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Sampled-data Iterative Learning Control for a Class of Nonlinear Networked Control Systems

Ya-Jun Pan, Horacio J. Marquez and Tongwen Chen

Abstract— **In this paper, a sampled-data iterative learning control (ILC) approach is proposed for a class of nonlinear networked control systems. The motivation of this approach is to deal with control problems when the environment is periodic or repeatable over iterations in a fixed finite interval. In the networked control systems (NCS), because of the existence of time delays and packet losses in input and output signal transmissions, remote stabilization of linear systems is not an easy task. Moreover, to track a desired trajectory through a remote controller is even more difficult. By assuming a partial prior knowledge on the transmission time delays, we successfully incorporate previous cycle based learning (PCL) method into the network based control for a general nonlinear system which satisfies global Lipschitz condition. The convergence property of this approach is proved: the tracking error tends to be zero as the number of iteration increases. Furthermore, the convergence in the iteration domain can also be ensured when there exists packet loss in both transmission channels.**

I. INTRODUCTION

Networked control systems are being studied in many application areas for good reasons including their low cost, reduced weight, system wiring and power requirements, simple installation, simple system diagnosis and maintenance, and higher reliability. In NCS, one feature is that the control loops are closed through a real-time communication channel which transmits signals from the sensors to the controller and from the controller to the actuators [1]-[3]. However, two main issues occur in NCS. The first is the network-induced delays, namely, sensor-to-controller delay and controller-to-actuator delay, that occur while exchanging data among devices connected to the shared medium. Such delays, either constant or time varying, may destabilize the system, or degrade the performance of control systems designed without considering the delays. The second is that some packets not only suffer transmission delay but, even worse, can be lost in the transmission channel which affects the performance of an NCS as well.

Besides the stability problem, designing a remote controller to achieve tracking of nonlinear systems is a challenging problem. Fortunately, for periodic systems, iterative learning control offers a systematic design that can improve tracking performance by iterations in a fixed time interval [4]-[5]. In the literature, many learning control approaches were proposed for: linear or nonlinear systems, time delay systems, cascade systems, etc. These approaches include: P-type learning, D-type learning, anticipatory learning, robust learning and optimal learning [6]-[7], etc. Specifically, in the case that the sum of the two channel delays is less than the magnitude of the fixed time interval of the learning system, previous cycle based learning (PCL) can be well incorporated into the NCS controller design since the nature of the PCL includes delay properties itself. As a result, the PCL with anticipatory learning can be applied to the NCS for the periodic tracking control tasks. This is one motivation of this paper. For a linear system with input time delay only, in [8], the authors proposed a sampled data iterative learning control to ensure the stability of the closed loop system which can be unstable if the control law is designed in the continuous time domain. For the application of the iterative learning control in nonlinear NCS, to achieve tracking control tasks, no results have been available in the literature yet, which also motivates the study of this paper. Furthermore, in practical, most networked control systems are implemented by the digital control techniques [9]. Hence the proposed control is investigated as a sampled-data approach.

In this paper, for a nonlinear system controlled over a network, a sampled-data PCL based learning control approach is proposed. The purpose of this approach is to deal with control problems when the environment is periodic over iterations in a finite interval. The nonlinear system satisfies the global Lipschitz condition. Due to the existence of time delays and packet losses in input and output signal transmissions, tracking a desired trajectory through a remote controller is not an easy task. The proposed control law is realized assuming that: (i) the sensor-to-controller time delay is measurable under the ideal condition that the clock can be synchronized in the sensor and controller sides; (ii) according to (i), we have partial knowledge on the controller-to-actuator time delay such that we can compensate the delay effect on the system performance by anticipatory steps ahead in the previous cycle learning control design. The convergence property of this approach is then rigorously proved. Notations: $\|\mathbf{x}\|_\infty$ is the sup-norm defined as $\|\cdot\|_\infty = max_{1\leq\ i\leq\ n}|x_i|$ and the induced matrix norm is $\|A\|_\infty = max_{1\leq\ i\leq\ n}\sum_{j=1}^{m}|a_{ij}|$ for the matrix $A \in \mathcal{R}^{n\times m}$; $\lambda_i(A)$ denotes the i-th eigenvalue of the matrix A; $D_{\mathbf{x}}f = \frac{\partial f(\mathbf{x},\mathbf{y})}{\partial \mathbf{x}}$ and $D_{\mathbf{y}}f = \frac{\partial f(\mathbf{x},\mathbf{y})}{\partial \mathbf{y}}$ are row vectors, where $f(\cdot)$ is a scalar function.

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II. Problem Formulation

Consider the following nonlinear system

$$\begin{cases} \dot{\mathbf{x}}_i(t) = \mathbf{f}(\mathbf{x}_i(t),t) + B(\mathbf{x}_i(t),t)\mathbf{u}_i(t-\tau_2(t)) \\ \mathbf{y}_i(t) = \mathbf{g}(\mathbf{x}_i(t),t), \end{cases} \quad (1)$$

where $\mathbf{x}_i(t) \in \mathcal{R}^n$ is the state vector, $\mathbf{y}_i(t) \in \mathcal{R}^p$ is the output vector, $\mathbf{u}_i(t-\tau_2(t)) \in \mathcal{R}^q$ is the input vector where $\tau_2(t)$ is the time delay from the controller to the actuator, and the subscript i denotes the operation cycle. $\mathbf{f}(\mathbf{x}_i,t)$, $B(\mathbf{x}_i,t)$ and $\mathbf{g}(\mathbf{x}_i,t)$ are known functions with respect to the corresponding arguments. $\mathbf{g}(\mathbf{x}_i,t)$ is continuously differentiable, $\forall(\mathbf{x}_i,t) \in \mathcal{R}^n \times \mathcal{R}^+$, $\mathcal{R}^+ \triangleq [0,\infty)$. $t \in [0,T]$ is the finite time for the periodic operation of the system. The set up of

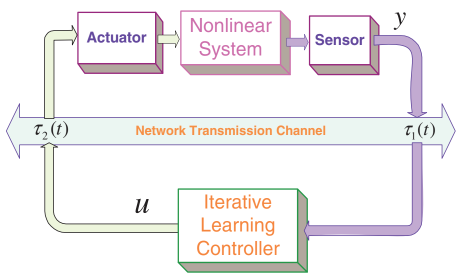

Fig. 1. Block diagram of the networked control systems

the control system in (1) is illustrated as in Fig.1. The sensor, actuator and the nonlinear system are remotely controlled by iterative learning controller that interchange measurement output and control signals through a communication network. In the sensor and controller sides, they are time driven and in the actuator side it is event-driven. In sending packets from one side to another, all information in one sampling is packaged in one packet. The objective of the controlled system is to track the desired trajectory $\mathbf{y}_d(t)$ in a finite time period $[0,T]$ which can be divided into K equal intervals with a certain sampling interval T_s. $\mathbf{y}_d(t)$ is realizable with a unique input bounded as $\|\mathbf{u}_d(\cdot)\| \leq \beta_{ud}$, where β_{ud} is a positive constant. The desired trajectory can be of the form

$$\begin{cases} \dot{\mathbf{x}}_d(t) = \mathbf{f}(\mathbf{x}_d(t),t) + B(\mathbf{x}_d(t),t)\mathbf{u}_d(t-\tau_2(t)) \\ \mathbf{y}_d(t) = \mathbf{g}(\mathbf{x}_d(t),t). \end{cases} \quad (2)$$

The following assumptions are first made in this paper.

Assumption 1: The resetting condition $\mathbf{x}_i(0) = \mathbf{x}_d(0)$ is satisfied for every ith iteration where i is a positive integer.

Assumption 2: The functions $\mathbf{f}(\mathbf{x},t)$, $B(\mathbf{x},t)$, $\mathbf{g}(\mathbf{x},t)$, $D_{\mathbf{x}}\mathbf{g}(\mathbf{x},t)$ and $D_t\mathbf{g}(\mathbf{x},t)$ are assumed to be globally uniformly Lipschitz in $\mathbf{x}$ on the finite period $[0,T]$, or

$$\|N(\mathbf{x}_1(t),t) - N(\mathbf{x}_2(t),t)\|_\infty \leq c_N\|\mathbf{x}_1(t) - \mathbf{x}_2(t)\|_\infty,$$

$\forall t \in [0,T]$, where c_N are positive constants for $N \in \{\mathbf{f}, B, \mathbf{g}, D_{\mathbf{x}}\mathbf{g}, D_t\mathbf{g}\}$.

Assumption 3: The functions $\mathbf{f}(\mathbf{x},t)$, $B(\mathbf{x},t)$, $\mathbf{g}(\mathbf{x},t)$, $D_{\mathbf{x}}\mathbf{g}(\mathbf{x},t)$ and $D_t\mathbf{g}(\mathbf{x},t)$ are assumed to be bounded as $\|N(\mathbf{x}(t),t)\|_\infty \leq \beta_N$, $\forall(\mathbf{x},t) \in \mathcal{R}^n \times \mathcal{R}^+$, $\mathcal{R}^+ \triangleq [0,T]$, where β_N are positive constants for $N \in \{\mathbf{f}, B, \mathbf{g}, D_{\mathbf{x}}\mathbf{g}, D_t\mathbf{g}\}$.

In this paper, we assume that the transmission time delay from the sensor to the controller, namely, $\tau_1(t)$ can be measured. This requires the clock synchronization of both sides. In practice, it is measured in the transmission channel by sending a local time signal t to the opposite side. On the controller side, it is measured by getting the difference between the current time (t) of the local controller side and the received signal delayed from the sensor side $(t-\tau_1(t))$. Furthermore, we assume that we have some knowledge on the time delay of the transmission channel from the controller to the actuator, $\tau_2(t)$, as shown in Assumption 4 below. This is reasonable because we can have some reference information from the measurement of $\tau_1(t)$ if the two transmission channels have similar settings while there are some differences between the two channel delays.

In [10], the delay is assumed to be a combination of two parts: a multiple of the sampling period and variation in the sampling instants. Similar application can be found in the work [11]. In this paper, as shown in the following Assumption 4, we still consider the time delay $\tau_2(t)$ in this format as in the literature, which better shows the nature of the time delay in sampled-data system. Then we address the problem of controller design with robustness to the delay variations in the sampling instants.

Assumption 4: The time delay can be represented as $\tau_2(t) = h_2T_s + \varepsilon_2(t)$, where $h_2 > 0$ is a known integer constant, T_s is the sampling time, and $\varepsilon_2(t)$ is unknown but is bounded as $0 \leq \varepsilon_2(t) < T_s$. $\tau_1(t)+\tau_2(t) < T$ is assumed to hold.

Note that Assumption 4 is made under the condition that the network is not busy and the transmission load is not very heavy. More discussions on the relationship between the message priority, transmission load and the variation of the time delay can be found in [1].

III. Sampled-data Iterative Learning Controller Design

In this section, we consider the controller design when there are no packet loss in signal transmission, i.e., the network load is not heavy. The following PCL based learning controller with delay compensation is designed for the periodic control task of the nonlinear system

$$\mathbf{u}_{i+1}(k) = \mathbf{u}_i(k) + \Gamma(k)\mathbf{e}_i(k+h_2+1), \quad (3)$$
$$k \in \{0,1,\cdots,K-h_2-1\},$$

where $\mathbf{u}_i(k) \triangleq \mathbf{u}_i(kT_s)$, $\mathbf{e}_i(k+h_2+1) = \mathbf{y}_d(k+h_2+1) - \mathbf{y}(k+h_2+1)$ and $\Gamma(k) \triangleq \Gamma(kT_s)$ is the control gain to be designed. The convergence property of the closed loop system is shown as in the following theorem.

Theorem 1: Assume that the system in (1) satisfies Assumptions 1-4. Under the learning control law in (3), if the following condition

$$\left\| I - \Gamma(k)\int_{(k+h_2)T_s+\varepsilon_2(k)}^{(k+h_2+1)T_s} D_x\mathbf{g}(\mathbf{x},s)B(\mathbf{x},s)ds \right\|_\infty \triangleq \upsilon(k) \leq \rho < 1 \quad (4)$$

is satisfied, then

$$\lim_{i\to\infty}\mathbf{x}_i(kT_s)=\mathbf{x}_d(kT_s),\quad \lim_{i\to\infty}\mathbf{y}_i(kT_s)=\mathbf{y}_d(kT_s),$$
$$\forall k\in\{h_2+1,h_2+2\cdots,K\}.$$

Proof: Denotes $\mathbf{g}_d\stackrel{\triangle}{=}\mathbf{g}(\mathbf{x}_d(t),t)$. At the ith iteration cycle and at the sampling instant $t=(k+h_2+1)T_s$, the tracking error $\mathbf{e}_i(k+h_2+1)$ can be represented as

$$\begin{aligned}
&\mathbf{e}_i(k+h_2+1)\\
=\;&\mathbf{g}(\mathbf{x}_d(k+h_2+1),k+h_2+1)\\
&-\mathbf{g}(\mathbf{x}_i(k+h_2+1),k+h_2+1)\\
=\;&\mathbf{g}(\mathbf{x}_d(k+h_2),k+h_2)-\mathbf{g}(\mathbf{x}_i(k+h_2),k+h_2)\\
&+\int_{(k+h_2)T_s}^{(k+h_2+1)T_s}\left[\dot{\mathbf{g}}(\mathbf{x}_d(s),s)-\dot{\mathbf{g}}(\mathbf{x}(s),s)\right]ds\\
=\;&\mathbf{e}_i(k+h_2)+\int_{(k+h_2)T_s}^{(k+h_2+1)T_s}\left[D_{\mathbf{x}_d}\mathbf{g}(\mathbf{x}_d(s),s)\dot{\mathbf{x}}_d(s)\right.\\
&+D_t\mathbf{g}(\mathbf{x}_d(s),s)-D_{\mathbf{x}}\mathbf{g}(\mathbf{x}(s),s)\dot{\mathbf{x}}(s)\\
&\left.-D_t\mathbf{g}(\mathbf{x}(s),s)\right]ds\\
=\;&\mathbf{e}_i(k+h_2)+\int_{(k+h_2)T_s}^{(k+h_2+1)T_s}\left[D_t\mathbf{g}_d-D_t\mathbf{g}\right.\\
&\left.+(D_{\mathbf{x}_d}\mathbf{g}_d)\mathbf{f}_d-(D_{\mathbf{x}}\mathbf{g})\mathbf{f}\right]ds\\
&+\int_{(k+h_2)T_s}^{(k+h_2+1)T_s}\left[(D_{\mathbf{x}_d}\mathbf{g}_d)B_d\mathbf{u}_d\right.\\
&\left.-(D_{\mathbf{x}}\mathbf{g})B\mathbf{u}_i(s-\tau_2(s))\right]ds.
\end{aligned}\tag{5}$$

As shown in Fig.2, we have $\mathbf{u}_i(t-\tau_2(t))=\mathbf{u}_i(k-1)$, $\forall t\in[(k+h_2)T_s,(k+h_2)T_s+\varepsilon_2(k))$ and $\mathbf{u}_i(t-\tau_2(t))=\mathbf{u}_i(k)$, $\forall t\in[(k+h_2)T_s+\varepsilon_2(k),(k+h_2+1)T_s)$. Denotes $\Delta\mathbf{z}_i=\mathbf{w}_d-\mathbf{w}_i$ where $\mathbf{w}$ is a variable. The equation in (5) becomes

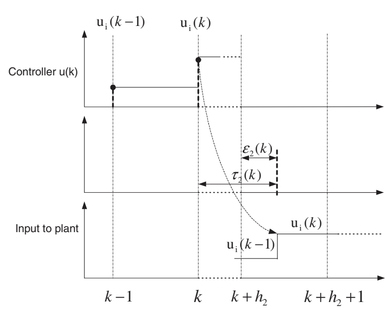

Fig. 2. Time diagram of the flow from controller to actuator in the case of no packet loss

$$\begin{aligned}
&\mathbf{e}_i(k+h_2+1)\\
=\;&\mathbf{e}_i(k+h_2)+\int_{(k+h_2)T_s}^{(k+h_2+1)T_s}\left[\Delta(D_t\mathbf{g}_i)\right.\\
&\left.+\Delta(D_{\mathbf{x}_i}\mathbf{g}\mathbf{f}_i)+\Delta(D_{\mathbf{x}_i}\mathbf{g}B_i)\mathbf{u}_d\right]ds\\
&+\int_{(k+h_2)T_s}^{(k+h_2)T_s+\varepsilon_2(k)}(D_{\mathbf{x}}\mathbf{g})Bds\Delta\mathbf{u}_i(k-1)\\
&+\int_{(k+h_2)T_s+\varepsilon_2(k)}^{(k+h_2+1)T_s}(D_{\mathbf{x}}\mathbf{g})Bds\Delta\mathbf{u}_i(k).
\end{aligned}\tag{6}$$

Substituting (6) into the controller in (3) and deriving $\Delta\mathbf{u}_{i+1}(k)=\mathbf{u}_d(k)-\mathbf{u}_{i+1}(k)$ as

$$\begin{aligned}
&\Delta\mathbf{u}_{i+1}(k)=\Delta\mathbf{u}_i(k)-\Gamma(k)\mathbf{e}_i(k+h_2+1)\\
=\;&\left[I-\Gamma(k)\int_{(k+h_2)T_s+\varepsilon_2(k)}^{(k+h_2+1)T_s}(D_{\mathbf{x}}\mathbf{g})Bds\right]\Delta\mathbf{u}_i(k)\\
&-\Gamma(k)\{\mathbf{e}_i(k+h_2)+\int_{(k+h_2)T_s}^{(k+h_2+1)T_s}\left[\Delta(D_t\mathbf{g}_i)\right.\\
&\left.+\Delta(D_{\mathbf{x}_i}\mathbf{g}\mathbf{f}_i)+\Delta(D_{\mathbf{x}_i}\mathbf{g}B_i)\mathbf{u}_d\right]ds\\
&+\int_{(k+h_2)T_s}^{(k+h_2)T_s+\varepsilon_2(k)}(D_{\mathbf{x}}\mathbf{g})Bds\Delta\mathbf{u}_i(k-1)\}.
\end{aligned}$$

Using the Lipschitz and boundedness properties in Assumptions 2 and 3, and $\|\Gamma(k)\|_\infty\le\beta_\Gamma$, then

$$\begin{aligned}
\|\Delta\mathbf{u}_{i+1}(k)\|_\infty\le\;&\upsilon(k)\|\Delta\mathbf{u}_i(k)\|_\infty\\
&+\beta_\Gamma\beta_g\|\Delta\mathbf{x}_i(k+h_2)\|_\infty\\
&+\beta_\Gamma\gamma_1\int_{(k+h_2)T_s}^{(k+h_2+1)T_s}\|\Delta\mathbf{x}_i(s)\|_\infty ds\\
&+\beta_\Gamma\gamma_2\|\Delta\mathbf{u}_i(k-1)\|_\infty,
\end{aligned}\tag{7}$$

where

$$\upsilon(k)=\left\|I-\Gamma(k)\int_{(k+h_2)T_s+\varepsilon_2(k)}^{(k+h_2+1)T_s}D_x\mathbf{g}(\cdot)B(\cdot)ds\right\|_\infty,$$

$$\gamma_1=\beta_{D_{\mathbf{x}}\mathbf{g}}(c_f+\beta_{\mathbf{u}_d}c_B)+c_{D_{\mathbf{x}}\mathbf{g}}(\beta_f+\beta_{\mathbf{u}_d}\beta_B)+c_{D_t\mathbf{g}},$$

$$\gamma_2=\beta_{D_{\mathbf{x}}\mathbf{g}}\beta_BT_s>\beta_{D_{\mathbf{x}}\mathbf{g}}\beta_B\varepsilon_2(k).$$

Now we derive the relationship between $\|\Delta\mathbf{x}_i(k)\|_\infty$ and $\|\Delta\mathbf{u}_i(k)\|_\infty$. From the system dynamics in (1),

$$\begin{aligned}
\Delta\mathbf{x}_i(t)\;=\;&\int_0^t\left[\Delta\mathbf{f}_i+B_i\Delta\mathbf{u}_i(s-\tau_2(s))\right.\\
&\left.+\Delta B_i\mathbf{u}_d(s-\tau_2(s))\right]ds,
\end{aligned}$$

it follows that

$$\begin{aligned}
\|\Delta\mathbf{x}_i(t)\|_\infty\;\le\;&\int_0^t\left[\gamma_3\|\Delta\mathbf{x}_i(s)\|_\infty\right.\\
&\left.+\beta_B\|\Delta\mathbf{u}_i(s-\tau_2(s))\|_\infty\right]ds,
\end{aligned}\tag{8}$$

where $\gamma_3=c_f+\beta_{\mathbf{u}_d}c_B$. Using $\mathbf{u}_i(t)=0$ and $\varepsilon_2(t)=0$ $\forall t<0$, the constant control input $\mathbf{u}_i(t-\tau_2(t))=\mathbf{u}_i(k)$ in each intervals $\forall t\in[(k+h_2)T_s+\varepsilon_2(k),(k+h_2+1)T_s+\varepsilon_2(k+1))$, the Bellman-Gronwall inequality and $0\le\varepsilon_2(\cdot)<T_s$, $\forall t\le(k+h_2+1)T_s$, we have

$$\begin{aligned}
\|\Delta\mathbf{x}_i(t)\|_\infty\;\le\;&\beta_B\int_0^{(k+h_2+1)T_s}e^{\gamma_3((k+h_2+1)T_s-s)}\\
&\|\Delta\mathbf{u}_i(s-\tau_2(s))\|_\infty ds\\
=\;&\frac{\beta_B}{\gamma_3}\sum_{j=0}^{k-1}e^{\gamma_3(k-1-j)T_s}\left[e^{\gamma_3(2T_s-\varepsilon_2(j))}\right.\\
&\left.-e^{\gamma_3(T_s-\varepsilon_2(j+1))}\right]\|\Delta\mathbf{u}_i(j)\|_\infty\\
&+\frac{\beta_B}{\gamma_3}\left[e^{\gamma_3(T_s-\varepsilon_2(k))}-1\right]\|\Delta\mathbf{u}_i(k)\|_\infty
\end{aligned}$$

$$
\begin{aligned}
&< \beta_B \frac{e^{2\gamma_3 T_s}-1}{\gamma_3} \sum_{j=0}^{k-1} e^{\gamma_3(k-1-j)T_s} \|\Delta \mathbf{u}_i(j)\|_\infty \\
&+\frac{\beta_B}{\gamma_3}\left(e^{\gamma_3 T_s}-1\right)\|\Delta \mathbf{u}_i(k)\|_\infty. \qquad (9)
\end{aligned}
$$

Similarly, $\forall t \leq (k+h_2)T_s$, we have

$$
\begin{aligned}
&\|\Delta \mathbf{x}_i(k+h_2)\|_\infty \\
&< \beta_B \frac{e^{2\gamma_3 T_s}-1}{\gamma_3} \sum_{j=0}^{k-2} \|\Delta \mathbf{u}_i(j)\|_\infty e^{\gamma_3(k-2-j)T_s} \\
&+\frac{\beta_B}{\gamma_3}(e^{\gamma_3 T_s}-1)\|\Delta \mathbf{u}_i(k-1)\|_\infty. \qquad (10)
\end{aligned}
$$

Substituting (9) and (10) into the inequality (7), we have

$$
\begin{aligned}
\|\Delta \mathbf{u}_{i+1}(k)\|_\infty &< \rho_1(k)\|\Delta \mathbf{u}_i(k)\|_\infty + \rho_2\|\Delta \mathbf{u}_i(k-1)\|_\infty \\
&+\rho_3 \sum_{j=0}^{k-2} \rho_4(k,j)\|\Delta \mathbf{u}_i(j)\|_\infty, \qquad (11)
\end{aligned}
$$

where

$$
\begin{aligned}
\rho_1(k) &= \upsilon(k) + \beta_B(e^{\gamma_3 T_s}-1)/\gamma_3, \\
\rho_2 &= \beta_\Gamma \gamma_2 + \beta_\Gamma \beta_g \beta_B (e^{\gamma_3 T_s}-1)/\gamma_3 \\
&\quad +\beta_\Gamma \gamma_1 T_s \beta_B (e^{2\gamma_3 T_s}-1)/\gamma_3, \\
\rho_3 &= \beta_\Gamma \beta_B (e^{2\gamma_3 T_s}-1)/\gamma_3, \\
\rho_4(k,j) &= \gamma_1 T_s e^{\gamma_3(k-1-j)T_s} + \beta_g e^{\gamma_3(k-2-j)T_s}.
\end{aligned}
$$

From $k=0$ to $k=K-h_2-1$ where $K=T/T_s$, (11) can be expressed as

$$
\boldsymbol{\phi}_{i+1} < H_i \boldsymbol{\phi}_i, \qquad (12)
$$

where $\boldsymbol{\phi}_i = [\|\Delta \mathbf{u}_i(0)\|_\infty, \|\Delta \mathbf{u}_i(1)\|_\infty, \cdots, \|\Delta \mathbf{u}_i(\chi)\|_\infty]^T$ with $\chi = K-h_2-1$, and

$$
H_i = \begin{bmatrix}
\rho_1(0) & 0 & 0 & \cdots & 0 \\
\rho_2 & \rho_1(1) & 0 & \cdots & 0 \\
\rho_3\rho_4(2,0) & \rho_2 & \rho_1(2) & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\rho_3\rho_4(\chi,0) & \rho_3\rho_4(\chi,1) & \rho_3\rho_4(\chi,2) & \cdots & \rho_1(\chi)
\end{bmatrix}. \qquad (13)
$$

Since H_i is a lower triangular matrix, all of its eigenvalues are the diagonal elements $\rho_1(k)$, $\forall k=0,1,\cdots,K-h_2-1$. Selecting the gain matrix Γ such that all the eigenvalues of H_i are within the unit circle, i.e., $\rho_1(k)<1$, the asymptotic convergence property of $\|\Delta \mathbf{u}_i(k)\|_\infty$ can be guaranteed along the iteration axis i. Then $\|\Delta \mathbf{u}_i(k)\|_\infty \to 0$ as $i \to \infty$, $\forall k=0,1,\cdots,K-h_2-1$. Note that if $0<\upsilon(k)\leq \rho<1$ is satisfied, it is possible to choose the sampling time T_s to be small such that $e^{\gamma_3 T_s}-1=0$ and as a result $\rho_1(k)<1$, $\forall k=0,1,\cdots,K-h_2-1$. Furthermore, from the controller structure in (3), $\|\Delta \mathbf{u}_i(k)\|_\infty \to 0$ implies that: $\mathbf{u}_i(k)\to \mathbf{u}_d(k)$. Then $\mathbf{x}_i(k)\to \mathbf{x}_d(k)$, $\mathbf{y}_i(k)\to \mathbf{y}_d(k)$ as $i\to\infty$, $\forall k=h_2+1,\cdots,K$. ■

IV. Effects of Packets Loss

In this section, we further discuss the situation when packets loss happen during the transmission. This phenomena occurs when the network is busy and under heavy load.

A. Single Packet Loss from Controller to Actuator Side

Here we start by assuming that there is one packet lost at time $t=kT_s$ during the ith iteration. In this case, the actuator side will not receive this packet. On the actuator side, since it is event driven, the packet sent at time $t=(k-1)T_s$ from the controller side will continue to be applied in the system before the packet sent at time $t=(k+1)T_s$ arrives. This procedure can be clearly seen in Fig.3.

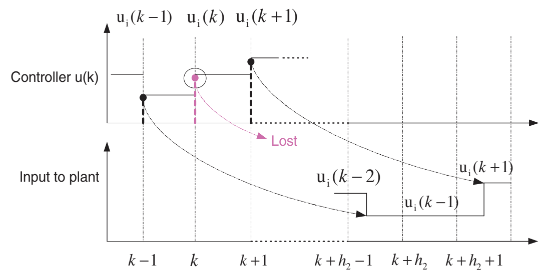

Fig. 3. Time diagram from controller to actuator with packet loss at $t=kT_s$ during the ith iteration

The learning controller is the same as in (3). For this case, from Fig.3, we have

$$
\begin{aligned}
\mathbf{u}_i(t-\tau_2(t)) = \mathbf{u}_i(k-1), \quad &\forall t\in[(k+h_2-1)T_s \\
&+\varepsilon_2(k-1),(k+h_2+1)T_s+\varepsilon_2(k+1)).
\end{aligned}
$$

Hence $\mathbf{e}_i(k+h_2+1)$ and $\mathbf{e}_i(k+h_2+2)$ are changed because the packet of $\mathbf{u}_i(k)$ is lost in the transmission channel from the controller to the actuator. The convergence property of this case can be concluded in the following theorem.

Theorem 2: Assume that there is one packet loss at time $t=kT_s$ during the ith iteration in the iteration domain. If the condition in (4) holds, then we still have

$$
\begin{aligned}
&\lim_{i\to\infty}\mathbf{x}_i(kT_s)=\mathbf{x}_d(kT_s), \quad \lim_{i\to\infty}\mathbf{y}_i(kT_s)=\mathbf{y}_d(kT_s), \\
&\qquad \forall k\in\{h_2+1,h_2+2\cdots,K\}.
\end{aligned}
$$

Proof: The proof can be composed of two parts due to packet loss effects on $\mathbf{e}_i(k+h_2+1)$ and $\mathbf{e}_i(k+h_2+2)$ during the ith iteration. As a result, the analysis on $\|\Delta \mathbf{u}_{i+1}(k)\|_\infty$ and $\|\Delta \mathbf{u}_{i+1}(k+1)\|_\infty$ are changed correspondingly (derivations are omitted due to page limit). Then the mapping matrix H_i is changed in the $(k+1)$st diagonal element and $(k+2)$nd rows if there is one packet loss at time $t=kT_s$ during the ith iteration. From $k=0$ to $k=K-h_2-1$, the mapping matrix H_i in (12) becomes $H_{i,k}$ in which the $(k+1)$st and $(k+2)$nd rows are different with H_i: $\boldsymbol{\phi}_{i+1}\leq H_{i,k}\boldsymbol{\phi}_i$, and $H_{i,k}$ is

$$
\begin{bmatrix}
\rho_1(0) & 0 & \cdots & \cdots & \cdots & 0 \\
\vdots & \ddots & \ddots & & & \vdots \\
\rho_3\rho_4(k,0) & \cdots & 1 & \ddots & & \vdots \\
\rho_3\rho_{4,l_{k+1}}(k+1,0) & \cdots & \rho_{2,l_{k+1}} & \rho_1(k+1) & \ddots & \vdots \\
\vdots & \vdots & \vdots & \ddots & \ddots & 0 \\
\rho_3\rho_4(K,0) & \cdots & \cdots & \cdots & \cdots & \rho_1(K)
\end{bmatrix},
$$

where

$$\rho_{2,l_{k+1}} = \beta_\Gamma\gamma_2 + \beta_\Gamma\beta_g\beta_B(e^{2\gamma_3 T_s} - 1)/\gamma_3 + \beta_\Gamma\gamma_1 T_s\beta_B(e^{3\gamma_3 T_s} - 1)/\gamma_3,$$
$$\rho_{4,l_{k+1}}(k+1, j) = \gamma_1 T_s e^{\gamma_3(k-j)T_s} + \beta_g e^{\gamma_3(k-1-j)T_s}.$$

Note that $H_{i,k}$ is also a lower triangular matrix, all its eigenvalues are the diagonal elements, with one element being "1" at $(k+1, k+1)$ and other $\rho_1(k) < 1$, $\forall k = 0, 1, \cdots, k-1, k+1, \cdots, K-h_2-1$. The asymptotic convergence property of $\|\Delta \mathbf{u}_i(k)\|_\infty$ can also be guaranteed along the iteration axis i because all eigenvalues of the mappings satisfy

$$\lambda_m \left[(\prod_{j=1}^{i-1} H_j) H_{i,k} (\prod_{j=i+1}^{\infty} H_j) \right] \longrightarrow 0, \quad m = 0, 1, \cdots, K-h_2-1. \tag{14}$$

Then $\|\Delta \mathbf{u}_i(k)\|_\infty \to 0$ as $i \to \infty$, $\forall k = 0, 1, \cdots, K-h_2-1$. Furthermore, from the controller structure in (3), $\|\Delta \mathbf{u}_i(k)\|_\infty \to 0$ also implies that $\mathbf{u}_i(k) \to \mathbf{u}_d(k)$, $\forall k = 0, 1, \cdots, K-h_2-1$. Then $\mathbf{x}_i(k) \to \mathbf{x}_d(k)$ and $\mathbf{y}_i(k) \to \mathbf{y}_d(k)$ as $i \to \infty$, $\forall k = h_2+1, \cdots, K$. ■

B. Single Packet Loss from Sensor to Controller Side

The difference between the packet loss in the two channels is that, from sensor to controller side the measured output signal is lost, while from controller to actuator side, the calculated control signal is lost. For convenience, assuming that the packet of $\mathbf{y}_i(k+h_2+1)$ during the ith iteration is missing in the memory of the controller side due to the data loss in the transmission from sensor to controller. In this case, the controller side will continue to use the output signal $\mathbf{y}_i(k+h_2)$ closest available signal previous to the current time stamp in the memory. Hence from (3), the controller is changed accordingly,

$$\mathbf{u}_{i+1}(k) = \mathbf{u}_i(k) + \Gamma(k)\mathbf{e}_i(k+h_2), \quad k \in \{0, 1, \cdots, K-h_2-1\}. \tag{15}$$

The convergence property is then concluded in the following Theorem, which is similar as Theorem 2.

Theorem 3: Assume that there is one packet loss at time $t = (k+h_2+1)T_s$ during the ith iteration in the iteration domain. If the condition in (4) holds, then we still have

$$\lim_{i\to\infty} \mathbf{x}_i(kT_s) = \mathbf{x}_d(kT_s), \quad \lim_{i\to\infty} \mathbf{y}_i(kT_s) = \mathbf{y}_d(kT_s), \quad \forall k \in \{h_2+1, h_2+2 \cdots, K\}.$$

C. Discussions on Multiple Packets Loss

From the observations on single packet loss in the two channels as shown in Theorem 2 and Theorem 3, we can find that multiple packets loss in both channels together will result in more "1" in the diagonal elements of the mapping matrix H_i $(i = 1, \cdots, \infty)$[1]. The structure of H_i remains the lower triangular form. This can be concluded from the proofs in Theorem 1 and Theorem 2 that only the past information previous to the time stamp of the lost packet will be used when the packet loss happens.

[1]There are multiple packets loss at $t = jT_s$ in the ith iteration as $j \in 0, 1, \cdots, K-h_2-1$.

Now review the structure of the serial product of the mapping matrix H_i in the ith iteration axis:

$$\Pi_{i=1}^{\infty} H_i = \begin{bmatrix} \Pi_{i=1}^{\infty}\rho_{i,1}(0) & 0 & \cdots & 0 \\ * & \Pi_{i=1}^{\infty}\rho_{i,1}(1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ * & \cdots & * & \Pi_{i=1}^{\infty}\rho_{i,1}(K-h_2-1) \end{bmatrix}. \tag{16}$$

From (16), $\Pi_{i=1}^{\infty} H_i$ is also a lower triangular matrix. The entries denoted as $(*)$ are multiple combination of the elements in $H_{i,j}$ and they are finite. However it's difficult to represent them mathematically to be a general form when $i \to \infty$. Hence we have the following Corollary, which further shows the impact of multiple packets loss on the system convergence rate.

Corollary 1: Define the packet loss rate as $0\% < P_{loss} \ll 100\%$ along the infinite iteration domain. Assume that the packet loss probability in the time period $\forall t \in [0, T]$ is uniformly distributed, we have the following convergence property: as $i \to \infty$, $\forall k = h_2+1, \cdots, K$, $\mathbf{y}_i(k)$ approaches $\mathbf{y}_d(k)$ with a slower convergence rate than the case without any packet loss.

V. Illustrative Examples

Consider the following nonlinear system of a single-link rigid robot which is controlled through some network

$$J\ddot{\theta} = -(0.5mgl + Mgl)sin(\theta) + u \tag{17}$$

where θ, $m = 1.5\ kg$, $M = 3\ kg$, $g = 9.8\ m/s^2$, $l = 0.5\ m$ and $J = Ml^2 + \frac{1}{3}ml^2 = 0.8333\ kg \cdot m^2$ denote the rotating angle, mass of the load, mass of the rigid link, gravitational acceleration, length and inertia of the robot link, respectively. u is the torque control input. The desired trajectory is $\theta_d = (\pi t^2)/6 - (\pi t^3)/27$ rad. According to (3), the iterative learning controller is

$$u_{i+1}(k) = u_i(k) + \gamma_1 \left[\theta_d(k+h_2+1) - \theta_i(k+h_2+1)\right] + \gamma_2 \left[\dot{\theta}_d(k+h_2+1) - \dot{\theta}_i(k+h_2+1)\right], \quad k \in \{0, 1, \cdots, K-h_2-1\},$$

where the control gain $\Gamma = [\gamma_1, \gamma_2]^T$ is designed to satisfy the condition in (4): $\upsilon = |1 - (T_s - \varepsilon_2)\Gamma\mathbf{b}| \leq \rho < 1$. Note that $0 \leq \varepsilon_2 < T_s$, hence it is easy to select Γ. In this simulation, $\Gamma = [1, 1]^T$ is chosen. The controller is set to $u_{i+1}(k) = u_i(k) + \Gamma(k)\mathbf{e}_i(K)$ when $k > K-h_2-1$.

Case 1: With numerical time delays $\tau_1 = \tau_2 = 4.5T_s + 0.45T_s sin(10\pi t)$

In this case, we have $h_2 = 4$. The maximum tracking error versus iterations is shown in Fig.4. From the evolution of the profile of the maximum tracking error, we can clearly observe the trend that it goes to zero asymptotically as iteration increases. Due to the influence of the lower triangular entries in the mapping matrix H_i as in (13), note that the maximum position tracking error is not monotonically decreasing but it satisfies asymptotical convergence property, which is consistent with Theorem 1.

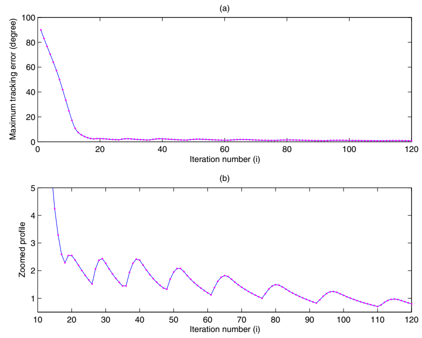

Fig. 4. The evolution of the maximum position tracking error versus iteration number: (a) Maximum tracking errors; (b) Zoomed view.

Case 2: With measured real time delays and packets loss

The time delay is measured using UDP protocol for transmissions in the Ethernet network. From Fig.5, we get that $h_2 = 4$. Correspondingly as in Case 1, the maximum tracking error versus iterations is shown in Fig.6.

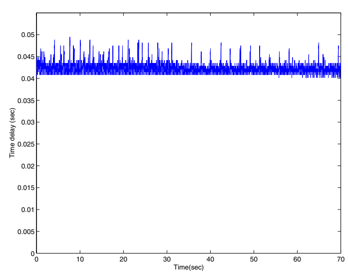

Fig. 5. The measured time delays when using UDP protocol in transmission.

Hence as long as the time delays satisfy Assumption 4, according to Theorem 1, the system position approaches the desired one from iteration to iteration. As shown in Fig.4 and Fig.6, the convergence is not monotonic but is asymptotic while there is a little difference in the convergent profiles due to the different types of time delays.

Furthermore, we consider the situation when packets loss rate through the time duration is $P_{loss} = 10\%$ throughout the iteration domain. As shown in Fig.6, the convergence property is retained while the convergence rate is slower than the case without any packet loss, which further verifies the results in Theorem 2, Theorem 3 and Corollary 1.

Furthermore, as shown in Fig.7, when $h_2 = 15$ and the controller is designed without h_2 steps compensation, then the system is not stable.

VI. Conclusions

For periodic control tasks of a class of nonlinear systems whose dynamics satisfy the global Lipschitz condition, a sampled-data iterative learning control approach is proposed

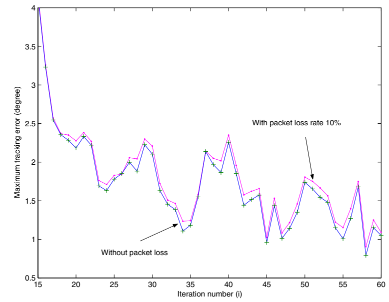

Fig. 6. The evolution of the maximum position tracking error versus iteration number when there are packets loss in transmission.

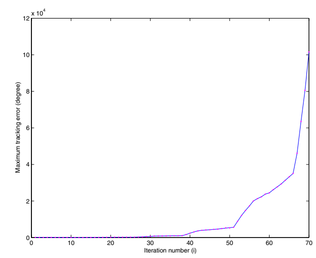

Fig. 7. The maximum tracking errors with no compensation in the controller.

with network transmission. By virtue of a PCL based learning law and partially known information on the time delays, the tracking error approaches zero as the number of the iterations increases. This property holds when there are packets dropout with certain rate in the data transmission. The proposed approach can eliminate the influence of time delays by compensation in the learning law, ensuring convergence.

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