Reliability Evaluation of Phased-Mission Systems Using Stochastic Computation

Peican Zhu, Jie Han, Member, *IEEE*, Leibo Liu, Member, *IEEE*, and Fabrizio Lombardi, Fellow, *IEEE*

-

***Abstract*—A phased-mission system (PMS) usually consists of several non-overlapping phases of tasks. All phases are required to be accomplished sequentially for a successful mission. Different features must be considered in the reliability evaluation of a PMS, including the dependence among the phases with respect to a common component and the different system topologies for the phases. To overcome the limitation of existing approaches, a stochastic computational approach is used for efficiently analyzing the reliability of a non-repairable PMS. Stochastic logic models are proposed to analyze the common components in the different phases. In the stochastic analysis, the signal probabilities of the basic components are encoded as non-Bernoulli sequences of random permutations with fixed numbers of 1s and 0s. Thus, the proposed stochastic approach can be used to evaluate a PMS under any distribution. Based on the generated stochastic sequences for the basic components and the system topology, the failure probability of the PMS can be efficiently predicted which avoids cumbersome analyzing process. Several case studies are evaluated to show the accuracy and efficiency of the stochastic approach. Compared with a combinatorial analysis, the accuracy of the stochastic analysis varies with the length of the stochastic sequences. However, it is shown that the stochastic analysis is more efficient than a Monte Carlo simulation at the same execution complexity in the number of runs.**

***Index Terms*—Phased-mission systems, stochastic computation, non-Bernoulli sequence, stochastic logic, reliability analysis.**

ACRONYM

FTA	fault tree analysis
DFT	dynamic fault tree
FDEP	functional dependency gate
PAND	priority AND gate
WSP	warm spare gate
CSP	cold spare gate
PCCF	probabilistic common cause failure
pdf	probability density function
cdf	cumulative density function
BDDs	binary decision diagrams
MC	Monte Carlo
PMS	phased-mission system
UAV	unmanned autonomous vehicle

NOTATION

H	the number of phases in the PMS
h	the index of a phase in the PMS
τ	mission time
τ_h	mission time of phase h
A, B	basic components
$A_j(h)$/ $B_j(h)$	the jth basic component of phase h
$Q_h(\cdot)$	logic structure derived from the system topology of phase h
$\phi(h)$	the basic component set of phase h
λ	failure rate of exponential distribution
$p(t_i)$	failure probability in the time interval $[t_i, t_i + \Delta t]$
P	overall failure probability of the PMS
$S(t_i)$	binary sequence at time t_i
$S_{i,j}(h)$	the jth bit in the stochastic sequence for the failure probability of component A_i or B_i at the end of phase h
$S'_{i,j}(h+1)$	the jth bit for the stochastic sequence for the failure probability of component A_i or B_i at phase $h+1$
L	sequence length for the stochastic approach
N	the number of 1s in the stochastic sequence

I. INTRODUCTION

A phased-mission system (PMS) undergoes different scenarios for which failure criteria vary throughout the entire mission time. For each phase, the system has different reliability requirements as indicated by its topology. The topology often varies from phase to phase and a system is usually decomposed into multiple non-overlapping phases. For the success of a PMS, all phases are required to be successfully and sequentially completed [1]. Many practical systems operate in this sequential manner, such as an aircraft flight, a nuclear power plant, aerospace and distributed computing systems [2]-[8]. For example, an aircraft mission of an unmanned autonomous vehicle (UAV) has a number of phases, including taxing, take-off, climbing to the required altitude, and cruising, descending and landing phases. The mission can fail in any of these phases and the PMS must be evaluated to obtain the failure probability of each phase. The PMS achieves an overall mission success only if every phase successfully completes the task. Hence, the overall mission failure is obtained by a logic OR of the failures of all phases [8].

The failure conditions of each phase must be identified; the system topology is usually modeled by a fault tree to indicate the combinations of component failures [9]. Hence, the technique of fault tree analysis (FTA) proposed in the 1960s

Peican Zhu is currently with the School of Computer Science and Technology, Northwestern Polytechnical University (NWPU), Xi'an, Shaanxi, China.

Jie Han is with the Department of Electrical and Computer Engineering, University of Alberta, Edmonton, AB T6G 1H9, Canada (e-mail: jhan8@ualberta.ca).

Leibo Liu is with the Institute of Microelectronics, Tsinghua University, Beijing, China (e-mail: liulb@tsinghua.edu.cn).

Fabrizio Lombardi is with the Department of Electrical and Computer Engineering, Northeastern University, Boston, MA, USA (e-mail: lombardi@ece.neu.edu).

[10], has been applied to evaluate the reliability of a PMS. However, dynamic behaviors, such as functionally dependent and priority relationships, also exist in a PMS. These behaviors are usually modeled by dynamic gates, e.g. the functional dependency (FDEP) and priority AND (PAND) gates. If dynamic relationships among the basic components are considered, dynamic fault trees (DFTs) are used to model these behaviors. The analysis of the reliability of a PMS is more challenging than a single-phased system because various factors (such as different system topologies and the dependence among different phases incurred by common components) must be considered.

Several approaches have been proposed to evaluate the reliability of a PMS. These approaches are mainly classified into two classes: analytical and simulation-based approaches [11], [12]. The analytical approaches can be differentiated into three categories: combinatorial methods [7], [13], state-space based methods [14]-[17] and phase modular methods [18]-[20]. A combinatorial method can handle any failure distribution and provide the exact failure probability using, for example, a binary decision diagram (BDD). However, it may be difficult to derive the exact analytical expression as function of the basic components' failure distributions for a system with a large number of basic components. The BDD based combinatorial approaches of [7] and [21] are only applicable to a PMS with static phases. Furthermore, dynamic relationships (such as functional dependency and priority relationships) usually exist among the basic components in a PMS. Hence, the reliability evaluation becomes even more challenging when dynamic gates are included in the PMS. For a state-space based method, a large complexity is usually encountered when analyzing complex systems due to the state-space explosion problem. The phase-modular methods of [18], [19] can model dynamic relationships, but a Markov chain analysis is required for the dynamic modules. The application of a Markov chain analysis is however limited when a basic component's failure behavior is not exponentially distributed. Several recent approaches have also been proposed for investigating a general PMS with propagated failures [22], [23], multimode state or failure scenarios [24] and repairable components [25].

Simulation based approaches can be found in [11], [12], [26] and Monte Carlo (MC) simulation [26] can also be used to evaluate the failure probability of a PMS. However, a large sample size is usually required for obtaining a stable accuracy due to a slow convergence. Hence, a long simulation time is often incurred for MC based approaches.

Recently, stochastic computational approaches have been proposed for analyzing logic circuits [27], [28] and calculating the failure probability of DFTs [29], [30]. [28] has shown that the use of non-Bernoulli sequences of random permutations of fixed numbers of 1s and 0s as initial inputs leads to an efficient and accurate evaluation. Furthermore, a stochastic model for PAND gates is proposed for an efficient analysis of DFTs [30]. The capability of a stochastic approach is shown by analyzing DFTs with non-exponentially distributed components using non-Bernoulli sequences to encode failure probabilities [30].

In this paper, a stochastic computational approach is proposed for analyzing a PMS. Initially, the topology of each phase in a PMS is modeled by either a static fault tree, or a DFT. A stochastic computational model is then proposed for evaluating the output failure probability of a PMS as a function of the failure probabilities of its components. Non-Bernoulli sequences of random permutations of fixed numbers of 1s and 0s are used to encode the failure probabilities of the basic components. It is shown that a PMS is efficiently evaluated by using the non-Bernoulli sequences. Due to the stochastic sequence's capability of preserving signal correlation, repeated common components in different phases (as frequently encountered in a PMS) are also analyzed. A PMS with dynamic phases can be analyzed using the proposed approach by utilizing stochastic models of PAND and FDEP gates. Finally, due to the direct encoding of failure probabilities into non-Bernoulli sequences, both exponential and non-exponential (such as Weibull) distributions can be analyzed by the stochastic model, as shown by several case studies.

The remainder of the paper is organized as follows. Section II presents the preliminaries, including the fundamentals of stochastic computation, the definition of a PMS and some hypotheses considered in this work. Section III presents the proposed stochastic computational model of a PMS. A flow of the stochastic analysis of a general PMS is described next in Section IV. Two case studies are presented in Section V. Finally, Section VI concludes the paper.

II. Preliminaries

A. Stochastic computational approach

In 1960s, stochastic computation was proposed for reliable circuit design [31]. In stochastic computation, a number of bits are set to specific values, which is referred to as a stochastic sequence. For example, the number of 1s is assigned to indicate the probability of 1s in a stochastic sequence. Stochastic sequences are efficiently processed by logic gates and the obtained stochastic sequence gives the output signal probability. Fig. 1 shows the stochastic logic gates used in this paper.

Stochastic computation has the advantages of hardware simplicity and fault tolerance. Boolean logic operations are transformed into probabilistic computations in the real domain [27]. As discussed in [28], the number of 1s in the output sequence is not deterministic but probabilistic due to inevitable stochastic fluctuations; however, the use of non-Bernoulli sequences of randomly permuted 1s and 0s as initial inputs can reduce the stochastic fluctuation [28]. In Fig. 1(a-d), a sequence of 10 bits is utilized to show the encoding and computing process in stochastic computation; a longer sequence is usually required for achieving a higher accuracy.

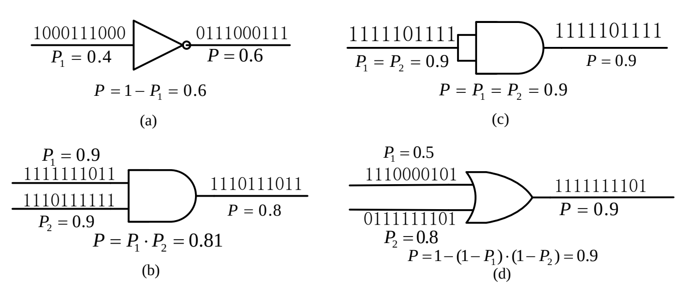

Fig. 1 Stochastic logic gates. (a) An inverter. (b) An AND gate with statistically independent inputs. (c) An AND gate with totally correlated inputs. (d) An OR

gate with statistically independent inputs.

Stochastic logic circuits can handle correlated signals (usually caused by reconvergence of fanout signals) because signal dependencies are inherently maintained and propagated (as shown in Fig. 1(c)). This is an advantageous property for handling the common components in the different phases of a phased-mission system (PMS).

Similar to [30], the mission time τ is discretized; it is divided into H phases, and the mission time of phase h is denoted by τ_h, $h \in \{1, 2, \cdots, H\}$. Each τ_h is further divided into M equal time intervals. The failure probability at a specific time is determined by the cumulative density function (*cdf*) derived from the given probability density function (*pdf*) of a basic component. A failure probability is more precisely estimated at a specific mission time with a larger M, which however incurs a longer run time. Hence, M is determined by a tradeoff between accuracy and evaluation efficiency. The discretization provides a relatively accurate estimate of the failure probability of a basic component by using an appropriate M.

For simplicity, the time interval $[t_i, t_i + \Delta t]$ is referred in this paper as time i. The failure probability of a basic component in a selected time interval $[t_i, t_i + \Delta t]$ is considered constant as the value at the beginning of the time interval, i.e., the failure probability is given by $p = F(t_i)$ for any time in this time interval. The failure probabilities for two adjacent time intervals (i.e., time $i-1$ and time i) are given by p_{i-1} and p_i respectively. Given a non-Bernoulli sequences of L bits, the number of 1s in the sequences for the two failure probabilities are given by $N(p_{i-1}) = L \cdot p_{i-1}$ and $N(p_i) = L \cdot p_i$ respectively. If the non-Bernoulli sequence for the probability at time $i-1$ is represented as $S(p_{i-1})$, then the sequence $S(p_i)$ for the probability at time i can be obtained by randomly assigning $\Delta N = N(p_i) - N(p_{i-1}) = L \cdot [p_i - p_{i-1}]$ 1s to replace the 0s in $S(p_{i-1})$. The relationship between two non-Bernoulli sequences for two adjacent time intervals (i.e., i and $i-1$) is given by

$$S(p_i)\ AND\ S(p_{i-1}) = S(p_{i-1}). \tag{1}$$

Due to the non-reparability, the 1s in $S(p_{i-1})$ remain as 1s in $S(p_i)$; then the mutual set in both sequences is given by $S(p_{i-1})$.

B. A phased-mission system (PMS)

A PMS is usually defined as follows [8].

1) It consists of multiple non-overlapping phases and the operation of phases is performed in a sequential order;
2) The topology of the system usually varies from phase to phase, i.e., different failure criteria apply to each phase;
3) All phases must be successfully completed for the mission to be successful.

A component can fail at any time during the mission time and the state of a component may be critical for a specific phase. Furthermore, the failure of a component resulting in the failure of the PMS may have occurred during previous phases. The mission can fail in any of the phases; hence, the evaluation of the PMS must calculate the failure probability occurred in each phase.

The system topology is usually modeled as a fault tree to show the combinations of component failures. Let $Q(\cdot)$ be the logic operation derived from the system topology; then, the structure function at phase h is denoted by $Q_h(\cdot)$. The technique of fault tree analysis (FTA) [10], [32] is utilized to calculate the failure probability during phase h as

$$p_h = Q_h(S_i(h)), \tag{2}$$

where $S_i(h)$ indicates the stochastic sequence generated based on the failure probability of component $B_i(h)$, i.e., $p_i(h)$ $B_i(h) \in \phi(h)$, $\phi(h)$ denotes the set of basic components at phase h, and p_h is the failure probability of the sub-system of phase h.

To compute the overall mission failure probability of the PMS, P, a conventional phase-OR model is adopted, as shown in Fig. 2. This model, after slight modifications, is also applicable if other relationships are considered for the phase failures, such as the AND/OR logic relationship [21]. In the structure in Fig. 2, the output failure probability of the PMS is determined by the stochastic sequences for the failure probabilities of different phases. As the failures are exclusive events at different phases, P is given by the sum of the failure probabilities of all phases [8], i.e.,

$$P = \sum_{h=1}^{H} p_h. \tag{3}$$

where $h \in \{1, 2, \cdots, H\}$ and H is the number of phases of the PMS.

Then, the reliability of a PMS, r, i.e., the probability that all phases are successful, is given by

$$r = 1 - P. \tag{4}$$

C. Assumptions

The following assumptions are applicable in the analysis of this paper.

- The quantization level of a component is indicated by a binary variable x, $x \in \{0,1\}$, where 0 indicates the fault-free scenario;
- All basic components are fault-free at the beginning of the mission time;
- The basic components are assumed to be non-repairable [8]. If a component fails, then the variable that indicates the status of the basic component, remains;
- For a PMS, the state of a component at the beginning of a phase is the same as the state at the end of the previous phase [7];
- The duration of each phase in the PMS is known for the investigation.

III. Proposed Stochastic Models for a PMS

A PMS consisting of H phases is usually represented by H fault trees (Fig. 2). For a binary PMS, both the system and the components can only have two states: success or failure. For the success of the PMS, all phases are required to be successfully completed. The failure of any phase results in the failure of the PMS. Let the stochastic sequence $S(h)$ encode the failure probability of phase h, $p(h)$, and $S_j(h)$ denote the value of the jth bit. If $S_j(h)$ = 1 (i.e., the failure of phase h), then $S_j(PMS)$ = 1 where $S(PMS)$ is the stochastic sequence for the failure probability of the overall PMS. The calculation of the failure probability of the PMS (as in formula (3)) can be efficiently implemented by an OR gate; thereafter, the reliability of the PMS is obtained by inverting the stochastic sequence for the failure probability of the PMS.

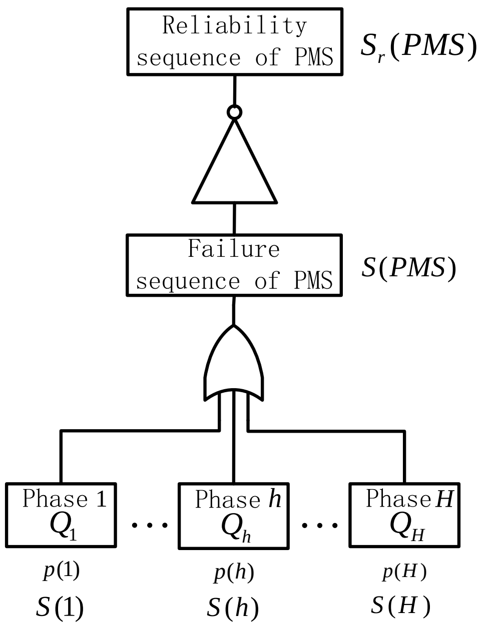

Fig. 2 A general structure of a phased-mission system (PMS) consisting of H phases with different system topologies. $S(h)$ is a stochastic sequence for the failure probability of phase h, $p(h)$. Q_h denotes the structural function at phase h, $1 \leq h \leq H$. H is the total number of phases of the PMS. $S(PMS)$ denotes the stochastic sequence for the failure probability of the PMS. $S_r(PMS)$ denotes the stochastic sequence for the reliability of the PMS.

Each of the fault trees derived from the system topology (indicated by Q_h) is used to model the failure condition of a phase. Fig. 3 illustrates a general fault tree structure of phase h, $h \in \{1, 2, \cdots, H\}$, for a PMS. Each phase has a different system topology, so let Q_h denote the structure function at phase h. If there are n basic components at phase h, let $A_j(h)$ indicate the jth component at phase h.

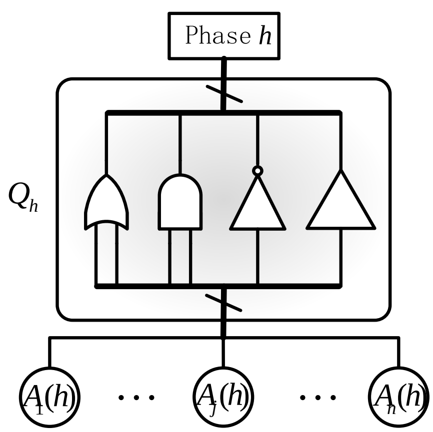

Fig. 3 A general fault tree structure of phase h for a phased-mission system (PMS) consisting of H phases with different system topology at each phase. $A_j(h)$ denotes the jth component at phase h ($1 \leq h \leq H$). H is the number of phases in the PMS. n is the number of basic events in phase h.

The structure function at phase h, Q_h, is constructed using the stochastic logic gates of Fig. 1 according to the relationships between the basic components. Then, the output stochastic sequence for the failure probability of phase h is obtained by first determining the input sequences for the basic components. Let $\boldsymbol{x_h}$ denote the states of basic components at the end of phase h, i.e., $\boldsymbol{x_h} = (x_1(h), x_2(h), \cdots, x_n(h))$, where $x_j(h)$ denotes the state of the basic component $A_j(h)$, $j \in \{1, 2, \cdots, n\}$. Then, $Q_h(\boldsymbol{x_h})$ gives the state of the system at the end of phase h. If $Q_h(\boldsymbol{x_h}) = 0$, phase h is successfully completed. Otherwise, the system fails by the end of phase h.

In a general PMS, common components are often encountered in different phases. For example, the engine of an unmanned autonomous vehicle (UAV) must function in most phases. However, some of the components might appear in all phases while some of them are just used in a few specific phases. The distribution of the common components is illustrated in Fig. 4, where $\phi(j)$ and $\phi(k)$ denote the basic component sets of phases j and k respectively, $j, k \in \{1, 2, \cdots, H\}$. For any phases, say j and k, $\phi(j) \cap \phi(k) \neq \emptyset$ indicates that common components exist at phases j and k (as in Fig. 4(a)). If the failure can be masked and does not necessarily cause a system failure, it is referred to as case 1. If the failure of a common component directly results in the failure of a specific phase, it is referred to as case 3. As in Fig. 4(b), $\phi(j) \cap \phi(k) = \emptyset$ indicates the case that there is no common component at phases j and k. This case is referred to as case 2 in this paper.

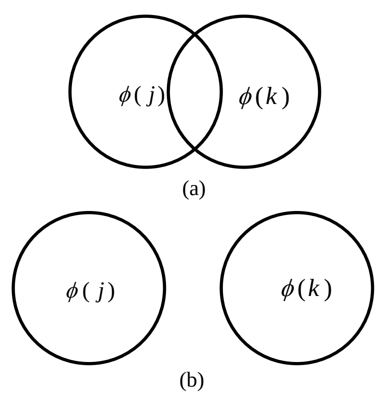

Fig. 4 Distribution of the common components for phases j and k, $j, k \in \{1, 2, \cdots, H\}$. (a) Common components exist in phases j and k; (b) No common component at phases j and k. $\phi(j)$ or $\phi(k)$ denotes the basic component set of phase j or k. H is the number of phases in the PMS.

A. Case 1

Assume that component A_i exists in phase k and also in phase j, i.e., $A_i \in \phi(j) \cap \phi(k)$. Fig. 5 shows an example of $A_i \in \phi(j) \cap \phi(k)$, where j and k are two consecutive or disjoint phases ($j < k$). In Fig. 5 (and all subsequent figures, wherever applicable), $A_i(j)$ denotes the ith component at phase j with $i \in \{1, 2, \cdots, m\}$, $j \in \{1, 2, \cdots, k\}$ and $B_l(k)$ denotes the lth component at phase k with $l \in \{1, 2, \cdots, n\}$. m and n are the total numbers of basic components for phase j and k respectively. If the failure of the common component A_i does not directly cause the failure of phase j, the failure of A_i is masked by other components (for instance by an AND operation); so this component is not required to survive prior to phase k (as an example of case 1).

For simplicity, let $j = h$ and $k = h + 1$. Since, the failure of the component A_i may occur during phase $h + 1$ or a phase before $h + 1$, the stochastic sequence for the failure probability of A_i at phase $h+1$, $S_i'(h+1)$, is found by considering $S_i(h)$ (the stochastic sequence at the end of phase h) and $S_i(h+1)$ (the stochastic sequence for the failure probability of component B_i during $h+1$). Then the sequence $S_i'(h+1)$ is obtained using the stochastic model in Fig. 6.

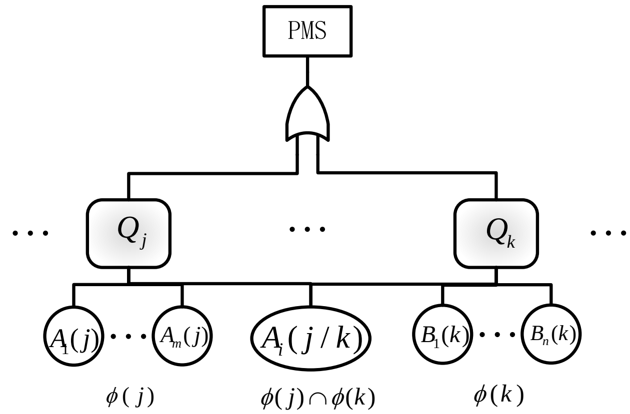

Fig. 5 Example of case 1: $\phi(j) \cap \phi(k) \neq \emptyset$ and $A_i(j/k) \in \phi(j) \cap \phi(k)$, where $A_i(j/k)$ is one of the common components.

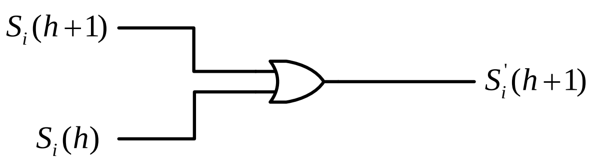

Fig. 6 A stochastic logic model for computing the failure probability of component A_i for cases 1 and 2.

Let $S_{i,j}(h)$, $S_{i,j}(h+1)$ and $S_{i,j}'(h+1)$ be the values of the jth bit of the corresponding stochastic sequences. If $S_{i,j}(h) = 1$ (so component A_i fails at the end of phase h), then $S_{i,j}'(h+1) = 1$ due to the assumption of non-reparability of the PMS. This corresponds to the case when component A_i fails before phase $h+1$. Otherwise, whether component A_i fails or not is determined by $S_{i,j}(h+1)$ for the failure probability during phase $h+1$, i.e., $p_{A_i}(h+1)$. This indicates that $S_{i,j}(h+1)$ is selected as the value of $S_{i,j}'(h+1)$ if $S_{i,j}(h) = 0$. This can be efficiently implemented by a stochastic OR gate as shown in Fig. 6.

B. Case 2

For case 2, there is no common component in any of the two phases j and k, i.e., $\phi(j) \cap \phi(k) = \emptyset$ (as in Fig. 4(b)); then whether the components of phase j or k fail or not cannot result in the failure of phase j or k (with $j < k$). Fig. 7 shows an example of case 2. With the stochastic logic models for phases j and k (i.e., Q_j and Q_k), the failure probabilities of phases j and k are determined by the failure probabilities (encoded into non-Bernoulli sequences) of the input components of phases j and k respectively.

Again, let $j = h$ and $k = h+1$. The failure of phase k is determined by the failure of basic events in $\phi(k)$. For a basic component B_i, it may fail before or during phase $h+1$. Both scenarios need to be considered for the failure of phase k. Hence, the stochastic sequence for the failure probability of component B_i at phase $h+1$, $S_i'(h+1)$, is obtained by considering $S_i(h)$ and $S_i(h+1)$, the stochastic sequences for the failure probabilities of component B_i at the end of phase h and during phase $h+1$ respectively, are as follows. Then, the failure probability of B_i is found by using the stochastic model of Fig. 6 (same as for case 1).

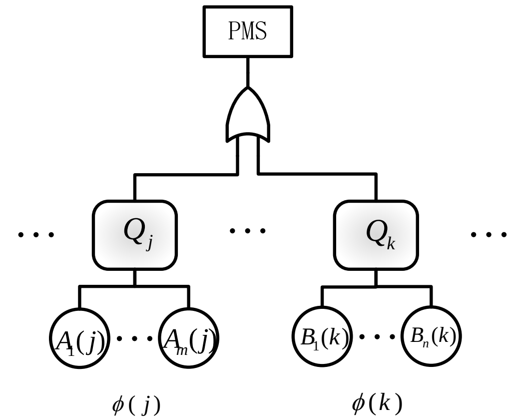

Fig. 7 Example of case 2 (corresponding to Fig. 4(b)). $\phi(j) \cap \phi(k) = \emptyset$.

C. Case 3

For case 3, assume that component A_i appears in both phases j and k, i.e., $A_i \in \phi(j) \cap \phi(k)$. Let again $j = h$ and $k = h+1$. The failure of A_i directly causes the failure of phase h, so the failure probability at the end of phase $h+1$ is given by the failure probability during phase $h+1$ with the failure before phase $h+1$ excluded, i.e., $S_i'(h+1) = S_i(h+1)$ implemented by a buffer as in Fig. 8.

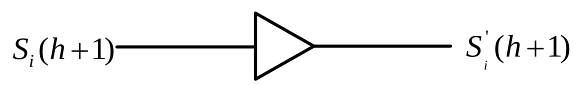

Fig. 8 A stochastic logic model for computing the failure probability for case 3 of component A_i.

Furthermore, if the failure probability for an intermediate time point during phase $h+1$ is of interest, the following analysis is applicable. Let $\tau(h+1)$ denote the intermediate mission time during phase $h+1$; then, the intermediate failure probability of the overall PMS during phase $h+1$ is derived by replacing the stochastic sequence $S_i(h+1)$ (for $p_i(h+1)$) with $S_i(\tau(h+1))$ (for $p_i(\tau(h+1))$).

Hence in the proposed approach, a stochastic model of the PMS can be constructed for the relationship among basic components using stochastic logic gates. Two stochastic models (Figs. 6 and 8) are utilized to evaluate the effect of common components in different phases and to obtain the reliability of each phase from the failure probabilities of the basic components. The failure probability of the overall PMS is encoded in the statistics, i.e., the proportion of number of 1s, in the output sequence of the stochastic analysis; the reliability of the PMS can then be determined by inverting the stochastic sequence indicating its failure probability (Fig. 2).

IV. Evaluation Procedure

For a general PMS, the process of evaluating its overall reliability is as follows.

Step 1: Construct the PMS using stochastic logic gates;

Step 2: Determine the common components in different phases and find whether the failure of the common components can directly cause the failure of the phase being investigated;

Step 3: Compute the failure probabilities at different mission times based on the provided *pdfs* and *cdfs*;

Step 4: Encode the basic modules' failure probabilities at different time steps into non-Bernoulli sequences following the algorithm provided in Section II A and Section III;

Step 5: If a common component exists, the stochastic sequences must be generated using the stochastic model in Fig. 6 or Fig. 8, as determined by whether a common component exists and whether the failure of the common component can result in the failure of the PMS;

Step 6: Derive the overall failure probability at different time steps by propagating the non-Bernoulli sequences through the stochastic models. The reliability can be obtained by inverting the stochastic sequence indicating the failure probability of the PMS (using the model in Fig. 2).

A flowchart illustrating the above evaluation procedure is given in Fig. 9.

In a DFT, the priority AND (PAND) gate can be utilized if required. This may occur for an input (to indicate the firing of a basic event in a predetermined order) and the output (to indicate whether a failure occurs [8]) as a priority relationship. A dynamic functional dependency (FDEP) gate can further be utilized to model the behavior among the components. However, it is cumbersome to analyze a system using a combinatorial analysis. A stochastic model can instead be utilized to replace the dynamic gates with static gates. For systems with perfect fault coverage, the FDEP can be treated as an OR gate [33] [34]. For the PAND gate in a PMS, the stochastic model of [30] can be used to model the priority relationship. Signal correlations are easily handled by the proposed stochastic model [30]; moreover, repeated events and events with non-exponential distributions are modeled efficiently. Stochastic computational models for the so-called Spare gate (such as warm (WSP) and cold (CSP) standby) have been presented in [35]. If the basic events suffer from (probabilistic) common cause failures, then these cases can be easily addressed by a stochastic approach, as shown by the simulation results in [35].

V. Case studies

In this section, several case studies are presented to show the efficiency of the proposed stochastic method. The results are compared with the combinatorial analysis of [13] and the Monte Carlo (MC) simulation of [15]. The proposed stochastic approach is not limited to a particular failure distribution of the basic component. Hence, both exponential and non-exponential distributions are investigated to show the capability of the stochastic approach in handling the general cases. All simulations are run on a computer with a 3.10 GHz i3-2100 microprocessor and 6 GB memory.

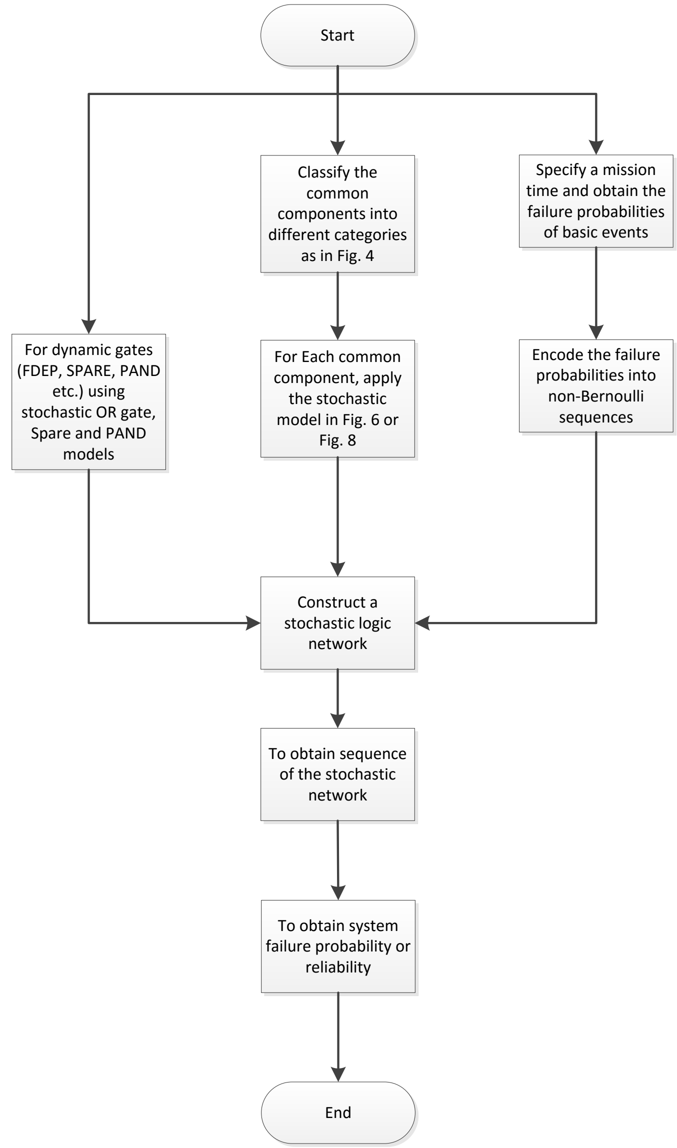

Fig. 9 A flowchart illustrating the evaluation procedure using the stochastic approach.

The mission time τ is divided into H phases and each phase is further divided into M equal intervals. Let $p(i)$ be the failure probability of a basic component at time i, then the failure probability of an output event for the system is given by a vector $\boldsymbol{F} = (F(1), F(2), \cdots, F(M))$, i.e., $F(j) = Q(p(i))$, where $Q(\cdot)$ is the logic operation determined from the system topology. Let $\boldsymbol{F}_{MC}$, $\boldsymbol{F}_A$ and $\boldsymbol{F}_S$ denote the failure probability vectors obtained by MC simulation, an accurate analysis and the stochastic approach respectively. Then, $\Delta\boldsymbol{F}_{MC-A} = \boldsymbol{F}_{MC} - \boldsymbol{F}_A$ denotes the difference between the failure probability vectors of the MC simulation and the accurate analysis, and $\Delta\boldsymbol{F}_{S-A} = \boldsymbol{F}_S - \boldsymbol{F}_A$ indicates the difference between the failure probability vectors of the stochastic approach and the accurate analysis. In the simulations of the following case studies (and for all related figures and tables as applicable), N denotes the number of simulation runs for the MC method, while L denotes the sequence length for the stochastic approach.

Similar to [30], the norms, $||\cdot||_1$, $||\cdot||_2$ and $||\cdot||_\infty$, are calculated to measure the differences of failure probability vectors that reveal the accuracy of different approaches. For a vector $\boldsymbol{x}$, the norms are defined as $\|\boldsymbol{x}\|_1 = \sum_{i=1}^{n}|x_i|$, $\|\boldsymbol{x}\|_2 = \sqrt{\sum_{i=1}^{n}|x_i|^2}$ and $\|\boldsymbol{x}\|_\infty = \max_{1\le i\le n}|x_i|$.

A. Example 1

A PMS with non-exponentially distributed basic events is analyzed first using the stochastic approach. It becomes challenging when the failures of basic components are not exponentially distributed for reliability evaluation. In this section, it is shown that this issue is effectively addressed by the stochastic approach. A Weibull distribution is considered for a PMS with non-exponentially distributed basic components. Furthermore, the reliability of the PMS at any time during the entire mission time can be found.

Example 1 deals with a non-repairable PMS consisting of four elements for a mission time of τ = 630 hours [13]. The system is successful only if the three phases perform without failure; the durations of the three phases are τ_1 = 160 hours, τ_2 = 200 hours and τ_3 = 270 hours. The structure of the stochastic PMS is shown in Fig. 10.

The parameters for each element of Example 1 in each phase are given in Table 1 [13]. Except for element 3 at phase 3 and element 4 at phase 1, which follow Weibull distributions, all other elements fail exponentially.

For an exponential distribution, the probability density function (*pdf*) and the cumulative density function (*cdf*) are given by

$$f(t) = \lambda\, e^{-\lambda t}, \tag{5}$$

and

$$F(t) = \int_0^t f(u)\, du = 1 - e^{-\lambda t}, \tag{6}$$

where t is the specified mission time and λ is the (constant) failure rate of a basic component for an exponential distribution.

The *pdf* and *cdf* of a Weibull distribution are given by

$$f(t) = \frac{k}{\lambda}\left(\frac{t}{\lambda}\right)^{k-1} e^{-(t/\lambda)^k} \tag{7}$$

and

$$F(t) = 1 - e^{-(t/\lambda)^k} \tag{8}$$

respectively, where k and λ are the shape and scale parameters respectively.

For the stochastic approach and the combinatorial analysis of [13], the reliabilities of the components at each phase and the reliability of the overall PMS are given in Table 2. The results of the MC simulation and the stochastic approach follow approximately a Gaussian distribution for a large number of simulation runs, or a large sequence length [28]. For a confidence level of 95%, the error is given by $E = \frac{z_c}{\mu}\sqrt{\frac{v}{m}}$, where μ denotes the accurate mean, v is the variance of the distribution of the results, m is the number of simulation runs or sequence length, and the parameter z_c is equal to 1.96. Hence, the error decreases with the increase of m. For the input parameters shown in Table 1, the accurate mean of the system's reliability is found to be 0.8188. For the stochastic approach and the MC simulation, the standard deviations are obtained as 0.0004, and 0.0013, respectively, for L or N = 100k. Then, at a confidence level of 95%, the errors for the stochastic approach and the MC simulation are 0.0003% and 0.0010% respectively. Hence, the stochastic approach is more accurate than the MC simulation. This occurs because deterministic numbers of 1s are generated for the non-Bernoulli sequence encoding the signal probabilities for the stochastic approach. As analyzed in [28], the use of a non-Bernoulli sequence of a fixed number of 1s and 0s results in the same mean but a smaller variance than using Bernoulli sequences as initial inputs.

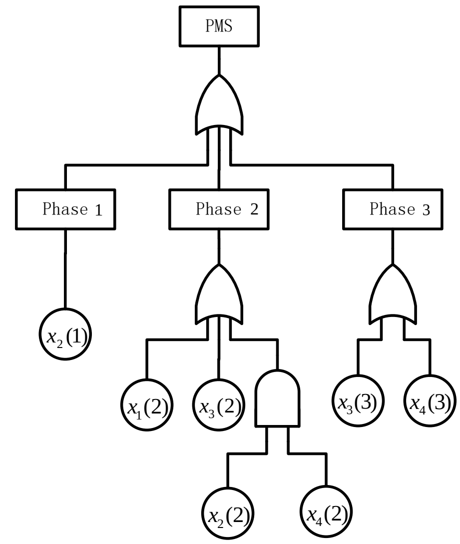

Fig. 10 A non-repairable PMS of Example 1 consisting of three phases and four components [13]. $x_i(j)$ denotes the ith component at phase j with $i \in \{1, 2, 3, 4\}$ and $j \in \{1, 2, 3\}$.

Table 1 Input parameters for Example 1 [13] with exponentially and Weibull distributed basic components. $\boldsymbol{k}$ and $\boldsymbol{\lambda}$ denote the shape and scale parameters for the Weibull distribution.

Component	Phase 1	Phase 2	Phase 3
x_1	0.0002	0.0001	0.00015
x_2	0.0001	0.0001	0.0001
x_3	0.00025	0.0001	$1/\lambda$ = 0.0001, k = 2
x_4	$1/\lambda$ = 0.0001, k = 1.5	0.0002	0.0001

Table 2 Reliabilities of PMS at different phases for Example 1 (*N* or *L* = 100k)

Components	Methods		
	combinatorial [13]	Monte Carlo	Stochastic
x_1 (h = 1)	0.9685	0.9672	0.9685
x_1 (h = 2)	0.9802	0.9789	0.9803
x_1 (h = 3)	0.9603	0.9600	0.9603
x_2 (h = 1)	0.9841	0.9837	0.9841
x_2 (h = 2)	0.9802	0.9804	0.9803
x_2 (h = 3)	0.9734	0.9726	0.9374
x_3 (h = 1)	0.9608	0.9613	0.9608
x_3 (h = 2)	0.9802	0.9813	0.9803
x_3 (h = 3)	0.9971	0.9974	0.9971
x_4 (h = 1)	0.9980	0.9982	0.9980
x_4 (h = 2)	0.9608	0.9586	0.9610
x_4 (h = 3)	0.9734	0.9726	0.9374
Overall Reliability	0.8188	0.8169	0.8190

As shown by the results in Table, the found reliability of the PMS by using the stochastic approach is very close (or equal) to the probability obtained from the combinatorial analysis,. For a reasonable sequence length, the stochastic approach provides a very accurate result, and it is more accurate than MC simulation.

For Example 1, the failure probability and reliability obtained by the stochastic approach are plotted in Fig. 11 for a mission time of 630 hours.

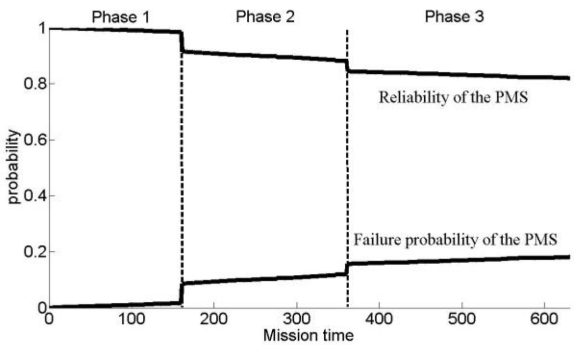

Fig. 11 Failure probability and reliability obtained by the stochastic approach for Example 1 with a sequence length of 10k bits.

Next, $\|\cdot\|_1$, $\|\cdot\|_2$, and $\|\cdot\|_\infty$ of the differences in the reliability vectors obtained by the stochastic and MC approaches compared to the accurate analysis are shown in Table 3. As per the results in Table 3, the disparity in the failure probability vectors of the stochastic and accurate analysis decreases, and so does the disparity in the failure probability vectors of the MC simulation and the accurate analysis. This indicates that the stochastic fluctuations in both approaches decrease with the increase of the sequence length and the number of simulation runs. However, the stochastic approach is more accurate and more efficient, as indicated by the smaller norm values and the smaller average run time for each sequence length or sample size (i.e., L/N in Table 3). The relationship between the run time of the two approaches are further illustrated in Fig. 12.

Table 3 Norms of the differences in the failure probability vectors obtained by the stochastic approach and Monte Carlo (MC) simulation [15] compared to an accurate analysis [36] for the PMS in Example 1.

		***L/N* = 1k**	***L/N* = 10k**	***L/N* = 100k**
$\|\Delta \boldsymbol{F}_{MC-A}\|_1$		4.4711	0.9999	0.4660
$\|\Delta \boldsymbol{F}_{MC-A}\|_2$		0.2444	0.0448	0.0195
$\|\Delta \boldsymbol{F}_{MC-A}\|_\infty$		0.0189	0.0033	0.0012
$\|\Delta \boldsymbol{F}_{S-A}\|_1$		0.6992	0.3157	0.1950
$\|\Delta \boldsymbol{F}_{S-A}\|_2$		0.0369	0.0157	0.0115
$\|\Delta \boldsymbol{F}_{S-A}\|_\infty$		0.0043	0.0013	0.0008
Average run time (s)	**Stochastic**	0.0556	0.3425	7.6772
	MC	0.2977	2.4170	23.302

As shown by these simulation results, the stochastic approach can evaluate a PMS with non-exponentially distributed components at a high accuracy. As the encoding of a failure probability into a stochastic sequence is not limited to exponential distributions, a PMS with non-exponentially distributed basic components can be efficiently evaluated by the proposed stochastic approach using a reasonable sequence length. Hence, the proposed stochastic approach is applicable to both exponential and non-exponential distributions for a PMS analysis.

As shown for the average run time, MC requires a longer simulation time than the stochastic analysis. For an accurate analysis, the exact expression is derived and the required computation time is very small (for example, the calculation time is 0.000797 second for Example 1). The runtime of the stochastic analysis may appear to be longer than an accurate analysis; however for an accurate analysis, the time to identify and decompose reconvergent fanouts in a PMS, and the time for dealing with dynamic behaviors and repeated components at different phases, are not included. If factors affecting system topology (such as common cause failures) are taken into account, the exact expression must be derived again. For the stochastic analysis, only minor changes (such as adding an extra module) need to be made; moreover, the execution of the stochastic approach can be parallelized to further reduce the run time due to the bit-wise independence of the random binary bit streams.

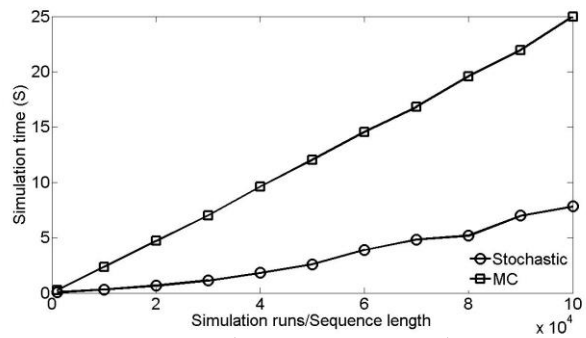

Fig. 12 The average run time for 10 simulation runs of Example 1 using the stochastic approach and Monte Carlo (MC) simulation.

B. Example 2

Example 2 is adapted from [20], [37] and deals with the non-repairable PMS shown in Fig. 13(a). This PMS is simulated to assess the efficiency of the stochastic approach for a total mission time of 500 hours. The PMS consists of three consecutive non-overlapping phases and eight components. Furthermore, several common components exist in the PMS. The PMS contains a WSP and a dynamic PAND gate in phase 1 and an FDEP gate in phase 3. Furthermore, the components of a system are considered to be subject to common cause failures (CCFs) that could be caused by events, such as earthquakes and design errors. Such CCFs usually occur on a probabilistic basis. At phase 1, a probabilistic common cause failure (PCCF) is considered to occur to the basic component G with a probability of 0.015. The probability that the affected basic component G fails due to a CCF, is assumed to be 0.8, i.e., $\gamma = 0.8$.

A stochastic model can be constructed for the PMS shown in Fig. 13(a) using stochastic logic gates (as shown in Fig. 13(b)). For systems with perfect fault coverage, the FDEP is treated as an OR gate [33], [34]; moreover, the stochastic models in [30] and [35] are utilized for modeling the PAND gate, the WSP and PCCFs. Table 4 shows the exponentially-distributed failure rates of the components at different phases. Assume the durations of the three phases are τ_1 = 100 hours, τ_2 = 250 hours, and τ_3 = 150 hours. The failure probabilities for the

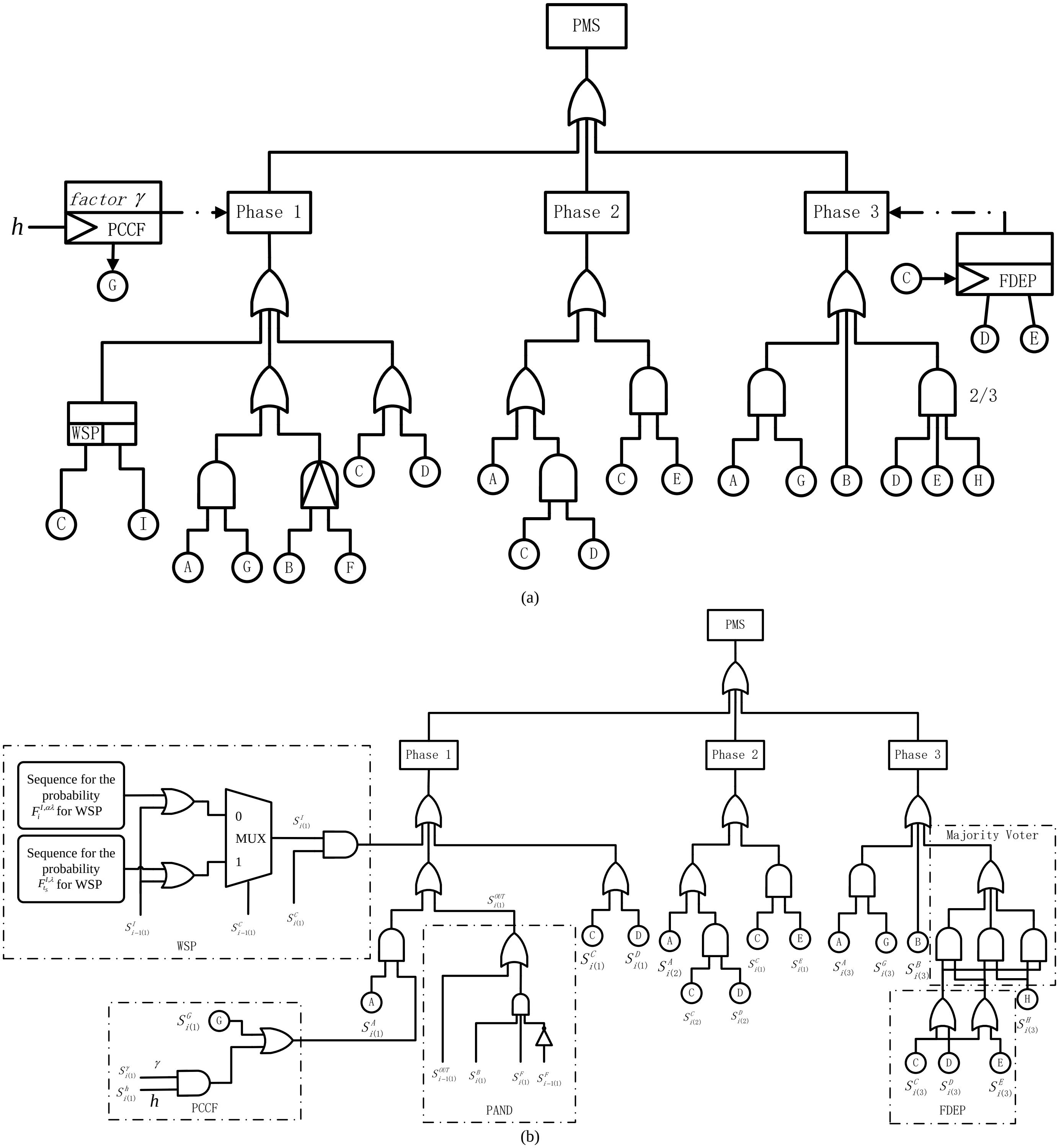

Fig. 13 (a) A PMS of three phases (adapted from [20], [37]) and (b) the stochastic model for the PMS in (a).

components at different time points can be computed using (6).

Both the stochastic computational approach and MC simulation [15] are applied to find the failure probability/reliability of the PMS at any mission time. The accuracy of the stochastic approach is compared with the MC simulation. The differences in the reliability vectors for the stochastic and the MC approaches with respect to the accurate analysis are given in Table 5. The disparity is indicated by the norm values. The average run times of the stochastic approach and MC method are also shown.

As per the norms in Table 5, the proposed stochastic approach requires a shorter simulation time than the MC approach for the same value of sequence length/simulation runs. Therefore, the stochastic approach is more efficient than the MC method, and provides a very accurate result with a reasonable sequence length.

The failure probability for the PMS of Example 2 is plotted in Fig. 14 for a total mission time of 500 hours using the stochastic approach, while the difference between the obtained failure probability vectors for the stochastic and MC methods is calculated for different sequence lengths or numbers of simulation runs. $\|\cdot\|_1$, $\|\cdot\|_2$, and $\|\cdot\|_\infty$ of the differences in the failure probability vectors obtained by the proposed stochastic and MC approaches are 0.7963, 0.0372 and 0.0041, respectively, for using a sequence length or sample size of 10k bits.

Table 4 Input failure parameters (10^{-3}/hour) for Example 2 with exponentially distributed failures for the basic components.

Basic components		Phase 1 τ_1 = 100 hours	Phase 2 τ_2 = 250 hours	Phase 3 τ_3 = 150 hours
A		1.0	2.0	1.5
B		0.5	1.0	1.0
C		3.0	2.0	1.0
D		3.0	2.0	1.0
E		1.5	1.0	2.0
F		0.5	1.0	1.0
G		2.0	1.0	1.0
H		1.0	2.0	2.0
I	**spare**	1.0	Does not appear at these phases	
	working	2.0		

Table 5 Norms of the differences in the failure probability vectors obtained by the proposed stochastic approach and Monte Carlo (MC) simulation [15] for the PMS of Example 2 compared with an accurate analysis [36].

		N/L = 1k	N/L = 10k	N/L = 100k
$\|\Delta \boldsymbol{F}_{MC-A}\|_1$		5.0121	1.1826	0.5426
$\|\Delta \boldsymbol{F}_{MC-A}\|_2$		0.2523	0.0599	0.0257
$\|\Delta \boldsymbol{F}_{MC-A}\|_\infty$		0.0259	0.0078	0.0030
$\|\Delta \boldsymbol{F}_{S-A}\|_1$		1.1054	0.5356	0.2165
$\|\Delta \boldsymbol{F}_{S-A}\|_2$		0.0569	0.0214	0.0099
$\|\Delta \boldsymbol{F}_{S-A}\|_\infty$		0.0060	0.0025	0.0011
Average run time (s)	**Stochastic**	0.8355	6.1081	63.508
	MC	2.6053	26.196	237.33

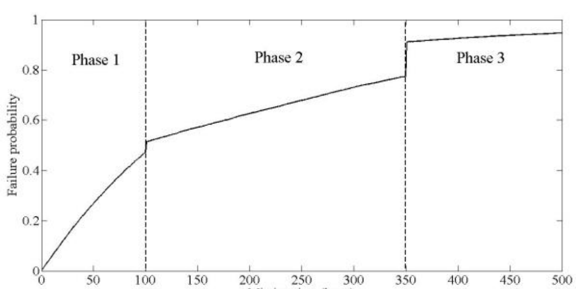

Fig. 14 Failure probability of a PMS consisting of three stages for a mission time of 500 hours obtained by the proposed stochastic approach with a sequence length of 10k bits (the durations of the three phases are τ_1 = 100 hours, τ_2 = 250 hours, and τ_3 = 150 hours).

As can be seen from these results, the PMS (inclusive of static and dynamic gates, such as PAND, spare and FDEP gates) can be efficiently evaluated by the proposed stochastic approach, using the stochastic PAND, spare gate and FDEP models. Hence, a general PMS with dynamic behaviors can be evaluated by the stochastic approach, even under the effect of PCCFs. Moreover, the failures of common components in different phases can be effectively evaluated by the proposed stochastic models.

VI. CONCLUSION

A stochastic model has been proposed for the analysis of phased-mission systems (PMSs). A PMS consisting of H phases is represented by H fault trees with each of them modeling the failure conditions of a phase. The fault tree for each phase can be constructed using stochastic logic gates. An OR gate model has been utilized to calculate the output stochastic sequence to indicate the failure probabilities of the H phases, i.e., for the entire system. Based on this analysis, the common components in different phases have been considered to determine whether their failure can cause the failure of the corresponding phase; different stochastic models have been proposed to compute the failure probabilities of the components for each phase. Furthermore, if dynamic behaviors (such as functional dependency and priority relationships) are included for the relationships between components, stochastic models for dynamic gates, such as the priority AND (PAND) and the functional dependency gates (FDEP), can be utilized for stochastic analysis of a system. Hence, the stochastic model of a general PMS can be constructed with stochastic logic gates.

A general PMS has been evaluated approximately and efficiently using non-Bernoulli sequences of random permutations of fixed numbers of 1s and 0s as initial input probabilities. The accuracy of the stochastic analysis is affected by the simulated sequence length. In the case studies considered, it is shown that the accuracy of the stochastic approach is better than Monte Carlo simulation with the same number of runs. Furthermore, the stochastic approach is capable of considering any failure distribution of the basic components; both exponential and Weibull distributions of the basic components have been analyzed for the case studies considered.

REFERENCES

[1] L. Xing, S. V. Amari, "Reliability of Phased-Mission Systems," K. B. Misra, editor. *Handbook of performability engineering*. Springer; 2008. p. 349–68 Chapter 23.

[2] A. K. Somani, J. A. Ritcey, S. H. L. Au, "Computationally efficient phased-mission reliability analysis for systems with variable configurations," *IEEE Transactions on Reliability*, 41(4):504–11, 1992.

[3] L. Xing, "Reliability evaluation of phased-mission systems with imperfect fault coverage and common-cause failures," *IEEE Transactions on Reliability*, 56(1):58–68, March, 2007.

[4] A. Pedar, V. V. S. Sarma, "Phased-mission analysis for evaluating the effectiveness of aerospace computing-systems," *IEEE Transactions on Reliability*, R-30(5):429–37, 1981.

[5] H. S. Winokur JR., L. J. Goldstein, "Analysis of mission-oriented systems," *IEEE Transactions on Reliability*, R-18(4):144–8, 1969.

[6] J. L. Bricker, "A unified method for analyzing mission reliability for fault tolerant computer systems," *IEEE Transactions on Reliability*, R-22(2):72–7, 1973.

[7] X. Zang, H. Sun, K. S. Trivedi, "A BDD-based algorithm for reliability analysis of phased-mission systems," *IEEE Transactions on Reliability*, 48(1):50–60, 1999.

[8] R. Remenyte-Prescott, J. D. Andrews, P. W. H. Chung, "An efficient phased-mission reliability analysis for autonomous vehicles," *Reliability Engineering & System Safety 2010*; 95(3): 226–35, March, 2010.

[9] J. D. Esary, and H. Ziehms, "Reliability of phased-missions," *Reliability and Fault-Tree Analysis*, 1975, pp. 213 – 236 (Society for Industrial Applied Mathematics, Philadelphia, Pennsylvania).

[10] C. A. Clifton Ericson, II, "Fault tree analysis – a history," *In Proceedings of the 17th International System Safety Conference*, August 16–21, 1999.

[11] R. E. Altschul, P. M. Nagel, "The efficient simulation of phased fault trees," *Proceedings of the Annual Reliability and Maintainability Symposium*, pp. 292–296, January, 1987.

[12] F. A. Tillman, C. H. Lie, C. L. Hwang, "Simulation model of mission effectiveness for military systems," *IEEE Transactions on Reliability*, R-27:191–4, 1978.

[13] L. Xing and G. Levitin, "BDD-based reliability evaluation of phased-mission systems with internal/external common-cause failures," *Reliability Engineering & System Safety* 112 (2013): 145-153.

[14] A. Bondavalli, S. Chiaradonna, F. D. Giandomenico, I. Mura, "Dependability modeling and evaluation of multiple-phased systems using DEEM," *IEEE Transactions on Reliability*, 53(4): 509–22 Dec, 2004.

[15] J. B. Dugan, "Automated analysis of phased-mission reliability," *IEEE Transactions on Reliability*, 40(1): 45–5255, April, 1991.

[16] I. Mura, A. Bondavalli, "Markov regenerative stochastic Petri nets to model and evaluate phased-mission systems dependability," *IEEE Transactions on Computers*, 50(12): 1337–51, Dec, 2001.

[17] M. K. Smotherman, K. Zemoudeh, "A non-homogeneous Markov model for phased-mission reliability analysis," *IEEE Transactions on Reliability*, 38(5): 585–90, Dec, 1989.

[18] L. Meshkat, L. Xing, S. Donohue, Y. Ou, "An overview of the phase-modular fault tree approach to phased-mission system analysis," *Proceedings of the International Conference on Space Mission Challenges for Information Technology*, pp. 393–398, 2003.

[19] Y. Ou, J. B. Dugan, "Modular solution of dynamic multi-phase systems," *IEEE Transactions on Reliability*, 53(4): 499–508, 2004.

[20] L. Xing, L. Meshkat, and S. Donohue, "An efficient approach for the reliability analysis of phased-mission systems with dependent failures," *In Proc. of the 8th Intl. Conf. on Probabilistic Safety Assessment and Management (PSAM8)*, New Orleans, LA, USA, May 14–19, 2006.

[21] L. Xing, and J. B. Dugan, "Analysis of Generalized Phased-Mission System Reliability Performance, and Sensitivity," *IEEE Transactions on Reliability*, 51(2): 199-211, 2002.

[22] G. Levitin, L. Xing, S. V. Amari, Y. Dai, "Reliability of non-repairable phased-mission systems with propagated failures," *Reliability Engineering & System Safety* 119(2013): 218–28, 2013.

[23] R. Peng, Q. Zhai, L. Xing, J. Yang, "Reliability of demand-based phased-mission systems subject to fault level coverage," *Reliability Engineering & System Safety* 121(2014): 18-25, 2014.

[24] Y. Mo, L. Xing, J. B. Dugan, "MDD-Based Method for Efficient Analysis on Phased-Mission Systems with Multimode Failures Systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 44(6): 757-769, 2014.

[25] J. Lu, X. Wu, "Reliability evaluation of generalized phased-mission systems with repairable components," *Reliability Engineering & System Safety* 121(2014): 136-145, 2014.

[26] R. K Durga, *et al*, "Dynamic fault tree analysis using Monte Carlo simulation in probabilistic safety assessment," *Reliability Engineering & System Safety 2009*; 94 (4): 872–83.

[27] H. Chen, J. Han, "Stochastic Computational Models for Accurate Reliability Evaluation of Logic Circuits," *Proc. Great Lakes Symp. VLSI (GLVLSI)*, Providence, RI, USA, pp. 61-66, 2010.

[28] J. Han, H. Chen, J. Liang, P. Zhu, Z. Yang and F. Lombardi, "A Stochastic Computational Approach for Accurate and Efficient Reliability Evaluation," *IEEE Transactions on Computers*, 63(6): 1336-1350, 2014.

[29] H. Aliee and H. R. Zarandi, "A Fast and Accurate Fault Tree Analysis Based on Stochastic Logic Implemented on Field-Programmable Gate Arrays," *IEEE Trans on Reliability*, 62(1): 13-22, March, 2013.

[30] P. Zhu, J. Han, L. Liu and M. J. Zuo, "A Stochastic Approach for the Analysis of Fault Trees with Priority AND Gates," *IEEE Transactions on Reliability*, 63(2): 480-494, June, 2014.

[31] B. R. Gaines, "Stochastic Computing Systems," *Advances in Information Systems Science*, Vol. 2, pp. 37-172, 1969.

[32] M. Stamatelatos, and W. Vesely, "Fault tree handbook with aerospace applications," Volume 1.1, pp. 1–205. *NASA Office of Safety and Mission Assurance*, 2002.

[33] A. Ejlali, and S. Miremadi, "FPGA-based Monte Carlo simulation for fault tree analysis," *Microelectronics Reliability*, 44(6): 1017–1028, 2004.

[34] G. Merle, J.-M. Roussel, and J.-J. Lesage, "Improving the Efficiency of Dynamic Fault Tree Analysis by Considering Gates FDEP as Static," in *Proc. Eur. Safety Rel. Conf. 2010 (ESREL2010)*, Rhodes, Greece, 2010, pp 845-851.

[35] P. Zhu, J. Han, L. Liu, and F. Lombardi, "A Stochastic Approach for the Analysis of Dynamic Fault Trees with Spare Gates under Probabilistic Common Cause Failures." *IEEE Transactions on Reliability*, 64(3): 878-892, 2015.

[36] S. Amari, G. Dill, and E. Howald, "A new approach to solve dynamic fault trees," *in Proc. Int. Annu. IEEE Rel. Maintainability Symp.*, pp. 374–379, 2003.

[37] L. Xing, A. Shrestha, L. Meshkat, and W. Wang, "Incorporating Common-Cause Failures into the Modular Hierarchical Systems Analysis," *IEEE Transactions on Reliability*, 58(1): 10-19, March, 2009.