Robust stabilization of nonlinear interconnected systems with application to an industrial utility boiler

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Abstract

This paper presents a new scheme for robust stabilization of nonlinear interconnected systems, based on linear matrix inequalities (LMIs). The fact that the improvement in stability is significant and the controller uses only the output information of plant leads to the name robust output feedback control. The control design is formulated as a convex optimization problem, which makes it computationally tractable, when the problem size increases. The controller concept is then evaluated on a natural circulation drum boiler (utility boiler), where the nonlinear model describes the complicated dynamics of the drum, downcomer, and riser components. The linearized system is non-minimum phase and has two poles at the origin, which are major sources of interaction, bandwidth limitation and instability. Simulation results are presented which show the effectiveness of the proposed control against instabilities following sudden load variations. The control is also effective for steady state operation.

Keywords: LMI applications; Output feedback; Nonlinear systems; Boiler systems; Control applications; Robustness

1. Introduction

Interconnected systems can be found in diverse fields such as power systems, space structures, robotics, and manufacturing processes. During the last few decades, many researchers in the field of power systems have been devoted to decentralized robust control strategies (Illic & Zaborszky, 2000; Kundur, 1994; Siljak, 1978, 1991; Siljak, Stipanovic, & Zecevic, 2002; Zecevic, Neskovic, & Siljak, 2004). This is due to its simplified design procedure, failure tolerance capability, less communication overhead and its wide acceptance by the operators in industry. There are also spectacular changes in the power plants caused by increasing demand for rapid changes in power generation. This deregulation leads to more rigorous requirements on the control systems for the processes. Also in a deregulated environment, systems tend to be more stressed and the load distribution is in fact impossible for the operators to foresee. Under these situations, it is necessary to develop robust control strategies that can keep the processes operating well under large changes in operating conditions and can defend them against both large and small disturbances, whenever they may arise.

Recently, there has been a strong research effort towards the development of decentralized robust control strategies, including decentralized turbine/governor control (Jain & Khorrami, 1997; Jiang, Chai, Dorsey, & Qu, 1997; Qu, Dorsey, Bond, & McCalley, 1992; Siljak et al., 2002; Wang, Hill, & Guo, 1998) and decentralized excitation control (Cai, Qu, & Dorsey, 1996; Chapman, Illic, King, Eng, & Kaufman, 1993; Guo, Wang, & Hill, 2000; King, Chapman, & Illic, 1994; Xie, Xie, Wang, & Guo, 2000; Zecevic et al., 2004). A linear feedback law based on the solution of parameterized Riccati equations for each subsystem was developed in Wang et al. (1998). These ideas have been extended to robust exciter control based on the concept of direct feedback linearization, which transforms the original nonlinear model into a linear one. This kind of transformation makes the control design straightforward, but the
real-time implementation becomes complicated by the fact that the resulting controller is nonlinear. There are also several results introduced in Cai et al. (1996), Jiang et al. (1997); and Cai, Qu, Gan, and Dorsey (2000), where linear feedback control law is based on appropriately chosen Lyapunov functions to produce lower bounds for local gains. This method, however, incorporates a quadratic term in the model which cannot be properly used in analysis. In Cai et al. (1996), this issue was resolved by introducing a partial linearization, which leads to eliminating the quadratic term from the model. An efficient approach based on linear matrix inequalities (LMIs) was then proposed in Zecevic et al. (2004), which does not require any approximations and removes most of the restrictions in previous design methods.

The main objective of this paper is to apply the multifaceted LMI tools (Boyd, Ghaoni, Feron, & Balakrishnan, 1994; Ghaoui & Niculesu, 2000) to the design of a robust decentralized output feedback control strategy, and its application to an industrial utility boiler. A method of designing a decentralized static state feedback control strategy based on LMIs has been developed in Siljak et al. (2002), Siljak and Zecevic (2004), Zecevic et al. (2004). In Siljak and Zecevic (2004), new ideas were also presented for broadening the scope of control design under decentralized information structure constraints. Both structural and algebraic features of decentralized feedback are considered, with LMIs as a common tool.

From an application perspective, since boilers are so common there are many modeling efforts. There are results available in literature (Bouamama, Medjaher, Samantaray, & Staroswiecki, 2006), which deals with a tool called bond graph modeling. This can be used to represent the model structures of such processes along with their control system instrumentation. The structural analysis of this model can be used to obtain fault signatures and to identify the hardware redundancies in the sensor placement. For control purpose, there are also black box models of reasonable complexity that describe the system well in specific operating conditions (Li, Chen, Marquez, & Gooden, 2006). Since black box models are only valid for specific operating points, a good nonlinear process model is important for model-based control.

In this paper, a new algorithm has been proposed for designing dynamic output feedback controllers. An important distinction of this algorithm within the vast literature on robust control is that the method determines a linear controller which stabilizes the system, and, at the same time, maximizes the class of uncertain perturbations which can be tolerated by the closed loop system. These perturbations may arise within the subsystem itself and also in the interconnections. Unlike previous work, there is no assumption that all the states are available for feedback (Siljak & Stipanovic, 2000; Siljak et al., 2002), the perturbations satisfy matching conditions (Leitmann, 1993), or the LTI part of the nonlinear model is stable (Kokotovic & Arcak, 1999). It is shown that by relaxing these assumptions, an algorithm can be developed to solve not only this complex problem, but also to extend its applicability to large scale interconnected systems. To this end, a new approach is presented for designing decentralized dynamic output feedback control strategy for large scale systems. It requires no communication between the subsystem controllers, and also maximizes the interconnection bounds to increase the robustness of the closed loop system against uncertain perturbations. In addition to this, the control law guarantees stability of the overall closed loop system when subsystems are disconnected and again connected in various ways to perform programmed tasks. This attractive feature of connective stability and no additional requirement of observer design can be a possible source of attraction to both theorists and practitioners. The design method has also an added flexibility of accommodating various design constraints involving pole placements, bounds on nonlinear terms, and the structure of controller matrices, to mention a few. All controllers are linear; so their implementation is straightforward and bears low computational cost.

The name of the proposed control is robust output feedback control. The term robust output feedback is one that requires careful explanation. About 30% of the emergency shutdowns in pressurized water reactor (PWR) plants are caused by poor level control of the steam water level. This is due to the complicated shrink and swell dynamics, which makes the control problem difficult. It is responsible for a non-minimum phase behavior which changes significantly with the operating conditions. In this paper, the control design is based on the complex nonlinear model for steam generation systems (Astrom & Bell, 2000), which captures the key dynamical properties over a wider operating range. The physical parameters of the model are obtained from Syncrude Canada Ltd. (SCL) integrated energy facility located in Mildred Lake, Alberta, Canada. The state variables of the model are: the total volume of water in the drum \((V_w)\), the drum pressure \((p)\) which represents the total energy, the steam-mass fraction in the riser \((a_r)\) which captures the distribution of steam and water, and the steam volume in the drum \((V_{sd})\). The deviation of the drum level from its normal operating point can be measured by differential pressure transmitter, but the volume of steam, volume of water and steam-mass fraction are not measurable quantities. Moreover, the linearized system is non-minimum phase and has two poles at the origin (associated with water dynamics and pressure dynamics), which are major sources of interaction, bandwidth limitation and instability. Hence, from an industrial point of view, there is a need for a control strategy which maintains stability and acceptable performance under normal operating conditions, which is easily implementable and which is without the additional cost of an observer. In a highly nonlinear interconnected system such as the one described here, guaranteeing a satisfactory performance over a wide range of operating condition is a difficult task. There is always a disturbance waiting in the
wings that will defeat any reasonable control strategy. The 6.306 MPa header in 280 MW Syncrude utility plant exhibits oscillatory modes during sudden load variations in the system. The load change causes drum pressure to rise or fall abruptly, which also leads to shrinking and swelling in the drum level. The reason can be attributed to the nonlinear characteristics of the plant, and the coupling effects which originate from overlooking the multivariable nature of the plant. Under deregulation, what is required is a control strategy that will counteract disturbances, maintain the drum level and drum pressure at its set point and also bring robustness of the system against uncertain nonlinearities and neglected dynamics. The proposed control robustly stabilizes the system by output feedback in this sense.

This paper uses LMIs as the main tool for the controller design. From a control engineering point of view (Ghaoui & Niculesu, 2000; Skogestad & Postlethwaite, 2005), one of the main advantages of using LMIs is that they can be used to solve problems which involve several matrix variables, constraints, and, moreover different structures can be imposed on these matrix variables (in decentralized control design). Also, they are flexible and it is relatively straightforward to pose a variety of linear, quadratic or nonlinear optimization problems as LMI problems. This can be done by using the well known Schur’s complement method, S-procedure, and other variable transformation techniques. Very often LMI methods can be applied in cases where customary methods either fail, or struggle to find a solution. Moreover, it is possible to unite many previous results in a common framework like μ optimal control, multiobjective $H_2/H_\infty$ optimization, and robust model predictive control (RMPC), to mention just a few.

The proposed method has a number of advantages, among which the following points can be singled out:

1. The control design is devised as a convex optimization problem, which is computationally tractable, and guarantees the existence of a solution.
2. The proposed algorithm is applicable to cases where all the states are not available for feedback. The controller is designed without the additional cost of an observer. It also maximizes the class of perturbations that can be tolerated by the closed loop system. Moreover, the control law results in a connectively stable system, thus overcoming the barrier of instability caused by sudden structural perturbations.
3. Additional pole placement constraints avoid fast controller dynamics and greatly influences the transient characteristics such as maximum overshoot, frequency of oscillatory modes, delay time, rise time and settling time. This is a crucial step in controller design.
4. The controller concept is evaluated on a real-world problem (utility boiler with natural circulation). The controller is linear; so its implementation is straightforward and cost effective. The control design algorithm can also be applied to many other practical systems like $n$ interconnected generators with steam valve control (Siljak et al., 2002), two-link robot manipulator (Green & Sasiadek, 2004), and two coupled inverted penduli (Siljak & Stipanovic, 2000), to mention just a few.

The paper is organized as follows. In Section 2, the problem is precisely stated and the controller parameters are designed: sufficient conditions are provided involving LMI feasibility problems. The result is then generalized to multiple interconnected subsystems in Section 3. The application of the proposed control to an industrial utility boiler with natural circulation is given in Section 4, with particular attention devoted to the computation of quadratic bounds for the nonlinearities. In Section 5, simulation results are presented, which show the effectiveness of the proposed control against sudden load variations. In all cases, a nonlinear model of the boiler is used, which capture the key dynamical properties of risers, downcomers and the boiler drum. Section 5 also includes comparison of the proposed LMI design with static and dynamic state feedback control. Finally, Section 6 makes some concluding remarks.

2. Output feedback stabilization in the LMI framework

Consider a general nonlinear plant (Siljak et al., 2002; Zecevic et al., 2004)

$$\dot{x}(t) = Ax(t) + Bu(t) + Gh(t, x), \quad y(t) = C x(t),$$

(1)

where $x(t) \in \mathbb{R}^n$ is the state of the system, $u(t) \in \mathbb{R}^m$ is the input vector, $y(t) \in \mathbb{R}^p$ is the output vector, $A$, $B$ and $C$ are constant $n \times n$ and $n \times m$ and $p \times n$ matrices. $h_1: \mathbb{R}^{n+1} \rightarrow \mathbb{R}^m$ is a piecewise continuous nonlinear function in both arguments $t$ and $x$, satisfying $h_1(t, 0) = 0$. This means that the origin is the equilibrium point of the system. If it is not the case, then a new system with equilibrium point at the origin can be defined by performing a change of variables. It is assumed that $(A, B)$ is stabilizable, $(C, A)$ is detectable and the uncertain term $h_1(t, x)$ is bounded by a quadratic inequality (Siljak et al., 2002; Zecevic et al., 2004)

$$h_1^T(t, x)h_1(t, x) \leq x^T x^T H_1^T H_1 x,$$

(2)

where $x \geq 0$ is a scalar parameter and $H_1$ is a known constant matrix. The parameter $x$ can be termed as a degree of robustness, because maximization of $x$ leads to increased robustness of the closed loop system against uncertain perturbations. These perturbations may arise due to uncertain structural reconfigurations, changing operating conditions, and load variations. If there exists a dynamic output feedback controller of the form

$$\dot{x}_h(t) = A_h x_h(t) + B_h y(t), \quad u(t) = C_h x_h(t) + D_h y(t),$$

(3)
where the elements of $h$ satisfy asymptotic stability of origin. The asymptotic stability of (6) can be established via a suitable example in Remark 2.4. This is also explained with a suitable example in Remark 2.4. Lyapunov’s theorem (Marquez, 2003) can be used to establish the existence of a stabilizing output feedback controller. Theorem 2.1.

Theorem 2.1. If there exists $\gamma, X_1, Y_1, \tilde{A}_1, \tilde{B}_1, \tilde{C}_1, \tilde{D}_1, \tilde{Y}_1, \tilde{G}$, such that the following optimization problem is feasible:

$$
\min_{X_1, Y_1, \tilde{A}_1, \tilde{B}_1, \tilde{C}_1, \tilde{D}_1, \gamma} \gamma
data$$

$$
\begin{bmatrix}
X_1 & I \\
I & Y_1
\end{bmatrix} > 0,
$$

$$
\begin{bmatrix}
AX_1 + X_1A^T + B\tilde{C}_1 + (B\tilde{C}_1)^T \\
Y_1A + \tilde{B}_1C + A^TY_1 + C^T\tilde{B}_1^T \\
\end{bmatrix} A + \tilde{A}_1^T + B\tilde{D}_1C + G X_1H_1^T < 0.
$$

where $\tilde{X}$ is a symmetric positive definite matrix ($\tilde{P} > 0$) and this allows to define a collection of sets (Zecevic et al., 2004)

$$
\pi(r) = \{ \tilde{x} : \epsilon(\tilde{x}) \leq r \},
$$

where by denoting the largest set that satisfies $\pi(r) \subset \Omega$ by $\pi(r_0)$, it now follows that solutions $\tilde{x}(t; t_0, \tilde{x}_0)$ of (6) originating at $\tilde{x}_0 \in \pi(r_0)$ must remain in the set $\pi(r_0)$ at all times. More precisely, $\tilde{x}_0 \in \pi(r_0)$ implies $\tilde{x}(t; t_0, \tilde{x}_0) \in \pi(r_0)$ for all $t \geq t_0$, and $\pi(r_0)$ is an invariant set. In this way, the controller obtained stabilizes the closed loop system locally and $\pi(r_0)$ can be represented as an estimate of the region of attraction (Khalil, 2002; Marquez, 2003). This is physically shown in Fig. 1 for a two-dimensional case, and with a Lyapunov function $v(x) = x_1^2 + x_2^2$. The radius of the disk in Fig. 1 can be calculated from the minimum of $v(x)$ at the very edge of the condition $x_2 = \pm x_2$. Hence, in general, $\pi(r_0)$ can be viewed as an ellipsoidal approximation of the domain of attraction inside the set $\Omega$. The following theorem provides sufficient conditions for the existence of a stabilizing output feedback controller.

**Theorem 2.1.** If there exists $\gamma, X_1, Y_1, \tilde{A}_1, \tilde{B}_1, \tilde{C}_1, \tilde{D}_1$, such that the following optimization problem is feasible:

$$
\begin{bmatrix}
X_1 & I \\
I & Y_1
\end{bmatrix} > 0,
$$

$$
\begin{bmatrix}
AX_1 + X_1A^T + B\tilde{C}_1 + (B\tilde{C}_1)^T \\
Y_1A + \tilde{B}_1C + A^TY_1 + C^T\tilde{B}_1^T \\
\end{bmatrix} A + \tilde{A}_1^T + B\tilde{D}_1C + G X_1H_1^T < 0.
$$

where $\tilde{X}$ is a symmetric positive definite matrix ($\tilde{P} > 0$) and this allows to define a collection of sets (Zecevic et al., 2004)

$$
\pi(r) = \{ \tilde{x} : \epsilon(\tilde{x}) \leq r \},
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where by denoting the largest set that satisfies $\pi(r) \subset \Omega$ by $\pi(r_0)$, it now follows that solutions $\tilde{x}(t; t_0, \tilde{x}_0)$ of (6) originating at $\tilde{x}_0 \in \pi(r_0)$ must remain in the set $\pi(r_0)$ at all times. More precisely, $\tilde{x}_0 \in \pi(r_0)$ implies $\tilde{x}(t; t_0, \tilde{x}_0) \in \pi(r_0)$ for all $t \geq t_0$, and $\pi(r_0)$ is an invariant set. In this way, the controller obtained stabilizes the closed loop system locally and $\pi(r_0)$ can be represented as an estimate of the region of attraction (Khalil, 2002; Marquez, 2003). This is physically shown in Fig. 1 for a two-dimensional case, and with a Lyapunov function $v(x) = x_1^2 + x_2^2$. The radius of the disk in Fig. 1 can be calculated from the minimum of $v(x)$ at the very edge of the condition $x_2 = \pm x_2$. Hence, in general, $\pi(r_0)$ can be viewed as an ellipsoidal approximation of the domain of attraction inside the set $\Omega$.
then the system in (6) is asymptotically stable for all nonlinearities satisfying the quadratic constraint in (5).

**Proof.** Please see Appendix.

**Corollary 2.1.** As $v(\dot{x})$ is radially unbounded, under the condition of Theorem 2.1, if the quadratic constraint in (5) holds globally ($\forall x \in \mathbb{R}^n$), then the system in (6) is globally stabilized.

**Remark 2.1.** Assume that the controller $K(s) = D_k + C_k(sI - A_k)^{-1}B_k$ is strictly proper ($D_k = 0$) and that $A$ does not have any fast mode, i.e., any eigenvalue with large real part. Then, from the identity

$\text{Trace}(\hat{A}) = \text{Trace}(A) + \text{Trace}(A_k)$,

$A_k$ has fast modes if and only if $\hat{A}$ has fast modes. Therefore, fast controller dynamics can be prevented by constraining the closed loop eigenvalues, e.g., in a common region of small radius disk and a conic sector for bounding the maximum overshoot. From an application perspective, this is valuable for real-time digital implementation of the controller. One way of simultaneous tuning the stability and transient behavior is therefore to combine the above LMIs with pole placement objectives. Considering these issues, and starting from the pole placement constraints of Chilali and Gahinet (1996), some variable transformations and mathematical calculations show that

$$
\begin{bmatrix}
F_{11} + (2z_{\min})X_1 & F_{12} + (2z_{\min})I \\
F_{22} + (2z_{\min})Y_1
\end{bmatrix} < 0,
$$

$$
\begin{bmatrix}
-\omega_{\max}X_1 & -\omega_{\max}I \\
-\omega_{\max}I & -\omega_{\max}Y_1
\end{bmatrix}
\begin{bmatrix}
A + \hat{B}_1C & \hat{A} \\
\hat{A}_1 & Y_1A + \hat{B}_1C
\end{bmatrix}
\begin{bmatrix}
-\omega_{\max}X_1 & -\omega_{\max}I \\
-\omega_{\max}I & -\omega_{\max}Y_1
\end{bmatrix} < 0,
$$

$$
\begin{bmatrix}
D_{11} & D_{12} \\
D_{22}
\end{bmatrix} < 0,
$$

where

$$
D_{11} = D_{22} \triangleq \begin{bmatrix}
(\sin \theta) F_{11} & (\sin \theta) F_{12} \\
(\sin \theta) F_{12}^T & (\sin \theta) F_{22}
\end{bmatrix},
$$

$$
D_{12} \triangleq \cos \theta
\begin{bmatrix}
A_{1} + B_{1}C_{1} - X_{1}A_{1}^T - (B_{1}C_{1})^T & A + B_{1}D_{1}C - \hat{A}_1^T \\
\hat{A}_1 - A_{1} - C_1 \hat{B}_1^T & Y_1A + \hat{B}_1C - A_{1}^T - C_1 \hat{B}_1^T
\end{bmatrix},
$$

and $F_{11}$, $F_{12}$, $F_{22}$ are defined in Appendix (Eq. (58)). These regions are the intersection of three elementary LMI regions: an $z_{\min}$ stability region $\text{Re}(s) \leq z_{\min}$, a disk of radius $\omega_{\max}$ and the conic sector $\mathcal{F}(0, 0, \theta)$. Confining the closed loop eigenvalues to this region ensures a minimum decay rate $z_{\min}$, a minimum damping ratio $\zeta = \cos \theta$, and a maximum undamped natural frequency $\omega_{\max}$. This in turn bounds the delay time, the rise time, the maximum overshoot, and the frequency of oscillatory modes.

**Remark 2.2.** Theorem 2.1 provides only sufficient conditions for the solution of controller synthesis problem. The conservativeness and computational burden of the proposed approach strongly depend on the choice of the system parameterizations and of performance requirements.

**Remark 2.3.** The number of LMI decision variables increases in dynamic controller design. For the static state feedback, the number of LMI decision variables are $n \times (\frac{n+1}{2} + m)$, while for the dynamic case this number increases to $n \times (2n + 1 + p + m) + m \times p$. An increase in the number of decision variables implies an increase of the off-line computational effort required to solve the LMIs. It is worth to notice that the presence of controller dynamic equations also require more on-line computations. As a summary, as far as stability is concerned, state feedback control offers best possible performance with low computational cost, provided all states are measurable.

**Remark 2.4.** Dynamic controllers offer additional degrees of freedom compared to static ones. If used appropriately, the additional freedom can lead to increased robustness properties. Consider the system (Zecevic et al., 2004)

$$
\dot{x}_1 = 3x_1 + x_1 x_2 + x_2 + u,
$$

$$
\dot{x}_2 = -x_1 + x_2^2 + u.
$$

It has an unstable equilibrium point at the origin, and a nonlinear term $h_1(x)$ satisfying

$$
h_1^T(x)h_1(x) = x^T \begin{bmatrix} x_2 & 0 \\ 0 & x_2^2 \end{bmatrix} x.
$$

The system contains an additional pair of equilibria: an unstable one at $[0.145 - 0.382]^T$, and a stable one at $[6.853 - 2.618]^T$. The system trajectory is shown in Fig. 2. If a region is selected as

$$
\Omega_1 = \{x : x_1 \in \mathbb{R}, |x_2| \leq 1\},
$$

then

$$
h_1^T(x)h_1(x) \leq x^T x, \quad \forall x \in \Omega_1.
$$

Solving the LMI optimization (Theorem 2.1 without pole placement constraints) using $H_1 = I$ gives $z = 2.5$. This shows that the closed loop system remains locally stable for any nonlinearity $h_1(x)$ satisfying

$$
h_1^T(x)h_1(x) \leq 2^2 x^T H_1 x = 6.25 x^T x, \quad \forall x \in \Omega_1.
$$

By comparing (14) and (15), it can be concluded that the system is capable of tolerating significant nonlinear perturbations in $h_1(x)$, provided that the initial conditions remain within the set $\rho(0) \subset \Omega_1$. However, this is obtained at the expense of very fast controller dynamics, which is undesirable. By choosing an admissible region $\mathcal{D}(4.45, 0.707, 6.3)$ of the closed loop eigenvalues, $z$ is calculated to be 1.781. The closed loop eigenvalues are $-4.455 + 4.4545i$, $-4.455 - 4.4545i$, $-5.9167 + 0.5413i$, $-5.9167 - 0.5413i$. For the same configuration, static state feedback controller
design (Zecevic et al., 2004) produces \( z = 1.6 \), and dynamic output feedback design gives \( z = 1 \). This implies that dynamic state feedback control is capable of tolerating more uncertainty in \( h_1(x) \) than static control strategy. Figs. 3 and 4 show the dynamics of states for full state feedback and output feedback control design. Hence, this controller can be applied to cases where all the states are not available for feedback, and fast controller dynamics can be prevented by choosing an admissible region in the left half of the complex plane.

Now consider a larger region \( \Omega_2 \), defined by

\[
\Omega_2 = \{ x : x_1 \in \mathfrak{R}, |x_2| \leq 1.6 \}. 
\]  

When the LMI optimization is repeated with \( H_1 = 1.6I \), \( z = 1 \) for static controller and \( z = 1.14 \) for dynamic controller. Hence, \( h_1(x) \) satisfies (17) with no margin of uncertainty in the static case. The number of decision variables for the static feedback controller design is \( 2 \times (1^2 + 1) = 5 \). For the dynamic controller, this is increased to \( 2 \times (5 + 2 + 1) + 1 \times 2 = 18 \) for full state feedback and \( 2 \times (5 + 1 + 1) + 1 \times 1 = 15 \) for output feedback. Dynamic controllers have computational complexity, but it also offers more physical insight into
controller design. An intuitive explanation of this fact is that the transient behavior of the overall system is affected considerably by a dynamic controller. In real-time implementation, the dynamics of the controller filters the measurement noise, and fast controller dynamics can be avoided by choosing a disk of smaller radius in the design phase.

3. Generalization to multiple interconnected subsystems

From the viewpoint of boilers, robotics, vehicle platooning, it is necessary to extend the preceding analysis to models that include decentralized control. To that effect, consider the interconnected system (Siljak et al., 2002; Zecevic et al., 2004)

\[ \dot{x}_i = A_i x_i + B_i u_i + G_i h_i(t, x), \]
\[ y_i = C_i x_i, \quad i = 1, 2, 3, \ldots, n, \]

(18)

where \( x_i \in \mathbb{R}^{m_i} \) are the states of subsystems, \( u_i \in \mathbb{R}^{m_i} \) are the control inputs and \( h_i : \mathbb{R}^{m+1} \rightarrow \mathbb{R}^{m_i} \) are the interconnections. It is assumed that pairs \((A_i, B_i)\) are stabilizable, \((C_i, A_i)\) are detectable, and all nonlinear functions \( h_i(t, x) \) satisfy the quadratic constraints (Siljak et al., 2002)

\[ h_i^T(t, x) h_i(t, x) \leq \bar{z}_i^2 x_i^T H_i^T H_i x, \]

(19)

where \( x = [x_1^T \ x_2^T \ \ldots \ x_n^T]^T, \ \bar{z}_i > 0 \) are interconnection parameters and \( H_i \) are fixed known matrices. Following the discussions of the previous section, the closed loop system for the \( i \)th subsystem (as in (6)) can be written as

\[ \dot{x}_i = \hat{A}_{iD} x_i + G_{Ni} h_i, \quad i = 1, 2, \ldots, n, \]

(20)

where \( x_i = [x_i^T \ x_{i2}^T \ \ldots \ x_{ik_i}^T]^T \), \( x_{ik} \) represent controller states for the \( i \)th subsystem and

\[ \hat{A}_{iD} = A_i + B_i D_i C_i, \quad B_{iC_k} C_i, \quad G_{Ni} = G_{C_k}. \]

(21)

\( A_{ik_i}, B_{ik_i}, C_{ik_i}, C_i, \) and \( D_{ik_i} \) are the state space matrices of the controller for \( i \)th subsystem. Defining \( x_N = [x_1^T \ x_{i2}^T \ x_{i2}^T \ \ldots \ x_{ik_i}^T x_{ik_i}^T]^T \), the quadratic inequality of the \( i \)th subsystem (as in (5)) can be rewritten as

\[ h_i^T(t, x) h_i(t, x) \leq \bar{z}_i^2 x_i^T H_i^T H_i x = \bar{z}_i^2 x_i^T H_i^T H_i x_N, \]

(22)

where the elements of \( H_{ik_i} \) corresponding to \( x_{ik_i}, x_{ik_2}, \ldots, x_{ik_j} \) are zero. The overall interconnected system can be written in compact form as

\[ \dot{x}_N = \hat{A}_D x_N + G_D h(t, x_N), \]

(23)

where \( \hat{A}_D = \text{diag}(\hat{A}_{iD_1}, \ldots, \hat{A}_{iD_n}), \ G_D = \text{diag}(G_{N1}, \ldots, G_{Nn}) \) and \( h = [h_1^T \ h_2^T \ \ldots \ h_n^T]^T. \) The nonlinear function of the overall system \( h(t, x) \) is bounded by a quadratic inequality

\[ h^T(t, x) h(t, x) \leq \sum_{i=1}^{n} \bar{z}_i^2 x_i^T H_i^T H_i x_N \]

\[ = x_N^T \sum_{i=1}^{n} \frac{1}{\bar{z}_i^2} H_i^T H_i x_N. \]

The following design algorithm provides sufficient conditions for the existence of decentralized controllers for large scale interconnected systems.

**Theorem 3.1.** If there exists \( \gamma_i, X_{i1}, Y_{i1}, \hat{A}_i, \hat{B}_i, \hat{C}_i, \hat{D}_i, \)
\( i = 1, 2, 3, \ldots, n, \) such that the following optimization problem is feasible:

\[ \min_{\gamma_i} \sum_{i=1}^{n} \gamma_i, \]

s.t.

\[ \Pi_{2p}^T \gamma_i \Pi_{2p} > 0, \]

\[ \left[ \begin{array}{c} \Pi_{2p}^T \hat{A}_{N1} \Pi_{2p} + \Pi_{2p}^T G_D \Pi_{2p} H_N^T \Pi_{2p} \Pi_{2p}^T \gamma_i \Pi_{2p} \Pi_{2p}^T \gamma_i \Pi_{2p} \Pi_{2p}^T \gamma_i \Pi_{2p} \\ -I \\ 0 \end{array} \right] < 0, \]

\[ \Pi_{2p}^T \hat{A}_{N1} \Pi_{2p} + 2\gamma_i \Pi_{2p}^T Y_{i1} \Pi_{2p} < 0, \]

\[ \left[ \begin{array}{c} -\omega_{\max} \Pi_{2p}^T Y_{i1} \Pi_{2p} \\ -\omega_{\max} \Pi_{2p}^T Y_{i1} \Pi_{2p} \end{array} \right] < 0, \]

\[ \delta_{11}, \delta_{12}, \delta_{22}, \]

where

\[ H_N^T = [H_{N1}^T, \ldots, H_{Nn}^T], \]

\[ \gamma_D = \text{diag}(\gamma_1, \gamma_2, \ldots, \gamma_n), \]

\[ \omega_{\min} = \text{diag}(\omega_{\min_1}, \omega_{\min_2}, \ldots, \omega_{\min_n}), \]

\[ \delta_{11} = \cos^2 \theta \Pi_{2p}^T \hat{A}_{iD_1} \hat{A}_{iD_1} \Pi_{2p}, \]

\[ \delta_{12} = \cos \theta \Pi_{2p}^T \hat{A}_{iD_1} \hat{B}_{iD_1} \Pi_{2p}, \]

then the interconnected system in (23) is asymptotically stable for all nonlinearities satisfying the quadratic constraint in (24).

**Proof.** Please see Appendix.

Minimizing \( \sum \gamma_i \) leads to increasing robustness of the closed loop system against uncertain structural perturbations, changing operating conditions and load fluctuations. This is by recognizing the well known fact that interconnections can also be considered as perturbations of the subsystem dynamics.

**Corollary 3.1.** Under the conditions of Theorem 3.1, if the quadratic constraint in (24) holds globally (\( \forall x \in \mathbb{R}^n \)), then the system in (23) is globally stabilized.

**Remark 3.1.** It is clear that the stability of (23) is guaranteed for any nonlinearity satisfying (24). However, it is important to note that an inherent trade-off exists between the size of the region of attraction and the robustness of system. It is true that a large value of \( \bar{z}_i \) implies a greater degree of robustness. On the other hand, it is also true that the value of \( \bar{z}_i \) obtained from the LMI
optimization decreases as the region $\Omega$ becomes larger. Remark 2.4 clearly illustrates this effect.

**Remark 3.2.** The proposed method guarantees connective stability of the overall large scale system, i.e., overall stability can be maintained when the subsystems are disconnected and again connected in various ways to perform programmed tasks. It can be seen that the formulation of the constraints in (24) or (22) on the interconnection terms $h_i(t, x)$ includes the case when any and/or all $h_i(t, x)\equiv 0$, guaranteeing that the overall closed loop system in (23) is connectively stable (Siljak, 1991; Siljak & Stipanovic, 2000). This is because $h_i(t, x)\equiv 0$ implies that the $i$th subsystem is disconnected, but (24) is still satisfied and the controller designed based on Theorem 3.1 guarantees overall closed loop stability.

**Remark 3.3.** Due to the requirement of block diagonal Lyapunov function in decentralized design, there may be problems of infeasibility and loss of robustness of the Lyapunov function in decentralized design, there may be load operating conditions. However, as optimization in Theorem 3.1 are computed for normal operating conditions. This approach can lead to an increase in boiler efficiency while considerably reducing the production of NOx.

This paper deals with designing a robust control strategy for water side of the boiler. In the steam–water system, preheated water ($q_r$) from the economizer is fed into the steam drum, which then flows through the downcomers into the mud drum. The mud drum distributes the water to risers and the heat $Q$ supplied to risers cause boiling of the water. Gravity forces the saturated steam to rise, causing a circulation in the riser–drum–downcomer loop. The saturated steam ($q_s$) is then separated from the water and exits the steam drum into the primary and secondary superheaters. A schematic diagram of Syncrude utility boiler is shown in Fig. 5. Let $V$ denote the volume, $\rho$ the specific density, $u$ the specific internal energy, $h$ the specific enthalpy, $t$ the temperature, $p$ the drum pressure and $q$ the mass flow rate. Moreover, let subscripts $s, w, f$ and $m$ refer to steam, water, feedwater and metal, respectively. Subscript $t$ denotes the total steam, $d$ the drum and $r$ the riser. The total mass of the metal tubes and the drum is $m_t$, and the specific heat of the metal is $C_p$.

The global mass and energy balances are given by (Astrom & Bell, 2000)

$$\frac{d}{dt} [\rho_r V_{st} + \rho_w V_{wt}] = q_f - q_r,$$

$$\frac{d}{dt} [\rho_s h_i V_{st} + \rho_w h_w V_{wt} - p V_t + m_t C_p t_m] = Q + q_f h_f - q_s h_s,$$

where $V_{st}$ and $V_{wt}$ represents the total steam volume and the total water volume, respectively. The total volume of the boiler drum, downcomers, and risers is $V_t = V_{st} + V_{wt}$.

Riser dynamics: The global mass balance for the riser section can be written as (Astrom & Bell, 2000):

$$\frac{d}{dt} [\rho_r \bar{z}_r V_r + \rho_w (1 - \bar{z}_r) V_r] = q_{dc} - q_r,$$

where $q_r$ is the total mass flow rate out of the risers and $q_{dc}$ is the total mass flow rate into the risers (downcomer flow rate). $V_r$ represents the riser volume and the total amount of the steam in risers is governed by the average volume fraction (Astrom & Bell, 2000):

$$\bar{z}_r = \frac{1}{2z} \int_0^{2z} f(\xi) d\xi = \frac{\rho_w}{\rho_w - \rho_s} \times \left[ 1 - \frac{\rho_s}{(\rho_w - \rho_s)z_r} \ln \left( 1 + \frac{\rho_w - \rho_s}{\rho_s} z_r \right) \right],$$

where $z_r$ is the steam-mass fraction in the riser. Finally, the global energy balance for the riser section is given by (Astrom & Bell, 2000)

$$\frac{d}{dt} \left[ \rho_r h_r \bar{z}_r V_r + \rho_w h_w (1 - \bar{z}_r) V_r - p V_r + m_t C_p t_m \right] = Q + q_{dc} h_w - (\bar{z}_r h_r + h_w) q_f,$$

Drum dynamics: Let $V_{sd}$ and $V_{wd}$ be the volume of steam and water under the liquid level, and the steam flow rate through the liquid surface in the drum be $q_{sd}$. The mass balance for the steam under the liquid level is governed by

$$\frac{d}{dt} \left[ \rho_r h_r \bar{z}_r V_r + \rho_w h_w (1 - \bar{z}_r) V_r - p V_r + m_t C_p t_m \right] = Q + q_{dc} h_w - (\bar{z}_r h_r + h_w) q_f,$$
(Astrom & Bell, 2000)
\[
\frac{d}{dt}(\rho_s V_{sd}) = \dot{z}_s q_f - q_{sd} - q_{cd},
\]
where \(q_{cd}\) is the condensation flow which can be expressed as
\[
q_{cd} = \frac{h_u - h_f}{h_c} q_f + \frac{1}{h_c} \left( \rho_s V_{sd} \frac{dh_t}{dt} + \rho_w V_{wd} \frac{dh_w}{dt} \right) - (V_{sd} + V_{wd}) \frac{dp}{dt} + \dot{m}_d c_p \frac{dt}{dt}.
\]
The empirical model of \(q_{sd}\) is represented by (Astrom & Bell, 2000)
\[
q_{sd} = \frac{\rho_s}{\rho_f} (V_{sd} - V_{sd}^0) + \dot{z}_s q_{de} + \dot{z}_t (q_{de} - q_r),
\]
where \(V_{sd}^0\) denotes the volume of steam in the drum in the hypothetical situation, when there is no condensation of steam in the drum and \(T_d\) is the residence time of the steam in the drum. The constant \(\beta\) is user defined and is approximately 0.3.

4.2. Simplification of the nonlinear model to standard form

In this section, the conversion of the nonlinear model given by (25)–(27), (29) and (30) to the form in (1) has been carried out. The state variables of the system are: total water volume \((V_{wt} = x_1)\), drum pressure \((p = x_2)\), steam mass fraction in the riser \((x_r = x_3)\), and steam volume in the drum \((V_{sd} = x_4)\). The control variables are feedwater flow rate \((q_f = u_1)\), steam flow rate \((q_r = u_2)\) and the heat supplied to the risers \((Q = u_3)\). From (25), straightforward calculation shows that
\[
\frac{d}{dt} \left[ \rho_s (V_{r} - x_1) + \rho_w x_1 \right] = u_1 - u_2,
\]
\[
\left( \rho_w - \rho_s \right) \frac{dx_1}{dt} + \left[ (V_{r} - x_1) \frac{\partial \rho_s}{\partial x_2} \right] \frac{dx_2}{dt} = u_1 - u_2,
\]
\[
\left( \rho_w - \rho_s \right) \frac{dx_1}{dt} + \left[ (V_{r} - x_1) \frac{\partial \rho_s}{\partial x_2} + (V_{r} - x_1) \rho_s \frac{\partial h_s}{\partial x_2} \right] \frac{dx_2}{dt} + x_1 \left( \frac{\partial h_u}{\partial x_2} + \rho_w \frac{\partial h_w}{\partial x_2} \right) \frac{dx_2}{dt} - V_r \frac{dx_2}{dt} + m_i C_p \frac{dt}{dt} \frac{dx_2}{dt} = u_1 + u_1 h_f - u_2 h_s,
\]
\[
e_{21} \frac{dx_1}{dt} + e_{22} \frac{dx_2}{dt} = u_3 + u_1 h_f - u_2 h_s,
\]
where \(e_{21} = \rho_u h_u - \rho_h h_s\) and \(e_{22} = x_1 (h_u \frac{\partial h_u}{\partial x_2} + \rho_w \frac{\partial h_w}{\partial x_2}) - V_r + m_i C_p \frac{\partial h_u}{\partial x_2} + (V_{r} - x_1) (h_u \frac{\partial h_u}{\partial x_2} + \rho_w \frac{\partial h_w}{\partial x_2})\). Following along the same lines, multiplying (27) by \((x_3 h_c + h_w)\) and subtracting (29) from (27)
\[
e_{32} \frac{dx_2}{dt} + e_{33} \frac{dx_3}{dt} = u_3 - x_3 h_c q_{dc},
\]
where
\[
e_{32} = \left( \rho_w \frac{\partial h_w}{\partial x_2} - x_3 \rho_v \frac{\partial h_v}{\partial x_2} \right) (1 - \varphi_3) V_r - V_r + \left( 1 - x_3 \right) \frac{\partial h_v}{\partial x_2} + \rho_v \frac{\partial h_v}{\partial x_2},
\]
\[
e_{33} = \left( 1 - x_3 \right) \rho_v + x_3 \rho_v h_v \frac{\partial \dot{z}_v}{\partial x_3}. \tag{35}
\]

Finally, the drum dynamics is governed by
\[
e_{43} \frac{dx_3}{dt} + e_{44} \frac{dx_4}{dt} = \frac{\rho_v}{T_d} (V_d^0 - x_4) + \frac{h_f - h_m}{h_c} u_1, \tag{36}
\]
where
\[
h_c = h_l - h_m, \quad e_{44} = \rho_r, \quad \text{and} \quad e_{43} = 1.3 \times \rho_r (\rho_c - \rho_v) V_r \frac{\partial \dot{z}_v}{\partial x_3}. \]

Starting from the nonlinear equations (32)–(34), and (36), some mathematical calculations and simplifications give
\[
\dot{x}_1 = \frac{(e_{22} - e_{12} h_f) u_1 + (e_{12} h_f - e_{22}) u_2 + e_{12} u_3}{e_{11} e_{22} - e_{12} e_{21}},
\]
\[
\dot{x}_2 = \frac{(e_{11} h_f - e_{21}) u_1 + (e_{21} - e_{11} h_f) u_2 + e_{11} u_3}{e_{11} e_{22} - e_{12} e_{21}},
\]
\[
\dot{x}_3 = \frac{-h_c q_d x_3 + e_{21} e_{32} - e_{11} e_{32} h_f}{e_{33} e_{12} e_{33} e_{32}} + \frac{(e_{11} e_{32} - e_{21} e_{32}) u_2 + (e_{11} e_{22} - e_{12} e_{21}) u_3}{e_{11} e_{22} - e_{12} e_{21}},
\]
\[
\dot{x}_4 = \frac{e_{43} h_c q_d x_3 - \rho_x x_4 + \rho_v V_d^0}{e_{33} e_{44} e_{33}} + \frac{e_{43} h_c}{e_{11} e_{22} - e_{12} e_{21}} e_{33} e_{44} + \frac{(h_f - h_m)(e_{11} e_{22} - e_{12} e_{21}) e_{33} - e_{21} e_{32} e_{33} h_c}{(e_{11} e_{22} - e_{12} e_{21}) e_{33} e_{44}} U_1. \tag{37}
\]

For Syncrude utility plant, some parameters of this nonlinear model are obtained from steam table at a saturation pressure of 7018.6 kPa. The steam tables are approximated by quadratic functions, and the parameters are shown in Table 1. At steady state, \( u_1 = u_2 = 58 \) in pu, \( u_3 = 84 \) in pu, \( x_1 = V_{\text{vol}} = 57.5 \) m\(^3\), \( x_3 = p = 7018 \) kPa, \( x_3 = \alpha_r = 0.05 \), \( x_4 = V_{\text{ad}} = 5 \) m\(^3\), and total volume of water and steam \( V_f = 85 \) m\(^3\). The construction parameters of the nonlinear model are: riser volume \( V_r \), downcomer volume \( V_d \), drum area \( A_d \), total metal mass \( m_t \) and mass of the riser \( m_r \).

The parameters \( e_{11}, e_{12}, \ldots, e_{43} \) are the functions of states \( x_1, \ldots, x_4 \), and the properties of steam and water, namely, specific enthalpy, specific density, etc. However, they are not functions of \( u_1, u_2, \) and \( u_3 \). This provides an advantage of bringing the nonlinear model in the form of (1). The following expansion brings the model in the standard form:

\[
\dot{x} = \left[ \frac{\partial f}{\partial x} \right]_X x_0 + \left[ \frac{\partial f}{\partial u} \right]_X (x - x_0) + \frac{1}{2!} \left[ \frac{\partial^2 f}{\partial x^2} \right]_X (x - x_0)^2 + \cdots
\]

where \( X_0 \) represents the equilibrium values, \( x^T = [x_1^T, x_2^T, X^T] \), and \( u^T = [u_1^T, u_2^T, u_3^T] \). The Taylor series expansion in (38) gives the following input matrix \( B \):

\[
B = \left[ \frac{\partial f}{\partial x} \right]_X x_0 + \left[ \frac{\partial f}{\partial u} \right]_X (x - x_0) + \frac{1}{2!} \left[ \frac{\partial^2 f}{\partial x^2} \right]_X (x - x_0)^2 + \cdots.
\]

This is calculated from \( \left[ \frac{\partial f}{\partial u} \right]_X x_0 \). Moreover, as none of the terms \( e_{11}, e_{12}, \ldots, e_{43} \) are functions of \( u_1, u_2, \) and \( u_3 \), \( \left[ \frac{\partial f}{\partial u} \right]_X x_0 = \left[ \frac{\partial f}{\partial u} \right]_X X_0 = \cdots = 0 \). Similarly, state matrix \( A \) matrix in (1) can be calculated from \( \left[ \frac{\partial f}{\partial x} \right]_X x_0 \). With \( \Delta x = (x - x_0) \), (38) can be expressed as

\[
\Delta \dot{x} = \left[ \frac{\partial f}{\partial x} \right]_X \Delta x + \left[ \frac{\partial f}{\partial u} \right]_X \Delta u
\]

\[
+ \frac{1}{2!} \left[ \frac{\partial^2 f}{\partial x^2} \right]_X \Delta x^2 + \frac{1}{3!} \left[ \frac{\partial^3 f}{\partial x^3} \right]_X \Delta x^3 + \cdots.
\]

Table 1

| Parameters of the nonlinear model at a saturation pressure of 7018.6 kPa |
|-------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( t_c \)   | \( h_w \)   | \( h_r \)   | \( \rho_v \) | \( \rho_w \) | \( \frac{\partial h_r}{\partial \xi_2} \) | \( \frac{\partial h_r}{\partial \xi_3} \) | \( \frac{\partial \rho_v}{\partial \xi_2} \) | \( \frac{\partial \rho_v}{\partial \xi_3} \) |
| 285.9       | 1267.8      | 2772.3       | 739.9        | 36.6          | 51.9          | -13.3          | -18.1          | 5.9            | 11.53         |
with $G$ as the identity matrix. Starting from (28), some calculations show that

$$\frac{\partial \xi}{\partial x_3} = \frac{\rho_w}{2\rho_s} \left[ (1 + x_3) \rho_s - \rho_w x_3 \right].$$

(39)

Hence, from (37)

$$-\frac{h_3 q_{dc} x_3}{e_{33}} = - \frac{2 \rho_s q_{dc} x_3}{V_t \rho_s [(1 + x_3) \rho_s + x_3 (\rho_s - \rho_w)]} x_0$$

$$+ \frac{2 \rho_s q_{dc} x_3}{V_t \rho_s [(1 + x_3) \rho_s + x_3 (\rho_s - \rho_w)]} \Delta X_3$$

$$+ \frac{2 \rho_s^2 q_{dc} x_3}{V_t \rho_s [(1 + x_3) \rho_s + x_3 (\rho_s - \rho_w)]^2} \Delta X_3^2.$$  

(40)

Also, $-\frac{\rho_s x_4}{e_{43} e_{44}} = - \frac{\Delta x_3}{x_0}$ and

$$\frac{e_{43} h_3 q_{dc} x_3}{e_{33} e_{44}} = \frac{q_{dc} x_3}{\rho_s} \left[ \frac{1.3 (\rho_s - \rho_w) x_3}{\rho_s [(1 + x_3) \rho_s + x_3 (\rho_s - \rho_w)]} x_0 \right.$$

$$+ \frac{q_{dc} x_3^2}{\rho_s} \left[ \frac{1.3 (\rho_s - \rho_w)^2}{\rho_s [(1 + x_3) \rho_s + x_3 (\rho_s - \rho_w)]^2} \right] \Delta X_3$$

$$+ \frac{q_{dc} x_3^2}{\rho_s} \left[ \frac{2.6 (\rho_s - \rho_w) x_3}{\rho_s [(1 + x_3) \rho_s + x_3 (\rho_s - \rho_w)]} \right] \Delta X_3^2$$

$$+ \frac{x_3 (\rho_s - \rho_w)}{[(1 - x_3) \rho_s + x_3 \rho_w]} \left[ \frac{1}{(1 - x_3) \rho_s + x_3 \rho_w} \right] \Delta X_3^2.$$

(41)

The higher order terms can be neglected as the variation of steam-mass fraction from the normal operating point is 5–6%. In real-time application, the measured variables of the utility boiler are drum pressure ($y_1 = x_2$) and drum level $y_2$, where the drum level measured from its normal operating level is given by (Astrom & Bell, 2000)

$$y_2 = \frac{(V_{wd} + V_{sd})}{A_d} = I_w + I_r.$$  

(42)

The term $I_w$ represents level variations caused by changes of the amount of water and $I_r$ represents variations caused by the steam in the drum. $V_{dc}$ represents the downcomer volume and the water volume $V_{wd}$ is governed by

$$V_{wd} = V_{wt} - V_{dc} - (1 - \bar{x}_o) V_r.$$  

(43)

With some simplifications, the output vector $y$ can be obtained as

$$\begin{bmatrix}
y_1 \\
y_2
\end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\
1 & 0 & 1 & \frac{1}{A_d} \frac{\rho_w}{2 \rho_s} [(1 + x_3) \rho_s + x_3 \rho_w] \end{bmatrix} x_0 \begin{bmatrix} \Delta x_1 \\
\Delta x_2 \\
\Delta x_3 \\
\Delta x_4
\end{bmatrix}. $$

(44)

4.3. Computation of quadratic bounds for the nonlinearities

The pole zero (PZ) map of the linearized system is shown in Fig. 6. It has two poles at origin and it contains a RHP zero at 0.021. These are sources of interaction, bandwidth limitation and instability. It has been found that other nonlinear terms obtained by Taylor’s expansion of (37) are very small compared to (40) and (41). Therefore, they are neglected while computing the quadratic bounds for nonlinearities. As the steam-mass fraction is always less than 100%, a region $\Omega$ is defined as

$$\Omega = \{ x : x_1, x_2, x_4 \in \mathbb{R}, |x_3| \leq 1 \}. $$

(45)

Using (45), (40) and (41), it can be verified that the nonlinear function $h_3(x)$ is bounded by

$$b_1^T(x) h_1(x) \leq \left[ \frac{1.3 (\rho_s - \rho_w) q_{dc}}{\rho_s [(1 - x_3) \rho_s + x_3 \rho_w]} \left( \frac{x_3 (\rho_s - \rho_w)}{[(1 - x_3) \rho_s + x_3 \rho_w]} \right) \right]^2 \Delta x_3^2.$$
It should be noted at this point that the bound in (46) reflects the pre-fault state of the system and the nominal load values. With that in mind, the objective of LMI-based control is to maximize parameter $\alpha$ so that inequality

$$h^T(t, x) h_1(t, x) \leq \alpha^2 x^T H_1^TH_1 x$$

holds for a variety of operating conditions and load variations. The physical interpretation of this inequality is shown in Fig. 7 for one-dimensional case and for nominal load condition. The shaded region $(-z H_1 x \leq h_1(x) \leq z H_1 x)$ increases as $\alpha$ is maximized.

5. Simulation results

The controller is designed by solving the linear objective minimization problem of Theorem 2.1. The admissibility region is assumed to be $D(x_{\min} = 0.01, \zeta = 0.70, \omega_{\max} = 30)$, and is shown in Fig. 8 by a $\Pi$ shaped region of the left-half plane (LHP).

The original system has two poles at the origin, one associated with water dynamics and the other with pressure dynamics. The pole associated with pressure dynamics is at the origin because the steam flow is chosen as a control variable. The stabilizing effects of three different kinds of controllers, static state feedback, dynamic state feedback and dynamic output feedback, are shown in Figs. 9 and 10. The degrees of robustness $\alpha$ obtained with static state feedback, dynamic state feedback and dynamic output feedback controllers are 1.16, 1.28 and 1.04, respectively. This shows that the degree of robustness with output feedback is less than full state feedback. However, the performance is acceptable. Fig. 11 shows the disturbance rejection capability of the output feedback controller caused by sudden load variation in the system. Due to sudden decrease in load, the pressure in the drum rises, which increases the volume of water due to increased condensation. Steam quality $x_r$ at the riser outlet also increases rapidly. The volume of the steam $V_{sd}$ decreases due to the increased pressure which causes condensation of the steam. This shows that there exists a strong interaction between different state variables. The time scale also shows the effect of RHP zero which limits the closed loop bandwidth. The controller is also capable of attenuating the effect of sudden increase in load. Pole placement constraints help to reduce the initial peaking caused by load variations.
It should be noted that the control design does not consider uncertainties in the $A$ matrix ($A + \Delta A$), and the nominal controller is obtained with residence time of steam in the drum $T_d = 12$ s. To evaluate the robustness of the proposed control with respect to this time constant, an additional set of simulations are performed with $10 \leq T_d \leq 15$. Three representative responses are shown in Fig. 12, corresponding to sudden load variation. The results suggest that the proposed control performs well for the entire range of realistic values for $T_d$.

For a complete evaluation of the proposed control, it is also necessary to consider its practical implementation. It can be argued, for example, that the size and inertia of steam valves make it difficult to implement a control that is fast enough to maintain stability following a sudden load change (Siljak et al., 2002). In view of that, the LMI-based design along with pole placement constraints avoid fast controller dynamics and can be useful in this context. However, it should be stressed that to achieve a good performance, the choice of an appropriate $x_{\text{min}}$ stability region, $\Re(s) \leq x_{\text{min}}$, a disk of radius $\omega_{\text{max}}$, and the conic sector $\mathcal{H}(0, 0, \theta)$ has to be done very carefully and several attempts are needed if the exact region of closed loop eigenvalues are not known a priori.

6. Conclusions

This paper presents a new method of designing a dynamic output feedback control strategy for nonlinear interconnected systems, based on LMIs. The contribution of the paper stems from applicability of the approach to systems, where all the states are not available for feedback. The primary reason for selecting this type of control is also the underlying system model, which is seen to adapt to the requirements of LMI optimization. It is useful to know that the LMI formulation maximizes the interconnection bounds, thereby increasing the robustness of the closed loop system against uncertain perturbations. In addition to this, the control law results in a connectively stable system, thus overcoming the barrier of instability caused by sudden structural perturbations.

Simulation results show that the output feedback controller can also achieve good performance, but the degree of robustness is less than the static or dynamic state feedback control. However, the stabilization effect of the controller is good both during the normal and perturbed conditions, which makes it practically implementable. The proposed approach can also control the transient characteristic of the closed loop system by pole placement.
constraints. This helps in reducing the initial peaking caused by load variations and avoiding very fast controller dynamics.

Current work is concentrated on developing a theoretical framework that leads to the movement of the pole associated with the pressure dynamics to the LHP. The integration of turbine and governor models with the present configuration will be done and the design of decentralized reduced order controllers for each individual subsystem will be investigated. Preliminary results obtained along these lines are encouraging.

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Appendix

S-procedure (Boyd et al., 1994): The S-procedure is a technique which enables control engineers to combine several quadratic inequalities into one single inequality. There are many situations in control engineering where it is necessary to ensure that a single quadratic function of \( x \in R^n \) is such that (Skogestad & Postlethwaite, 2005)

\[
F_0(x) < 0, \quad F_0(x) \triangleq x^T A_0 x + 2b_0 x + c_0,
\]

whenever certain other quadratic functions are positive semi-definite, i.e.,

\[
F_i(x) \geq 0, \quad F_i(x) \triangleq x^T A_i x + 2b_i x + c_i, \quad i \in \{1, 2, \ldots, q\}.
\]

Now, consider the case \( i = 1 \) for simplicity. Hence, it is necessary to ensure that \( F_0(x) < 0 \) for all \( x \) such that \( F_1(x) \geq 0 \). In this case, if there exists a positive scalar, \( \tau > 0 \), such that \( F_{\text{aux}}(x) \triangleq F_0(x) + \tau F_1(x) < 0, \forall x \) subject to \( F_1(x) \geq 0 \), then the goal is automatically achieved.

Proof of Theorem 2.1. It is well known that a sufficient condition of the asymptotic stability of origin is the derivative of \( r(x) \) to be negative along the solutions of (6). This condition can be expressed by a pair of inequalities

\[
P > 0, \quad \dot{x}^T A^T P \dot{x} + h^T G_1^T P \dot{x} + \dot{x}^T P \dot{x} + x^T P G_1 h < 0.
\]

The S-procedure (Boyd et al., 1994) implies that when (5) is satisfied, condition (50) is equivalent to the existence of matrix \( P \) and a number \( \tau > 0 \) such that \( P > 0 \) and

\[
\begin{bmatrix}
\dot{A}^T P + P \dot{A} + \tau^2 H^T H & PG_1 \\
G_1^T P & -\tau I
\end{bmatrix} < 0.
\]

It is again equivalent to the existence of matrix \( Y \) which satisfies

\[
Y > 0, \quad (52)
\]

\[
\begin{bmatrix}
\dot{A} Y + Y \dot{A}^T & G_1 \\
G_1^T & -I
\end{bmatrix} < 0,
\]

where \( Y = \tau P^{-1} \) and \( \gamma = \frac{1}{\tau} \). It is well known that \( \gamma \) can be termed as degree of robustness; therefore, \( \gamma \) is a measure of the degree of robustness. The LMI in (52) and (53) cannot be used to find the controller because it is not affine in controller parameters \( A_k, B_k, C_k \) and \( D_k \). Hence a variable transformation is necessary. If \( A \) and \( A_k \) are \( n \times n \) and \( n_k \times n_k \), partitioning \( Y \) and \( Y^{-1} \) as

\[
Y = \begin{bmatrix} X_1 & M_1 \\ M_1^T & V_1 \end{bmatrix}, \quad Y^{-1} = \begin{bmatrix} Y_1 & N_1 \\ N_1^T & U_1 \end{bmatrix},
\]

where \( X_1 \) and \( Y_1 \) are \( n \times n \) and symmetric, \( M_1 \) and \( N_1 \) are \( n \times n_k \) and \( Y > 0 \) implies \( X_1 > 0, Y_1 > 0 \). From \( Y^{-1} Y = I \), it can be inferred that

\[
Y^{-1} \begin{bmatrix} X_1 \\ M_1 \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix},
\]

which leads to \( Y^{-1} \Pi_1 = \Pi_2 \), where,

\[
\begin{align*}
\Pi_1 & \triangleq \begin{bmatrix} X_1 & I \\ M_1^T & 0 \end{bmatrix}, \\
\Pi_2 & \triangleq \begin{bmatrix} Y_1 & N_1 \\ N_1^T & U_1 \end{bmatrix} \begin{bmatrix} M_1 & 0 \end{bmatrix} = \begin{bmatrix} I & Y_1 \\ 0 & N_1^T \end{bmatrix}
\end{align*}
\]

Pre- and post-multiplying (52) by \( \Pi_2^T \) and \( \Pi_1 \), respectively, and (53) by diag(\( \Pi_2^T, I, I \)) and diag(\( \Pi_2, I, I \)), respectively,

\[
\Pi_2^T Y \Pi_2 > 0,
\]

\[
\begin{bmatrix}
\Pi_2^T \dot{A} \Pi_2 + \Pi_2^T Y A^T \Pi_2 & \Pi_2^T G_1 \\
G_1^T \Pi_2 & -I
\end{bmatrix} < 0.
\]

Now, straightforward calculation shows that

\[
\Pi_2^T Y \Pi_2 = \begin{bmatrix} X_1 & I \\ I & Y_1 \end{bmatrix}
\]

and

\[
\Pi_2^T \dot{A} \Pi_2 = \begin{bmatrix} A X_1 + B \hat{C}_1 & A + B \hat{D}_1 C \\ \hat{A}_1 & Y_1 A + \hat{B}_1 C \end{bmatrix},
\]

where

\[
\begin{align*}
\hat{A}_1 & \triangleq Y_1^T (A + BD_k C) X_1 + N_1 B_k C X_1 + Y_1^T B C_k M_1^T + N_1 A_k M_1^T, \\
\hat{B}_1 & \triangleq Y_1 B D_k + N_1 B_k, \\
\hat{C}_1 & \triangleq D_k C X_1 + C_k M_1^T, \\
\hat{D}_1 & \triangleq D_k.
\end{align*}
\]
Hence,
\[ \Pi_2^T \hat{A} \Pi_2 + \Pi_2^T Y \hat{A} \Pi_2 = \begin{bmatrix} \mathcal{F}_{11} & \mathcal{F}_{12} \\ \mathcal{F}_{12}^T & \mathcal{F}_{22} \end{bmatrix}, \]
where
\[ \mathcal{F}_{11} \triangleq A X_1 + X_1 A^T + B \hat{C}_1 + (B \hat{C}_1)^T, \quad \mathcal{F}_{12} \triangleq A + \hat{A}_i + B \hat{D}_i C, \]
\[ \mathcal{F}_{22} \triangleq Y_1 A + \hat{B}_i C + \hat{A}_i Y_1 + C^T \hat{B}_i. \] (58)

Also,
\[ \Pi_2^T G_1 = \begin{bmatrix} I & 0 \\ Y_1^T & N_1 \end{bmatrix} \begin{bmatrix} G \\ 0 \end{bmatrix} = \begin{bmatrix} G \\ Y_1 G \end{bmatrix} \]
and
\[ \Pi_2^T Y H^T = \Pi_2^T H^T = \begin{bmatrix} X_1 & M_i \\ I & 0 \end{bmatrix} \begin{bmatrix} H_i^T \\ 0 \end{bmatrix} = \begin{bmatrix} X_i H_i^T \end{bmatrix}. \]

This variable transformation makes the LMI in (52) and (53) affine in controller parameters. Since \( X_1 \) and \( Y_1 \) are symmetric matrices, \( N_1 \) and \( M_1 \) can be chosen square and non-singular such that \( N_1 M_1^T = I - Y_1 X_1 \). Now, singular value decomposition gives
\[ [\Sigma \Phi \Phi^T] = \text{svd}(I - Y_1 X_1). \]
Therefore, \( N_1 \) and \( M_1 \) can be calculated as
\[ N_1 M_1^T = \Sigma \Phi \Phi^T, \quad N_1 = \Sigma A^{1/2}, \quad M_1 = \Phi A^{1/2}. \] (59)
This allows to calculate \( A_k, B_k, C_k \) and \( D_k \) from (57):
\[ D_k = \hat{D}_i, \]
\[ C_k = (\hat{C}_i - \hat{D}_i C X_1)(M_1^T)^{-1}, \]
\[ B_k = (N_1)^{-1}(\hat{B}_i - Y_1 B \hat{D}_i), \]
\[ A_k = (N_1)^{-1}(\hat{A}_i - Y_1 A X_1 - Y_1 B \hat{C}_i - N_1 B_k C X_1 (M_1^T)^{-1}). \]
\[ \Box \]

Proof of Theorem 3.1. The mathematical treatment follows along the same lines as in Theorem 2.1. From (52) and (53), it is easy to say that the sufficient conditions of the closed loop system in (23) to be asymptotically stable under the quadratic constraint in (24) are
\[ Y_D > 0, \] (60)
\[ \begin{bmatrix} \hat{A}_i Y_D + Y_D \hat{A}_i^T & G_D & Y_D H_i^T \\ -I & 0 \\ -\gamma_D I \end{bmatrix} < 0. \] (61)

Similar to Theorem 2.1, defining new variables as in (54) and (55),
\[ Y_i \triangleq \begin{bmatrix} X_i & M_i \\ M_i^T & V_i \end{bmatrix}, \quad Y_i^{-1} \triangleq \begin{bmatrix} Y_i & N_i \\ N_i^T & U_i \end{bmatrix}, \quad \Pi_2 \triangleq \begin{bmatrix} I & Y_1 \\ 0 & N_1^T \end{bmatrix}, \quad \hat{A}_i \triangleq Y_i^T (A_i + B_i D_{ki} C_i) X_i + N_i B_{ki} C_i X_i, \]
\[ + Y_i^T B_i C_i M_i^T + N_i A_{ki} M_i^T, \]
\[ \hat{B}_i \triangleq Y_i B_i D_{ki} + N_i B_{ki}, \quad \hat{C}_i \triangleq D_{ki} C_i X_i + C_{ki} M_i^T, \]
\[ \hat{D}_i \triangleq D_{ki}. \] (62)

where \( X_i \) and \( Y_i \) are \( n_i \times n_i \) and symmetric matrices, \( M_i \) and \( N_i \) are \( n_i \times n_i \) and \( Y_i > 0 \) implies \( X_i > 0, Y_i > 0 \). Hence, for the overall system
\[ Y_D = \text{diag}(Y_1, Y_2, \ldots, Y_n), \quad \Pi_2 = \text{diag}(\Pi_{21}, \Pi_{22}, \ldots, \Pi_{2n}), \]
and straightforward calculation shows that
\[ \Pi_2^T Y_D \Pi_2 = \text{diag} \left( \begin{bmatrix} X_1 & I \\ I & Y_1 \end{bmatrix}, \ldots, \begin{bmatrix} X_n & I \\ I & Y_n \end{bmatrix} \right), \]
\[ \Pi_2^T G_D = \text{diag} \left( \begin{bmatrix} G_1 \\ Y_1 G_1 \end{bmatrix}, \ldots, \begin{bmatrix} G_n \\ Y_n G_n \end{bmatrix} \right), \]
\[ \Pi_2^T \hat{A}_i Y_i \Pi_2 = \begin{bmatrix} A_i X_i + B_i \hat{C}_i & \hat{A}_i \\ Y_1 A_i + \hat{B}_i C_i \end{bmatrix}, \]
\[ \Pi_2^T \hat{A}_i X_i \Pi_2 \]
\[ = \text{diag} \left( \begin{bmatrix} (\mathcal{F}_{11})_i & (\mathcal{F}_{12})_i \\ (\mathcal{F}_{12})_i^T & (\mathcal{F}_{22})_i \end{bmatrix}, \ldots, \begin{bmatrix} (\mathcal{F}_{11})_n & (\mathcal{F}_{12})_n \\ (\mathcal{F}_{12})_n^T & (\mathcal{F}_{22})_n \end{bmatrix} \right), \]
where
\[ \hat{A}_i \triangleq (\hat{A}_D Y_D + Y_D\hat{A}_D^T), \]
\[ (\mathcal{F}_{11})_i \triangleq A_i X_i + X_i A_i^T + B_i \hat{C}_i + (B_i \hat{C}_i)^T, \]
\[ (\mathcal{F}_{12})_i \triangleq A_i + \hat{A}_i^T + B_i \hat{D}_i C_i, \]
\[ (\mathcal{F}_{22})_i \triangleq Y_i A_i + \hat{B}_i C_i + \hat{A}_i^T Y_i + C_i^T \hat{B}_i. \]
Pre- and post-multiplying (60) by \( \Pi_2^T \) and \( \Pi_2^T \), respectively, and (61) by \( \text{diag}(\Pi_{21}, I, I) \) and \( \text{diag}(\Pi_{22}, I, I) \), respectively, and adding pole placement constraints, the result follows. \[ \Box \]

References


