Abstract

The main idea in life extending control (LEC), also known as damage mitigating control, is to redesign the controller in order to achieve a better tradeoff between structural durability and dynamic performance in a system. This task involves both damage dynamics modeling and LEC design. In this paper, we propose a new hierarchical LEC structure and apply it to a typical boiler system. There are two damage models in this structure: Model I is for on-line LEC; Model II is for on-line life prediction for critical components of the system. For model I, we incorporate the improved rainflow cycle counting method and a continuous-time damage modeling approach. While for model II, we choose a method based on the P-K theory which involves the mean stress effect. Finally, an optimal LEC scheme is proposed and simulation results show that the designed LEC substantially reduces the accumulated damage with a minimum loss of dynamic performance.

1 Introduction

During the first half of the 19-th century, development of steam engines led to increasing sources of repeated stress on metal parts and structural elements. Shortly thereafter, “unexplainable” fractures – particularly in locomotive axles – became of great concern to engineers. About the middle of the century, experiments of Woehler and others showed the importance of the number of repetitions of stress (rather than duration of time) in causing failure of metals under relatively low stress [1]. Since then over 20,000 papers on fatigue analysis have been published throughout the world [2]; and a large amount of experiments have been done to provide useful data under various conditions. The data constitutes the basis of fatigue analysis and design. But the main objective of early studies was to understand material properties in order to design reliable systems.

Since the 1970’s, fatigue research has also been used as the basis for the development of life-prediction systems. Many methods for estimating fatigue life were proposed on which life-monitoring systems could be developed [3, 4, 5].

However, most control systems are designed focusing on stability and performance only, and assuming that the materials involved have invariant properties. In other words, the control design process ignores the effects of aging, fatigue, and damage in the materials. Systems designed this way may lose performance or even stability in the long term, yielding high risks in system operations.

In 1991, Noll and co-workers [6] pointed out the need for addressing the trade-off between system performance and durability (of critical components); this formed the basis for the so-called damage mitigating control, or life extending control (LEC). LEC is an area of research involving the integration of two distinct disciplines: system sciences and mechanics of materials. Although a significant amount of research has been conducted in each of the individual areas of control and diagnostics, and analysis and prediction of materials damage, integration of these two has not received much attention [7]. At present, there is little available literature which directly discusses the combination of the two disciplines.

An important step of LEC is to construct damage models for critical components. Most fatigue/damage models from material science are cycle-dependent; but cycle-dependent models are not suitable for on-line control implementation. In 1994, Ray and co-worker [8] proposed a continuum fatigue/damage modeling method for use in LEC. But because the rainflow cycle counting is used in this method, it is not very suitable
for real-time control. We find that an improved rain-
flow method proposed by Dowling [9] is more suitable
for real-time LEC.

Another important issue in the LEC research is to de-
sign a life extending controller. The challenge for this
is mainly in how to handle the two conflicting objec-
tives of reducing damage and improving dynamic perfor-
mance of the system. Ray [10] suggested an open-
loop control policy and simulation results showed that
it was possible to get a good compromise. Further work
on LEC has been reported in [11, 12, 13], with different
applications in rocket engines and fossil power plants
in the framework of feedforward and feedback control.
Zhang and Ray [14] have demonstrated the efficacy of
LEC on a laboratory test apparatus where peak stresses
in a critical component were reduced to increase its
structural durability. In our study of boiler-turbine
systems, we adopt the framework of optimal control
design theory to design a life extending controller.

In our research, a boiler-turbine system in an indus-
trial co-generation system at Syncrude Canada’s util-
ity plant in Fort McMurray is the object of study. A
new LEC closed-loop scheme is proposed in Section 2.
Modeling methods of damage dynamics of the turbine
(the critical component) are discussed in Section 3. In
Section 4, an optimal life extending controller is de-
signed for the boiler-turbine system and simulation re-
results are presented. The detailed model information
on the plant is given in the appendix.

2 LEC Closed-Loop Structure

In many industrial cases such as the boiler-turbine sys-
tem at hand, control systems are already in operation
for good dynamic performance. An important and
practical question is: How to enhance the structural
durability with existing control systems still in place?
Figure 1 is our proposed structure for LEC, in which
the existing controller is kept in place for the plant,
but a damage model and LEC are introduced on top

![Figure 1](image1.png)

**Figure 1:** A schematic diagram of LEC philosophy

at a higher level, which interact with both the existing
controller and the plant. The damage models are used
to estimate the accumulated damage and the damage
rate within the system, based on measurable variables;
the LEC is used to maintain the accumulated damage
and damage rate within prescribed limits, while trading
off little dynamic performance. This way we gain
improved component durability and longer service life
without affecting much of the routine operation.

Incorporating the above idea, we propose the new LEC
strategy shown in Figure 2, where the inner feedback

![Figure 2](image2.png)

**Figure 2:** A new LEC closed-loop scheme setup

loop represents the existing control system, designed
for system dynamic performance, damage models I and
II, together with the LEC in the outer feedback loop,
are introduced new. For the signals involved in Fig-
ure 2, \( r(t) \) is the reference input, \( y(t) \) the plant output,
\( \delta(t) \) the damage variable (used for LEC feedback), and
\( L(t) \) the output of damage model II for life prediction.
Note that the controller is kept in place, and no changes
are necessary; the LEC is introduced for reducing dam-
age level of critical components in the system. Damage
model I is used for on-line LEC feedback; while dam-
age model II for life prediction. Both models should
be suitable for on-line implementation. This control
structure has the following features:

- **Hierarchical:** The LEC is at a higher level with
  the existing controller in place; it provides an addi-
tional control signal to enhance durability and
  safety within the system. (This would be wel-
comed by practitioners and operators for ease in
implementing the LEC.)

- **Good dynamic performance:** The LEC is designed
to have minimum effect on the dynamic perfor-
mance already achieved by the existing control
system.

- **Optimal tradeoff:** We target optimal tradeoff be-
tween component durability and dynamic perfor-
mance.

3 Modeling Damage Dynamics

In this section we discuss ways to model damage.
3.1 Damage Model I

In the literature, the stress-fatigue life curves (S-N curves) of many materials are available. S-N curves are the basis for fatigue modeling. Most experimental data is generated by applying constant amplitude loads to test specimens. However, real-life components always work under fluctuated loads; so the task of a model is to predict the life of a component subjected to varying amplitude loads using constant amplitude load test data. Palmgren in 1924 proposed a linear damage rule which was further developed by Miner in 1945 [15]. This Palmgren-Miner (P-M) method can be described by the following formula

\[ D = \sum_i D_i = \sum_i \frac{n_i}{N_i}, \]  

(1)

where \( D \) is the damage variable (cumulative damage), \( n_i \) is the number of load cycles at the (constant) stress level \( \sigma_i \), and \( N_i \) is the total number of cycles to failure at the same stress level. Although the P-M method gives only approximate estimation of damage, it has found many applications because of its linear simplicity. The main disadvantage of the P-M method is that it doesn’t account for the sequential effect.

To improve the P-M rule, Marco and Starkey [15] proposed a non-linear cumulative damage rule:

\[ D = \sum_i D_i = \sum_i \left( \frac{n_i}{N_i} \right)^P, \]  

(2)

where the exponent \( P_i \) is a function of stress level \( \sigma_i \). This approach is of current research interest and has applications in design.

Most fatigue damage models from material science such as the ones in equations (1) and (2) are cycle-dependent; but cycle-dependent models are not suitable for on-line control implementation. Ray in 1994 [8] proposed a continuous-time fatigue model as follows.

Consider a critical component subjected to cyclic loading; let \( \epsilon_r \) be the total strain corresponding to the reference stress \( \sigma_r \) at the starting point \( R \) of a given reversal as determined from the rainflow cycle counting method, and \( \epsilon \) and \( \sigma \) be the strain and stress of the point on the same reversal with \( R \) respectively. We define

\[ \Delta \epsilon = |\epsilon - \epsilon_r|, \]

\[ \Delta \sigma = |\sigma - \sigma_r|. \]

The relationship between the stress \( \sigma \) and the strain \( \epsilon \) is given by Ellyin in [16]

\[ \frac{\Delta \sigma}{2} = \frac{\Delta \epsilon}{2E} + \left( \frac{\Delta \epsilon}{2K'} \right)^{\frac{1}{n'}}, \]  

(3)

where \( E \) is the modulus of elasticity, \( K' \) the cyclic strength coefficient and \( n' \) the cyclic strain-hardening coefficient, all determined experimentally based on material properties.

According to the rainflow method, \( \sigma_r \) is either a local minimum or maximum; so \( \frac{d \sigma_r}{dt} = 0 \). Further, since it is assumed that no damage occurs during unloading, the damage rate can be made equal to zero when \( \sigma < \sigma_r \). When \( \sigma \geq \sigma_r \), we have

\[ \frac{d \delta_e}{dt} = 2 \times \frac{d}{d\sigma} \left( \left( \frac{\sigma - \sigma_r}{\sigma_f - \sigma_m} \right)^{-\frac{1}{b}} \right) \times \frac{d\sigma}{dt}, \]  

(4)

\[ \frac{d \delta_p}{dt} = 2 \times \frac{d}{d\sigma} \left( \left( \frac{1}{\epsilon_f}(\sigma - \sigma_3) \right)^{\frac{1}{m}} \times (1 - \frac{\sigma_m}{\sigma_f})^{-\frac{1}{b}} \right) \times \frac{d\sigma}{dt}. \]  

(5)

Here \( \frac{d\sigma}{dt} \) can be found from direct measurements of the strain rate by specific gauges, or by the finite element analysis (FEA). When both are not available, Lu and Wilson suggested an alternate formula [17]:

\[ \sigma(\tau) = \frac{E\alpha}{1 - \nu} (T_m(\tau) - T(\tau)) \]

with \( \nu \) the Poisson’s ratio, \( \alpha \) the coefficient of linear expansion, \( T \) the temperature of the component at the critical point, and \( T_m \) the mean temperature. Thus the total damage rate is the weighted sum of the elastic damage rate and the plastic damage rate:

\[ \frac{d\delta}{dt} = w \frac{d\delta_e}{dt} + (1 - w) \frac{d\delta_p}{dt}. \]

The weighting function \( w \) is based on the elastic and plastic strain amplitudes

\[ w = \frac{\epsilon_e - \epsilon_{re}}{\epsilon_e - \epsilon_r} = \frac{\Delta \epsilon_e}{\Delta \epsilon}, \]

where \( \epsilon_{re} \) the elastic part of \( \epsilon_r \), is defined as

\[ \epsilon_{re} = \frac{\sigma_r}{E}. \]

Therefore, the accumulated damage \( D \) can be found by

\[ D = \sum_{k=1}^{N} \delta(k). \]

We notice that this continuous-time model involves the use of the rainflow cycle counting method to specify the starting point of one cycle to be the reference point \( R \). The rainflow method of cycle counting derived its name from an analogy used by Matsuishi and Endo in their early work on this subject. Several algorithms [18] are available to perform the counting; however, to eliminate counting half cycles, the start point should be at the strain value of greatest magnitude. Thus they all require that the entire load history be known before the counting process starts. As a result, they are not suitable for on-line control. For this reason, this
continuous-time model we discussed is not suitable for on-line control [19].

An improved rainflow cycle counting method, called the “one pass” method, was proposed by Dowling [9], which allows taking any point as a temporal start point during the counting process; if a certain condition is satisfied, the starting point should be moved. This improved method can give the same result as the conventional rainflow method with the added advantage of causal implementation. Thus, incorporating this method in Ray’s continuous model allows on-line implementation of LEC.

3.2 Damage Model II
The task of damage model II is to predict the remaining life of the component monitored. Different from model I, model II is not part of the LEC feedback loop. Thus there are no strict requirements on its real-time properties, but this model should be more accurate than model I. The following equations can be used to estimate the remaining life period of the component monitored:

\[ \epsilon_a = \frac{\sigma_f'}{E} (2N)^b + \epsilon_f'(2N)^c, \]

\[ \epsilon_a = \frac{\sigma_a}{E'} + \left( \frac{\sigma_a}{K'} \right)^{\frac{1}{n'}}. \]

The shortcoming of this method is that it does not include the mean stress effect. To overcome this, Dowling [5] derived a formula by an analogous method to the modified Goodman diagram:

\[ \epsilon_a = \frac{\sigma_f'}{E} \left( 1 - \frac{\sigma_a}{\sigma_f} \right) (2N)^b + \epsilon_f' \left( 1 - \frac{\sigma_a}{\sigma_f} \right) (2N)^c, \]

\( \epsilon_a \) here being the stress amplitude.

4 LEC Design for a Boiler-Turbine System
In this section, an optimal life extending controller is designed for a boiler-turbine system based on a linearized model. A linearized description of such a plant is given in appendix. In brief, this system has three inputs: the feedwater flow \( (u_1) \), fuel flow \( (u_2) \) and attemperator spray flow \( (u_3) \), and three outputs: the control drum level \( (y_1) \), drum pressure \( (y_2) \) and steam temperature \( (y_3) \).

According to the structure of Figure 2, the setup of the closed-loop system shown in Figure 3 is used. The purpose of the controller \( K \) is to make the boiler generate enough steam, maintain the steam pressure of a header in this system to its setpoint and the steam temperature to its setpoint. In the diagram, \( X \) is the plant state vector; \( \hat{X} \) is the estimated plant state vector; \( Y \) is the plant output vector; \( U_r \) is the reference input vector; \( U_{lec} \) is the output variable from LEC; and \( U \) is the control input vector. Because not all state variables are measurable, the state estimator is needed. The damage state vector \( v(t) \) is used to indicate the damage level at one or more critical points.

The material type of the turbine blade is known to be ASTM A-516 Gr.70. For damage modeling, the experiment constants in equations (4) and (5) can be found in appropriate tables:

\[ \nu = 0.27, \quad E = 209.5 \times 10^3, \quad \alpha = 0.00001, \]

\[ K' = 1193 \text{ MPa}, \quad n' = 0.202, \quad \epsilon_f' = 859, \]

\[ \epsilon_f' = 0.219, \quad b = -0.108, \quad c = -0.5. \]

Thus model I can be constructed according to equations (4) and (5), and model II according to equation (6).

To design the LEC via optimization, we formulate the LEC design problem as follows:

Given the plant dynamics:

\[ \dot{x} = Ax(t) + Bu(t), \quad x(t_0) = x_0, \quad (7) \]

\[ y(t) = Cx(t) + Du(t), \]

the damage dynamics:

\[ \frac{dv}{dt} = h(v(t), x(t), t); v(t_0) = v_0. \quad (8) \]

and the cost Function:

\[ J = \int (Y^T Q Y + U^T R U + v^T S v) dt, \]

where \( Q, R \) and \( S \) are the weighting matrices for outputs, inputs, and damage variables, respectively, design a damage feedback controller to minimize \( J \) subject to equations (7) and (8).

In fact, two different cost functions are used to design controllers to show the effect of LEC. The first one is given by

\[ J_1 = \int (q_1 y_1^2 + q_2 y_2^2 + q_3 y_3^2 + U^T RU) dt, \]

Figure 3: The closed-loop diagram with LEC
where $y_1, y_2, y_3$ are the plant outputs (thus $J_1$ does not account for the damage variables); the second is

$$ J_2 = \int (q_1 y_1^2 + q_2 y_2^2 + q_3 y_3^2 + q_4 \nu^2 + U^T RU) dt, $$

which includes the damage rate ($\nu$), the output of damage model I.

Because the damage dynamics represented by Equation (8) is nonlinear, linearization should be performed first at the normal load conditions (appendix). Controllers 1 and 2 are designed by minimizing $J_1$ and $J_2$, respectively, using the MATLAB function lqry based on the $LQ$ theory, setting all weightings to unity. The controllers are of high-order (with 19 states). Because of the way cost functions are defined, controller 2 is an LEC; while controller 1 is not. Simulations are done using a linearized boiler model under normal load conditions (appendix) and the nonlinear model of the boiler-turbine system; the results with the nonlinear model are shown in Figures 4 to 7. It is clear that the LEC achieves its objective of significantly reducing the accumulated damage while not affecting the performance too much in the system.

![Figure 4: Step responses of the drum level](image)

**5 Conclusions**

The contribution of this paper is three-fold:

- we proposed a hierarchical structure for life extending control;
- we discussed modeling of damage dynamics in connection with an industrial boiler-turbine system;
- we proposed an LQ based optimal LEC design, and applied it to the boiler-turbine system, yielding promising results in simulation.

![Figure 5: Step responses of the drum pressure](image)

![Figure 6: Step responses of the steam temperature](image)

**References**


Figure 7: Comparison of the accumulated damage


Appendix

The linearized plant model:

\[
\begin{bmatrix}
  y_1 \\
  y_2 \\
  y_3 \\
\end{bmatrix} =
\begin{bmatrix}
  g_{11} & g_{12} & 0 \\
  g_{21} & g_{22} & g_{23} \\
  g_{31} & g_{32} & g_{33} \\
\end{bmatrix}
\begin{bmatrix}
  u_1 \\
  u_2 \\
  u_3 \\
\end{bmatrix}
\]

where

\[
\begin{align*}
  g_{11} &= -0.00016s^2 + 0.000052s + 0.000014 \\
  g_{12} &= \frac{0.0031s - 0.000032s}{s^2 + 0.0215s} \\
  g_{21} &= \frac{-0.0573}{s + 0.0018} \\
  g_{22} &= \frac{0.312}{s + 0.0157} \\
  g_{23} &= \frac{0.0853s^2 + 0.02924s + 0.000131}{s^2 + 0.03528s + 0.000142} \\
  g_{31} &= \frac{-0.00118s + 0.000139}{s^2 + 0.01852s + 0.000091} \\
  g_{32} &= \frac{0.448s + 0.0011}{s^2 + 0.0127s + 0.000095} \\
  g_{33} &= \frac{0.582s - 0.0243}{s^2 + 0.1076s + 0.00104} \\
\end{align*}
\]

The operating conditions:

\[
\begin{bmatrix}
  u_{10} \\
  u_{20} \\
  u_{30} \\
\end{bmatrix} = \begin{bmatrix}
  63.84 \\
  3.399 \\
  0.1652 \\
\end{bmatrix},
\begin{bmatrix}
  y_{10} \\
  y_{20} \\
  y_{30} \\
\end{bmatrix} = \begin{bmatrix}
  1.0 \\
  6306 \\
  500 \\
\end{bmatrix}
\]

The original controller \( K \):

\[
\begin{bmatrix}
  206.94 & -0.0919 & -0.290 \\
  2.50 & 0.0055 & 0.0004 \\
  12.08 & 0.0099 & -0.0642 \\
\end{bmatrix} + \begin{bmatrix}
  2.20 & 0.0022 & -0.0021 \\
  0.027 & 0.0002 & 0 \\
  0.1355 & 0.0003 & -0.009 \\
\end{bmatrix} \frac{1}{s}
\]