Decentralized robust output feedback control for control affine nonlinear interconnected systems

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A B S T R A C T

In this paper, a scheme for decentralized robust control for control affine nonlinear interconnected systems using linear matrix inequalities (LMIs) is presented. Based on the Lyapunov theory, sufficient conditions for closed-loop stability of a nonlinear system, reference input tracking, and disturbance attenuation over its operating-range are obtained. Then, achieving these sufficient conditions are formulated as local LMI optimization problems. By solving the appropriately defined local problems, the obtained sufficient conditions are satisfied, the closed-loop stability, input tracking, and disturbance attenuation over the operating-range of the system are guaranteed. The proposed control is linear and whose implementation is straightforward. An example is given to illustrate the proposed methodology.

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1. Introduction

During the last two decades, robust output-feedback controller design has been solved for uncertain linear and nonlinear systems and numerous papers dealing this problem have been published [2,4,7,11,17,19,25]. Recently the versatile tools of LMI has been used to design robust (static and dynamic) output feedback controllers [1,9,24,25]. The static output feedback problem is one of the most important open questions in control engineering [23]. It is shown that many of robust static output feedback controller design problems can be reduced to the problem of finding an optimization problem under a biaffine matrix inequality (BMI) constraint [19]. Most of the strategies to solve optimization problems under BMI constraints use iterative algorithms in which a set of LMI problems, are repeated until certain convergence criteria are met [9,19]. However, iterative algorithms suffer from two major problems, convergence of the algorithm and finding good initial values for designing parameters. So, presenting a non-iterative strategy to design robust output feedback controllers is important. Control of the interacting multivariable processes can be realized either by centralized MIMO controllers or by a set of SISO local controllers. For MIMO industrial processes, decentralized control is preferred from the viewpoints of implementation, requiring fewer parameters to tune and loop failure tolerance of the resulting control system [3,16,20]. In designing decentralized controllers for nonlinear systems, the nonlinearity of the system and the interaction between subsystems can be dealt with the same framework of true uncertainties of the system [13,25]. Indeed, nonlinear industrial processes are affected by various kinds of external disturbances and it is important to study these disturbances and try to develop robust decentralized control designs that can protect the system against disturbances [21]. Much research effort has been devoted to the problem of designing feedback laws for the purpose of internally stabilizing a nonlinear system and minimizing the L2-gain between disturbance input and regulated output [6,10,15].

Because the implementation of linear controllers is straightforward and cost effective, most widely used controllers in modern industry are linear controllers like PID controllers. So, designing linear decentralized robust controllers for industrial systems which are mostly nonlinear in nature is beneficial. In this direction, in [21] using linear matrix inequalities, a decentralized state feedback is designed for a specific class of nonlinear interconnected systems. The nonlinearity of the system is considered as uncertainty. The control design is formulated as a convex optimization problem. By solving the defined LMI optimization problem, the closed-loop nonlinear system can be stabilized. The proposed methodology is a non-iterative method [21]. However, it is not always possible or economically feasible to measure all state variables. Therefore, designing decentralized output feedback controllers is important. In [25], an approach to the design of static output feedback controller based on linear matrix inequalities is proposed. The proposed method handle large-scale problems with
additive nonlinearities. The resulting control is robust with respect to uncertainties, and can incorporate several types of information structure constraints. However, the proposed method is applicable to a very restricted class of nonlinear systems. Following using LMI tools to design robust output feedback controllers for nonlinear systems, in this paper, for control affine nonlinear systems, designing a linear decentralized robust output feedback controller is proposed. Using the Lyapunov theory, sufficient conditions for closed-loop stability are obtained. Based on them, designing a decentralized controller is formulated as solving local LMI optimization problems. It is shown by solving the appropriately defined local LMI optimization problems, the closed-loop nonlinear system is stable over its operating-range provided several sufficient conditions are satisfied. To guarantee tracking of reference inputs, the closed-loop nonlinear system is approximated by its linear closed-loop block-diagonal part \[5\]. By defining an appropriate Lyapunov function for the approximation error dynamics, it is shown by solving the same defined local LMI optimization problems for the closed-loop stability, the approximation error can be minimized. Indeed, sufficient conditions are obtained for the existence of a parameterized output feedback controller such that the \(L_2\)-gain from disturbances to outputs can be made arbitrarily small by increasing gains of local controllers \[6\]. In fact, in solving the local LMI optimization problems, if the achieved local gains are small enough, this means the derivative of the defined Lyapunov function is negative enough, then the effect of the external disturbances can be attenuated. Based on this concept, sufficient conditions for disturbance attenuation are derived which can be satisfied by solving the former local LMI optimization problems. The designed controller is obtained directly with no need for tuning parameters, trial and error procedures, and iteration. By minimizing gains in local LMI optimization problems, the resulting control maximizes the class of nonlinearity perturbations that can be tolerated by the closed-loop system \[21\].

The rest of the paper is organized as follows: In Section 2, problem formulation is given. Section 3 gives sufficient conditions for closed-loop stability. In addition, based on the obtained sufficient conditions and using the variable transformation introduced in \[18\], the problem of designing local dynamic controllers for isolated linear subsystems is formulated as local optimization problems with LMI constraints. Section 4 investigates the problem of disturbance attenuation and reference input tracking for the nonlinear system and a method to design a decentralized PI controller which is widely used in industrial processes is given. In Section 5, an example is given to illustrate the proposed method. Section 6 contains the conclusions and final remarks.

### 2. Problem formulation

Consider a control affine nonlinear system defined as

\[
\begin{align*}
\dot{x}(t) &= f(x) + g(x)u(t) + h(x)d(t), \\
y(t) &= Cx(t),
\end{align*}
\]

where \(x \in \mathbb{R}^n\), \(u \in \mathbb{R}^m\), \(y \in \mathbb{R}^m\) and \(d \in \mathbb{R}^d\) are the state, the control input, the measurable output and the external disturbance vectors, respectively. It is assumed that \(f(0) = 0\), and the equilibrium point of interest, not necessarily stable for \(u = 0\), is the origin. Suppose that the linearized system about its equilibrium point has the state space matrices \(A, B\), and \(C\), the pair \((A, C)\) is observable, and the matrix \(C\) is full rank. If the assumptions are not valid, it is possible to change the matrices \(A, C\), slightly such that these conditions are satisfied. Then, it is possible to find a similarity transformation based on the observability matrix of the pair \((A, C)\) to transform the system into the output-decentralized form where the transformed output matrix is a block-diagonal matrix \[12, 14\]. Since the rank of the observability matrix is equal to \(n\), there are \(n\) linearly independent row vectors in the observability matrix. The matrix \(C\) is full rank and by selecting the numbers \(n_i, i = 1, \ldots, N\) such that \(\sum_{i=1}^{N} n_i = n\), and rearranging \(n\) linearly independent row vectors, the similarity transformation can be given by

\[
T = \begin{bmatrix}
\hat{c}_1 \\
\hat{c}_{A1} \\
\vdots \\
\hat{c}_{Am} \\
\hat{c}_{A_{m-1}} \\
\hat{c}_{A_{m-1}}^{-1}
\end{bmatrix}^{-1}
\]

where \(\hat{c}_i\) is the \(i\)-th row of the matrix \(\hat{C}\) \[12\]. Similarity transformations do not affect output feedback controllers. Hence, without loss of generality in the rest of the paper, it is assumed that the output matrix, \(C\) is a block-diagonal matrix. Then, \(f(x)\) and \(g(x)\) are decomposed as follows:

\[
\begin{align*}
\hat{f}(x) &= Ax(t) + f(x), \\
\hat{g}(x) &= B + g(x),
\end{align*}
\]

where \(A\) and \(B\) are the block-diagonal parts of the transformed matrices of \(\hat{A}\), \(\hat{B}\). The matrices \(B\) and \(C\) are full rank and the pairs \((A, B)\) and \((A, C)\) are controllable and observable, respectively. Again, if the assumptions are not valid, because the decomposition in Eq. (3) is arbitrary, it is always possible to change the matrices \(A, B, C\) such that these conditions and Eq. (3) are satisfied. Let

\[
\begin{align*}
\hat{x} &= \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}, \\
u &= \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{bmatrix}, \\
y &= \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}, \\
A &= \begin{bmatrix} A_{11} & 0 & \cdots & 0 \\ 0 & A_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_{NN} \end{bmatrix}, \\
B &= \begin{bmatrix} B_{11} & 0 & \cdots & 0 \\ 0 & B_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & B_{NN} \end{bmatrix}, \\
C &= \begin{bmatrix} C_{11} & 0 & \cdots & 0 \\ 0 & C_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & C_{NN} \end{bmatrix},
\end{align*}
\]

and over the operating-range we have

\[
\begin{align*}
f^T(x)f(x) &\leq \xi^T F^T F \xi, \\
g^T(x)g(x) &\leq G^T G,
\end{align*}
\]

where \(F = [f_i]\) and \(G = [g_i]\) are appropriately defined constant matrices. Afterward, the system can be decomposed into \(N\) local nonlinear subsystems given by

\[
\begin{align*}
\dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t) + f_i(x_i) + g_i(x_i)u_i(t), \\
y_i(t) &= C_i x_i(t),
\end{align*}
\]

where \(x_i \in \mathbb{R}^{n_i}, u_i \in \mathbb{R}^{m_i}, y_i \in \mathbb{R}^{m_i}, \sum_{i=1}^{N} n_i = n,\) and \(\sum_{i=1}^{N} m_i = m\). Indeed, \(f_i\) and \(g_i\) are the \(i\)-th row blocks of \(f\) and \(g\), respectively and the pairs \((A_i, B_i)\) and \((A_i, C_i)\) are controllable and observable, respectively. It shows that the dynamic nonlinear system is composed of a number of linear subsystems coupled together by means of
interconnections with nonlinearities. The objective is to design a local controller \( K_i(s) \) of order \( n_b = n_i \) with the following state space equations

\[
\dot{x}_i(t) = A_{ii}x_i(t) + B_{ii}u_i(t) + B_{ci}x_c(t) + D_{ii}u_i(t), \\
u_i(t) = C_{ii}x_i(t) + D_{ii}u_i(t),
\]

for the \( i \)-th isolated local linear subsystem

\[
\dot{y}_i(t) = C_{ii}x_i(t),
\]

such that by applying the decentralized controller

\[
K_i(s) = \text{diag}(K_i(s)),
\]

to the original nonlinear system given in Eq. (1), the equilibrium point of the closed-loop system is stable over its operating-range and the effect of the external disturbances is attenuated.

3. Sufficient conditions for closed-loop stability

In this section, based on the Lyapunov theory, sufficient conditions for closed-loop stability of nonlinear system (1) are obtained. The nonlinear system is given by the following state space equations

\[
\dot{x}(t) = Ax(t) + Bu(t) + f(x) + g(x)u(t), \\
y(t) = Cx(t).
\]

Assume for each isolated subsystem, a local dynamic controller is designed. Designing a dynamic controller for the \( i \)-th local subsystem can be converted into designing a static controller

\[
K_i = \begin{bmatrix} D_{ii} & C_{ii} \\ B_{ii} & A_{ii} \end{bmatrix},
\]

for the augmented \( i \)-th subsystem given by the following matrices

\[
\tilde{A}_i = \begin{bmatrix} A_{ii} & 0 \\ 0 & 0 \end{bmatrix}, \quad \tilde{B}_i = \begin{bmatrix} B_{ii} & 0 \\ 0 & I \end{bmatrix}, \quad \tilde{C}_i = \begin{bmatrix} C_{ii} & 0 \\ 0 & I \end{bmatrix}.
\]

Applying the decentralized controller to system (12), the closed-loop system is given by

\[
\dot{x} = \tilde{A}_i\tilde{x}(t) + \tilde{f} + \tilde{g}\tilde{C}\tilde{x}(t),
\]

with

\[
\tilde{x} = \begin{bmatrix} x_1 \\ x_3 \\ x_n \\ x_n \end{bmatrix}, \quad \tilde{A} = \begin{bmatrix} A_{11} & 0 & \cdots & 0 \\ 0 & I & \cdots & 0 \\ 0 & 0 & \cdots & A_{nn} \\ 0 & 0 & \cdots & 0 \end{bmatrix},
\]

\[
\tilde{B} = \begin{bmatrix} B_{11} & 0 & \cdots & 0 \\ 0 & I & \cdots & 0 \\ 0 & 0 & \cdots & B_{nn} \\ 0 & 0 & \cdots & I \end{bmatrix},
\]

\[
\tilde{C} = \begin{bmatrix} C_{11} & 0 & \cdots & 0 \\ 0 & I & \cdots & 0 \\ 0 & 0 & \cdots & C_{nn} \\ 0 & 0 & \cdots & I \end{bmatrix}, \quad \tilde{g} = \begin{bmatrix} g_{11}(x) & 0 & \cdots & g_{1n}(x) \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ g_{n1}(x) & 0 & \cdots & g_{nn}(x) \end{bmatrix}.
\]

Let

\[
E_i = \begin{bmatrix} I_{n_i} & 0 \\ 0 & 0 \end{bmatrix},
\]

\[
E = \text{diag}(E_i),
\]

\[
F_g = \begin{bmatrix} F_{g_1} & 0 \\ 0 & 0 \end{bmatrix}, \quad F = \begin{bmatrix} F_{g_1} \\ 0 \end{bmatrix},
\]

and

\[
T_\phi = \begin{bmatrix} C_{g_1} & 0 \\ 0 & 0 \end{bmatrix}. \quad T_\phi = [T_{\phi_1}].
\]

From Eqs. (6) and (7) and (20)–(25) we have

\[
f^T(x)f(x) = f^T(x)f(x) \leq x^TP_\phi x,
\]

\[
g^T(x)g(x) \leq T_\phi g,
\]

\[
E f^T(x) = f(x),
\]

\[
E g(x) = g(x),
\]

\[
E = E^T = E^2,
\]

\[
\mathcal{F} = FE,
\]

\[
\mathcal{C} = T_\phi E,
\]

\[
0 \leq (f^T(x) - x^TP_\phi f(x) - f^T(x)Pk - x^TP_\phi f(x) + x^TPEP_\phi x),
\]

\[
0 \leq (x^T\mathcal{C}^T\mathcal{C}^Tf^T(x)g(x)K^TCKx - x^T\mathcal{C}^T\mathcal{C}^Tf^T(x)EPx - x^TPEP_\phi x + x^TPEP_\phi x),
\]

\[
0 \leq (x^T\mathcal{C}^T\mathcal{C}^Tg^T(x)g(x)K^TCKx - x^T\mathcal{C}^T\mathcal{C}^Tg^T(x)EPx - x^TPEP_\phi x + x^TPEP_\phi x),
\]

\[
0 \leq (x^T\mathcal{C}^T\mathcal{C}^Tf^T(x)P_\phi x + x^TPEg(x)K^TCKx + x^TPEP_\phi x + x^TPEP_\phi x),
\]

\[
0 \leq (x^T\mathcal{C}^T\mathcal{C}^Tg^T(x)g(x)K^TCKx - x^T\mathcal{C}^T\mathcal{C}^Tg^T(x)EPx - x^TPEP_\phi x + x^TPEP_\phi x),
\]

\[
0 \leq (x^T\mathcal{C}^T\mathcal{C}^Tf^T(x)P_\phi x + x^TPEg(x)K^TCKx + x^TPEP_\phi x + x^TPEP_\phi x),
\]
and

\[ V(x) \leq \dot{x}^T (A_i^TP + P A_i) + \frac{1}{2} P F P + \frac{1}{2} F^T F + 2 P E P + C_i^T \Sigma_i^T \Sigma_i C_i^T) x. \]  

(37)

From Eq. (37), it is clear if we can find \( P > 0 \) and \( \gamma \leq 1 \), such that

\[ (A_i^TP + P A_i) + \frac{2}{\gamma} P E P + \frac{1}{\gamma} F^T F + \frac{1}{\gamma} C_i^T \Sigma_i^T \Sigma_i C_i^T < 0. \]  

(38)

the equilibrium point of system (1) is stabilized. Moreover, minimizing \( \gamma \) in Eq. (38), maximize the nonlinearity that can be tolerated by the system. In fact \( \gamma \) is a parameter to evaluate the degree of robustness of the system to uncertainty due to the nonlinearity [21]. By defining

\[ P = \gamma P, \]  

(39)

condition (38) is equivalent to

\[ (A_i^TP + P A_i) + \frac{2}{\gamma} P E P + \frac{1}{\gamma} F^T F + \frac{1}{\gamma} C_i^T \Sigma_i^T \Sigma_i C_i^T < 0. \]  

(40)

On other hand, by virtue of the Bounded Real Lemma [18,22], the \( H_{\infty} \) norm of the following transfer matrix

\[ F G C_k \]  

(41)

is less than \( \gamma \) if a matrix \( \tilde{P} > 0 \) can be found such that condition (40) is satisfied [18]. So, the closed-loop stability of nonlinear system (1) over its operating-range can be obtained by minimizing the \( H_{\infty} \) of transfer matrix (41). Since, the matrices \( A_i \) and \( E \) are block-diagonal, we have [22]

\[ \left\| \begin{bmatrix} F \\ G C_k \\ C_{k} \end{bmatrix} (sl - A_i)^{-1} \sqrt{2E} \right\|_{\infty} \leq \sqrt{N_{\max}} \left\| \begin{bmatrix} F_i \\ G C_k \\ C_{k} \end{bmatrix} (sl - A_{ik})^{-1} \sqrt{2E} \right\|_{\infty} \]  

(42)

where \( F_i \) and \( C_k \) are the \( i \)-th column blocks of \( F \) and \( C_k \), respectively.

Now, we conclude, minimizing \( \gamma_i \) such that

\[ \left\| \begin{bmatrix} F_i \\ G C_k \\ C_{k} \end{bmatrix} (sl - A_{ik})^{-1} \sqrt{2NE} \right\|_{\infty} < \gamma_i, \]  

(43)

\[ \gamma_i < 1, \]  

(44)

guarantees the closed-loop stability of the equilibrium point of system (1) over its operating-range. Using the Bounded Real Lemma, solving the following local optimization problem with matrix inequality constraints guarantees satisfaction of (43) and (44)

\[ \min_{\gamma_i} \gamma_i \]  

(45)

\[ \left[ \begin{array}{ccc} A_{i}^T P_i + P_i A_i & \sqrt{2NP_i E_i} & F_i^T C_{k} \Sigma_i^T C_k \cr \ast & -\gamma_i I & 0 \\ \ast & \ast & -\gamma_i I \end{array} \right] < 0, \]  

(46)

\[ \gamma_i < 1, \]  

(47)

\[ P_i > 0. \]  

(48)

So far, we have shown by solving the local matrix inequality optimization problems in (45)–(47), the equilibrium point of the system is stable over the operating-range. In solving local problem (45)–(47) for an isolated subsystem, it is clear if small values for \( \gamma_i \) can be obtained, the nonlinearity perturbations that can be tolerated by the closed-loop system can be maximized. Indeed, the proposed method has the advantage that the true model uncertain-
ties can be dealt within the same framework such that the decentralized controllers obtained can be robust with respect to system uncertainties as well.

Now, we design a local linear output feedback controller to solve local problem (45)–(47). This local problem cannot be solved by the LMI tools to find the controller matrices, because it is not affine in controller parameters \( A_i, B_i, C_i \) and \( D_{ci} \). Hence, a variable transformation is necessary. Since \( n_i = n_i \) is the number of states of the \( i \)-th subsystem (size of \( A_i \)), the matrix \( P_i \) and \( P_i^{-1} \) can be partitioned as

\[ P_i = \begin{bmatrix} X_i & N_i \\ N_i^T & * \end{bmatrix}, \quad P_i^{-1} = \begin{bmatrix} X_i & M_i \\ M_i^T & * \end{bmatrix}. \]  

(48)

where \( X_i \) and \( Y_i \) are \( n_i \times n_i \) and symmetric. From

\[ P_i P_i^{-1} = I, \]  

(49)

we infer

\[ P_i \begin{bmatrix} X_i \\ M_i^T \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix}, \]  

(50)

which leads to

\[ P_i \Pi_i = \Pi_{i^2}, \]  

(51)

\[ \Pi_i = \begin{bmatrix} X_i & I \\ 0 & N_i^T \end{bmatrix}, \]  

(52)

and

\[ \Pi_{i^2} = \begin{bmatrix} I \\ 0 \end{bmatrix}. \]  

(53)

We define the change of controller variables as follows [18]:

\[ \tilde{A}_i = N_i A_i M_i^T + N_i B_{ci} C_i X_i + Y_i B_{ci} C_{ci} M_i^T + Y_i (A_i + B_{ci} D_{ci} C_{ci}) X_i, \]  

(54)

\[ \tilde{B}_i = N_i B_{ci} + Y_i B_{ci} D_{ci}, \]  

(55)

\[ \tilde{C}_i = C_{ci} M_i^T + D_{ci} C_{ci} X_i, \]  

(56)

\[ \tilde{D}_i = D_{ci}. \]  

(57)

Then, we pre and post-multiply \( P_i \) by \( \Pi_i^T \) and \( \Pi_i \) and (46) by \( \text{diag}(\Pi_i^{1/2}, I, I, I) \) and \( \text{diag}(\Pi_i, I, I, I) \), respectively, we get

\[ \Pi_i^T P_i \Pi_i = \Pi_i^T \Pi_i = \begin{bmatrix} X_i & I \\ I & Y_i \end{bmatrix}, \]  

(58)

\[ \Pi_i^T P_i \tilde{A}_i \Pi_i = \Pi_i^T \tilde{A}_i \Pi_i = \begin{bmatrix} A_i X_i + B_{ci} \tilde{C}_i & (A_i + \tilde{B}_i \tilde{D}_i C_{ci}) \\ \tilde{A}_i & Y_i A_{ci} + \tilde{B}_i \tilde{C}_i \end{bmatrix}, \]  

(59)

\[ \Pi_i^T P_i E_i = \Pi_i^T E_i = \begin{bmatrix} I \\ Y_i \end{bmatrix} \]  

(60)

\[ \Pi_i^T F_i^T = \begin{bmatrix} X_i F_i & 0 \\ F_i & 0 \end{bmatrix}, \]  

(61)

\[ \Pi_i^T C_{ki}^T \Sigma_i^T C_{k} = \begin{bmatrix} X_i C_i D_{ci}^T C_{ki}^T + M_i C_{ki} C_{ki}^T & 0 \\ C_i D_{ci}^T C_{ki}^T & 0 \end{bmatrix} = \begin{bmatrix} \tilde{C}_i C_{ki}^T & 0 \\ C_i D_{ci}^T C_{ki}^T & 0 \end{bmatrix}. \]  

(62)

If \( M_i \) and \( N_i \) have full row rank, and if \( \tilde{A}_i, \tilde{B}_i, \tilde{C}_i, \tilde{D}_i, X_i, \) and \( Y_i \) are given, we can always compute controller matrices. Using Eq. (55)–(59), it is simple to show that by solving the following optimization problem with LMI constraints, the local optimization problem with matrix inequalities given in (45)–(47) is solved:
closed-loop system. All closed-loop nonlinear system will be guaranteed. Consider the
tracking for the closed-loop linear part, input tracking for the over-
designing a decentralized controller to guarantee reference input
is possible to approximate the closed-loop original nonlinear sys-
tem by the closed-loop linear part of the system [3,5]. Then, by
approximation be
e(t) = \tilde{x}(t) - \hat{x}(t),

then
\dot{\tilde{e}}(t) = \overline{A}_d e(t) + \tilde{f} + \tilde{g} \overline{K} \overline{C} \tilde{x}(t).

Now, consider the following Lyapunov function given by
\nu(e, \tilde{x}) = e^T P e + \tilde{x}^T P \tilde{x},

where the matrix P is the same as the Lyapunov matrix used in
Section 3. Then, the derivative of the Lyapunov function is as
follows:
\dot{v}(e, \tilde{x}) = e^T P e + e^T \tilde{f} + \tilde{x}^T P \tilde{x} + \tilde{x}^T P \tilde{x}.

Following the discussion given in this section by solving the fol-
lowing LMI optimization problem
\begin{align*}
\begin{bmatrix}
A_i X_i + X_i A_i^T + B_i \overline{C}_i + (B_i \overline{C}_i)^T \\
A_i + (A_i + B_i \overline{D}_i \overline{C}_i)^T \\
\sqrt{2}N_i \\
F_i X_i \\
G_i \overline{C}_i
\end{bmatrix} & < 0,
\begin{bmatrix}
X_i & I \\
Y_i & 0
\end{bmatrix} > 0,
\gamma_i < 1.
\end{align*}

Using Eqs. (60)–(66), it can be shown
\dot{v}(e, \tilde{x}) \leq e^T (\overline{A}_d P + P \overline{A}_d) e + \overline{f}^T P e + \overline{x}^T \overline{C} \overline{R}^T \overline{L} \overline{x} + 2 \overline{F}^T F \overline{x} + 2 \overline{P} \overline{E} P \overline{E}^T (\overline{C} \overline{R}^T \overline{L} \overline{C} \overline{K} \overline{C} \overline{K}) \overline{x}.

From Eq. (38) we have
\begin{align*}
\overline{A}_d P + P \overline{A}_d + 2 \overline{P} \overline{E} P & < \frac{1}{2} (\overline{F} \overline{F}^T + \overline{C} \overline{R}^T \overline{L} \overline{C} \overline{K} \overline{C} \overline{K}).
\end{align*}

By substituting Eq. (75) in (74), the derivative of the Lyapunov func-
tion for error dynamics given in (74) is negative definite if
\begin{align*}
\left(2 - \frac{1}{2}\right) \overline{F} \overline{F}^T + \left(2 - \frac{1}{2}\right) \overline{C} \overline{R}^T \overline{C} \overline{K} \overline{C} \overline{K} < 0.
\end{align*}

This means if
\gamma < \frac{1}{\sqrt{2}}.

Remark 1. For a class of nonlinear systems with \( g = 0 \), it is simple
to show that the derivative of the Lyapunov function given in (19)
can be written as
\begin{align*}
V(\tilde{x}) = \tilde{x}^T \left(\overline{A}_d^T P + P \overline{A}_d\right) \tilde{x} + \tilde{f}^T P \tilde{x} + \tilde{x}^T P \tilde{x}.
\end{align*}

Following the discussion given in this section by solving the fol-
lowing LMI optimization problem
\begin{align*}
\begin{bmatrix}
A_i X_i + X_i A_i^T + B_i \overline{C}_i + (B_i \overline{C}_i)^T \\
A_i + (A_i + B_i \overline{D}_i \overline{C}_i)^T \\
\sqrt{2}N_i \\
F_i X_i \\
G_i \overline{C}_i
\end{bmatrix} & < 0,
\begin{bmatrix}
X_i & I \\
Y_i & 0
\end{bmatrix} > 0,
\gamma_i < 1.
\end{align*}
the error dynamics given in (71) is stable. According to the discussion given in this section, we conclude by solving local LMI optimization problems (60), we can approximate closed-loop system (68) by system (69). Then, by designing appropriate local controllers, for example local PID controllers to guarantee trajectory tracking for the linear part of the system, it is possible to have a good reference input tracking for the closed-loop overall nonlinear system. It is also possible to design an extra controller in an outer control loop for the linear part of the closed-loop system as will be explained later.

Now, we assume the closed-loop system can be approximated by system (69), then, the closed-loop original system given in (1) under the decentralized controller can be approximated by the following state space equations

\[
\begin{align*}
\dot{x}(t) &= (A + BC) x(t) + h(x) d(t), \\
y(t) &= \overline{C} x(t),
\end{align*}
\]

(78)

where

\[
\overline{C} = \begin{bmatrix}
C_{11} & 0 & \ldots & 0 \\
\vdots & \ddots & \vdots & \vdots \\
0 & \ldots & C_{NN} & 0
\end{bmatrix},
\]

(79)

\[
h(x) = \begin{bmatrix}
h_1(x) \\
0 \\
\vdots \\
h_N(x) \\
0
\end{bmatrix},
\]

(80)

and \(h_i(x)\) is the \(i\)-th row block of \(h(x)\).

Let over the operating-range of the system

\[
h^t h \leq \gamma_i^2 I.
\]

(81)

According to the definition in (79) for the matrix \(\overline{C}\), we can show that

\[
\overline{C} = \overline{C} E.
\]

(82)

Since

\[
h^t h \leq \gamma_i^2 E,
\]

(83)

then, the \(L_2\) gain of the transfer function form the disturbance vector to the output vector, \(G_{xd}\) satisfies the following condition [10]:

\[
||G_{xd}||_{\infty} \leq ||E(sI - \overline{A}_d)^{-1} \gamma_i E||_{\infty}.
\]

(84)

Since, the matrix \(\overline{C}\) is a constant matrix, by minimizing the following norm:

\[
||G_{xd}||_{\infty} \leq ||E(sI - \overline{A}_d)^{-1} \gamma_i E||_{\infty} < 1,
\]

(85)

the effect of the external disturbance will be attenuated. It is equivalent to finding a symmetric matrix \(P\) (the matrix \(P\) is the same as the matrix \(P\) given in Section 3) such that

\[
\overline{A}_d^t P + P \overline{A}_d + PEP + \gamma_i^2 E < 0.
\]

(86)

Substituting from Eq. (38) in the above equation, we have

\[
-PEP - \frac{1}{\gamma_i^2} C^t R C C^t K K^t F + \gamma_i^2 E < 0.
\]

(87)

This condition may be satisfied if \(\gamma\) is small enough. In fact, since

\[
-PEP - \frac{1}{\gamma_i^2} C^t R C C^t K K^t F - \frac{1}{\gamma_i^2} F^t F + \gamma_i^2 E
\]

\[
\leq - \frac{1}{\gamma_i^2} C^t R C C^t K K^t F - \frac{1}{\gamma_i^2} F^t F + \gamma_i^2 E
\]

\[
\leq - \frac{\lambda_{min}(C^t R C C^t K K^t F + F^t F)}{\gamma_i^2} I + \gamma_i^2 E,
\]

(88)

then, if

\[
\gamma < \sqrt{\lambda_{min}(C^t R C C^t K K^t F + F^t F)}.
\]

(89)

condition (86) is satisfied. This means that by solving local LMI optimization problems (60), it is possible to attenuate the effect of the external disturbances. If \(\gamma \to 0\), the function given in (87) is negative enough to ignore the effect of the external disturbance.

Up to now, we have shown by solving the local LMI optimization problems in (60), the closed-loop system is stable and the effect of the external disturbances is attenuated. The derived conditions may be conservative and we do not need such conservatism. However, it gives us an idea that by solving local problems (60), it is possible to achieve closed-loop stability and acceptable disturbance rejection. In minimizing \(\gamma_i\)'s in local optimization problems which is equivalent to increasing gains of local controllers, there are some practical considerations. For example if there is a right hand transmission zero in a channel of the system, there is closed-loop bandwidth limitation and it is not possible to derive very small value for the related \(\gamma_i\).

4.1. Decentralized PI controller design

PI control has been one of the most commonly used schemes for single-input single-output (SISO) processes in various industries, particularly process control industries because of its simplicity, robustness, and near-optimal performance. In this section, we propose a scheme for decentralized PI controller design. In order to do this, first for the \(i\)-th isolated subsystem a local controller, \(k_{in}(s)\) by solving LMI local optimization problem (60) will be designed. The designed decentralized controller guarantees closed-loop stability of the equilibrium point of the nonlinear system and approximates it well with a diagonal linear system. Then, as shown in Fig. 1 for the \(i\)-th stabilized local isolated subsystem a PI controller, \(k_{out}(s)\) in an outer loop will be designed. Then, the closed-loop subsystem has the following transfer function

\[
\frac{g_i(s)k_{in}(s)k_{out}(s)}{1 - g_i(s)k_{in}(s) + g_i(s)k_{in}(s)k_{out}(s)}.
\]

(90)

On the other hand, if we design a single local controller, \(k_i(s)\) for the \(i\)-th isolated subsystem as shown in Fig. 2, the closed-loop isolated \(i\)-th subsystem has the following transfer function

\[
\begin{align*}
\frac{g_i(s)k_{in}(s)k_{out}(s)}{1 - g_i(s)k_{in}(s) + g_i(s)k_{in}(s)k_{out}(s)}
\end{align*}
\]

(91)

Fig. 1. Multi-loop controller.

Fig. 2. Single-loop controller.
Equating two transfer functions (90) and (91) we have
\[ k_i(s) = \frac{-k_{in}(s)}{1 - g_i(s)/k_{out}(s)}. \]  

This transfer function, like a PI controller, is a proper transfer function and has a pole at the origin. Now, we approximate the designed controller \( k_i(s) \) in (92) by a local PI controller.

### 5. Example

In this section, the nonlinear model of a quadruple-tank process derived in [18] is considered as an example to illustrate the proposed methodology. The process consists of four interconnected water tanks and two pumps. Its inputs are the voltages to the two pumps and the outputs are the water levels in the lower two tanks. The linearized model of the quadruple-tank process has a multivariable zero, which can be located in either the left or the right half-plane by simply changing a valve. It is shown that the valve positions of the process uniquely determine if the system is minimum phase or non-minimum phase. The nonlinear model equations are given as follows:

\[
\begin{align*}
\frac{dh_1}{dt} &= \frac{a_1}{A_1} \sqrt{2gh_1} + \frac{a_3}{A_3} \sqrt{2gh_3} + \eta_1 k_1 v_1, \\
\frac{dh_2}{dt} &= \frac{a_2}{A_2} \sqrt{2gh_2} + \frac{a_4}{A_4} \sqrt{2gh_4} + \eta_2 k_2 v_2, \\
\frac{dh_3}{dt} &= \frac{a_3}{A_3} \sqrt{2gh_3} + \frac{a_1}{A_1} \sqrt{2gh_1} + \eta_2 k_2 v_2, \\
\frac{dh_4}{dt} &= \frac{a_4}{A_4} \sqrt{2gh_4} + \frac{a_2}{A_2} \sqrt{2gh_2} + \eta_1 k_1 v_1, \\
y_1(t) &= k_1 h_1, \\
y_2(t) &= k_2 h_2,
\end{align*}
\]  

where \( A_1 = A_3 = 28 \text{ cm}^2, A_2 = A_4 = 32 \text{ cm}^2 \) are cross-sections of the tanks, \( a_1 = a_3 = 0.071 \text{ cm}^2, a_2 = a_4 = 0.057 \text{ cm}^2 \) are cross-sections of the outlet holes, \( k_1 = 0.5 \text{ V/cm}, g = 981 \text{ cm/s}^2, \) and \( \eta_1, \eta_2 \in (0, 1) \) [8].

Suppose the operating-range of the system is as follows:
\[
\begin{align*}
0 &\leq h_i \leq 20, \quad i = 1, 2, \\
0 &\leq h_i \leq 6, \quad i = 3, 4.
\end{align*}
\]  

It is shown that the system is non-minimum phase for
\[
0 \leq \eta_1 + \eta_2 \leq 1.
\]  

and minimum phase for
\[
1 \leq \eta_1 + \eta_2 \leq 2.
\]  

We consider these two cases separately. Since, our objective is to compare the achieved results with the results given in [8], we use the same parameters and operating point given in [8] i.e.
\[
\begin{align*}
h_1 &= 12.4 \text{ cm}, \quad h_2 = 12.7 \text{ cm}, \quad h_3 = 1.8 \text{ cm}, \\
h_4 &= 1.4 \text{ cm}, \quad v_1 = 3 \text{ V},
\end{align*}
\]  

\[
\begin{align*}
v_2 &= 3 \text{ V}, \quad \eta_1 = 0.7, \quad \eta_2 = 0.6, \quad k_1 = 3.33 \text{ cm}^3/\text{Vs}, \\
k_3 &= 3.35 \text{ cm}^3/\text{Vs}
\end{align*}
\]  

for the minimum phase case. By introducing \( x_1 = h_1 - 12.4, \) \( x_2 = h_2 - 12.7, \) \( x_3 = h_3 - 1.4, \) \( u_1 = v_1 - 3, \) \( u_2 = v_2 - 3, \) the system given in (93) can be written as follows:
\[
\begin{align*}
x_1 &= -0.1123 \sqrt{x_1} + 12.4 + 0.1123 \sqrt{x_2} + 1.8 + 0.0833(u_1 + 3), \\
x_2 &= -0.1123 \sqrt{x_2} + 1.8 + 0.0479(u_2 + 3), \\
x_3 &= -0.0789 \sqrt{x_3} + 12.7 + 0.0789 \sqrt{x_4} + 1.4 + 0.0628(u_2 + 3), \\
x_4 &= -0.0789 \sqrt{x_4} + 1.4 + 0.0312(u_1 + 3).
\end{align*}
\]  

The operating-range of the transformed system is as follows:
\[
\begin{align*}
-12.4 &\leq x_1 \leq 7.6, \\
-1.8 &\leq x_2 \leq 4.2, \\
-12.7 &\leq x_3 \leq 7.3, \\
-1.4 &\leq x_4 \leq 4.6.
\end{align*}
\]  

and over this operating-range with a good approximation we have
\[
\begin{align*}
\hat{x}_1 &= 0.00073x_1^2 - 0.017x_1 - 0.0047x_2^2 + 0.049x_2 + 0.0833u_1, \\
\hat{x}_2 &= 0.0047x_2^2 - 0.049x_2 + 0.0479u_2, \\
\hat{x}_3 &= 0.00052x_3^2 - 0.011x_3 - 0.0035x_4^2 + 0.0374x_4 + 0.0628u_2, \\
\hat{x}_4 &= 0.0035x_4^2 - 0.0374x_4 + 0.0312u_1, \\
y_1(t) &= 0.5x_1, \\
y_2(t) &= 0.5x_2.
\end{align*}
\]  

For this system we have
\[
\begin{align*}
A &= \begin{bmatrix} -0.017 & 0.0492 & 0 & 0 \\ 0 & -0.0492 & 0 & 0 \\ 0 & 0 & -0.011 & 0.0374 \\ 0 & 0 & 0 & -0.0374 \end{bmatrix}, \\
B &= \begin{bmatrix} 0 \\ 0 \\ 0.0479 \\ 0 \end{bmatrix}, \\
C &= \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \end{bmatrix}, \\
F &= \begin{bmatrix} 0.011 & 0 & 0 & 0 \\ 0 & 0.0036 & 0 & 0 \\ 0 & 0 & 0.011 & 0 \\ 0 & 0 & 0 & 0.004 \end{bmatrix}, \\
G &= \begin{bmatrix} 0.0833 & 0 \\ 0 & 0.0628 \end{bmatrix}.
\end{align*}
\]  

The specific structure of the matrix \( B \) is selected to have controllable and observable subsystems. Solving local LMI optimization problems for the isolated subsystems
\[
A_{11} = \begin{bmatrix} -0.017 & 0.0492 \\ 0 & -0.0492 \end{bmatrix}, \quad B_{11} = \begin{bmatrix} 0 \\ 0.0479 \end{bmatrix}, \quad C_{11} = \begin{bmatrix} 0.5 & 0 \end{bmatrix}.
\]  

and
\[
A_{22} = \begin{bmatrix} -0.011 & 0.0374 \\ 0 & -0.0374 \end{bmatrix}, \quad B_{22} = \begin{bmatrix} 0 \\ 0.0312 \end{bmatrix}, \quad C_{22} = \begin{bmatrix} 0.5 & 0 \end{bmatrix}.
\]  

and removing the fast dynamics from the local controllers we get
\[
\begin{align*}
k_{in}(s) &= \frac{-0.17337(s + 0.05171)}{(s + 0.09068)}, \\
k_{in}(s) &= \frac{-0.34977(s + 0.03621)}{(s + 0.07045)}.
\end{align*}
\]
In designing the local controllers the pairing $u_1 \rightarrow y_2$ and $u_2 \rightarrow y_1$ is used. So, because of the structure of the linear part of the system the closed-loop linear part of the system has the following form

$$ T_{in}(s) = \begin{bmatrix} T_{in1} & 0 \\ 0 & T_{in2} \end{bmatrix} = \begin{bmatrix} \frac{-g_1 k_{n1}(s)}{T_{g1} k_{n1}} & 0 \\ 0 & \frac{-g_2 k_{n2}(s)}{T_{g2} k_{n2}} \end{bmatrix} = \begin{bmatrix} T_{in1} & 0 \\ 0 & T_{in2} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{109} $$

In order to have good reference input tracking we design local PI controllers for $T_{in}$, $i = 1, 2$ as

$$ k_{out1}(s) = \frac{5(s + 0.05)}{s}, \tag{110} $$

$$ k_{out2}(s) = \frac{5(s + 0.04)}{s}. \tag{111} $$

Now, we use the suggested method in subSection 4.1, to design single PI controllers for the nonlinear system in (93) as follows:

![Fig. 3.](image-url) Solid: the approximated local PI controller, dashed: the single-loop controller $k_i(s)$.

![Fig. 4.](image-url) Solid: the approximated local PI controller, dashed: the single loop controller $k_i(s)$.
Figs. 3 and 4 show the singular values of designed local single controllers \( k_i(s), i = 1, 2 \) in Fig. 2 and the approximated local PI controllers. From the figures we observe the local PI controllers approximate the local controllers well. By applying the local PI controllers to the nonlinear system given in (93), Fig. 5 shows that the system outputs track the reference inputs over the operating-range well. Fig. 6 shows the related control signals.

In [8], the local PI controllers designed for the local linearized model about the operating point given in 97, 98, are given by

\[
\begin{align*}
    k_{11}(s) &= \frac{-0.8668s + 0.0217}{s}, \\
    k_{12}(s) &= \frac{-1.749s + 0.0287}{s}.
\end{align*}
\]

Suppose that the system is at the steady state condition with \( y_1 = 8 \text{ cm} \) and \( y_2 = 5 \text{ cm} \), then at time \( t = 300 \text{ s} \) a disturbance, \( d \) is applied to the closed-loop system as follows:

\[
\frac{dh_1}{dt} = -\frac{a_1}{A_1} \sqrt{2gh_1} + \frac{a_3}{A_1} \sqrt{2gh_3} + \frac{\eta_1 k_1}{A_1} u_1 + d.
\]

It is clear for the quadruple tank system we should have \( u_i \geq 0, i = 1, 2 \). Fig. 9 shows the responses of the closed-loop system.
under the decentralized controller to different disturbance signals \((d = 0.2; d = 0.23; d = 0.25)\). As it is observed from the figure, the disturbance is rejected well for \(d \leq 0.25\). But by increasing the disturbance signal more, the closed-loop system can not reject the disturbance completely. Fig. 10 shows the related control signals which are greater than zero. Now, consider the following operating point [8]:

\[
\begin{align*}
&h_1 = 12.6 \text{ cm}, \\
&h_2 = 13 \text{ cm}, \\
&h_3 = 4.8 \text{ cm}, \\
&h_4 = 4.9 \text{ cm}, \\
&v_1 = 3.15 \text{ V}. \\
&v_2 = 3.15 \text{ V}, \\
&\eta_1 = 0.43, \\
&\eta_2 = 0.34, \\
&k_1 = 3.14 \text{ cm}^2/\text{V} \text{ s}, \\
&k_2 = 3.29 \text{ cm}^3/\text{V} \text{ s}. \\
\end{align*}
\]

(115)

For \(\eta_1 = 0.43, \eta_2 = 0.34\), the system has non minimum phase characteristics [8]. By introducing \(x_1 = h_1 - 12.6, x_2 = h_3 - 4.8, x_3 = h_2 - 13, x_4 = h_4 - 4.9, u_1 = v_1 - 3.15, u_2 = v_2 - 3.15\), the system given in (93) can be written as follows:

\[
\begin{align*}
&x_1 = -0.1123\sqrt{x_1} + 12.6 + 0.1123\sqrt{x_2} + 4.8 + 0.0482(u_1 + 3.15), \\
&x_2 = -0.1123\sqrt{x_2} + 4.8 + 0.0775(u_2 + 3.15), \\
&x_3 = -0.0789\sqrt{x_3} + 13 + 0.0789\sqrt{x_4} + 4.9 + 0.035(u_2 + 3.15), \\
&x_4 = -0.0789\sqrt{x_4} + 4.9 + 0.0559(u_1 + 3.15). \\
\end{align*}
\]

(118)

The operating-range of the transformed system is as follows:

\[
\begin{align*}
&12.6 \leq x_1 \leq 7.4, \\
&4.8 \leq x_2 \leq 1.2, \\
&13 \leq x_3 \leq 7, \\
&-4.9 \leq x_4 \leq 1.1. \\
\end{align*}
\]

(119)

Fig. 8. Control signals of the closed-loop nonlinear system to a step on the first input, solid: the new controller, dashed: the controller designed in [8].

Fig. 9. The response of the closed-loop system to the disturbances for \(d = 0.2\) (dashed), 0.23 (dashed-dot) and 0.25 (solid).
and over this operating-range with a good approximation we have

\[ x_1 = 0.00073x_1^2 - 0.016x_1 - 0.0047x_2^2 + 0.02x_2 + 0.0482u_1, \]
\[ x_2 = 0.0047x_2^2 - 0.02x_2 + 0.0775u_2, \]
\[ x_3 = 0.00052x_3^2 - 0.011x_3 - 0.0033x_4^2 + 0.014x_4 + 0.035u_2, \]
\[ x_4 = 0.0033x_4^2 - 0.014x_4 + 0.0559u_1, \]
\[ y_1(t) = 0.5x_1, \]
\[ y_2(t) = 0.5x_3. \]

For this system we have

\[
A = \begin{bmatrix}
-0.016 & 0.02 & 0 & 0 \\
0 & -0.02 & 0 & 0 \\
0 & 0 & -0.011 & 0.014 \\
0 & 0 & 0 & -0.014
\end{bmatrix},
\]
\[
B = \begin{bmatrix}
0 & 0 & 0 & 0.0775 \\
0 & 0 & 0 & 0 \\
0.0559 & 0 & 0 & 0
\end{bmatrix},
\]
\[
C = \begin{bmatrix}
0.5 & 0 & 0 & 0
\end{bmatrix}.
\]

(120)

Fig. 10. Control signals in the response of the closed-loop system to the disturbances for \( d = 0.2 \) (dashed), 0.23 (dashed-dot) and 0.25 (solid).

Fig. 11. Reference input tracking over the operating-range (solid: the outputs, dashed: the reference inputs.)
\[
F = \begin{bmatrix}
0.01 & 0 & 0 & 0 \\
0 & 0.008 & 0 & 0 \\
0 & 0 & 0.011 & 0 \\
0 & 0 & 0 & 0.004
\end{bmatrix}, \quad G = \begin{bmatrix}
0.0482 & 0 \\
0 & 0.035
\end{bmatrix}.
\]

(121)

Since the system for the valve positions in (117) is non minimum phase, we can see over operating-range (119), the smallest right-hand plane (RHP) zero is at \( z = 0.011 \). The limitations RHP zeros impose on the MIMO systems are similar to those for SISO systems, although often not quite so serious because they only apply in particular directions [22]. In solving the local LMI optimization problems for the two isolated subsystems, although the isolated subsystems are not non-minimum phase, but since the overall system is non-minimum phase, the local controllers should be designed such that the bandwidth limitation of the overall system is satisfied. By solving the local optimization problems with appropriate constraints on the closed-loop loops and removing the fast dynamics from the local controllers, we get

\[
k_{\text{in}1}(s) = \frac{-0.112(s + 0.07727)}{(s + 0.04259)},
\]

(122)

\[
k_{\text{in}2}(s) = \frac{-0.50215(s + 0.09628)}{(s + 0.08086)}.
\]

(123)

Following the given method for the minimum phase case, we design the following local PI controllers for the closed-loop linear part of the system as follows:

\[
k_{\text{out}1}(s) = \frac{-8.74s - 0.1226}{s},
\]

(124)

\[
k_{\text{out}2}(s) = \frac{0.119 + 0.0034}{s}.
\]

(125)

Now, we use the suggested method in subSection 4.1, to design single PI controllers for the nonlinear system in (93) as follows:
By applying the local PI controllers to the nonlinear system given in (93), Fig. 11 shows that the system outputs track the reference inputs over the operating-range well. Fig. 12 shows the related control signals.

In [8], the local PI controllers designed for the local linearized model about the operating point given in (115) and (116), are given by

\[ k_{p1}(s) = \frac{-0.9791 - 0.0166}{s}, \]

\[ k_{p2}(s) = \frac{0.05976s + 8.25e^{-4}}{s}. \]

We can observe from the figures, the response for the proposed method has less settling time but a little larger overshoot. However, the control signals for the new controller are less aggressive and smaller than the controller signals for the controller in [8].

Suppose the system is at the steady state condition with \( y_1 = 8 \) cm and \( y_2 = 8 \) cm, then at time \( t = 500 \) s a disturbance, \( d \) is applied to system under the as given in (114). Fig. 15 shows the response of the closed-loop system under the decentralized controller to different disturbance signals \( (d = 0.05; d = 0.1; d = 0.15) \). As it is observed from the figure, the disturbance is rejected well for \( d < 0.15 \). But by increasing the disturbance signal more, the closed-loop system can not reject the disturbance because the related control signals will be negative. Fig. 16 shows the related control signals which are greater than zero.
6. Conclusion

This paper proposes a method for decentralized robust control of control affine nonlinear systems using linear matrix inequalities (LMIs). Sufficient conditions for closed-loop stability, reference input tracking and external disturbance attenuation over the operating-range of the closed-loop system are derived. It is shown satisfying these sufficient conditions can be formulated as local optimization problems with LMI constraints. By solving the appropriately defined local optimization problems, the derived sufficient conditions are satisfied. The designed controller is a linear controller whose implementation is straightforward and cost effective. The example given in the paper illustrates the effectiveness of the proposed methodology.

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