Solving Traveling Salesman Problems via a Parallel Fully Connected Ising Machine

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Outline

• Motivation
• Preliminaries of Ising Machines
• Improved Parallel Annealing Algorithm for Solving Traveling Salesman Problems
  • An Exponential Temperature Function
  • A Dynamic Offset
  • A Clustering Approach
• Experimental Results and Evaluation
• Conclusion
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Motivation

- Combinatorial optimization (CO) problems
  - Drug discovery
  - Internet of Things
  - Machine learning

- Non-deterministic polynomial time (NP)-hard

The Ising machine
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The Ising Model

The Hamiltonian of an $N$-spin Ising model [1]:

$$H(\sigma_1, ..., \sigma_N) = -\sum_{i,j} J_{ij} \sigma_i \sigma_j - \sum_i h_i \sigma_i$$

$\sigma_i$ ($\sigma_j$): the state of the $i$th ($j$th) spin (upward +1 and downward -1)

$J_{ij}$: the interaction value between the $i$th spin and $j$th spin

$h_i$: the external field of the $i$th spin
Solving CO Problems via an Ising Machine

A CO problem → Mapping to Ising model → Finding ground state of the Ising energy → Getting the optimal solution

Initial configuration

Annealing process

Optimal solution

The limitation of the conventional annealing algorithms:

Only one spin can be updated per iteration for the fully connected Ising model. It decreases the annealing speed.

How to achieve parallel spin update?
Parallel Annealing

- Each spin is connected with all the other spins in the other layer but has no interaction with the spins in the same layer.
- The interactions between $\sigma_i^R$ and $\sigma_i^L$ are called self-interactions ($\omega_i$).
- All spins in the right and left layers are updated simultaneously per iteration.
- The Hamiltonian ($H_P$) [1]:

$$H_P = -\sum_{i,j} J_{ij} \sigma_i^L \sigma_j^R - \frac{1}{2} \sum_i h_i (\sigma_i^L + \sigma_i^R) + \omega_i \sum_i (1 - \sigma_i^L \sigma_i^R)$$

Is it efficient for solving constrained combinatorial optimization problems?

Max-cut Problems & Traveling Salesman Problems

Max-cut Problems

\[ H_{\text{Max-cut}} = \frac{1}{2} \sum_i \sum_j W_{ij} - \frac{1}{2} \sum_i \sum_j W_{ij} \sigma_i \sigma_j \]

Traveling Salesman Problems (TSPs)

\[ H_{\text{TSP}} = \frac{A}{4} \sum_{k \neq l} \sum_i W_{kl} \sigma_{ik} \sigma_{il} + \frac{A}{2} \sum_{k \neq l} \sum_{i \neq l} W_{kl} \sigma_{ik} \sigma_{il} + \frac{B}{4} \sum_i \sum_k \sum_l \sigma_{ik} \sigma_{il} + \frac{(n-2)B}{2} \sum_{i \neq j} \sum_k \sigma_{ik} \sigma_{jk} + \frac{C}{4} \sum_i \sum_k \sum_j \sigma_{ik} \sigma_{jk} + \frac{(n-2)C}{2} \sum_{i \neq j} \sum_k \sigma_{ik} \sigma_{jk} + \frac{A}{4} \sum_k \sum_l W_{kl} + \left( \frac{n^3}{4} - n^2 + n \right) (B + C) \]

Energy landscape of solving CO problems via Ising machine

Constant

Interaction

External field
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Temperature Function

• For conventional parallel annealing algorithm [1]:

\[ T_s = \frac{1}{\beta_0 \ln(1 + s)} \]

\( T_s \) approaches a value that makes \( P_i \) to be low but not zero. Spins can flip at the end of annealing.

• For the improved parallel annealing algorithm:

\[ T_s = T_{\text{init}} r^{s-1} \]

\( T_s \) approaches a value that makes \( P_i \) to be zero. Spins can not flip at the end of annealing.

Dynamic Offset

- The Ising model is stuck in a local minimum and hard to escape when $T_s$ is low.
- A dynamic offset $\Delta T$ is introduced.
  - $T_s$ increases by $\Delta T$ when there is no flip among the spins.
  - $\Delta T$ resets to zero when the configuration of spins changes.
- The Ising model can escape from a local minimum quickly and have a higher probability to find a better solution.
Improved Parallel Annealing for TSPs

1. Initialize spin configuration, temperature, and dynamic offset
2. For $s = 1$ to iteration do
   (update left layer if $s$ is odd, right layer if $s$ is even)
3. Update temperature value using an exponential temperature function
4. Calculate energy variation and spin-flip probability of each spin
5. Determine new state of each spin
6. Update dynamic offset
7. End for

The solution quality decreases when the number of cities in the TSP is large.
A Clustering Approach [2]

The K-medoids Clustering

Step 1
1. For $i = 1$ to $M$
2. $D_i = \sum_{j=1}^{M} W_{ij}$
3. End for
4. Choose $k$ cities with the first $k$ smallest $D$ as the central points

$M$: the number of cities for clustering
$W$: the distance matrix
$k$: the number of groups
The K-medoids Clustering

Step 2

1. **For** \( i = 1 \) to \( M \)

2. Assign the \( i \)th city to the closest central point

3. **End for**

Assign all points into groups
The K-medoids Clustering

Step 3
1. For each group
2. For each cities in the group
3. \[ d_i = \sum_j W_{ij} \]
4. End for
5. Choose the city with the smallest \( d \) as the new central points of the current group
6. End for

Update new central points
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Penalty Parameter Setting

The effect of the penalty parameters $B$ and $C$ ($B=C$) on the quality of solutions: (a) for 12 cities from gr431, (b) for 14 cities from burma14, and (c) for 16 cities from ulysses16. The blue shadow area indicates the results do not meet constraints.

- The found distance is shorter when the penalty parameter is smaller.
- Too small penalty parameter values produce results that do not meet constraints.

\[ B = C = 1 \times \max\{W\} \]
Dynamic offset setting

The effect of $T_{inc}$ on the quality of solutions: (a) for the benchmark burma14, (b) for the benchmark ulysses16, and (c) for the benchmark ulysses22.

- The solution quality tends to be stable when $\frac{\max\{\abs(J)\}}{90} \leq T_{inc} \leq \frac{\max\{\abs(J)\}}{10}$.
- The traveling distance increases when $T_{inc} < \frac{\max\{\abs(J)\}}{90}$ for burma14.

$T_{inc} = \frac{\max\{\abs(J)\}}{90}$
Parallel annealing algorithms, including the improved parallel annealing (IPA) and momentum annealing (MA) [1], have better performance than the conventional annealing algorithm, i.e., digital annealing (DA) [3].

- The IPA can find a shorter travel distance than MA.
- The found distance is shorter after using the clustering approach.


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• Parallel spin update improves the speed of annealing as all spins can be updated in every iteration.
• The improved parallel annealing algorithm achieves higher solution quality after applying an exponential temperature function, a dynamic offset, and a clustering approach.
• The runtime of IPA is $44.4 \times$ faster than the DA and $19.9 \times$ faster than the MA.
• Parallel annealing algorithms show potential in the development of energy-efficient systems.
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Thank you!

Questions?

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