Brief paper

Input-to-state stabilization for nonlinear dual-rate sampled-data systems via approximate discrete-time model

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\textbf{ABSTRACT}

The problem of state feedback stabilization of nonlinear sampled-data systems is considered under the "low measurement rate" constraint. A dual-rate control scheme is proposed that utilizes a numerical integration scheme to approximately predict the current state. Given an approximate discrete-time model of a sampled nonlinear plant and given a family of controllers that stabilizes the plant model in input-to-state sense, we show that under some standard assumptions the closed loop dual-rate sampled data system is input-to-state stable in the semiglobal practical sense.

\textbf{1. Introduction}

The prevalence of digital controllers and the fact that most systems of interest in control systems are often nonlinear, motivate the area of nonlinear sampled-data control systems. Significant progress has been made in recent years (Chen & Francis, 1991; Li, Shah, & Chen, 2002; Nesic, Teel, & Kokotovic, 1999; Nesic & Teel, 2001; Nesic & Laila, 2002; Nesic & Teel, 2004, 2006; Polushin & Marquez, 2004). There are two main approaches for the design of digital controllers (see Nesic and Teel (2001)): continuous-time design (CTD) and discrete-time design (DTD). The first one involves digital implementation of a continuous-time stabilizing control law. The second approach consists of discretizing the plant model and then designing a discrete-time controller. Both of these approaches are essentially single-rate, i.e. sampling rates of the input and the measurement channels are assumed to be equal. For single-rate sampled-data systems, Nesic and Laila (2002) investigated input-to-state (ISS) property proposed by Sontag (1989, 1998). They showed that if a digital controller input-to-state stabilizes the approximate discrete-time plant model, then it would also input-to-state stabilize the exact discrete-time model. However, this result requires fast sampling which means that it may not be implementable in cases when the required sampling period is too small to be realized with the available hardware. In practical applications, hardware restrictions on input and measurement sampling rate can be essentially different. For example, the D/A converters are generally faster than the A/D converters, so the measurement sampling rate is often made slower than that of the input. In such cases, it makes sense to configure the control system so that several sample rates co-exist to achieve better performance.

In this paper, we address the problem of sampled-data stabilization of nonlinear systems under the "low measurement rate" constraint. In this case, the single rate method (Nesic & Laila, 2002) may lead to unstable closed loop performance (see example in Section 3). We address the design of dual-rate controllers containing a fast-rate model to estimate the intersample states based on the DTD method. We show that under some standard assumptions the closed loop dual-rate sampled data system is input-to-state stable in the semiglobal practical sense. We emphasize that the result is prescriptive since it can be used as a guide when designing controllers based on an approximate discrete-time plant model.

The paper is organized as follows. After a brief description of problem statement and relevant definitions and notations, the
main result is presented and illustrated via an example. Finally, the paper is closed with conclusions in Section 4.

2. Statement of the problem

The following notations will be used in the sequel. Denote $\mathbb{Z}^+$ as the sets of nonnegative integer numbers. A continuous function $\alpha : R_{\geq 0} \rightarrow R_{\geq 0}$ is said to belong to class $K$ if $\alpha(0) = 0$ and it is strictly increasing. Also, a continuous function $\beta : R_{\geq 0} \times R_{\geq 0} \rightarrow R_{\geq 0}$ is said to belong to class $KL$ if for each fixed $t \geq 0$, $\beta(s, t)$ tends to $K$ and for each fixed $x \geq 0$, $\beta(x, t)$ decreases to zero as $t \rightarrow \infty$.

The Euclidean norm of a vector is denoted as $| \cdot |$. For a function $w : R_{\geq 0} \rightarrow R^a$, we denote $w[i] := \{w(t) : t \in [iT, (i + 1)T], i \in \mathbb{Z}^+\}$ with the norm $\|w[i]\|_\infty = \text{esssup}_{t \in [iT, (i + 1)T]}|w(t)|$ and $w(i)$ is the value of $w(\cdot)$ at $t = iT$, $i \in \mathbb{Z}^+$.

Definition 1. The system $x(i + 1) = F_T(x(i), w[i])$ is said to be input-to-state stable if there exist $\beta \in KL$ and $\gamma \in K$ such that for any positive real numbers $(\Delta_1, \Delta_2)$ there exist $T^* > 0$ such that for all $(x(0))$, $\|w\|_\infty \leq \Delta_2$ and $T \in (0, T^*)$, the solution of the system satisfy $|x(t)| \leq \beta(|x(0)|, iT) + \gamma(\|w\|_\infty)$, $\forall t \in \mathbb{Z}^+$.

Consider the nonlinear continuous-time plant:

$$\dot{x}(t) = f(x(t), u(t), w(t))$$

where $x \in R^n$, $u \in R^m$ and $w \in R^p$ are respectively the state, control input and exogenous disturbance and $f$ is locally Lipschitz. Let

$$x(i + 1) = F_T^p(x(i), u(i), w[i])$$

be the exact discrete-time model of (1) with the sampling period $T > 0$. We assume that the input sampling period is equal to the sampling period of system (2), that is $\bar{T} = T$. Suppose that due to physical constraints, we cannot sample the measurement as fast as we wish. Without loss of generality, let the measurement sampling $T_m$ be a multiple of $T$, i.e. $T_m = kT$ for some integer $k \geq 1$. For this setting we consider a dual-rate control scheme and for such a scheme to work, we need a model with fast sampling rate for the nonlinear plant. We emphasize that the exact discrete-time model $F_T^p$ in (2) is unknown in most cases. Hence, let $F_T^p(x(i), u(i), w[i])$ be a family of approximate discrete-time model of (2). We assume that the approximate model with zero disturbance

$$x(i + 1) = F_T(x(i), u(i), w[i])$$

(3)

corresponding to the sampling period $T$ is available, parameterized by the modeling parameter $h > 0$. The parameter $h$, which represents the integration period of the numerical integration used to generate the family of the approximate models, may be different from the sampling period $T$. Then the idea is the following: to compensate for the lack of information about the states which are fed to the fast-rate controller, we introduce a periodic switch which connects to the actual state $x$ at times $kTl$ and connects to the estimate of the state at $t = kTl + jT, j \in \{1, 2, \ldots, l - 1\}$, which is reconstructed by the zero disturbance model with periodically updated initialization at sampling instant $i = kTl$ by the actual state. Thus the output of the switch is a fast rate signal given by

$$x_c(i + 1) = \begin{cases} x(i + 1), & i = kTl, k \in \mathbb{Z}^+ \\ F_T(x(i), u(i), w[i]) & \text{with initialization} \\ x(kl) = x(kl), & \text{otherwise.} \end{cases}$$

(4)

The controller depends on the switch output $x_c(i)$ and is implemented using a zero-order hold. We consider a dynamic feedback controller:

$$z(i + 1) = G_{r,b}(x_c(i), z(i))$$

$$u(i) = U_{T,b}(x_c(i), z(i))$$

(5)

(6)

where $z \in R^m$ and $G_{r,b}, U_{T,b}$ are zero at zero. To summarize, the dual-rate control scheme uses a fast-rate approximate model, a fast-rate controller and a periodic switch.

To shorten our notation, we denote $\tilde{x} := (x^T, z^T)^T, w[i] := w[i]$ and $F_{T,b}^p(x, w[i]) := [F_{T,b}^p(x, U_{T,b}(x, z), w[i])]$. We now introduce the following definitions.

Definition 2. The system $x(i + 1) = F_{T,b}^p(\tilde{x}(i), w[i])$ is equi-Lipschitz Lyapunov-ISS if there exist functions $\alpha_1, \alpha_2, \alpha_3 \in KL, \gamma \in K$ and for any positive real numbers $(\Delta_1, \Delta_2)$ there exist $T^* > 0$ such that for each fixed $T \in (0, T^*)$ there exists $h^* \in (0, T^*)$ such that for all $\tilde{x} \in R^{2n}, \|w\|_\infty \leq \Delta_2$ and $h \in (0, h^*)$ there exists a function $V_{T,b} : R^m \rightarrow R_{\geq 0}$ with the following properties:

$$\alpha_1(|\tilde{x}|) \leq V_{T,b}(\tilde{x}) \leq \alpha_2(|\tilde{x}|)$$

$$V_{T,b}(F_{T,b}^p(\tilde{x}, w[i])) - V_{T,b}(\tilde{x}) \geq -\alpha_3(|\tilde{x}|) + \gamma(\|w\|_\infty)$$

(7)

(8)

and, for all $\tilde{x}_1, \tilde{x}_2 \in B(\Delta_1)$, there exists $M > 0$ such that $|V_{T,b}(\tilde{x}_1) - V_{T,b}(\tilde{x}_2)| \leq M|\tilde{x}_1 - \tilde{x}_2|$.

Definition 3. $F_T^p(x, u, w)$ is said to be one-step consistent with $F_T(x, u, w)$ if for any positive real numbers $(\Delta_1, \Delta_2, \Delta_3)$ there exist a $KL$-class function $\rho(\cdot)$ and $T^* > 0$ such that for each fixed $T \in (0, T^*)$, there exists $h^* \in (0, T^*)$ such that for all $x \in B(\Delta_1), u \in B(\Delta_2), \|w\|_\infty \leq \Delta_3$ and $h \in (0, h^*)$.

Definition 4. The control law $(G_{r,b}, U_{T,b})$ is said to be uniformly locally Lipschitz if for any $\Delta_1 > 0$ there exist $L_1, L_2 > 0$ and $T^* > 0$ such that for each fixed $T \in (0, T^*)$ there exists $h^* \in (0, T^*)$ such that for all $\xi_1, \xi_2 \in B(\Delta_1)$ and $h \in (0, h^*)$. We have $|G_{r,b}(\xi_1) - G_{r,b}(\xi_2)| \leq L_1|\xi_1 - \xi_2|, |U_{T,b}(\xi_1) - U_{T,b}(\xi_2)| \leq L_2|\xi_1 - \xi_2|$, where $\xi := (x^T, z^T)^T$.

3. Main result

In this section, we state and prove our main result. We consider a dual-rate control scheme that is based on a fast numerical integration approximation to predict the interstates between samples. Our result specifies conditions which guarantee that the dual-rate controller input-to-state stabilizes the closed-loop sampled-data system in the semiglobal practical sense. More precisely, we address the stabilization problem under the following assumptions.

Assumption. (1) $F_{T,b}^p(\tilde{x}, w[i])$ is equi-Lipschitz Lyapunov-ISS. (2) $F_T^p(x, u, w)$ is one-step consistent with the exact discrete-time model $F_T(x, u, w)$. (3) The controller (5) and (6) is uniformly locally Lipschitz.

Remark 1. By Assumption 3 and the property that $U_{T,b}$ is zero at zero, we have that given positive numbers $(\Delta_1, \Delta_2, \delta)$ there exist $T^* > 0, h^* > 0$ such that for all $\xi := (x^T, z^T)^T \in B(\Delta_1)$ and $h \in (0, h^*)$, $|U_{T,b}(\xi)| \leq \Delta_2$ holds. That is, the output of the controller is locally uniformly bounded (see Khalil (1996)).

Theorem 1. Under Assumption 1–3, there exist $\beta \in KL$ and $\gamma \in KL$ such that the following holds. Given any positive real numbers $(\Delta_3, \Delta_4, \delta)$, there exists $T^* > 0$ such that for each $T \in (0, T^*)$ there exists $h^* \in (0, T^*)$ such that for all $\|x(0)\| \leq \Delta_3, \|w\|_\infty \leq \Delta_4$ and all $h \in (0, h^*)$, the exact closed loop discrete-time model (2) and (4)-(6) satisfies $|\tilde{x}(i)| \leq \beta(|\tilde{x}(0)|, iT) + \gamma(\|w\|_\infty) + \delta$.

We begin with the following claims.

Claim 1. Consider the exact closed loop discrete-time model (2) and (4)–(6). Given any strictly positive real numbers \(D_1, D_2, \epsilon\), there exists \(T_1 > 0\) such that for any fixed \(T \in (0, T_1)\) there exists \(h_1 \in (0, T]\) such that for all \(h \in (h_1, h_2]\), \(\|x(0)\| \leq D_1\) and \(\|w\| \leq D_2\), the following holds: if \(\max_{\delta=1, \ldots, \epsilon} |\tilde{x}(\delta)| \leq D_1\) for some \(k \in \{0, 1, \ldots\}\) then the exact discrete-time state of the plant satisfies: \(|x(k) - x_k| \leq T + \epsilon + \delta\|w\|_\infty\) for some \(\lambda > 0\).

**Proof.** Let \((D_1, D_2, \epsilon)\) be given. Define \(\Delta_1 = D_1 + \epsilon + 1\). By Remark 1, for given \(D_2 > 0\) there exist \(T_{11} > 0\) and \(h_1 > 0\) such that \(|x_{i+1}(x_i, z)| \leq D_2\) for all \((x_i, z)^T \in \mathbb{B}(\Delta_1)\). Let \(L > 0\) be the Lipschitz constant of function \(f\). Also, let \(\lambda > 0\) be a number such that \(e^{-\lambda T} - 1 \leq \lambda T e^{-\lambda 2} - 1 \leq \lambda T e^{-\lambda 3} - 1 \leq \epsilon\). Finally, we define \(T_1 = \min\{T_{11}, T_{12}, T_{13}, 1/2D_1, 1\} + h_1\). Suppose \(T \in (0, T_1]\), \(h \in (0, h_1]\) and \(\max_{\delta=1, \ldots, \epsilon} |\tilde{x}(\delta)| \leq D_1\) for some \(k \in \{0, 1, \ldots\}\). Then we have \(|x_k| - \tilde{x}(\delta)| \leq T + \epsilon + \delta\|w\|_\infty\) for some \(\lambda > 0\).

Claim 4. Consider the exact closed loop model (2) and (4)–(6). There exists \(\gamma \in \mathcal{K}_\infty\) such that the following holds. For any strictly positive real numbers \((C_1, C_2, v)\) with \(C_2 \geq \alpha \gamma C_1\) (\(\gamma > C_1\)) and \(\gamma > \alpha \gamma\), there exists \(T_4 > 0\) such that for each \(T \in (0, T_4]\) there exists \(h_4 \in (0, T]\) such that for all \(h \in (h_4, h_2]\), \(|\tilde{x}(0)| \leq \alpha \gamma C_1\) \(\|w\|_\infty \leq C_2\) and all \(i \in \mathbb{Z}^+\) we have

\[\|\tilde{x}(i+1) - \tilde{x}(i)\| \leq \frac{T}{4} \frac{1}{\alpha \gamma} (\|w\|_\infty).\]

**Proof.** Let positive real numbers \((C_1, C_2, v)\) be given. Define \(\epsilon_2 = \frac{1}{2} \epsilon_1\), \(\epsilon_3 = \alpha \gamma C_1\), and \(\Delta = \alpha \gamma\). From Claim 1, let \((C_1, C_2, v)\) generate \(T_{14}, h_4\) and let \(\lambda > 0\) be as in Claim 1. Let \(T_{12}, h_3\) come from Claim 2 corresponding to \((C_2, C_2, v)\), and also the following holds: \(T_{24} h_4 \leq \epsilon_1\). Using Assumption 1 and \(\tilde{x}(0) \leq \alpha \gamma C_1\) we have \(\|w\|_\infty \leq C_2\). Now suppose \(T \in (0, T_4]\), \(h \in (0, h_3]\), and \(\max_{\delta=1, \ldots, \epsilon} |\tilde{x}(\delta)| \leq D_1\). Then we have \(|\tilde{x}(k)| \leq \epsilon_2 + \delta\|w\|_\infty\) for some \(k \in \{0, 1, \ldots\}\). From now suppose this to be true. Applying Claim 2, we have \(\|\tilde{x}(i+1) - \tilde{x}(i)\| \leq \Delta\). The proof of Claim 2 is complete.
the choice of $T_{43}$ and $h_{43}$, we have $\hat{\gamma}(\|w\|_{\infty}) + \frac{v}{2} \leq \frac{v}{2} + V_5(\hat{x}(i))$. Hence we have

$$V_{T,h}(\hat{x}(i + 1)) \geq \hat{\gamma}(\|w\|_{\infty}) + \frac{v}{2} \Rightarrow V_{T,h}(\hat{x}(i)) \geq \hat{\gamma}(\|w\|_{\infty}).$$

From the definition of $\hat{\gamma}(\cdot)$, we have $\text{Term} \,(*) \leq 0$ holds. By supposition $V_{T,h}(\hat{x}(i + 1)) \geq \hat{\gamma}(\|w\|_{\infty}) + \frac{v}{2}$, we have $\hat{x}(i + 1) \geq \alpha_2^{-1}(\frac{v}{2}) = 2\epsilon_2$. Then from the choice of $T_{42}$ and $h_{42}$, we obtain $\|F_{T,h}^2(\hat{x}(i), w[i])\| \geq \|\hat{x}(i + 1) - \hat{x}(i) + 1 - F_{T,h}^2(\hat{x}(i), w[i])\| \geq 2\epsilon_2 - \epsilon_2 = \epsilon_2$. Using our choice of $T_{45}$, it follows that

$$\alpha_2(\hat{x}(i)) \geq V_{T,h}(F_{T,h}^2(\hat{x}(i), w[i])) - T\hat{\gamma}(C_u) \geq \alpha_1((F_{T,h}^2(\hat{x}(i), w[i]))) - T\hat{\gamma}(C_u) \geq \alpha_1(\frac{1}{2}\alpha_2(\epsilon_2)) = \frac{1}{2}\alpha_2(\epsilon_2),$$

which implies $|\hat{x}(i)| \geq \alpha_2^{-1}(\frac{1}{2}\alpha_2(\epsilon_2)) = \epsilon_3 \geq \alpha_2^{-1}(4\mu_1)$ and then $\text{Term} \,(*) \leq 0$ holds. Moreover, from the choice of $T_{44}$ and $h_{44}$, we have $|\hat{x}(i)| \geq \epsilon_3 \Rightarrow \frac{1}{4}\alpha_2(|\hat{x}(i)|) + T\mu_2 \leq 0$. Hence, by supposition $V_{T,h}(\hat{x}(i + 1)) \geq \hat{\gamma}(\|w\|_{\infty}) + \frac{v}{2}$, we have $V_{T,h}(\hat{x}(i + 1)) - V_{T,h}(\hat{x}(i)) \leq -\frac{1}{4}\alpha_2(|\hat{x}(i)|)$. It remains to establish our initial claim: $|\hat{x}(i)| \leq C_4$ for any $i \in Z^+$. This claim follows by induction. Indeed, clearly it holds for $i = 0$, since $|\hat{x}(0)| \leq \alpha_1(\epsilon_1) \leq C_4$ by the definition of $\epsilon_0$. Then (10) holds for $i = 0$ from the deduction above. By Claim 3, we have $|\hat{x}(i)| \leq C_4$. That is, this claim holds for $i = 1$ as well. Then $|\hat{x}(i)| \leq C_4$, $i \in Z^+$ follows by induction. The proof of Claim 4 is complete. \hspace{1cm} \Box

**Proof of Theorem 1.** Now the proof of Theorem 1 can be finalized as follows. Let $(\Delta_3, \Delta_{w3}, \delta)$ be given and let all conditions in Theorem 1 hold. Let $\hat{x} \in \kappa_{\infty}$ come from Claim 4. We define $(C_3, C_3, v, \rho)$ as: $C_3 =: \Delta_3, v > 0$ is such that $\text{sup}_{(\epsilon_0,\epsilon_1)}([\alpha_1(\gamma(s) + \rho(\epsilon_0,\epsilon_1)) - \delta, C_3] = \max(\alpha^{-1}(\gamma(s) + \rho(\epsilon_0,\epsilon_1)), \alpha^{-1}(\epsilon_1)).$ From the choice of $(\Delta_3, C_3, v)$, we have $C_3 \geq \alpha^{-1}(\gamma(s) + \rho(\epsilon_0,\epsilon_1))$ and $\|x(0)\| \leq \alpha^{-1}(\epsilon_1) \leq \alpha_1(\epsilon_1)$ using Claim 4, let $(C_3, C_3, v)$ generate $T^* > 0, h^* > 0$ such that (10) holds. Let $D = C_3$ and $d = \gamma(\|w\|_{\infty}) + \nu$, then we have $\alpha_1(D) \geq d$. With the definition of $(D, d)$, we have all conditions of Claim 3 are satisfied. Therefore for all $h \in (0, h^*)$, $\|x(0)\| \leq \Delta_3$ and $\|w\|_{\infty} \leq \Delta_w$, we have $|\hat{x}(i)| \leq \beta(\|x(0)\|, \|x(0)\|, \|w\|_{\infty}) \leq \beta(\|x(0)\|, \|w\|_{\infty}) \leq \beta(\|x(0)\|, \|w\|_{\infty}) \leq \beta(\|x(0)\|, \|w\|_{\infty}) + \delta$, where $\gamma(s) := \alpha^{-1}(\hat{x}(i)) - \hat{x}(i)$. This completes the proof of Theorem 1. \hspace{1cm} \Box

**Remark 2.** Following the proof of Theorem 1, it is easy to see that if we assume $\alpha_0(\epsilon_0) \leq 1$ to the assumption of Practical Lyapunov-SS, that is, $V_{T,h}(F_{T,h}^2(\hat{x}, w[i])) - V_{T,h}(\hat{x}) \leq -\alpha_0(\epsilon_0) + T\hat{\gamma}(\|w\|_{\infty}) + T\delta(h)$ holds, then Theorem 1 still holds.

**Example.** Consider the continuous-time plant $\hat{x}(t) = x^3(t) + u(t) + w(t)$. Let $x(i+1) = F_{T}^x(x(i), w(i), u[i])$ be the exact discrete-time model of the continuous-time plant with the sampling period $T$. Let $f_{0}(x, u, w) \equiv x + h(x^3 + u) + \int_{iT}^{(i+1)T} w(s)ds$. Then we can generate its numerically integrated approximate model $F_{T,h}^1(\cdot, \cdot, \cdot)$ by

$f_{0}(k, x, u, w) := x + h(x^3 + u) + \int_{iT}^{(i+1)T} w(s)ds$

where $\delta$, $\mu_1$ represents the integration free, $\gamma$ is the sampling period and $N = \frac{1}{T}$. Moreover, the approximate model with disturbance free, which reconstructs the missing plant states between samples, is generated by $F_{T,h}^2(\cdot, \cdot, \cdot, \cdot)$.

**4. Conclusions**

In this paper, we concentrate on the problem of sampled-data input-to-state stability of nonlinear systems under low measurement constraint. Our approach to the solution of this problem employs a dual-rate scheme based on discrete-time
controller design, since the fast sampling results may not be implemented due to hardware limitations. The main idea is to introduce a controller that includes an approximate discrete-time model of the plant. The control action depends on the state of this model which is corrected from time to time using the low rate measurements of the actual state of the plant. We show that if one designs a discrete-time controller for an approximate discrete-time plant model so that the closed-loop system is input-to-state stable, then the input-to-state stability property will be preserved for the exact discrete-time plant model based on a dual-rate control scheme in a semi-globally practical sense.

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References


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