Decentralized robust control of a class of nonlinear systems and application to a boiler system

Batool Labibi a,*, Horacio Jose Marquez b, Tongwen Chen b

a Advanced Process Automation and Control (APAC) Research Group, K.N. Toosi University of Technology, Tehran 16315-1355, Iran
b Electrical and Computer Engineering Department, University of Alberta, Edmonton, Canada

ABSTRACT

This paper presents a scheme for designing a robust decentralized controller for an industrial utility boiler system. First, a new method for designing linear robust decentralized controllers for a class of nonlinear systems is presented. For systems in this class the feedthrough matrix, D, is not equal to zero. By using a descriptor system representation, the system is converted into a system with D = 0. Then, sufficient conditions for closed-loop stability, and asymptotic output tracking over the operating-range of the system are obtained. Satisfaction of these conditions is formulated as an optimization problem with LMI constraints. By solving the optimization problem, a decentralized controller for the system can be calculated. In order to control a utility boiler, an identified model of the utility boiler is considered. The identified model is linear, second order, and captures all the characteristics of the real system, including shrink and swell phenomenon, non-minimum phase behavior, and instability. Using the proposed methodology, a decentralized robust controller is designed for the identified model. Thereafter, the designed controller is applied to the simulation model of the real system. The simulation results show excellent input tracking and disturbance rejection.

1. Introduction

In the Syncrude Canada Ltd. (SCL) integrated energy facility located in Mildred Lake, Alberta, utility boilers are used to regulate the steam pressure. The key problem in the utility boilers in SCL, like many similar boilers available worldwide, is the design of an efficient and robust controller, so that the boiler can provide a continuous supply of steam at the desired pressure conditions. In addition, the amount of water in the steam drum must be maintained at the desired level to prevent overheating of drum components or flooding of steam lines [11]. However, it is difficult to achieve these objectives because of the high nonlinearity and uncertainty of boiler systems. Indeed, the control performance is strongly affected by load changes and large disturbances.

In modern industry, despite abundance of advanced nonlinear controllers and nonlinear nature of the industrial processes, linear controllers like PID controllers are still the most widely used controller type. The reason for this is the simplicity of implementation of linear systems. Control of the interacting multivariable systems can be realized either by centralized MIMO controllers or by a set of local controllers. To whatever extent, the decentralized control is more desirable from the viewpoints of implementation, requiring fewer parameters to tune, and loop failure tolerance of the resulting control system [10]. Consequently, in process control applications often linear decentralized controllers are used.

Control of linear and nonlinear systems deals with the tasks of stabilization, tracking and disturbance rejection or attenuation (and various combinations of them). However, for nonlinear systems these problems are more difficult to solve. A practical way to stabilize a nonlinear system uses linearization. The system is linearized about the desired equilibrium point and a feedback control law is designed for the linearized system. The validity of this idea comes from the Lyapunov’s indirect method [6]. Clearly, this approach is local. Another linearization idea deals with a specific class of nonlinear systems that can be transformed into linear systems via feedback and a change of variables [4]. The designed controller in this method is nonlinear and usually its implementation is difficult and of no appeal for industrial processes. So, designing a robust linear controller to control a nonlinear system over its operating-range is of great importance.

In industrial processes the tracking problem in the presence of disturbances is a general control problem. In fact, in industrial processes, the outputs must be able to follow the desired inputs while due to load changes, there are large disturbances. Achieving asymptotic output tracking in the presence of input disturbances is called asymptotic disturbance rejection [4]. A common method to asymptotic disturbance rejection is to minimize the error between outputs of the system and the model which generates the desired output.
In this paper, designing a decentralized robust controller to control the drum pressure and level of the industrial utility boilers in SCL is proposed. First, a new method for decentralized robust control of a specific class of nonlinear systems over the operating-range is proposed. This class includes systems such as boilers, RLC circuits with nonlinear resistors, turbofan engines and many other systems. Because for systems in this class, the feedthrough matrix is not equal to zero, a descriptor system representation for the system is introduced to convert the conventional system into a descriptor system with the feedthrough matrix equal to zero. Next, according to the Lyapunov theory for stability of descriptor systems, a sufficient condition for stabilization is obtained. Satisfying this condition is formulated as an optimization problem with matrix inequality constraints for a descriptor system. In order to achieve asymptotic disturbance rejection, the closed-loop linear block-diagonal part of the system is considered as the model to generate the desired inputs. In fact, the closed-loop nonlinear system is approximated with its closed-loop block-diagonal linear part. Then, based on the Lyapunov theory, a sufficient condition for stabilization of the approximation error dynamics is derived. It is shown that by solving the same optimization problem with the same matrix inequality constraints for stabilization, this condition can be satisfied as well. The proposed method is a non-iterative method and very easy to implement. Therefore, in order to design a decentralized controller for the utility boilers in SCL to control the drum level and pressure to achieve asymptotic output tracking, the identified nonlinear model for the utility boiler systems in SCL given in [5] is used. The identified nonlinear model is of order two and captures all the essential features of the actual boiler dynamics including nonlinearities, non-minimum phase behavior, instabilities and load disturbances. Then, by using the proposed method in the paper, a linear decentralized controller with the same order as the order of the system and a decentralized PI controller for the identified model are designed. The designed PI controller is applied to the SYNSIM simulation model to show the effectiveness of the proposed method. SYNSIM is a simulation package in SCL which is developed with the purpose of simulating the real system in SCL and is a general tool for stability analysis.

The rest of this paper is organized as follows: In Section 2, the problem of designing a decentralized robust controller for a specific class of nonlinear systems is considered. Sufficient conditions for closed-loop stability and asymptotic output tracking over the operating-range of the system are obtained. Then, the optimization problem with LMI constraints is obtained, such that by solving the problem, these conditions are satisfied. Section 3 deals with designing a decentralized controller and a decentralized PI controller for the nonlinear model of the boiler. The designed decentralized PI controller is applied to the SYNSIM simulation model. The simulation results illustrate excellent reference input tracking and disturbance rejection. Section 4 contains the conclusions and final remarks.

2. Decentralized robust controller design for a specific class of nonlinear systems

Consider a system given by the following state-space equations
\[
\dot{x}(t) = \begin{bmatrix} A & B \\ C & D \end{bmatrix} x(t) + \begin{bmatrix} E \\ F \end{bmatrix} u(t),
\]
\[
y(t) = \begin{bmatrix} G \\ H \end{bmatrix} x(t) + \begin{bmatrix} I \\ J \end{bmatrix} u(t),
\]
where \( x \in \mathbb{R}^n, u \in \mathbb{R}^m \) and \( y \in \mathbb{R}^m \) are the state, the control input, the measured output vectors respectively and the other matrices and vectors have appropriate dimensions. Suppose the block-diagonal part of the linearized model about its operating point is controllable and observable and has the state-space matrices \( (A, B, C) \). Then, the equations given in (1) can be written as
\[
\dot{x}(t) = Ax(t) + Bu(t) + f(x) + Hu(t),
\]
\[
y(t) = Cx(t) + h(x) + Du(t),
\]
with
\[
f(x) = f(x) - Ax(t),
\]
\[
B = B - H,
\]
\[
h(x) = h(x) - Cx(t).
\]
Assume over the operating-range of the system we have
\[
f(x)^T f(x) <= \gamma^2 x^T x,
\]
\[
h(x)^T h(x) <= \gamma^2 x^T x.
\]
It is assumed that the equilibrium point of the system, not necessarily stable, is or by change of variables can be transferred to the origin. For the given system in (1), the feedthrough matrix \( D \) is not equal to zero. In centralized controller design for a proper system with state-space matrices \( (A, B, C, D) \), without loss of generality, it can be assumed that the system is strictly proper and \( D = 0 \). In fact, by defining a fictitious output \( y(t) = Cx(t) \), the proper system can be converted into a strictly proper system. Then, after designing a controller \( (A_c, B_c, C_c, D_c) \), the matrices of the controller should be modified as
\[
\begin{align*}
\tilde{A}_c &= (A_c - B_c D (I + D_c D)^{-1} C_c), \\
\tilde{B}_c &= (B_c - B_c D (I + D_c D)^{-1} D_c), \\
\tilde{C}_c &= (I + D_c D)^{-1} C_c, \\
\tilde{D}_c &= (I + D_c D)^{-1} D_c.
\end{align*}
\]
There is no assumption on \( D \) except that the matrix \( (I + D_c D)^{-1} \) should exist. However, for decentralized control, this strategy is applicable only if the matrix \( D \) is a block-diagonal matrix with block dimensions compatible with subsystem dimensions. Because, in general and especially for the considered system in this paper, this condition is not the case, we present a new descriptor system representation for system (1) to convert it into a system with \( D = 0 \). In order to do it, a new state vector \( z(t) \) is defined as follows:
\[
z(t) = h(x) + Du(t).
\]
Then, the descriptor system representation for the system has the following state-space equations:
\[
E(x) = \tilde{A}x + \tilde{B}u + \tilde{f} + \tilde{H}u,
\]
\[
y(t) = Cx,
\]
with
\[
E = \begin{bmatrix} I_{n_x} & 0 \\ 0 & 0_{m,n_x} \end{bmatrix}, \quad \tilde{x}(t) = \begin{bmatrix} x(t) \\ z(t) \end{bmatrix}, \quad \tilde{A} = \begin{bmatrix} A & 0 \\ 0 & -I \end{bmatrix},
\]
\[
\tilde{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad \tilde{H} = \begin{bmatrix} H \\ D \end{bmatrix}, \quad \tilde{f} = \begin{bmatrix} f(x) \\ h(x) \end{bmatrix}, \quad \tilde{C} = \begin{bmatrix} C \\ I \end{bmatrix}.
\]
The objective in this paper is to design a linear decentralized controller \( K(z) \) of order \( n_z = n \) with the following state-space equations
\[
\dot{z}(t) = Ax_z(t) + Bu_z(t),
\]
\[
u(t) = Cz_x(t) + Dz_y(t),
\]
with \( (A_z, B_z, C_z, D_z) \) as block-diagonal matrices, for the linear block-diagonal part of the system \( (E, \tilde{A}, \tilde{B}, \tilde{C}) \) such that by applying the controller to the original nonlinear system, the closed-loop system is stabilized. To this end, before deriving a sufficient condition for closed-loop stability, we give some basic definitions for descriptor systems. Consider the descriptor system given by \( (E, \tilde{A}, \tilde{B}, \tilde{C}) \). The existence and uniqueness of solutions to this system is guaranteed
if the pair \((E, A)\) is regular, i.e., if \(\det(sE - A)\) is not identically zero. In addition, the system is called impulse-free if \(\text{deg}\ \det(sE - A) = \text{rank}(E)\) where \(s \in \mathbb{C}\), the set of complex numbers \([1]\). It is clear for the system given in \((7)\) and \((8)\) we have
\[
\det(sE - A) = \det(sI - A),
\]
which is not identically equal to zero. This means that the system is regular. For descriptor system \((7)\) and \((8)\) we have
\[
\text{deg} \ \det(sE - A) = \text{deg} \ \det(sI - A) = n = \text{rank}(E),
\]
which guarantees that the system is impulse-free. Moreover, the regular system is strongly controllable (s-controllable) if
\[
\text{rank}([sE - A \ B]) = n + m.
\]
For system \((7)\) and \((8)\) we have
\[
\text{rank}([sE - A \ B]) = \text{rank}([sI - A \ B]) + m.
\]
It follows that if the pair \((A, B)\) is controllable, then the descriptor system given in \((7)\) and \((8)\) is strongly controllable \([1]\). Observability is the dual of controllability, and thus similar concept exists for the observability of descriptor systems and we conclude if the pair \((A, C)\) is observable, then the descriptor system given in \((7)\) and \((8)\) is strongly observable \([1]\).

2.1. Sufficient condition for closed-loop stability

In this section, a sufficient condition for closed-loop stability of nonlinear system \((1)\) is obtained. The closed-loop descriptor system has the following state-space equations
\[
E_c\dot{x}_c(t) = A_c x_c + B_c \bar{U} C_c x_c, \quad y(t) = C_c x_c,
\]
with
\[
E_c = \begin{bmatrix} I_{n_c} & 0 & 0 \\ 0 & I_{n_x} & 0 \\ 0 & 0 & 0_{m_x}
\end{bmatrix}, \quad A_c = \begin{bmatrix} A & BD_c C & BC_c B_d \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 & 0 & -I \\ 0 & 0 & 0
\end{bmatrix},
\]
\[
A_c = \begin{bmatrix} D_c & C_c \\ B_c & A_c & B_d \end{bmatrix}, \quad C_c = \begin{bmatrix} C & 0 & I \\ 0 & I & 0
\end{bmatrix},
\]
\[
\bar{f}_c = \begin{bmatrix} f(x) \\ h(x)
\end{bmatrix}, \quad \bar{C}_c = [C \ 0 \ I].
\]

Let
\[
F_1 = \begin{bmatrix} I_{n_x} & 0 & 0 \\ 0 & I_{n_x} & 0 \\ 0 & 0 & 0_{m_x}
\end{bmatrix}, \quad F_2 = \begin{bmatrix} I_{n_x} & 0 & 0 \\ 0 & I_{n_x} & 0 \\ 0 & 0 & I_{m_x}
\end{bmatrix}
\]
with
\[
F_1 = F_1^T F_1 = F_1^2, \quad F_2 = F_2^T F_2 = F_2^2, \quad \bar{f}_c = F_2 f_c.
\]

The following Theorem gives a sufficient condition for closed-loop stability.

**Theorem 1.** Consider the system in \((14)\) and \((15)\). The system is stable if there exist solutions \(P \in R^{n_x \times m_x} \) and \(\bar{K} \) to the following system of inequalities:
\[
\bar{A}_c^T P + P^T \bar{A}_c + \bar{d}_f^2 F_1 + 2P^T F_1 P + \bar{C}_c^T \bar{K}^T \bar{K} \bar{C}_c < 0, \tag{19}
\]
\[
E_c^T P > 0, \tag{20}
\]
\[
E_c^T P = P^T E_c, \tag{21}
\]
where
\[
\bar{d}_f^2 = \gamma_f^2 + \gamma_h^2. \tag{22}
\]

**Proof.** Consider the following Lyapunov function for descriptor system \((14)\) and \((15)\) \([3]\)
\[
V(x_c) = \bar{x}_c(t)^T E_c X E_c \bar{x}_c = \bar{x}_c(t)^T E_c X E_c + E_0 Q \bar{x}_c(t), \tag{23}
\]
with
\[
X = X^T > 0, \tag{24}
\]
\[
E_c^T E_0 = 0, \tag{25}
\]
and any matrix \(Q\) of compatible dimensions. By defining
\[
P = X E_c + E_0 Q, \tag{26}
\]
the Lyapunov function is given by \([3]\)
\[
V(x_c) = \bar{x}_c(t)^T F_1^T P \bar{x}_c(t). \tag{27}
\]
It is clear that \(E_c^T P\) should be a positive semi-definite matrix. This means conditions \((20)\) and \((21)\) should be satisfied. The derivative of the Lyapunov function is given by
\[
\dot{V}(x_c) = x_c(t)^T E_c X E_c \dot{x}_c + x_c(t)^T E_c X \dot{x}_c = x_c(t)^T E_c X E_c + x_c(t)^T E_c \dot{X} E_c \dot{x}_c
\]
\[
= x_c(t)^T \bar{A}_c^T P + \bar{A}_c^T P \bar{X}_c + \bar{C}_c^T \bar{K}^T \bar{K} \bar{C}_c + \bar{f}_c^T \bar{f}_c
\]
\[
= x_c(t)^T \bar{A}_c^T P + \bar{A}_c^T P \bar{X}_c + \bar{C}_c^T \bar{K}^T \bar{K} \bar{C}_c + \bar{f}_c^T \bar{f}_c.
\]

Because
\[
P^T \bar{K} \bar{C}_c + \bar{C}_c^T \bar{K}^T P = P^T F_1 \bar{K} \bar{C}_c + \bar{C}_c^T \bar{K}^T F_1 \bar{P}
\]
\[
\leq P^T F_1 \bar{P} + \bar{C}_c^T \bar{K}^T \bar{K} \bar{C}_c, \tag{29}
\]
\[
\bar{f}_c^T \bar{f}_c = \bar{f}_c^T \bar{P} \bar{X}_c + \bar{X}_c^T \bar{P} \bar{f}_c \leq \bar{f}_c^T \bar{P} \bar{f}_c + \bar{X}_c^T \bar{P} \bar{P} \bar{f}_c
\]
\[
\leq \gamma_f^2 \bar{x}_c^T \bar{X}_c + \bar{x}_c^T \bar{P} \bar{P} \bar{X}_c \tag{30}
\]
then, the derivative of the Lyapunov function satisfies the following condition
\[
\dot{V}(x_c) \leq \bar{x}_c(t)^T \bar{A}_c^T P + \bar{A}_c^T P \bar{X}_c + \bar{d}_f^2 F_1 + 2P^T \bar{P} \bar{P} + \bar{C}_c^T \bar{K}^T \bar{K} \bar{C}_c \bar{x}_c, \tag{31}
\]
and it is negative definite if condition \((19)\) is satisfied, which completes the proof. \(\square\)

According to the result of Theorem 1, using the Schur complement Theorem designing a dynamic linear controller to stabilize the nonlinear system in \((1)\) over its operating-range can be converted into solving the following optimization problem to find the matrix \(P\) and the controller state-space matrices.
Until now, we have shown designing a decentralized controller to stabilize the original nonlinear system in (1) over its operating-range is converted into finding matrices $A_c, B_c, C, D_c, P_1, P_2, P_3$ such that the conditions in (33) and (39) for $\gamma \leq 1$ are satisfied. The problem in (33) is a bilinear matrix inequality problem and it is not affine in the state-space matrices of the controller. Hence, we should use change of variables proposed in [8]. By pre and post-multiplying matrix (33) by diag$(F_1, I, I, I)$ and diag$(F_1, I, I, I)$, where $F$ is an appropriate matrix which will be derived later, we get

$$F^T(A_c^T(A^T P + P^T A_c) F + 2F^T P F_2) \gamma_1 F_b^T F_1 + F^T C^T K^T H^T < 0. \tag{40}$$

Let $n$ be the number of states of the system (size of $A$), and let $n_c = n$ be the order of the controller, by partitioning $P_1$ and $P_1^{-1}$ as [8]

$$P_1 = \begin{bmatrix} \mathbf{Y} & \mathbf{N} \\ \mathbf{N}^T & * \end{bmatrix}, \quad P_1^{-1} = \begin{bmatrix} \mathbf{X} & \mathbf{M} \\ \mathbf{M}^T & * \end{bmatrix},$$

where $\mathbf{X}$ and $\mathbf{Y}$ are $n \times n$ and symmetric, positive definite and block-diagonal with block dimensions compatible with subsystem dimensions, from $P_1 P_1^{-1} = I$,

we infer

$$P_1 \begin{bmatrix} \mathbf{X} \\ \mathbf{M}^T \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix}. \tag{42}$$

which leads to

$$P_1^2 = P_1, \tag{44}$$

and

$$P_2 = \begin{bmatrix} I & \mathbf{Y} \\ 0 & \mathbf{N}^T \end{bmatrix}. \tag{46}$$

We define the change of controller variables as follows [8]:

$$\hat{A} = N A M^T + N B_c C X + Y B_c M^T + Y (A + B D_c) C X,$$

$$\hat{B} = N B_c + Y B D_c,$$

$$\hat{C} = C M^T + D_c C X,$$

$$\hat{D} = D_c.$$

By pre and post-multiplying $P_1$ by $P_1^T$ and $P_1$ we get

$$P_1^T P_1 P_1 = P_1^2 P_2 = \begin{bmatrix} \mathbf{X} & \mathbf{I} \\ \mathbf{I} & \mathbf{Y} \end{bmatrix}. \tag{48}$$

Now, let define the matrix $F$ as

$$F = \begin{bmatrix} P_1 & 0 \\ 0 & I \end{bmatrix}, \tag{49}$$

$\mathbf{A}_c$ partitioned as

$$\mathbf{A}_c = \begin{bmatrix} A_1 & A_2 \\ 0 & -I \end{bmatrix}, \quad A_1 = \begin{bmatrix} A + B D_c C & B C_c \\ B_c & A_c \end{bmatrix}, \quad A_2 = \begin{bmatrix} B D_c \end{bmatrix}, \tag{50}$$

then

$$F^T(A_c^T P + P^T A_c) F \begin{bmatrix} \Pi_1^T(\hat{A}_1^T P_1 + P_1 \hat{A}_1) \Pi_1 & \Pi_2^T(P_1 \hat{A}_2 - P_2^T) \\ \Pi_1^T(\hat{A}_2^T P_1 - P_2 \hat{A}_2) \Pi_1 & -P_3 - P_3^T \end{bmatrix}. \tag{51}$$

By introducing the new LMI variable

$$L = P_1 \Pi_1 = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}, \tag{52}$$

and considering

$$\Pi_1^T(\hat{A}_1^T P_1 + P_1 \hat{A}_1) \Pi_1 = \begin{bmatrix} \mathbf{A} + \mathbf{X}^T + \mathbf{B} \hat{C} + (\hat{B} \mathbf{C})^T & \mathbf{A} + \mathbf{X}^T + \mathbf{B} \hat{D} \mathbf{C} \end{bmatrix},$$

and

$$\Pi_2^T(\hat{A}_2^T P_1 - P_2 \hat{A}_2) \Pi_1 = \begin{bmatrix} \mathbf{B} \hat{D} \mathbf{C} \\ \mathbf{B} \end{bmatrix}. \tag{54}$$

$$F_1 F = \begin{bmatrix} \mathbf{X} & \mathbf{I} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{55}$$

$$F_2 F = \begin{bmatrix} \mathbf{I} & \mathbf{Y} & 0 \\ 0 & 0 & 0 \\ L_1 & L_2 & P_3 \end{bmatrix}, \tag{56}$$

and

$$\mathbf{R}_K C F = \begin{bmatrix} \mathbf{H} \hat{C} & \mathbf{H} \mathbf{D} & \mathbf{H} \hat{D} \\ 0 & \mathbf{D} \mathbf{C} & \mathbf{D} \mathbf{D} \end{bmatrix}. \tag{57}$$

the problem in (32)–(35) can be converted into
In the above equations, the LMI variables are shown by boldface letters, and the LMI variables $\bar{A}$, $\bar{B}$, $\bar{C}$, $D$, $X$, and $Y$ are considered to be block-diagonal matrices with dimensions compatible with subsystem dimensions. By solving optimization problem (58) with the LMI constraints, the controller construction proceeds as follows [8]: find nonsingular block-diagonal matrices $M$ and $N$ to satisfy
\[ MN^T = I - XY, \]
and define the controller by
\[ D_c = \bar{D}_c, \]
\[ C_c = (\bar{C} - D_cCX)M^{-T}, \]
\[ B_c = N^{-1} \bar{B} - YBD_cX. \]
\[ A_c = N^{-1} (\bar{A} - NB_cCX - YBC_cX - Y(A + BD_cX)M^{-T}]. \]

Remark 1. In optimization problem (58), the LMI variables $\bar{A}$, $\bar{B}$, $\bar{C}$, $D$, $X$, and $Y$ are taken to be block-diagonal. In designing decentralized controllers using non-iterative LMI based methods, selecting block-diagonal Lyapunov matrices, or block-diagonal LMI variables is a common practice [9]. In fact, the introduced conservatism due to the block-diagonal variables is the price that we pay for decentralization.

Remark 2. The LMI framework described in (58) seems to involve a rather large number of variables. In fact, in deriving the LMI framework from Eq. (19) by linearization procedure introduced in [8], only Schur complement, and congruence transformation are used. So, it should be stressed that the linearization procedure does not introduce any conservatism [8]. This means, the LMI frame in (58) is as conservative as the inequality in (19).

2.2. Asymptotic output tracking

In this section, we show by solving optimization problem (58), that asymptotic output tracking can be achieved. In asymptotic output tracking, a common method is to minimize the error between the output of the closed-loop system and the desired input in the presence of input disturbances. The desired input can be generated by a desired known model. We consider the block-diagonal part of the closed-loop transfer matrix of the system as the desired model. Then, by appropriately designing a controller for the block-diagonal part, and at the same time minimizing the error between this part and the overall closed-loop transfer matrix, it is possible to obtain asymptotic output tracking. Suppose the external disturbance is applied to the system through a vector $d(x)$ as follows:
\[ \dot{x}(t) = f(x) + Bu(t) + d(x)d, \]
\[ y(t) = h(x) + Du(t), \]
where $d$ is an external disturbance, the vector $d(x)$ is a vector of nonlinear functions and over the operating-range of the system we have
\[ d(x)^T d(x) \leq \gamma_0^2 x^T F_1 x. \]

Then, the closed-loop system has the following state-space equations
\[ E_c \dot{x}_c(t) = \bar{A}_c \bar{x}_c + f_c + \bar{H} \bar{K} C_c \bar{x}_c + d_c(x)d, \]
\[ y(t) = C_c \bar{x}_c, \]
with
\[ d_c(x) = \begin{bmatrix} d(x) \\ 0 \\ 0 \end{bmatrix}. \]

Now, consider the closed-loop system in (14) and the following approximated linear system
\[ E_c \dot{x}(t) = \bar{A}_c \bar{x}(t). \]

Let the error of approximation be
\[ e(t) = \bar{x}_c(t) - \bar{x}(t), \]
then
\[ E_c \dot{e}(t) = \bar{A}_c e(t) + \bar{f}_c + \bar{H} \bar{K} C_c \bar{x}_c(t). \]

Now, consider the following Lyapunov function given by
\[ V(e, \bar{x}_c) = e^T \hat{P} e + \hat{X}_c'^T \hat{P} \hat{X}_c, \]
where $P$ is defined in (38) and (39). Following the discussion given in the previous subsection, the derivative of the Lyapunov function satisfies the following equation:
\[ V(e, \bar{x}_c) \leq e^T \hat{A}_c \hat{P} e + \hat{P} \hat{A}_c + \hat{A}_c^T \hat{P} e + \hat{X}_c'^T \hat{P} \hat{X}_c + 2\gamma_1^2 F_1 \]
\[ + 2\gamma_2^2 F_2 P + 2\gamma_3 \hat{K}^T \bar{H} \hat{K} C_c \hat{x}_c. \]

It is clear that by solving the problem in (32)-(35) or equivalently the optimization problem in (58), we have
\[ \hat{A}_c^T \hat{P} + \hat{P} \hat{A}_c + 2\gamma_2^2 P < \frac{\gamma_1^2}{\gamma_2} F_1 - \frac{1}{\gamma_2} \hat{C}_c^T \hat{H}^T \hat{H} \hat{K} C_c \hat{x}_c. \]

By substituting Eq. (70) into (69), the derivative of the Lyapunov function in (69) is negative definite if
According to definitions (15) and (17) and the closed-loop system can be approximated fairly with its condition (72) is satisfied, the error dynamics in (67) is stabilized to show that

\[
\gamma < \frac{1}{2}.
\]  

(72)

This means that by solving optimization problem (58) that if condition (72) is satisfied, the error dynamics in (67) is stabilized to show that

\[
\gamma < \frac{1}{2}.
\]  

(72)

Now, we assume the closed-loop system can be approximated by the system in (65). Then the closed-loop original system under the decentralized controller can be approximated by the following state-space equations

\[
E_c \dot{x}_c(t) = \bar{A}_c x_c + \bar{d} d,
\]  

\[
y(t) = \bar{C}_c x_c.
\]  

(73)

Now, we consider the problem of disturbance attenuation. According to definitions (15) and (17)

\[
\bar{C}_c = C_c F_2.
\]  

(74)

Because

\[
\| \bar{d}^T d \| \leq \| \bar{d}^T \bar{C}_c F_2 \|.
\]  

(75)

then, the \( L_2 \) gain of the transfer function form the external disturbance to the output vector, \( G_{ed} \) satisfies the following condition [4]

\[
\| G_{ed} \|_{\infty} \leq \| \bar{C}_c F_2 (sE_c - \bar{A}_c) \| \gamma_d F_1 \|_{\infty}.
\]  

(76)

By combining Eqs. (33) and (79), condition (79) is satisfied if

\[
\gamma_d^2 F_1 + P^T F_1 P - 2 P^T F_2 P - \frac{\gamma_d^2}{7} \frac{1}{\gamma_d^2} C_c \bar{A}_c \bar{C}_c \| \gamma_d F_1 \|_{\infty} < 0.
\]  

(80)

According to the definition of the matrices \( F_1 \) and \( F_2 \) in (16) and (17)

\[
P^T F_2 P - 2 P^T F_1 P = P^T (F_1 - 2 F_2) P \leq 0.
\]  

(81)

So, it is clear that conditions (79) are satisfied if

\[
\gamma < \frac{\gamma_d}{\gamma_d^2}
\]  

(82)

\( P_1 = P_1^T > 0 \)

Based on the discussion given so far, it can be concluded by solving optimization problem (58), provided \( \gamma \) is small enough

\[
\gamma < \min \left( \frac{1}{2}, \frac{\gamma_d^2}{\gamma_d^2} \right).
\]  

(83)

the closed-loop system is stable with a good asymptotic output tracking in the presence external disturbances.

Up to now, we have shown by solving the local LMI optimization problems in (58), the closed-loop system is stable and the effect of the external disturbances is attenuated. The derived conditions may be conservative and we do not need such conservatism. However, it gives us an idea that by solving problem (58), it is possible to achieve closed-loop stability and acceptable disturbance rejection. So, we relax the condition \( \gamma < 1 \) and conclude by solving the following optimization problem it is possible to stabilize the system with good performance provided \( \gamma \) is small enough.

\[
\begin{aligned}
&\text{min } \gamma \\
&\text{subject to} \\
&\| F_2 (sE_c - \bar{A}_c) \| \gamma_d F_1 \|_{\infty} < \gamma.
\end{aligned}
\]  

(77)

By virtue of Bounded Real Lemma for descriptor systems [7], minimizing norm (78) is finding a matrix \( P \) as given in (38) and (39) such that

\[
\bar{A}_c^T P + P^T \bar{A}_c + \gamma_d^2 F_1 + P^T F_1 P < 0.
\]  

\( E_c^T P \geq 0. \)  

(79)

2.3. Decentralized PI design

In this section, the proposed method for designing a decentralized controller for the system given in (14) and (15) is modified to design a decentralized PI controller. Suppose the objective is designing a decentralized PI controller for the system \((A, B, C)\) can be converted into designing a static controller

\[
\bar{F} = [K_1 \ K_2].
\]  

(86)
for the augmented system with the state-space matrices \( \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \). Following the discussion leading to the matrix inequality problem given in (33), and by defining the matrices
\[
\bar{A} = \begin{bmatrix} A & 0 & 0 \\ C & 0 & I \\ 0 & 0 & -I \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} C & 0 & I \\ 0 & I & 0 \end{bmatrix},
\]
we can show by solving the following optimization problem with bilinear matrix inequality (BMI)

\[
\min \gamma \\
\begin{bmatrix} (\bar{A} + \bar{B} \bar{C})P + P(\bar{A} + \bar{B} \bar{C})^T - 2P^2F_2 \gamma P \gamma^T \bar{C} \bar{P} \bar{C}^T P^T \\
* & -\gamma I & 0 & 0 \\
* & * & -\gamma I & 0 \\
* & * & * & -\gamma I \end{bmatrix} < 0, \quad P = P^T > 0.
\]

with
\[
\bar{H} = \begin{bmatrix} H \\ 0 \\ D \end{bmatrix}.
\]

the designed PI controller stabilizes the closed-loop system with a good asymptotic tracking, provided \( \gamma \) is small enough. To solve the optimization problem given in (88), the improved iterative linear matrix inequality (ILMI) algorithm proposed in [2] can be used. According to the algorithm presented in [2], by an iterative procedure, in an iteration by assuming the matrix \( \bar{P} \) as a known matrix we derive the matrix \( P \). Then we can employ the matrix \( P \) to derive the matrix \( \bar{P} \) in the next step.

3. Utility boiler

The utility boilers in Syncrude Canada are water tube drum boilers which comprise two separate systems, the water side and the fire side of the boiler. In water side system, preheated water from the economizer is fed into the steam drum, then flows through the downcomers into the mud drum. The mud drum distributes the water to the risers, where the water is heated to saturation conditions. The saturated steam-water mixture then reenters the steam drum in which the steam is separated from the water and exits the steam drum into the primary and secondary super heaters. For proper functioning of the boiler systems, a steam pressure of the 6.306-MPa header must be maintained despite variations in the amount of steam demanded by users. The amount of water in the steam drum must be maintained at the desired level to prevent overheating of drum components or flooding of steam lines [11].

In this paper, the main objective is to design a controller to maintain the drum pressure and level. The system to be controlled is a 2 × 2 system. The input variables of the system are \( u_1 \) feedwater flow rate (kg/s); \( u_2 \) fuel flow rate (kg/s) and the principal output variables are \( y_1 \) drum level (m); \( y_2 \) drum pressure (kPa). In order to design a decentralized controller for the utility boiler system, the identified nonlinear model of the system obtained in [5] is used. According to the field experience, the utility boilers in SCL work mainly in three typical operating points [11], which are low load, normal load, and high load operating points. The parameters of the different operating points are listed in Table 1.

<table>
<thead>
<tr>
<th>Steam flow rate</th>
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3.1. The model

The identified model for the utility boiler is given as follows: [5]

\[
\begin{align*}
\dot{x}_1(t) &= u_1 - 0.03 \sqrt{x_2^2 - (6306)^2}, \\
\dot{x}_2(t) &= (-1.8506 \times 10^{-7} x_2 - 0.0024) \sqrt{x_2^2 - (6306)^2} - 0.0404u_1 + 3.025u_2, \\
y_1(t) &= 0.010157116x_1 + 1.8386 \times 10^{-4} \sqrt{x_2^2 - (6306)^2} - 0.001u_1 + 0.019814u_2 - 6.1982, \\
y_2(t) &= x_2.
\end{align*}
\]

The parameters of the different operating points for the identified model are listed in Table 2.

In addition, the following limit constraints exist for the three control variables:

\[
0 \leq u_1 \leq 120, \\
0 \leq u_2 \leq 7, \\
-0.017 \leq u_2 \leq 0.017.
\]

3.2. Controller design and simulation results

In this section a decentralized controller for the model given in (89) is designed such that the closed-loop system tracks the related inputs and has a good disturbance rejection. The model at normal load is considered as the nominal system and the change of the model at the two other operating points is considered as uncertainty. Selecting the normal load model as the nominal plant and the change of the model as a known matrix \( \Delta \), the model can be approximated as follows:

\[
\begin{align*}
\dot{x}_1(t) &= u_1 - 0.03 \sqrt{x_2^2 - (6306)^2}, \\
\dot{x}_2(t) &= (-1.8506 \times 10^{-7} x_2 - 0.0024) \sqrt{x_2^2 - (6306)^2} - 0.0404u_1 + 3.025u_2, \\
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y_2(t) &= x_2.
\end{align*}
\]

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The identified model for the utility boiler is given as follows: [5]
\[
\dot{x}_1(t) = 6.4457 \times 10^{-3} u_1 + 3.0263 \times 10^{-7} x_2^2 - 5.78094 \times 10^{-4} x_2,
\]
\[
\dot{x}_2(t) = -0.0110621 x_2 + 3.025 u_2 - 6.993 \times 10^{-9} x_2^2 + 5.876 \times 10^{-6} x_2^2 - 0.0404 u_1,
\]
\[
y_1(t) = 0.010157116 x_1 - 3.2 \times 10^{-7} x_2^2 + 5.4903 \times 10^{-4} x_2 - 0.001 u_1 + 0.019814 u_2,
\]
\[
y_2(t) = x_2.
\]  
(91)

over the operating-range
\[-18.8231 \leq u_1 \leq 14.5569,\]
\[-1.0259 \leq u_2 \leq 0.8139,\]
\[1.053 \leq x_1 \leq -22.469,\]
\[-188.1 \leq x_2 \leq 182.3.\]

Now, the equilibrium point of the model in (91) is the origin. The state-space matrices of the linearized model of (91) about the origin are as follows:

\[A_0 = \begin{bmatrix} 0 & -0.0006 \\ 0 & -0.0111 \end{bmatrix}, \quad B_0 = \begin{bmatrix} 0.0064 \\ -0.0404 \\ 3.0250 \end{bmatrix}, \quad C_0 = \begin{bmatrix} 0.0101 \\ 0.0005 \\ 0 \end{bmatrix}, \quad D_0 = \begin{bmatrix} -0.001 \\ 0.0198 \end{bmatrix}.\]  
(92)

It can be seen, the linearized system has a pole at the origin and a stable pole at \(-0.0111\) associated with water and pressure dynamics, respectively. The system has a transmission zero at \(0.0886\) which shows the non-minimum phase behavior of the system.
system. The unstable transmission zero limits the achievable bandwidth of the closed-loop system. It follows in designing the controller and solving problem (84), an LMI constraint should be added to restrict the closed-loop pole locations according to zero location [10]. Following the controller design methodology given in the previous section, for this system we have

$$A = \begin{bmatrix} 0 & 0 \\ 0 & -0.0111 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0064 & 0 \\ 0 & 3.025 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.0102 & 0 \\ 0 & 1 \end{bmatrix},$$

$$H = \begin{bmatrix} 0 & 0 \\ -0.004 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} -0.001 & 0.0198 \\ 0 & 0 \end{bmatrix},$$

$$f(x) = \begin{bmatrix} -5.7809 \times 10^{-4} x_1 + 3.0263 \times 10^{-7} x_2^2 \\ -6.993 \times 10^{-4} x_1^2 + 5.876 \times 10^{-8} x_2^2 \end{bmatrix},$$

$$h(x) = \begin{bmatrix} -3.2 \times 10^{-7} x_2^3 + 5.4903 \times 10^{-3} x_1 \\ 0 \end{bmatrix}. \quad (94)$$

By solving the optimization problem given in (84), the designed controller

$$K(s) = \begin{bmatrix} \frac{2.339e^{-5s} - 1293}{s + 0.8253} & 0 \\ 0 & \frac{0.009337s - 0.04828}{s + 0.817} \end{bmatrix},$$

stabilizes the original nonlinear system. But, in order to have a good reference input tracking a decentralized PI controller can be designed.

$$\gamma_i = 0.0016. \quad (97)$$

By solving the BMI problem in (88), we obtain

$$K(s) = \begin{bmatrix} 0.0074 & 0 & 0.0023 & 0 & 0 & 0 \\ 0 & 0.0256 & 0 & 0.0004 & 0 & 0 \\ 0.0023 & 0 & 0.0014 & 0 & 0 & 0 \\ 0 & 0.0004 & 0 & 0.0003 & 0 & 0 \\ 0.0001 & 0 & 0 & 0 & 0.8753 & 0 \\ 0 & 0 & 0.0003 & 0 & 0 & 0.8714 \end{bmatrix},$$

$$K(s) = \begin{bmatrix} -(182 + 0.91) & 0 \\ 0 & -(0.015 + 0.0041) \end{bmatrix}, \quad \gamma = 1.1. \quad (99)$$

and

$$\gamma_i = 0.0016. \quad (97)$$

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$$K(s) = \begin{bmatrix} -(182 + 0.91) & 0 \\ 0 & -(0.015 + 0.0041) \end{bmatrix}, \quad \gamma = 1.1. \quad (99)$$
The calculated matrix $P$ satisfies the condition given in (38) and (39). The value of $\gamma$ is a little greater than one. Despite of this value, because of the conservatism of the obtained sufficient conditions, simulation results of running the closed-loop system shows good input tracking and disturbance rejection. The designed controller is applied to the SYNSIM simulation model. In order to compensate the constraints given in (90) on control signals, as explained in [12], these constraints can be ignored at the design stage, and the effects of the constraints are compensated after the controller design using anti-windup bump-less transfer (AWBT) techniques [12]. Applying the designed decentralized controller with decentralized AWBT compensation to the nonlinear system, Figs. 1–3 show the responses of the closed-loop system in switching from normal load to high load, from high load to normal load, and from normal load to low load, respectively. These figures show good set point tracking of the closed-loop system. Figs. 4–6 show the related control signals and that the constraints given on control signals are satisfied.

3.2.1. Disturbance rejection

The most significant disturbances in the drum pressure and level control are changes in steam flow due to the variation in demand from the steam header. Suppose the closed-loop system operates at the normal load. Then, at $t = 1000$ s the steam flow increases up to 10% of its steady-state value at normal load. Fig. 7 shows the response of the closed-loop system to the increase in the steam flow. As it can be observed from Fig. 7, there is a strong shrink and swell effect in response of the level. The drum pressure decreases because of the increased steam flow. However, these changes are temporary and the controller controls the closed-loop system very well to maintain the drum level and pressure at their steady-state values. In order to do it, the control signals as shown in Fig. 8 are increased to compensate for the increase in steam flow. But the control signals still satisfy the limitations given in (90). These observations illustrate the effectiveness of the designed controller.

3.2.2. Comparison results

In [5], a new method based on IMC (internal model control) strategy is proposed to design a decentralized PI controller for nonlinear utility boilers in SCL. In this section, using the proposed method in [5] a decentralized PI controller is designed and applied to the SYNSIM simulation model. Figs. 9 and 10 compares the step responses and the related control signals for the designed
decentralized PI controllers in this paper and in [5] in switching from normal load to high load. It can be observed the new controller has improved the tracking problem. However, the control signal for pressure control in [5] is less than the control signal for the controller designed in this paper. Figs. 11 and 12 compare the disturbance rejection problem for both controllers. Again the new strategy shows a slight improvement in the disturbance rejection.

Fig. 7. Disturbance rejection at normal load (output signals).

Fig. 8. Disturbance rejection at normal load (control signals).

Fig. 9. Switching from normal load to high load, (solid: new controller, dashed: IMC based controller).

Fig. 10. Switching from normal load to high load, (control signals) (solid: new controller, dashed: IMC based controller).
4. Conclusion

In this paper, a method for robust decentralized controller design for a class of nonlinear systems is proposed. Systems in this class have non-zero feedthrough matrix $D$. By using a descriptor system representation for the system, it is possible to convert the system into a descriptor system with $D = 0$. Then, sufficient conditions for closed-loop stability and asymptotic output tracking over the operating-range of the system are obtained. Satisfaction of these conditions is formulated as an optimization problem with LMI constraints which by solving it a decentralized controller for the system can be calculated. By modifying the proposed methodology, it is possible to design a decentralized PI controller for the system. Then, for an identified nonlinear model of the utility boilers in SCL a decentralized PI controller is designed. Applying the designed controller to the SYNSIM simulation model shows good input tracking and disturbance rejection.

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References