Robust State Observer Design
with Application to an
Industrial Boiler System

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Abstract

A novel approach to the problem of robust state estimation in the presence of model uncertainty as well as plant disturbance and sensor noise is considered. Two new observer structures are discussed and shown to have certain advantages over the classical state space observer when studying robustness issues. The results are used to design an observer for a real industrial boiler system.

I Introduction

State space realizations are routinely used in science and engineering to model linear and nonlinear dynamical systems. Examples include state-feedback control (see, for example, [1], [3], [5], [15], or [26]), fault detection ([28], [6],[11]), and system monitoring. The state, however, is usually not available since rarely one can have a sensor on every state variable, and some form of reconstruction from the available measured output is required. In this case, an observer can be constructed using the mathematical model of the plant to obtain an estimate $\hat{x}$ of the true state $x$. This estimate can then be used as a substitute for $x$, as required.

Depending upon the characteristics of the plant model, this problem has a well established solution. However, even in the simplest case of finite dimensional linear time-invariant plant models, “true” complex systems encountered in industrial facilities invariably present a challenge. Indeed, nonlinear behavior, plant disturbance, sensor noise, and modeling errors will invariably lead to deviations from the true state unless due precautions are taken during the observer design. In the sequel we will refer to the robust observer design problem, defined loosely as the problem of designing observers which are insensitive to these effects.

The classical approach to state reconstruction consists of studying a free or unforced system subject to nonzero initial conditions, and design the observer gain to stabilize the error dynamics $\ddot{x} = \dot{x} - x$, thus achieving asymptotic convergence. See for example any of the standard reference on linear systems [1], [3], [5], [15], or [26]. Modeling errors are very difficult to incorporate in this setting. Much attention has been focused on this problem in recent years ([13], [31], [23], [20]), particularly in connection with the problem of residual generation in observer-based fault detections ([22], [8], [9], [14]). There are many ways of representing model uncertainties, thus leading to various approaches and results. References [31] and [23] consider linear time-invariant systems influenced by white noise, and represent model uncertainty as a perturbation in the state matrix $A$. Reference [13] consider a similar case in a deterministic setting. A rather different approach, very popular in the fault detection literature, consists of modeling uncertainties and unknown external excitations as unknown inputs ([22], [8], [14]). See also the recent survey [11] for an extensive account of available methods for robust state observation along with applications in fault detections.

In this paper our interest is in the application of state observers to monitor important (unmeasurable) variables in an industrial boiler system. To this end we first consider the robust observer design problem. We view systems as mappings from input-to-state and look for small observation errors in the presence of a very general class of model uncertainties as well as persistent excitation and measurement noise. We view an observer as a filter designed so that the error
dynamics \((\dot{x} - x)\) has some desirable frequency domain characteristics. To add additional degrees of freedom in our design we consider 2 alternative observer structures denoted dynamical observer and input-output observer, respectively. The dynamical observer is introduced first. We show that this new structure retains all of the characteristics of the popular (constant gain) state observers of the classical literature and in addition provides additional degrees of freedom in the design and thus can result in a significant performance improvement in the final design. It will also be shown that this structure contains the classical state space observer as a special case. The input-output observer structure is considered next. This observer is not new and was previously introduced by the first author in reference [20]. To give a complete coverage of the robust estimation problem, we summarize the essential features of this structure and highlight the differences with the dynamical observer of the previous section. In both cases, the synthesis of these two observers is discussed in detail. Our formulation is realistic, i.e., the class of perturbations considered is unstructured and accounts for effects such as parameter uncertainties, neglected high order dynamics, etc., that are always present when dealing with real physical systems such as those found in industry. Thus, our primary goal is to study under what conditions and to what extent observer design can compensate for the effect of this type of modeling errors.

Nonlinear systems are considered next. We notice that while observing the state of a linear time-invariant state space realization has a well established solution, nonlinear plants present a challenge and no universal solution to the (nonlinear) observer problem is known. We introduce a simple approach, based on the linear observers previously obtained, utilizing the concept of observer scheduling. The idea is to introduce an observer bank in which each observer is tuned to work near an specific operating point. A switching mechanism is then provided to select the particular observer as a source of the estimated state value.

In the second part of the paper we consider observer design of a utility boiler that is part of a cogeneration system owned and operated by Syncrude Canada Ltd. in Mildread Lake, Alberta. Linear time-invariant models of this boiler, derived about a given operating condition, proved to be accurate only within a neighborhood of those conditions. To account for this nonlinear behavior we introduce a technique, in the sequel referred to as observer scheduling, consistent of a bank of observers, each designed to operate under certain operating conditions, plus a scheduling mechanism that enables only one of those outputs at a given time. The result is a nonlinear observer, design based on a number of LTI designs. This technique is inspired by the popular gain scheduling technique used in control systems as well as in similar schemes commonly used to detect sensor and actuator faults in control systems.

The rest of the paper is organized as follows: Section II introduces the notation. In Section III and IV we introduce the dynamical observer and the input-output observer, respectively. In Section V we solve the synthesis problem for the observers of the previous sections and in Section VI we consider nonlinear systems and introduce the observer switching technique. In Section VII we introduce the model of an industrial boiler to be considered. In Section VIII we present the results of our design. Finally in Section IX we present the nonlinear extension of the previous results for the same boiler system. Section X contains conclusions and final remarks.

II Preliminaries and Notation

In the sequel \(\mathbb{R}\) represents the field of real numbers, \(\mathbb{R}^n\) the set of \(n\)-tuples of real numbers, and \(\mathbb{R}^{n \times p}\) the set of real matrices of order \(n\) by \(p\). We will consider a system \(\Sigma\), defined via a minimal
state space realization of the form
\[
\dot{x} = Ax + Bu, \quad A \in \mathbb{R}^{n \times n}, \quad B \in \mathbb{R}^{n \times p} \\
y = Cx + Du, \quad C \in \mathbb{R}^{q \times n}, \quad D \in \mathbb{R}^{q \times p}
\] (1)
(2)

To simplify our notation throughout most of our presentation we will assume that \( D = 0 \) in equation (2). In other words, we will replace (2) with
\[
y = Cx, \quad C \in \mathbb{R}^{q \times n}
\] (3)

Alternatively, \( \Sigma \) will be seen as a mapping from input-to-state and represented in transfer functions form as follows:
\[
\begin{align*}
P_0(s) &= (sI - A)^{-1}B \\
H_0(s) &= CP_0(s) = C(sI - A)^{-1}B
\end{align*}
\] (4)
(5)

These two mappings will be represented using a black box model, as shown in Figure 1. A key role in this representation is played by the state vector \( x \). The standard approach used to estimate the state, consists of employing the following structure:
\[
\dot{\hat{x}} = A\hat{x} + Bu + L(y - Cx) 
\] (6)

Equation (6) defines the usual constant gain state observer. It is well known that the observer error dynamics is given by,
\[
\dot{\hat{x}} - \dot{x} = (A - LC) \hat{x}
\]
thus (6) is sometimes called an asymptotic observer, since \( \hat{x} \) asymptotically converges to the true state \( x \), provided that the eigenvalues of the matrix \( A - LC \) lie in the left-half of the complex plane. This analysis, however, is affected by problems neglected in our analysis: first it ignores sensor noise and plant disturbance. Second, it assumes perfect modeling.

To account for these effects we now introduce the following notation. Let
\[
\begin{align*}
P(s) &= (sI - A)^{-1}B + \Delta = P_0(s) + \Delta \\
H(s) &= CP(s) = CP_0(s) + C\Delta = H_0(s) + C\Delta
\end{align*}
\] (7)
(8)
that can be represented as shown in Figure 2.

Thus the difference between the true state $x$ and that predicted by the model $P_0$ is assumed to be within the bound $\Delta$. The perturbation $\Delta$ is unknown, but satisfies a frequency dependent bound of the form

$$|\Delta(j\omega)| \leq l(j\omega) \quad \forall \omega \in \mathbb{R}$$

We will also assume the existence of exogenous disturbance $d$ and sensor noise $n$ satisfying:

$$\int_{-\infty}^{\infty} |d(t)|^2 dt < \infty$$

$$\int_{-\infty}^{\infty} |d(j\omega)|^2 |\tilde{W}(j\omega)|^2 d\omega < 1$$

The space of all measurable functions $d$ satisfying (10) is the so-called Lebesgue space $L_2$. Thus our assumption is that $d \in L_2$ and satisfies the inequality (11) for some function $\tilde{W}(\cdot)$, where $\tilde{W}(s) \in \mathbb{R}[s]^{q \times q}$ is analytic and has no zeros in the right half plane. Similar assumptions apply to the noise $n$. These assumptions are typical in the robust control literature and the $H_\infty$ framework.

## III Dynamical Observer

In this section we introduce the notion of dynamical observer. Define:

$$\dot{x} = A\hat{x} + Bu + C_1 x_1$$

$$\dot{x}_1 = A_1 x_1 + B_1 (y - C\hat{x})$$

where

- $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times p}$, and $C \in \mathbb{R}^{q \times n}$ are the state equations of the dynamical system $\Sigma$ defined by equations (1) and (2).
• $A_1 \in \mathbb{R}^{n_1 \times n_1}$, $B_1 \in \mathbb{R}^{n_1 \times q}$, and $C_1 \in \mathbb{R}^{p \times n_1}$ constitute a “filter” or “compensator” to be designed.

In the sequel we will refer to the system (12)-(13) as a dynamical observer. Before studying the properties of this structure we notice that the dynamical observer contains the classical state space observer as a special case. To see this notice that selecting the dynamical filter of equation (13) as a memoryless (or constant gain) filter given by

\[
\begin{align*}
A_1 &= I \\
x_1 &= -B_1 (y - C \bar{x})
\end{align*}
\]

we obtain $\dot{x}_1 = 0$ and thus the observer of equations (12)-(13) results in

\[
\begin{align*}
\dot{x} &= A\hat{x} + Bu + C_1 x_1 \\
x_1 &= -B_1 (y - C \bar{x})
\end{align*}
\]

which is identical to the state space observer (6) with $L = -C_1 B_1$.

We now look at the dynamical observer in more detail. To begin our analysis we re-write equations (12)-(13) as follows:

- **1st Observer Block (the plant):**

  \[
  \dot{\hat{x}} = A\hat{x} + Bu + z \\
  \hat{y} = C\bar{x}
  \]

- **2nd Observer Block (observer filter):**

  \[
  \dot{x}_1 = A_1 x_1 + B_1 (y - \hat{y}) \\
  z = C_1 x_1
  \]

Consider now the observer error $\bar{x} = \hat{x} - x$. We have:

\[
\begin{align*}
\dot{\bar{x}} &= \dot{\hat{x}} - \dot{x} = (A\hat{x} + Bu + z) - (Ax + Bu) \\
&= A(\bar{x} - x) + C_1 x_1
\end{align*}
\]  

(14)

also

\[
\dot{x}_1 = A_1 x_1 - B_1 C(\bar{x} - x)
\]  

(15)

Defining

\[
\bar{x} = \begin{bmatrix} \bar{x} \\ x_1 \end{bmatrix}
\]

equations (14) and (15) can be rewritten as follows:

\[
\dot{\bar{x}} = \begin{bmatrix} \hat{\bar{x}} \\ \dot{x}_1 \end{bmatrix} = \begin{bmatrix} A & C_1 \\ -B_1 C & A_1 \end{bmatrix} \bar{x} \triangleq A\bar{x}
\]  

(16)

We can now state the following theorem.
Theorem 1: Given a dynamical system of the form (1) and (2) and a dynamical observer of equations (12)-(13), the state \( \hat{x} \) asymptotically converges to the true state \( x \) if and only if all of the eigenvalues of the matrix \( \bar{A} \) lie in the open left half-plane.

Proof: Follows immediately from (16).

We now introduce the following notation:

\[
\Phi_0(s) = (sI - A)^{-1} \\
\hat{x}(s) = \Phi_0(s)\xi = \Phi_0(s)[Bu + z(s)] \\
\tilde{y} = C\hat{x}(s) \\
x_1(s) = G(s)e(s) = (sI - A_1)^{-1}B_1e(s) \\
e(s) = y(s) - \tilde{y}(s) \\
z(s) = C_1x_1(s)
\] (17)-(22)

Equations (17)-(22) can be represented as shown in Figure 3, where we have included the plant \( P \), containing an uncertainty block \( \Delta \) as well as noise \( n \) and disturbance \( d \).

Figure 3 is useful to visualize the dynamical observer structure. Here the 1st observer block consists of the plant model. The correction term \( z \) is the output of the 2nd observer block \( C_1Ge = C_1G(y - \tilde{y}) \), consistent of a dynamical system instead of the constant gain \( L \) used in the observer.
(6). It will be shown that the additional degrees of freedom in allowing $G$ to be a dynamical system can be used to improved the design.

We now define the feedback system $S_1$ as follows (see Figure 3):

$$S_1: \begin{cases} e = v_1 - C\Phi_0 \xi \\ \xi = v_2 + Ge \end{cases}$$

Clearly, $S_1$ corresponds to the feedback loop that defines the dynamical observer structure (12)-(13).

**Theorem 2** Given a system of the form (1) and (2) and a dynamical observer of equations (12)-(13), the state $\hat{x}$ asymptotically converges to the state $x$ if and only if a state space realization of the feedback system $S_1$ has an exponentially stable equilibrium point at the origin.

**Proof:** We have

- **1st Observer Block (the plant):**
  $$\begin{align*}
  \dot{x} &= A\hat{x} + I\xi \\
  \hat{y} &= C\hat{x} \\
  \xi &= v_2 + z
  \end{align*}$$

- **2nd Observer Block (observer filter $G$):**
  $$\begin{align*}
  \dot{x}_1 &= A_1 x_1 + B_1 e \\
  z &= C_1 x_1 \\
  e &= v_1 - \hat{y}
  \end{align*}$$

Therefore, we have

$$\begin{align*}
  \dot{x}_1 &= A_1 x_1 + B_1(v_1 - C\hat{x}) \\
  \dot{\hat{x}} &= A\hat{x} + I(v_2 + C_1 x_1)
  \end{align*}$$
or

\[
\begin{bmatrix}
\dot{\hat{x}} \\
\dot{\hat{x}}_1
\end{bmatrix} = \begin{bmatrix}
A & C_1 \\
-B_1C & A_1
\end{bmatrix}\begin{bmatrix}
\hat{x} \\
x_1
\end{bmatrix} + \begin{bmatrix}
v_2 \\
B_1v_1
\end{bmatrix}.
\]  

(23)

Setting \( v_1 = v_2 = 0 \) equation (23) has the form \( \dot{\hat{x}} = \hat{A}\hat{x} \) and we have that the origin is an exponentially stable equilibrium point if and only if all of the eigenvalues of the matrix \( \hat{A} \) have negative real part, and the result follows from Theorem 1.

**Theorem 3** Given a system of the form (1) and (2) there exist a dynamical observer of the form (12)-(13) such that \( \hat{x} \) asymptotically converges to \( x \) if and only if \((A, C)\) is a detectable pair.

**Proof:** It follows from Theorem 2 that \( \hat{x} \) exponentially converges to \( x \) if and only if a compensator \( G \) can be constructed to stabilize the system \( S_1 \). Thus, a stable dynamical observer exists if the system \( H_0 \) is given by

\[
\begin{cases}
\dot{\hat{x}} = Ax + I\xi \\
\hat{y} = C\hat{x}
\end{cases}
\]

can be stabilized via a compensator \( G \) of the form

\[
\begin{cases}
\dot{x}_1 = A_1x_1 + B_1\hat{y} \\
\xi = C_1x_1
\end{cases}
\]

It is well known in control theory (see for example [5]) that this is possible if and only if the state space realization of the subsystem \( H_0 \) is stabilizable and detectable. More precisely, if and only if the pair \((A, I)\) is stabilizable and the pair \((A, C)\) is detectable. The result thus follows since the pair \((A, I)\), is trivially seen to be always controllable (and so stabilizable).

**IV Alternative Formulation and Remarks**

The results of the previous section are very important. According to Theorems 1-3, any compensator \( G \) in feedback with the plant model \( C\Phi_0 \), that exponentially stabilizes the feedback system of Figure 4 constitutes an observer whose state \( \hat{x} \) exponentially converges to that of the plant \( x \). The result is significant for at least 2 reasons

(i) permits tackling the observer design problem as the design of the feedback system \( S_1 \), using any of the well established techniques available for feedback design. This property will allow us to study the robust observer design in the same framework as any other problem of robust control design. We will approach the design problem in the frequency domain as an \( H_\infty \) optimization problem, thus taking full advantage of powerful results in robust control theory.

(ii) The dynamical observer structure is more general than the structure of the classical observer of equation (6) used in the literature and allows more freedom and flexibility in the design. as will be seen in Section V.

We point out that the dynamical observer can be interpreted as having a generalized Luenberger form (see reference [17]). Indeed, defining

\[
\begin{align*}
\dot{\hat{z}} &= Dz + Ey + Fu, \\
\hat{x} &= Pz + Vy, \\
TA - DT &= EC, \quad PT + VC = I_n, \quad F = TB
\end{align*}
\]
Then
\[
D = \bar{A}, \quad E = \begin{bmatrix} 0 \\ B_1 \end{bmatrix}, \quad F = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad P = [I_n \ 0], \quad V = 0, \quad T = \begin{bmatrix} I_n \\ 0 \end{bmatrix}
\]
give the dynamical observer (12)-(13). Our formulation, however, permits a more transparent view of the observer properties in terms of feedback elements. It is also aimed at tackling observer design using well known techniques in the robust control literature, as we will see. See also reference [12] for an alternative approach.

Before proceeding, we notice that the dynamical observer structure introduced in the previous section is not the only possible formulation. Indeed, a slightly modified structure, introduced in reference [20], leads to a perhaps more intuitive interpretation of the observer problem. Consider the following alternative observer structure, in the sequel referred to as an input-output observer, to distinguish it from the dynamical observer of the previous section.

\[
\begin{align*}
\hat{x}(s) &= P_0(s)[u(s) + C_1G(s)e(s)] \\
e(s) &= y(s) + n(s) + d(s) - \hat{y}(s) \\
\hat{y}(s) &= C\hat{x}
\end{align*}
\]

which can be represented as shown in Figure 5. The only change, of course, is that in this formulation the plant model \(P_0(s) = (sI - A)^{-1}B = \Phi_0(s)B\) incorporates the matrix \(B\) of the state space realization of the plant. Thus, \(CP_0(s) = H_0(s)\) (see Section 2) which is the input-output model of the plant. The observer block \(G(s)\) is the observer filter, or generalized observer gain to be designed, and has properties identical to that in the previous section. Notice that, despite dealing with input-output models, this formulation retains the coordinate dependent structure of the state space realization. Define now the feedback systems \(S_2\) as shown in Figure 6:

\[
S_2 : \begin{cases} 
  e &= v_1 - CP_0\xi \\
  \xi &= v_2 + Ge
\end{cases}
\]

As in the previous section, \(S_2\) corresponds to the feedback loop that defines the input-output observer structure (24)-(26).

Theorems 4-6 given below are analogous to Theorem 1-3 in the previous section:

**Theorem 4** : Given a dynamical system of the form (1) and (2) and an input-output observer of equations (24)-(26), the state \(\hat{x}\) asymptotically converges to the true state \(x\) if and only if all of the eigenvalues of the matrix

\[
\bar{A} = \begin{bmatrix} A & BC_1 \\ -B_1C & A_1 \end{bmatrix}
\]

lie in the open left half-plane.

**Theorem 5** Given a system of the form (1) and (2) and an input-output observer of equations (24)-(26), the state \(\hat{x}\) asymptotically converges to the state \(x\) if and only if a state space realization of the feedback system \(S_2\) has an exponentially stable equilibrium point at the origin.

**Theorem 6** Given a system of the form (1) and (2) there exist an input-output observer of the form (24)-(26), such that \(\hat{x}\) asymptotically converges to \(x\) if and only if \((A, B)\) and \((A, C)\) constitute a stabilizable and detectable pair, respectively.
Figure 5: Input-Output Observer.

Figure 6: The system $S_2$. 
Proof: The proof is identical to that of Theorem 3, with the exception that in this case the subsystem $H_0$ is given by
\[
\begin{align*}
\dot{x} &= Ax + B\xi \\
\hat{y} &= C\hat{x}
\end{align*}
\]
Thus, in this case it is possible to stabilize $H_0$ if and only if the pair $(A, B)$ is stabilizable and the pair $(A, C)$ is detectable. This completes the proof.

V Robust Observer Design

In this section we study the robust observer design problem. More explicitly, we pursue the design of observers that are insensitive to the effect of noise, disturbances, as well as modeling error. To this end we begin by deriving the equations that determine the observer error.

Case 1: Dynamical Observer From Figure 3, we have that the state $x$ is given by:
\[
x = \Phi_0Bu + \Delta u
\]
and the reconstructed state $\hat{x}$ is
\[
\hat{x} = \Phi_0[Bu + C_1Ge]
\]
Thus,
\[
\hat{x} = \hat{x} - x = -\Delta u + \Phi_0C_1Ge
\] (28)
In equation (28), the error $e$ represents the “output error prediction,” given by $e = y_m - \hat{y}$, where $y_m$ is the “measured output.” We have:
\[
y_m = C\Phi_0Bu + C\Delta u + d + n
\] (29)
and
\[
\hat{y} = C\Phi_0[Bu + C_1Ge] \\
= C\Phi_0[Bu + C_1G(y_m - \hat{y})]
\]
Thus
\[
\hat{y} = (I + C\Phi_0C_1G)^{-1}[C\Phi_0Bu + C\Phi_0C_1Gy_m]
\] (30)
We can now the output error prediction. We have that
\[
e = y_m - \hat{y} \\
= y_m - (I + C\Phi_0C_1G)^{-1}[C\Phi_0Bu + C\Phi_0C_1Gy_m]
\]
\[
e = (I + C\Phi_0C_1G)^{-1}[y_m - C\Phi_0Bu]
\] (31)
and taking account of (29) we have that
\[
e = (I + C\Phi_0C_1G)^{-1}[C\Delta u + n + d]
\] (32)
Finally, substituting (32) in (28) we obtain
\[
\begin{align*}
\dot{x} &= -\Delta u + \Phi_0C_1G(I + C\Phi_0C_1G)^{-1}[C\Delta u + n + d] \\
&= -\Delta u + \Phi_0C_1G(I + C\Phi_0C_1G)^{-1}C\Delta u + \Phi_0C_1G(I + C\Phi_0C_1G)^{-1}(n + d).
\end{align*}
\]
A little algebra shows that $\tilde{x}_1$ and $\tilde{x}_2$ can be re-written as follows

$$\tilde{x}_1 = -(I + \Phi_0 C_1 GC)^{-1} \Delta u$$

$$\tilde{x}_2 = (I + \Phi_0 C_1 GC)^{-1} \Phi_0 C_1 G(n + d).$$

Thus, finally we obtain

$$\tilde{x} = (I + \Phi_0 C_1 GC)^{-1}[-\Delta u + \Phi_0 C_1 G(n + d)] \tag{33}$$

Equations (32) and (33) can be used to discuss the properties of the observer as well as discuss possible design strategies and constraints. Small estimation errors can be achieved by finding a compensator $G(s)$ that (a) internally stabilizes the observer, and (b) minimizes the error in equation (33) with respect to uncertainties of the form (9), and noise and plant disturbances of known frequency content of the form (11). In this context, observer design can be approached as any other problem of feedback design, and therefore using any of the standard design techniques available for control design such as $H_\infty$ ([32]), $L_1$ ([7]), Model Predictive Control ([18]-[4]) (in any of its many variations), etc. For example, neglecting modeling errors, robust estimation with respect to noise and disturbance signals in $L_2$ can be obtained by minimizing:

$$\|\Phi\|_\infty = \| W(I + \Phi_0 C_1 GC)^{-1} \Phi_0 C_1 G \|_\infty \tag{34}$$

where $W$ is a weighting function that is large in the frequency range where noise and disturbance need to be attenuated. Speed of response can also be directly addressed into the design by extending the system bandwidth. Alternatively, the design can be carried out considering the “output-error equation,” (32) with similar remarks.

Case 2: Input-Output Observer

If the input-output observer structure is used, then an analogous analysis shows that

$$e = y_m - \tilde{y} = (I + CP_0 C_1 G)^{-1}[C \Delta u + (n + d)], \tag{35}$$

and

$$\tilde{x} = (I + P_0 C_1 GC)^{-1}[-\Delta u + P_0 C_1 G(n + d)] \tag{36}$$

the synthesis problem is analogous to the previous case.

We notice here the possible advantage of both of these observers over the classical state observers. Alternative results for the observer (6) can be obtained by replacing the filter $G$ with the constant observer gain $L = B_1 C_1$. The ability of the resulting observer to reduce the effects of noise, disturbances and modeling errors is thus hampered by a much more limited freedom available to the designer to shape the filter $(I + \Phi_0 C_1 GC)^{-1}$. To see this consider observer design for a state space realization of order 2. To achieve robust reconstruction of the state it is useful to think of the observer as a frequency selective filter designed to shape the filter $(I + \Phi_0 C_1 GC)^{-1}$. Using the observer (6) will result in a filter of the same order of the state space realization (2nd order in this case) and thus will only achieve relatively poor frequency attenuation properties. Both observers discussed here can have arbitrarily large order and can be designed to minimize (34) using a suitably selected weighting function $W$. Notice also that the dynamical observer contains the observer (6) as a special case. Therefore, the observer (6) can never outperform the dynamical observer considered here.
VI Nonlinear Extension: Observer Scheduling

In this section we consider a simple extension of the results of the previous sections to the case of nonlinear plants. We introduce a simple approach, based on the linear observers of the previous section, utilizing the concept of observer scheduling. Inspired by the popular theory of gain scheduling, the idea, depicted in Figure 7 using 3 observers, is to introduce an observer bank in which each observer is tuned to work near a specific operating point. A switching mechanism is then provided to select the particular observer as a source of the estimated state value. This switching mechanism is based on the measured values of a variable (or set of variables) which should have a strong correlation with the internal state. Despite the similarities with gain scheduling, there is a fundamental difference between the observer scheduling discussed here and the gain scheduling technique used in control systems. In control systems the main problem associated with the gain scheduling technique is closed loop stability of the resulting feedback system. Indeed, most scheduling algorithms are implemented following extensive simulations and without proper proofs of stability, simply because stability of gain scheduling systems is very difficult to assess. The observer scheduling problem discussed here, however, is not affected by this problem. Indeed, each observer in the observer bank constitutes a linear feedback filter, that has no nonlinear terms and no model uncertainty of any sort, and it is therefore stable by construction. Convergence of the observer bank to the true state is, of course, a different matter. To analyze the estimation error associated with each observer in the observer bank we consider again the system of Figure 3, but this time we assume that the uncertainty block $\Delta(s)$ is replaced with a nonlinear uncertainty term $\Delta$. No particular structure is assumed here to represent the uncertainty, but $\Delta$ is assumed to be input-output stable, i.e. for any input $u \in \mathcal{L}_2$, $\Delta u \in \mathcal{L}_2$. Moreover, we assume that some estimate of the (nonlinear) $\mathcal{L}_2$ norm associated with $\Delta$ is available. In other words, we assume that for each input $u \in \mathcal{L}_2$,

$$\|\Delta u\|_{\mathcal{L}_2} \leq \gamma(\Delta) \|u\|_{\mathcal{L}_2}$$

where $\|u\|_{\mathcal{L}_2}$ is the $\mathcal{L}_2$ norm of the input function $u$ given by

$$\|u\|_{\mathcal{L}_2} = \sqrt{\int_0^\infty |u(t)|^2 \, dt}$$

and similarly for $\|\Delta u\|_{\mathcal{L}_2}$. The term $\gamma(\Phi)$ represents the $\mathcal{L}_2$-gain of the operator $\Phi : \mathcal{L}_2 \to \mathcal{L}_2$, defined as follows:

$$\gamma(\Phi) \triangleq \frac{\|\Delta u\|_{\mathcal{L}_2}}{\|u\|_{\mathcal{L}_2}}.$$

It is well known that for a linear time-invariant system $P : \mathcal{L}_2 \to \mathcal{L}_2$, we have that

$$\gamma(P) \triangleq \frac{\|Pu\|_{\mathcal{L}_2}}{\|u\|_{\mathcal{L}_2}} = \|P\|_{\infty}.$$ 

where $\|P\|_{\infty}$ is the H-infinity norm of the transfer function $P(s)$ of the system $P$.

We point out that the exact value of the norm $\gamma(\Delta)$ is inessential since underestimating $\gamma(\Delta)$ will affect the estimation error but not the stability of the filters in the filter bank. In practice, some estimate of this norm can be obtained via computer simulation and comparison of the nonlinear plant and the linear approximation. An explicit computational procedure, based on the use of the Hamilton-Jabobi equation and the nonlinear $\mathcal{L}_2$ theory, to estimate an upper bound of $\gamma(\Delta)$ was recently obtained in reference [16]. The number of observers in the observer bank is closely
related to the norm $\gamma(\Delta)$. Consider a nonlinear plant and its linearization about an equilibrium point. The accuracy of the linear approximation deteriorates as the region of operation about the equilibrium point becomes large. Thus, in general, the bound $\gamma(\Delta)$ will be valid in a compact region about the equilibrium point and will increase as the deviations from the equilibrium point become large. Overestimating the bound of $\gamma(\Delta)$ will therefore result in a larger number of observers in the observer bank in an attempt to reduce the (expected) estimation error.

The estimation error can now be easily computed as follows: Let $\delta = (\Delta)u$. Then the observer of Figure 3 can be represented as shown in Figures 8 where $\delta$ is an unknown (exogenous) signal that satisfies:

$$
\|\delta\|_2 \leq \gamma(\Delta) \|u\|_2
$$

With this interpretation, then for each $\delta$ we can find the Laplace transform $\delta(s) = L\{\delta(t)\}$ and the estimation error analysis of Section V is still valid, with the substitution $\delta(s) = \Delta u$. Thus, denoting by $\tilde{x}^i$ to the estimation error in the $i^{th}$ observer in the observer bank, we have that

$$
\tilde{x}^i = -(I + \Phi_0 C_1 G C)^{-1} \delta + (I + \Phi_0 C_1 G)^{-1} \Phi_0 C_1 G(n + d) = \tilde{x}^i_1 + \tilde{x}^i_2
$$

The term $\tilde{x}^i_1$ contains the modeling error and is bounded by

$$
\|\tilde{x}^i_1\|_2 \leq \gamma((I + \Phi_0 C_1 G C)^{-1}) \|\delta\|_2
$$

and since

$$
\gamma((I + \Phi_0 C_1 G C)^{-1}) = \|(I + \Phi_0 C_1 G C)^{-1}\|_\infty
$$

$$
\|\delta\|_2 \leq \gamma(\Delta) \|u\|_2
$$

we have that

$$
\|\tilde{x}^i_1\|_2 \leq \|(I + \Phi_0 C_1 G C)^{-1}\|_\infty \gamma(\Delta) \|u\|_2
$$

(38)

It then follows that the same synthesis procedure outlined in Section V can be used to synthesize each observer in the observer bank.

Assume now that $q$ observers are used in the observer bank The overall estimation error due to unmodeled dynamics by the observer bank is given by

$$
\|\tilde{x}_1\|_2 = \max\{\|\tilde{x}^1_1\|_2, \|\tilde{x}^2_1\|_2, \ldots, \|\tilde{x}^q_1\|_2\}
$$

An identical analysis can be done for the input-output observer structure.

**VII Case Study: The Plant Model**

In the remaining of the paper we present an application of the observer introduced in the previous section to an industrial boiler system. More explicitly; we consider a utility boiler which is part of the power generating station for the bitumen extraction and processing plant, owned and operated by Syncrude Canada Ltd in Mildred Lake, Alberta, Canada. The plant, which supplies 280MW of electric power as well as steam to the processing plant, utilizes a complex header system for steam distribution, which includes headers at four different pressure levels (6.306, 4.24, 1.068 and 0.372 MPa).
Figure 7: Observer scheduling scheme.

Figure 8: Modified Dynamical Observer Structure.
The 6.306 MPa header receives steam from three utility-type (UB) boilers burning refinery gas, three CO-type boilers burning coker off gas and refinery gas, and two once-through steam generators (OTSG). The steam is then distributed through the header system to several steam turbines to generate electricity. The overall plant, like many similar available worldwide, is thus a rather complex, nonlinear, interconnected system. A simple diagram of the utility plant is shown in Figure 9.

![Diagram of the utility plant](image)

Figure 9: A simple diagram for the Syncrude utility plant.

Each of the 3 steam, or “utility” boilers constitute an essential component of the plant, and will be the focus of our study. The utility boilers in this plant are watertube drum boilers, each producing 94.5Kg/sec of steam. This type of boiler usually comprises two separate systems. One system is the steam-water system, which is also called the water side of the boiler. In this system preheated water from the economizer is fed into the steam drum, then flows through the downcomers into the mud drum. The mud drum distributes the water to the risers, where the water is heated to saturation conditions. The saturated steam-water mixture then re-enters the steam drum in which the steam is separated from the water and exits the steam drum into the primary and secondary superheaters. In the two superheaters, the steam is further heated, and then is fed into the 6.306 MPa header. In between the two superheaters is an attemperator which regulates the temperature of the steam exiting in the secondary superheater by mixing water at a lower temperature with the steam from the primary superheater.

The other system is the fuel-air-flue gas system, which is also called the fire side of the boiler. In this system the fuel and air are thoroughly mixed and ignited in a furnace. The resulting combustion converts the chemical energy of the fuel to thermal or heat energy. The gases resulting from the combustion, known as the flue gases, pass through the superheaters, the risers, and the downcomers, and leave the boiler. A schematic diagram of this type of boiler is shown in Figure 10 (where the arrow points out the direction of the steam-water flow).

Several boiler models have been proposed in the recent years, e.g., [24, 10, 21], and there are several simulation packages for steam plants e.g., [27]. Simulation packages are different from
control-type models in that they give a much more accurate prediction of the plant behavior, at the cost of a much higher complexity that would make them inadequate for design purposes. Syncrude Canada Inc has available a simulation package, known as SYNSIM [25]. The SYNSIM model was developed with the purpose of simulating certain upset conditions that have been sporadically detected, as well as a general tool for stability analysis. The model has been extensively tested and correlation between measurements from the true plant outputs and predictions by SYNSIM are excellent. We emphasize that while SYNSIM constitutes an invaluable analysis tool, it cannot be used directly for observer design due to the excessively large complexity of the models used in the simulation. Two different control strategies for this plant, along with simulations using SYNSIM were reported in references [29] and [30]. Reference [29] considers the fundamental problem of regulating the 6.306 MPa header to maintain the pressure and temperature at the appropriate set points, despite disturbances and modeling errors, and shows that this can be accomplished by controlling the drum level, drum pressure, and steam temperature of the utility boiler, using the $H_\infty$ theory. Reference [30] on the other hand, considers the problem of improving plant stability by including a secondary controller, referred to as a stabilizer.

For monitoring purposes, in our problem, we are interested in the following variables (i) furnace gas temperature, (ii) superheater gas temperature, and (iii) economizer gas temperature. These three variables are available in the simulation package SYNSIM, but not in the real plant.
Thus we will endeavor to construct an observer to generate an estimate of these variable. In order to proceed, it is necessary to construct a mathematical model with furnace gas temperature, superheater gas temperature, and economizer gas temperature as three of the state variables. Unfortunately, no first-principles model was found to be suitable for this application. Therefore, we decided to use a very simple, crude, and possibly affected by large modeling error, state space representation of the boiler. Our model was obtained by simplifying the internal blocks in the SYNSIM model. After several iterations and consultation with plant personnel, a single-input–single output linear time-invariant third order model was obtained. Because the original boiler is nonlinear, no linear model can cover the entire operating range without significant deterioration. Therefore, three different operating points were chosen, resulting in three different set of parameters for the linear model, each to be used near the corresponding operating point. The model obtained is a state space realization of the form:

\[
\begin{align*}
\dot{x} &= Ax + Bu & A \in \mathbb{R}^{3 \times 3}, & B \in \mathbb{R}^{3 \times 1} \\
y &= Cx + Du & C \in \mathbb{R}^{1 \times 3}, & D \in \mathbb{R}
\end{align*}
\]

where

\[
\begin{align*}
u &: \text{steam load (in KPPH)}, \\
y &: \text{drum pressure},
\end{align*}
\]

The state vector \( x = [x_1 \ x_2 \ x_3]^T \) has the following form:

\[
\begin{align*}
x_1 &: \text{furnace gas temperature}, \\
x_2 &: \text{super heater gas temperature}, \\
x_3 &: \text{economizer temperature},
\end{align*}
\]

The following operating points were selected:

1- Low-load operating point (LOP): \( u = 350 \text{ KPPH} \).

2- Middle-load operating point (MOP): \( u = 550 \text{ KPPH} \).

3- High-load operating point (HOP): \( u = 700 \text{ KPPH} \).

resulting in the state space matrices \( A_i, B_i, \text{ and } C_i, D_i, i = 1, 2, 3 \) given in the appendix. Figures 11-13 comparison the model response to that predicted by SYNSIM following a steam load increase of 4% for each of the three models. From the figures we conclude that the model is adequate although modeling error is evident, particularly in the transient response. This is of course expected given the low order used in the model approximation.

VIII Design Results

In this section we study observer design for the utility boiler of Section VII. As mentioned, observer design reduces to finding a compensator \( G \) that stabilizes the observer and has some desired characteristics. In this paper we will follow the loop-shaping \( H_\infty \) approach introduced by McFarlane and Glover [19]. The essential features of this approach can be summarized as follows:

- It can incorporate both performance and robust stability requirements.
Figure 11: Drum pressure response to step change in steam load (LOP).

Figure 12: Drum pressure response to step change in steam load (MOP).
• It is conceptually and computationally simple and retains many of the features encountered in the design of classical PI loops using frequency domain techniques. This is important since experienced engineers working in industry can incorporate their knowledge of the plant and classical control design into modern, powerful techniques.

Given a plant model $H$, the design approach consists of three steps:

1. **Loop Shaping**: Using pre- and/or post-compensators $W_2$ and $W_1$, the singular values of the original plant $H$ are shaped to give the compensated plant $\tilde{H} = W_2HW_1$ a desired open-loop shape. This step contains all of the ingredients of the classical techniques. The shaping functions $W_1$ and $W_2$ are controlled by the designer and the properties of the resulting design depend upon these functions in an essential manner.

2. **Robust Stabilization**: A feedback compensator that stabilizes the ‘shaped’ plant is found. There are infinite many filters that stabilize the shaped plant. One such controller is singled out via $H_\infty$ optimization. More explicitly, for the shaped plant $\tilde{H}$, we solve the following $H_\infty$ optimization problem:

$$
\varepsilon_{\max}^{-1} = \inf_{\hat{K}} \text{stabilizing } \begin{bmatrix}
(I + \tilde{H}\hat{K})^{-1} & (I + \tilde{H}\hat{K})^{-1}\tilde{H} \\
\hat{K}(I + \tilde{H}\hat{K})^{-1} & \hat{K}(I + \tilde{H}\hat{K})^{-1}\hat{H}
\end{bmatrix}_\infty.
$$

This $H_\infty$-optimization problem can be solved explicitly without iteration, using only two Riccati equations.
3. The final feedback compensator is obtained as $K = W_1 \tilde{K} W_2$

To simplify the design, we designed an observer to minimize the output error estimation obtained in equation (35). This approach simplifies the design because the resulting “plant” is single input-single output and indirectly minimizes also the error in the estimate of the state. To proceed with the design we considered the LOP model and define weighting functions $W_2 = 0$ and

$$W_1 = 0.1 \left[ \frac{(s^2 + 0.005s + 0.00040)}{s(s + 0.0126)} \right]$$

Notice that an integrator was included in $W_1$ to ensure convergence. Using these weightings and following a model order reduction of the resulting controller, the following filter was obtained


with

$$A_G = \begin{bmatrix} 0.0001953 & -0.004298 \\ 0.0030622 & -0.067379 \end{bmatrix}, \quad B_G = \begin{bmatrix} -0.03515 \\ 0.12661 \end{bmatrix},$$

$$C_G = \begin{bmatrix} -0.024861 \\ -0.12576 \end{bmatrix}, \quad D_G = [0.23795]$$

A plot of the observer sensitivity $(I + CP_0 G)^{-1}$ is shown in Figure 14.

To evaluate our design, the observer was implemented with SYNSIM represented the real plant. Figures 15-17 show the response provided by SYNSIM and that predicted by the observer.
following a step change of 4% on the steam load. As can be seen the observer provides an accurate estimate of the steady state, although the transient response is affected by error. This is expected given the level of model uncertainty assumed during the design. Similar results were obtained at the other 2 operating points.

IX Nonlinear Extension: Observer Scheduling

In this section we consider the fact that the actual plant is nonlinear whereas the model used in the observer design and, consequently the observer itself, is linear. To show the nonlinear effects we consider the multi-step load change shown in Figure 18. Figure 19 shows the estimate of the furnace gas temperature predicted by the observer designed using the model at the LOP (denoted Observer 3 in the Figure). It is evident from Figure 19 that the quality of the prediction by the observer quickly deteriorates as the temperature increases. We now apply the observer scheduling procedure described in Section VII. In our case the observer bank consists of only three observers, one for each of the equilibrium points identified earlier. A switching mechanism is then provided to select the particular observer as a source of estimated state value. This switching mechanism is based on the measured values of a variable (or set of variables) which has strong correlation with the internal state. The scheduling algorithm was implemented as follows: the input and output to the observer bank are the steam load and drum temperature, respectively. The steam load was found experimentally to show a strong influence on the plant states and so was chosen as the switching parameter, or scheduling variable. The switching mechanism was implemented using logic blocks provided in the Simulink library. After experimentally comparing the individual
Figure 16: Superheater gas temperature with a 4% change in steam load at t=500 sec.

Figure 17: Economizer gas temperature with a 4% change in steam load at t=500 sec.
observer behavior with the plant dynamics in the entire operating region, switching points were selected. The comparison was done employing the simulation package SYNSIM representing the true plant and the prediction by the observer. The upper and lower limits in the operating region were steam load of 116 kg/sec and 260 Kg/sec, respectively with the 2 switching points selected at steam load of 150 kg/sec and 230 kg/sec, respectively. A dead band in the switching logic was implemented to prevent chattering due to noise and disturbance between 2 adjacent points. Figures 20-22 compare the three state variables obtained from SYNSIM with those predicted by the observer bank for the multistep input function of Figure 18. As can be seen from the figures, the observer bank provides a good (in loose term) estimate of the desired variables. Of the three state variables, the economizer gas temperature shows the most discrepancies with the true values given by SYNSIM. This is because of the strong nonlinear nature of the energy exchange mechanism between the boiler fed water and the exhaust gas. Of course, a better prediction can be achieved by raising the number of observers in the observer bank.

X Conclusions and Final Remarks

In this paper we have considered the robust state observer problem. In order to make our discussion as general as possible, two observer structures were considered, namely, the dynamical observer of Section III and the input-output observer of Section IV. Both observers are “dynamical,” since both contain a filter $G$ with no fixed structure, and both provide additional degrees of freedom to the designer with respect to the classical observer structure. There are, however, certain differences between these 2 cases:
Figure 19: Furnace Gas Temperature (plant and LOP observer Prediction).

Figure 20: Furnace Gas Temperature (plant Observer Bank).
Figure 21: Super Heater Gas Temperature (plant Observer Bank).

Figure 22: Economizer Gas Temperature (plant Observer Bank).
1. The dynamical observer retains the form of the classical observer (6). The only difference with the classical case is the “generalized filter” $G$ replacing the observer gain $L$. This difference, however, can be significant in terms of the performance of the actual design in the context considered here; namely robust observation with respect to disturbances, noise and modeling errors.

2. Besides additional degrees of freedom in the design with respect to the classical observers, our formulation of both the dynamical observer and the input-output observer emphasize looking at the observer design problem purely as a problem of feedback stabilization.

3. It was shown in Section III that the state reconstructed by the dynamical observer “asymptotically” converges to the true state, provided that the state space realization of the feedback system $S_1$ is asymptotically stable. As with the observer (6), stabilization is actually possible if and only if the state space realization of the original system is detectable.

4. The input-output observer of Section IV has properties similar to the dynamical observer in terms of robustness of the reconstructed state. As with the dynamical observer, the reconstructed state asymptotically converges to the true state if and only if the state space realization of the feedback system $S_2$ is asymptotically stable. Stabilization of this feedback system is, however, possible if and only if the state space realization of the original system is both stabilizable and detectable. The strengthening of the stabilizability condition are a product of the fact that the $B$ matrix in the state space realization of the original system is “absorbed” inside the feedback loop in Figure 5. In this respect, the dynamical observer structure is always preferable to the input-output observer.

5. The main importance of the input-output observer structure is that it gives a more pragmatic interpretation of the observer problem as one of feedback stabilization of the plant model, with the proper coordinate dependent state space realization preserved. Essentially, this result suggests that any observer can be seen as composed by a system model along with a compensator that achieves stabilization, a principle that has not been emphasized in the classical literature which is heavily focused on the stabilization of the observer error dynamics via the eigenvalues of the matrix $(A - LC)$. Besides the theoretical importance of this result, this concept has potentially important implications in observer design for nonlinear plants, a problem that remains largely unsolved in the systems literature. Research along this lines is currently underway by the authors.

The results were used to design an observer for an actual industrial system, and validated using a fairly elaborated simulation package (SYNSIM). As explained earlier, SYNSIM has been tested extensively, and correlation with the actual plant are considered excellent. It is also important to notice that no comparison with actual plant measurement is possible, since there are no existing measurement of the estimated variables at the plant. Overall, the results obtained were excellent.

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References


**Appendix**

\[
A_1 = \begin{bmatrix} 0.1320 & -0.2827 & 0.66350 \\ 0.1021 & -0.2099 & 0.46570 \\ 0.0137 & -0.0280 & 0.06080 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.0683 \\ 0.0433 \\ 0.0061 \end{bmatrix}
\]

\[
C_1 = \begin{bmatrix} -11.1366 & 21.6540 & -47.1108 \end{bmatrix}, \quad D_1 = -0.4757
\]

\[
A_2 = \begin{bmatrix} -0.7927 & 1.2592 & -4.9913 \\ -0.6775 & 1.0806 & -4.3029 \\ -0.0463 & 0.0746 & -0.3015 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.0470 \\ 0.0360 \\ 0.0015 \end{bmatrix}
\]
\[ C_2 = [101.6679 \ -162.7801 \ 642.5514], \quad D_2 = -0.5804 \]

\[ A_3 = \begin{bmatrix}
-0.4723 & 0.7708 & -4.8314 \\
-0.4056 & -0.6657 & -4.2173 \\
-0.0190 & 0.0317 & -0.2056
\end{bmatrix}, \quad B_3 = \begin{bmatrix}
0.0326 \\
0.0282 \\
0.0012
\end{bmatrix} \]

\[ C_3 = [99.7865 \ -160.2198 \ 937.7032], \quad D_1 = -0.0349 \]