Robust Output Feedback Stabilization of Nonlinear Interconnected Systems with Application to an Industrial Utility Boiler

Adarsha Swarnakar, Horacio Jose Marquez and Tongwen Chen

Abstract — This paper presents a new scheme for robust stabilization of nonlinear interconnected systems, based on linear matrix inequalities (LMIs). The fact that the improvement in stability is significant and the controller uses only the output information of the plant, leads to the name robust output feedback control. The control design is formulated as a convex optimization problem, which makes it computationally tractable, when the problem size increases. The controller concept is then evaluated on a natural circulation drum boiler (Utility boiler), where the nonlinear model describes the complicated dynamics of the drum, downcomer, and riser components. The system is non-minimum phase and has two poles at the origin, which are major sources of interaction, bandwidth limitation and instability. Simulation results are presented which show the effectiveness of the proposed control against instabilities following sudden load variations. The control is also effective for steady-state operation.

I. INTRODUCTION

During the last few decades, many researchers in the field of power systems are devoted to decentralized robust control strategies [1], [4], [6], [9], [13]. There are also dramatic changes in the power industry because of deregulation. One consequence of this fact is that the demands for rapid changes in power generation are increasing. This leads to more stringent requirements on the control systems for the processes. Also in deregulated environment, system tends to be more stressed and the load distribution is virtually impossible to anticipate. Under such circumstances, it is necessary to develop robust control designs that keep the processes operating well for large changes in the operating conditions and can protect it against both large and small disturbances, wherever they may arise.

Recently, there has been a strong research effort in literature towards development of decentralized robust control strategies [14], including decentralized turbine/governor control [4], [8], [9] and decentralized excitation control [1], [6], [11]. A linear feedback control based on the solution of parameterized Riccati equations for each subsystem is developed in [8]. These ideas have been extended to robust exciter control based on the concept of direct feedback linearization, which transforms the original nonlinear model into a linear one. After such a transformation, the control design becomes straightforward, but the implementation is complicated by the fact that the resulting controller is nonlinear. There are several results introduced in [9], [11], where linear feedback is used in conjunction with appropriately chosen Lyapunov functions to produce lower bounds for lower gains. A problem that arises in this context is associated with the quadratic term in the model, which cannot be properly incorporated into the analysis. In [11], this issue is resolved by performing a partial linearization, which amounts to discarding the problematic term from the model. An efficient approach based on LMIs is then presented in [1], which do not require any approximations.

The main objective of this paper is to apply the multifaceted tools of LMI [5], [10] for the design of robust output feedback control of nonlinear interconnected systems. A method of designing decentralized static state feedback control, based on LMI has been developed in [1], [2], [4]. In [2], new ideas are presented for broadening the scope of control design under information structure constraints. Both structural and algebraic enhancements of decentralized feedback are considered, with LMIs as a common tool. In this paper, we propose a new mathematical framework of designing a dynamic output feedback controller, based on LMI. As the controller is dynamic, it introduces extra degrees of freedom in the design and it also brings robustness of the system against uncertainties and neglected dynamics. This analysis is then extended to models that include decentralized control. To this end, we present a new approach of designing decentralized dynamic output feedback control for interconnected systems. All controllers are linear; so their implementation is straightforward and without exorbitant cost of additional observer design.

We have given the proposed control the name robust output feedback control. The term robust output feedback is one that requires careful explanation. About 30 percent of the emergency shutdowns in pressurized water reactor (PWR) plants are caused by poor level control of the steam water level. One reason is that the control problem is difficult because of the complicated shrink and swell dynamics. This creates a nonminimum phase behavior which changes significantly with the operating conditions. Our control design is based on the complex nonlinear model for steam generation systems [7], and the physical parameters of the model are obtained from Syncrude Canada Ltd. (SCL) located in Mildred Lake, Alberta, Canada. The state variables of the model are: the total volume of water in the drum \( V_{\text{drum}} \), the drum pressure \( p \) which represents the total energy, the steam-mass fraction in the riser \( \alpha_r \) which captures the distribution of steam and water, and the steam volume in the drum \( V_{\text{sd}} \). The deviation of the drum level from its normal operating point can be
measured by differential pressure transmitter, but steam mass fraction \( \alpha \) is not a measurable parameter. Moreover, the system is non-minimum phase and has two poles at the origin (associated with water dynamics and pressure dynamics), which are major sources of interaction, bandwidth limitation and instability. Hence, from industrial point of view, there is a need of control strategy which maintains stability and performance under the normal operating conditions, and is without the additional cost of an observer. No control strategy can claim to be globally stable. There is always a disturbance waiting in the wings that will defeat any reasonable control strategy. The 6.306 MPa header in Syncrude utility plant exhibits oscillatory modes during sudden load variations in the system. The load change causes drum pressure to rise or fall abruptly, which also leads to shrinking and swelling in the drum level. Under this deregulation what is required is a control strategy that will counteract disturbances, maintains the drum level. Under this deregulation what is required is a control strategy that will counteract disturbances, maintains the drum level and drum pressure at its set point, and also exhibits oscillatory modes during sudden load variations in

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + Gh_1(t, x), \\
y(t) &= Cx(t),
\end{align*}
\]

where \( x(t) \in \mathbb{R}^n \) is the state of the system, \( u(t) \in \mathbb{R}^m \) is the input vector, \( y(t) \in \mathbb{R}^p \) is the output vector, \( A, B \) and \( C \) are constant \( n \times n \) and \( n \times m \) and \( p \times n \) matrices. \( h_1 : \mathbb{R}^{n+1} \rightarrow \mathbb{R}^n \) is a piecewise continuous nonlinear function in both arguments \( t \) and \( x \), satisfying \( h_1(t, 0) = 0 \). We assume that \((A, B)\) is stabilizable, \((C, A)\) is detectable and the uncertain term \( h_1(t, x) \) is bounded by a quadratic inequality \([1, 4]\)

\[
h_1^T(t, x)h_1(t, x) \leq \alpha^2 x^TH_1^TH_1x,
\]

where \( \alpha > 0 \) is a scalar parameter and \( H_1 \) is a constant matrix. If there exists a dynamic output feedback controller

\[
\begin{align*}
\dot{x}_k(t) &= A_kx_k(t) + B_ky(t), \\
u(t) &= C_kx_k(t) + D_ky(t),
\end{align*}
\]

the closed loop system takes the form

\[
\begin{align*}
\dot{x}(t) &= \begin{bmatrix} A + BD_kC & BC_k \\ B_kC & A_k \end{bmatrix} \begin{bmatrix} x(t) \\ x_k(t) \end{bmatrix} \\
&\quad + \begin{bmatrix} G & 0 \end{bmatrix} h(t, X), \\
y(t) &= \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x_k(t) \end{bmatrix},
\end{align*}
\]

where \( X(t) = \begin{bmatrix} x^T(t) \\ x_k^T(t) \end{bmatrix} \). The nonlinear function \( h(t, X) \) satisfies

\[
h^T(t, X)h(t, X) \leq \alpha^2 X^T(t) \begin{bmatrix} H_1^TH_1 & 0 \\ 0 & 0 \end{bmatrix} X(t) = \alpha^2 X^T(t)H^T(t)X(t),
\]

and the closed loop system can be rewritten as

\[
\begin{align*}
\dot{X}(t) &= \hat{A}X(t) + G_1h(t, X), \\
y(t) &= \hat{C}X(t),
\end{align*}
\]

We will use Lyapunov’s theorem \([3]\) to establish stability. The asymptotic stability of \((6)\) can be established by using a Lyapunov function \( V(X) = X^TPX \), where \( P \) is a symmetric positive definite matrix \((P > 0)\).

Theorem 2.1: If there exists \( \gamma, X_1, Y_1, \hat{A}_1, \hat{B}_1, \hat{C}_1, \hat{D}_1 \), such that the following optimization problem is feasible

\[
\begin{align*}
\min_{x_1, y_1, \hat{A}_1, \hat{B}_1, \hat{C}_1, \hat{D}_1, \gamma} & \quad \gamma \\
s.t. & \quad \begin{bmatrix} X_1 & I \\ I & Y_1 \end{bmatrix} > 0, \\
& \quad \begin{bmatrix} \mathcal{F}_{11} & \mathcal{F}_{12} \\ \mathcal{F}_{22} & Y_1G \end{bmatrix} \begin{bmatrix} X_1H_1^T \\ -I \\ 0 \\ -\gamma I \end{bmatrix} < 0,
\end{align*}
\]

where \( \mathcal{F}_{11} \triangleq AX_1 + X_1A^T + B\hat{C}_1 + (B\hat{C}_1)^T, \mathcal{F}_{12} \triangleq A + \hat{A}_1^T + B\hat{D}_1C, \mathcal{F}_{22} \triangleq Y_1A + \hat{B}_1C + A^TY_1 + C^T\hat{B}_1^T \), then the system in \((6)\) is asymptotically stable for all nonlinearities satisfying the quadratic constraint in \((5)\).

Proof: It is well known that a sufficient condition for stability is the derivative of \( V(X) \) to be negative along the
solutions of (6). Formally, this condition can be expressed as a pair of inequalities $P > 0$, and
\[ X^T A^T P X + h^T G_1^T P X + X^T P A X + X^T P G_1 h < 0. \]

The $S$-procedure [10] implies that when (5) is satisfied, then the above condition is equivalent to the existence of a matrix $P$ and a number $\tau > 0$, such that $P > 0$ and
\[
\begin{bmatrix}
A^T P + P A + \tau \alpha^2 H^T H & PG_1 \\
G_1^T P & -\tau I
\end{bmatrix} < 0.
\]

This is further equivalent to the existence of matrix $Y$ which satisfies
\[
Y > 0,
\]
\[
\begin{bmatrix}
\hat{A} Y + Y A^T & Y H^T \\
G_1^T & -I & 0 \\
HY & 0 & -\gamma I
\end{bmatrix} < 0,
\]
where $Y = \tau P^{-1}$ and $\gamma = \frac{1}{2\tau}$. This LMI cannot be used to find the controller because it is not affine on controller parameters $A_k, B_k, C_k$ and $D_k$. Hence a variable transformation is necessary. If $A$ and $A_k$ are $n \times n$ and $n_k \times n_k$, partition $Y$ and $Y^{-1}$ as
\[
Y = \begin{bmatrix} X_1 & M_1 \\ M_1^T & V_1 \end{bmatrix}, \quad Y^{-1} = \begin{bmatrix} Y_1 & N_1 \\ N_1^T & U_1 \end{bmatrix},
\]
where $X_1$ and $Y_1$ are $n \times n$ and symmetric, $M_1$ and $N_1$ are $n \times n_k$ and $Y > 0$ implies $X_1 > 0$, $Y_1 > 0$. From $Y^{-1}Y = I$, it can be inferred that
\[
Y^{-1} = \begin{bmatrix} X_1 & I \\ M_1^T & 0 \end{bmatrix}, \quad Y^{-1} = \begin{bmatrix} I & Y_1 \\ 0 & N_1 \end{bmatrix},
\]
which leads to $Y^{-1} \Pi_1 = \Pi_2$, where,
\[
\Pi_1 \triangleq \begin{bmatrix} X_1 & I \\ M_1^T & 0 \end{bmatrix}, \quad \Pi_2 \triangleq \begin{bmatrix} I & Y_1 \\ 0 & N_1 \end{bmatrix}.
\]

Pre and post multiplying (8) by $\Pi_2^T$ and $\Pi_2$, respectively, and (9) by diag$(\Pi_2^T, I, I)$ and diag$(\Pi_2, I, I)$ respectively, we get
\[
\begin{bmatrix}
\Pi_2^T \hat{A} Y \Pi_2 + \Pi_2^T Y \hat{A}^T \Pi_2 & \Pi_2^T G_1 \\
G_1^T \Pi_2 & -I & 0 \\
HY \Pi_2 & 0 & -\gamma I
\end{bmatrix} > 0,
\]
\[
\begin{bmatrix}
\Pi_2^T G_1 \\
\Pi_2^T Y H^T
\end{bmatrix} < 0.
\]

Straightforward calculation shows that
\[
\Pi_2^T \hat{A} Y \Pi_2 = \begin{bmatrix} X_1 & I \\ I & Y_1 \end{bmatrix},
\]
\[
\Pi_2 \hat{A} Y \Pi_2 = \begin{bmatrix} A X_1 + B \hat{C}_1 & A + B \hat{D}_1 \hat{C}_1 \\ \hat{A}_1 & Y_1 A + \hat{B}_1 \hat{C}_1 \end{bmatrix},
\]
where
\[
\hat{A}_1 \triangleq Y_1^T (A + B D_k C) X_1 + N_1 B_k C X_1 + Y_1^T B C_k M_1^T + N_1 A_k M_1^T, \quad \hat{B}_1 \triangleq Y_1 B D_k + N_1 B_k,
\]
\[
\hat{C}_1 \triangleq D_k C X_1 + C_k M_1^T, \quad \hat{D}_1 \triangleq \hat{D}_k.
\]

Also, $\Pi_2^T \hat{A} Y \Pi_2 + \Pi_2^T Y \hat{A}^T \Pi_2 = \begin{bmatrix} \mathcal{F}_{11} & \mathcal{F}_{12} \\ \mathcal{F}_{12} & \mathcal{F}_{22} \end{bmatrix}$,
\[
\Pi_2^T G_1 = \begin{bmatrix} G \end{bmatrix}, \quad \Pi_2^T Y H^T = \begin{bmatrix} X_1 H_1^T \end{bmatrix}.
\]

Since $X_1$ and $Y_1$ are symmetric matrices, $N_1$ and $M_1$ can be chosen square and non-singular such that $N_1 M_1^T = I - Y_1 X_1$. Using singular value decomposition, $[\Sigma \Lambda \Omega^T] = \text{svd}(I - Y_1 X_1)$. This gives $N_1 M_1^T = \Sigma \Delta \Omega^T$, $N_1 = \Sigma \Delta^\perp$, and $M_1 = \Omega \Lambda^\perp$. It allows us to calculate $A_k, B_k, C_k$ and $D_k$ from (15).

**Corollary 2.1:** Under the condition of Theorem 2.1, if the quadratic constraint in (5) holds globally, then the system in (6) is globally stabilized.

**Remark 2.1:** Assume that $K(s)$ is strictly proper ($D_k = 0$) and that $A$ does not have any fast mode, i.e., any eigenvalue with large real part. Then from the identity $\text{Trace}(A) = \text{Trace}(A) + \text{Trace}(A_k)$, we see that $A_k$ has fast modes if and only if $A$ has fast modes. Consequently, fast controller dynamics can be prevented by constraining the closed loop eigenvalues, e.g., in a disc centered at the origin and with appropriately small radius. This ability is valuable in the perspective of real-time digital implementation of the controller. One way of simultaneous tuning the stability and transient behavior is therefore to combine the above LMIs with pole placement objectives. Considering these issues, and starting from the pole placement constraints of [12], we get after some transformations and mathematical calculations:
\[
\begin{bmatrix}
\mathcal{F}_{11} + (2\alpha_{\text{min}}) X_1 & \mathcal{F}_{12} + (2\alpha_{\text{min}}) I \\ \mathcal{F}_{22} + (2\alpha_{\text{min}}) Y_1
\end{bmatrix} < 0,
\]
\[
\begin{bmatrix}
-(\omega_{\text{max}}) \Pi_2^T \hat{A} Y \Pi_2 & \Pi_2^T \hat{A} Y \Pi_2 \\ -\Pi_2^T \hat{A} Y \Pi_2 & \Pi_2^T \hat{A} Y \Pi_2
\end{bmatrix} < 0,
\]
\[
\begin{bmatrix}
\mathcal{D}_{11} & \mathcal{D}_{12} \\ \mathcal{D}_{12} & \mathcal{D}_{22}
\end{bmatrix} < 0,
\]
where
\[
\mathcal{D}_{11} \triangleq \begin{bmatrix} (\sin \theta) \mathcal{F}_{11} & (\sin \theta) \mathcal{F}_{12} \\ (\sin \theta) \mathcal{F}_{12} & (\sin \theta) \mathcal{F}_{22} \end{bmatrix},
\]
\[
\mathcal{D}_{12} \triangleq \begin{bmatrix} (\cos \theta) \Pi_2^T \hat{A} Y \Pi_2 - \Pi_2^T \hat{A} Y \Pi_2
\end{bmatrix},
\]
\[
\mathcal{D}_{22} \triangleq \begin{bmatrix} (\sin \theta) \mathcal{F}_{11} & (\sin \theta) \mathcal{F}_{12} \\ (\sin \theta) \mathcal{F}_{12} & (\sin \theta) \mathcal{F}_{22} \end{bmatrix}.
\]

$\Pi_2^T \hat{A} Y \Pi_2$ and $\Pi_2^T \hat{A} Y \Pi_2$ are calculated in (13) and (14) respectively. These regions are the intersection of three elementary LMI regions: an $\alpha_{\text{min}}$ stability region $\text{Re}(\alpha_{\text{min}}) \leq \alpha_{\text{min}}$, a disc of radius $\omega_{\text{max}}$ and the conic sector $S(0, 0, \theta)$. Confining the closed loop eigenvalues to this region ensures a minimum decay rate $\alpha_{\text{min}}$, a minimum damping ratio $\zeta = \cos \theta$, and a maximum undamped natural frequency $\omega_{\text{max}}$. This in turn bounds the maximum overshoot, the frequency of oscillatory modes, the rise time, the delay time and the settling time.

**Remark 2.2:** For the static state feedback, the number of LMI decision variables are $n \times \left(\frac{n+1}{2} + m\right)$, while for the dynamic case, this number increases to $n \times (2n + 1 + p + m) + m \times p$. An increase in the number of decision variables implies an increase of the off-line computational effort required.
to solve the LMIs. The number \( n \) of states depends on the plant we deal with. It is worth to notice that the presence of controller dynamic equations also require more on-line computations. As a summary, as far as stability is concerned, state feedback control offers best possible performance with low computational cost, provided all states are measurable. This result is reminiscent. Also, this method is not applicable to deal with a nonlinear system with time varying part.

III. GENERALIZATION TO MULTIPLE SUBSYSTEMS

From the standpoint of power systems, robotics, it is useful to extend the preceding analysis to models that include decentralized control. To that effect, let us consider the interconnected system [1], [4],

\[
\dot{x}_i = A_i x_i + B_i u_i + G_i h_i(t, x), \\
y_i = C_i x_i, \quad i = 1, 2, 3, \ldots, n
\]

which is composed of \( n \) LTI subsystems

\[
\dot{x}_i = A_i x_i + B_i u_i, \quad i = 1, 2, 3, \ldots, n
\]

where \( x_i \in \mathbb{R}^{n_i} \) are the states, \( u_i \in \mathbb{R}^{m_i} \) are the inputs and \( h_i : \mathbb{R}^{n_i+1} \rightarrow \mathbb{R}^{n_i} \) are the interconnections. We will assume that pairs \( (A_i, B_i) \) are stabilizable, \( (C_i, A_i) \) are detectable and require that all nonlinear functions \( h_i(t,x) \) satisfy the quadratic constraints [4]

\[
h^T_i(t,x)h_i(t,x) \leq \alpha_i x^T H^T_i H_i x,
\]

where \( x = [x_1^T \ x_2^T \ \ldots \ x_n^T]^T \), \( \alpha_i > 0 \) are interconnection parameters and \( H_i \) are fixed matrices. Following the discussions of the previous section, we can write for the \( i \)th subsystem, the closed loop takes the form

\[
\dot{x}_i = \hat{A}_i x_i + G_N h_i, \quad i = 1, 2, \ldots, n
\]

where \( \hat{A}_i = [A_i + B_i D_i C_i - B_i C_i A_i] \), \( G_N = [G_i \ 0] \).

\( A_k, B_k, C_k \) and \( D_k \) are the state space matrices of the controller for \( i \)th subsystem. Defining \( x_N = [x_1^T \ x_2^T \ x_3^T \ \ldots \ x_n^T]^T \), we can write for the \( i \)th subsystem,

\[
h^T_i(t,x)h_i(t,x) \leq \alpha_i x^T H^T_i H_i x = \alpha_i^2 x^T \sum_{i=1}^{n} H^T_i H_i x_N,
\]

where the elements of \( H_i x_N \) corresponding to \( x_1, x_2, \ldots, x_{n-i} \) are zero. The overall interconnected system can be written in compact form as

\[
\dot{x} = \hat{A} x + G_D h(t,x),
\]

where \( \hat{A} = \text{diag} (\hat{A}_1, \ldots, \hat{A}_n) \), \( G_D = \text{diag} (G_N, \ldots, G_N) \) and \( h = [h_1^T \ \ldots \ h_n^T]^T \). The nonlinear function of the overall system \( h(t,x) \) is bounded by a quadratic inequality

\[
h^T(t,x)h(t,x) \leq x_N \sum_{i=1}^{n} \gamma_i^2, \quad \gamma_i = \frac{1}{\alpha_i}.
\]

As in Theorem 2.1 defining new variables for the overall system, we can write \( Y_D = \text{diag} (Y_1, Y_2, \ldots, Y_n) \), \( \Pi_{2D} = \text{diag} (\Pi_{21}, \ldots, \Pi_{2n}) \). Straightforward calculation shows that

\[
\Pi_{2D} Y_D \Pi_{2D} = \text{diag} \left( \begin{bmatrix} X_{i1} & I \\ X_{i1} & Y_{i1} \end{bmatrix}, \ldots \right),
\]

\[
\Pi_{2D} G_D = \text{diag} \left( \begin{bmatrix} G_1 & \cdots & G_n \\ G_1 & \cdots & Y_{1n} \end{bmatrix}, \ldots \right),
\]

\[
\Pi_{2D}^T \hat{A}_N D_{2D} = \text{diag} \left( \begin{bmatrix} (F_{11})_1 & (F_{12})_1 \\ (F_{11})_1 & (F_{22})_1 \end{bmatrix}, \ldots \right),
\]

where \( \hat{A}_N \) and \( D_i \), \( i = 1, 2, 3, \ldots, n \), such that the following optimization problem is feasible

\[
\min_{i=1}^{n} \gamma_i \text{ s.t. } \Pi_{2D} Y_D \Pi_{2D} > 0,
\]

\[
\Pi_{2D}^T \hat{A}_N D_{2D} + 2\alpha_{\min, 0} \Pi_{2D} Y_D H_N^T < 0,
\]

\[
-\omega_{\max} \Pi_{2D} Y_D < 0,
\]

\[
D_{11}, D_{22} < 0,
\]

where \( H_N = [H_{1n}^T, \ldots, H_{nn}^T], \gamma_D = \text{diag} (\gamma_1, \gamma_2, \ldots, \gamma_n), \)

\( \alpha_{\min} = \text{diag} (\alpha_{\min, 1}, \alpha_{\min, 2}, \alpha_{\min, m}), \)

\( D_{11}, D_{22} = \sin \theta_i \left[ \begin{bmatrix} (F_{11})_i & (F_{12})_i \\ (F_{11})_i^T & (F_{22})_i \end{bmatrix}, \ldots \right], \)

\( D_{12} = \cos \theta_i \left[ \begin{bmatrix} (F_{11})_i \hat{A}_N D_{2D}, Y_{1i} \hat{A}_N D_{2D} \end{bmatrix}, \ldots \right], \)

then the interconnected system in (19) is asymptotically stable, for all nonlinearities satisfying the quadratic constraint in (20).

**Proof:** From (8) and (9), we can say that the sufficient conditions of the closed loop system (19) to be asymptotically stable under the quadratic constraint (20) are

\[
Y_D > 0, \quad \left[ \begin{array}{cccc} \hat{A} & Y_D A_D^T & G_D & Y_D H_N^T \\ Y_D & -I & 0 & -\gamma_D I \\ & & & 0 \end{array} \right] < 0.
\]

The rest of the proof along with pole placement constraints are similar to Section II, and is therefore omitted. ■

**Corollary 3.1:** Under the conditions of Theorem 3.1, if the quadratic constraint in (20) holds globally, then the system in (19) is globally stabilized.
Remark 3.1: It follows that stability of (19) is guaranteed for any nonlinearity satisfying (20). In this context, it is important to note an inherent trade-off that exists between the size of the region of attraction and the robustness of the system. Namely, whenever \( \alpha > 1 \), inequality (20) clearly holds even if there is a degree of uncertainty in \( h(t,x_N) \). In that sense, a larger \( \alpha \) implies a greater degree of robustness. On the other hand, it is also true that the value of \( \alpha \) obtained from the LMI process decreases as the region \( \Omega \) becomes larger.

Remark 3.2: Decentralized designs based on LMI generally require block diagonal Lyapunov functions. Such a constraint is often restrictive, and can significantly degrade the robustness of the closed loop system. In some cases, it may even lead to infeasibility of the optimization. The interior point algorithm, which solves the linear objective minimization problem in Matlab, cannot be used for more than 20 matrix variables.

IV. APPLICATION TO AN INDUSTRIAL UTILITY BOILER

To illustrate how the proposed LMI approach can be applied to a real world problem, we consider a nonlinear dynamic model for natural circulation drum-boilers [7]. The state variables of the system are: total water volume \( (V_{wt} = x_1) \), drum pressure \( (p = x_2) \), steam-mass fraction in the riser \((\alpha_r = x_3)\), and steam volume in the drum \((V_{sd} = x_4)\). \( \alpha_r \) is a measurable parameter. The control variables are feedwater flow rate \((u_1)\), steam flow rate \((u_2)\) and fuel flow rate \((u_3)\). At steady state, \( u_1 = u_2 = 58 \) in p.u., \( u_3 = 8.4 \) in p.u., \( V_{sd} = 5 \) m\(^3\), \( V_{wt} = 57.5 \) m\(^3\), \( p = 6.523 \) MPa, \( \alpha_r = 0.05 \), and total volume of water and steam \( V_i = 80 \) m\(^3\). Starting from the nonlinear equations of [7], we get after some mathematical calculation and simplifications:

\[
\dot{x}_1 = (e_{22} - c_{12}h_f)u_1 + (e_{12}h_s - e_{22})u_2 - e_{12}u_3,
\]
\[
\dot{x}_2 = (e_{11}h_f - c_{21})u_1 + (e_{21} - e_{11}h_i)u_2 + e_{11}u_3,
\]
\[
\dot{x}_3 = h_qd_x - e_{33} + \frac{e_{21}e_{32}u_2 + (e_{12}e_{22} - e_{21}e_{21})u_4}{e_{33}e_{34}} - \frac{e_{12}e_{21}e_{33}}{e_{33}e_{34}},
\]
\[
\dot{x}_4 = \frac{e_{43}h_xq_d x_3}{e_{33}e_{34}} - \frac{\rho_s x_4}{e_{44}T_d} + \frac{\rho_s V_{d}^0}{e_{44}T_d} + \frac{e_{43}u_4}{e_{33}e_{34}}.
\]

Here \( h_s = h_s - h_{w_s}, e_{11} = \rho_s - \rho_w, e_{21} = \rho_w h_w - \rho_s h_s, e_{44} = \rho_s \) and

\[
e_{12} = x_1 \frac{\partial \rho_s}{\partial x_2} + (V_i - x_1) \frac{\partial \rho_s}{\partial x_2},
\]
\[
e_{22} = x_1 \left( h_w \frac{\partial \rho_s}{\partial x_2} + \rho_w \frac{\partial h_w}{\partial x_2} \right) - V_i + m_T C_p \frac{\partial V}{\partial x_2} + (V_i - x_1) \left( h_s \frac{\partial \rho_s}{\partial x_2} + \rho_s \frac{\partial h_s}{\partial x_2} \right),
\]
\[
e_{33} = (1 - x_3) \rho_s + x_3 \rho_w h_r \frac{\partial \alpha_r}{\partial x_3},
\]
\[
e_{32} = \left( \frac{\partial h_w}{\partial x_2} - x_3 h_c \frac{\partial \rho_w}{\partial x_2} \right) \left( (1 - \bar{\alpha}_r) V_r - V_r + \frac{1}{x_3} \frac{\partial h_s}{\partial x_2} + \rho_s \frac{\partial h_s}{\partial x_2} \right) \frac{\partial \alpha_r}{\partial x_2},
\]
\[
e_{43} = 1.3 x_3 (\rho_s - \rho_w) V_r \frac{\partial \alpha_r}{\partial x_3}.
\]

\( \bar{\alpha}_r, h, \rho \) and \( q_d \) represents average steam volume ratio, specific enthalpy, specific density and downcomer flow rate respectively. Subscripts \( s \) stands for steam and \( w \) stands for water. The parameters of the nonlinear model are [7]: drum volume \( V_d \), riser volume \( V_r \), downcomer volume \( V_{dc} \), area of drum \( A_d \), total metal mass \( m_s \), riser mass \( m_r \), residence time of steam \( T_d \). The additional parameters are obtained from steam table at the saturation pressure. The state space model has poles at 0, 0, -0.045 and -0.083. It contains a RHP zero at 1.021. As the steam mass fraction is always less than 100 percent, we define a region \( \Omega \).

\[
\Omega = \{ x : x_1, x_2, x_4 \in \mathbb{R}, \ x_3 \leq 1 \}.
\]

Straightforward but tedious calculation shows that the nonlinear function \( h_1(x) \) can be bounded by

\[
h_1(x)h_1(x) \leq \left[ \frac{1.3(\rho_s - \rho_w)q_d}{\rho_s(1 - x_3)\rho_s + x_3\rho_w} \right] + \left[ \frac{x_3\rho_s - \rho_w}{(1 - x_3)\rho_s + x_3\rho_w} \right]^2 x_3^2 + \left[ \frac{2\rho_s q_d(\rho_s - \rho_w)}{V_r \rho_s - x_3 \rho_w} \right]^2 x_3^2.
\]

where subscript \( x_0 \) represents the operating point.

The controller is designed by solving the linear objective minimization problem of Theorem 2.1. The admissibility region is assumed to be \( D(\alpha_{\min} = 1, \zeta = 0.707, \omega_{\max} = 30) \). The stabilizing effect of three different kinds of controllers: static state feedback, dynamic state feedback and dynamic output feedback are shown in Fig. 1 and Fig. 2. The degree of robustness \( \alpha \) obtained with static state feedback, dynamic state feedback and dynamic output feedback controllers are 1.16, 1.28 and 1.04 respectively. This shows that the degree of robustness with output feedback is less than full state feedback. However the performance is acceptable.

Fig. 3 shows the disturbance rejection capability of the output feedback controller caused by sudden load variation in the system. Due to sudden decrease in load, the pressure in the drum rises, which increases the volume of water due to increased condensation. Steam quality \( \alpha_r \) at the riser outlet also increases rapidly. The volume of the steam \( V_{sd} \) decreases due to the increased pressure which causes condensation of the steam. This shows that there exists a strong interaction between different state variables. The time scale also shows the effect of RHP zero which limits the closed loop bandwidth. The controller is also capable of attenuating the effect of sudden increase in load. Pole placement constraints help to reduce the initial peaking caused by load variations. Corresponding to sudden load variations, Fig. 4 shows the robustness of the proposed control with respect to varying residence time of steam in the drum (10 sec \( \leq T_d \leq 15 \) sec).
The results suggest that the proposed control performs well for the entire range of realistic values for $T_d$.

V. CONCLUSIONS

A new method is presented for designing a dynamic output feedback control strategy for nonlinear interconnected systems, based on LMI. The contribution of the paper stems from applicability of the approach to systems, where all the states are not available for feedback. Our primary reason for selecting this type of control was also the underlying system model, which was seen to conform to the requirements of LMI optimization. Simulation results show that output feedback controller can also achieve good performance, but the degree of robustness is less than the static or dynamic state feedback control. However, the stabilization effect of the controller is good both during the normal and perturbed conditions, which makes it practically implementable. Our approach can also control the transient characteristic of the closed loop system by pole placement constraints. This helps in reducing the initial peaking caused by load variations and avoiding very fast controller dynamics.

REFERENCES


