sum_{k=1}^{K} p_k \geq P\). Since the values of \(r\) and \(\eta_k\) are increasing and they are bounded, thus the algorithm converges. Therefore, the proposed algorithm can be summarized as

\textbf{Algorithm:} Iterative computation of the optimal ZF precoders to maximize the fairness metric under the total sum power constraint:

1) Initialize \(r, \eta_k = 0, k = 1, \ldots, K\)
2) For \(k = 1, \ldots, K\), increase \(\eta_k\) till \(R_k = r\)
3) Update \((S_k, p_k)\) using (26) and (27) with the above values of \(\eta_k\), for \(k = 1, \ldots, K\)
4) If \(P > \sum_{k=1}^{K} p_k\), then increase \(r\) and goto Step 2.

Notice that \(\eta_k\) is a one-to-one function of \(R_k\), and thus there is a specific solution for \(\eta_k\) when \(R_k = r\).

\textbf{IV. NUMERICAL RESULTS}

The results of numerical evaluation of achievable rates and fairness involving a Monte Carlo approach are discussed in this section. The sum rates are compared under throughput and fairness criteria. The number of transmit antennas \(N_t\) is assumed to be \(KN_r\). In Fig. 1, sum rates are compared for total sum power constraint \(P = 0\) dB and \(P = 10\) dB and as a function of the number of receive antennas \(N_r\). The system which is simulated includes \(K = 20\) users and the fairness metric is multiplied by the number of users \(K\) to obtain the sum rate. The simulation results are averaged over high number of channel matrices generated. The elements of the channel matrices \(H_k, k = 1, \ldots, K\) are randomly generated as independent, zero mean and unit variance circularly symmetric complex Gaussian random variables. In Fig. 2, we consider the maximization of throughput and fairness for different power limits \(P\). The results for different values of \(N_r\) are considered. It is shown that the difference between throughput and fairness is almost independent of the value of \(P\) as it just scales the resulting power. The system has been evaluated as a function of the number of users served in Fig. 3.

\textbf{V. CONCLUSIONS}

We have considered the optimization of multiuser ZF precoding for MIMO Gaussian broadcast channels where the
base station and the users are both equipped with multiple antennas. The objectives are maximum throughput ($\sum_k R_k$) and maximal fairness ($\min_k R_k$) subject to the total sum power constraint, where $R_k$ is the rate for user $k$. The structure of the optimal ZF precoder is presented. The approach in this paper proves that the conventional block diagonalization scheme in [9] is indeed optimal multiuser ZF precoding under total sum power constraint. The main contribution of this paper can be summarized as the extension of the result in [1] to the system with multiple antennas per user and subject to the total sum power constraint. Future work should focus on the systems under individual per-antenna power constraints. Also, the results in this paper can be extended to non-linear precoding schemes such as ZF-DPC [2] or Tomlinson-Harashima [18].

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