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Solving Traveling Salesman Problems via a Parallel Fully Connected Ising Machine

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Outline

- Motivation
- Preliminaries of Ising Machines
- Improved Parallel Annealing Algorithm for Solving Traveling Salesman Problems
  - An Exponential Temperature Function
  - A Dynamic Offset
  - A Clustering Approach
- Experimental Results and Evaluation
- Conclusion

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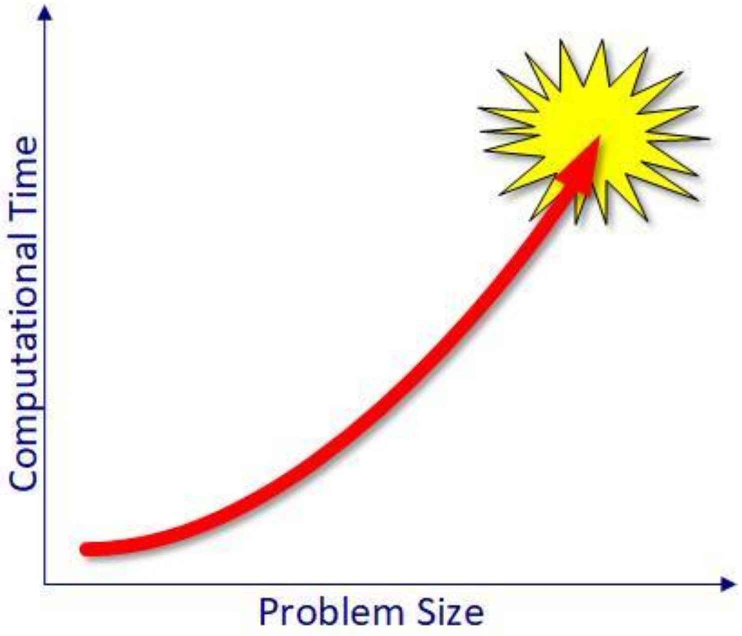
Motivation

- Combinatorial optimization (CO) problems

Drug discovery

Internet of Things

Machine learning



- Non-deterministic polynomial time (NP)-hard

The Ising machine

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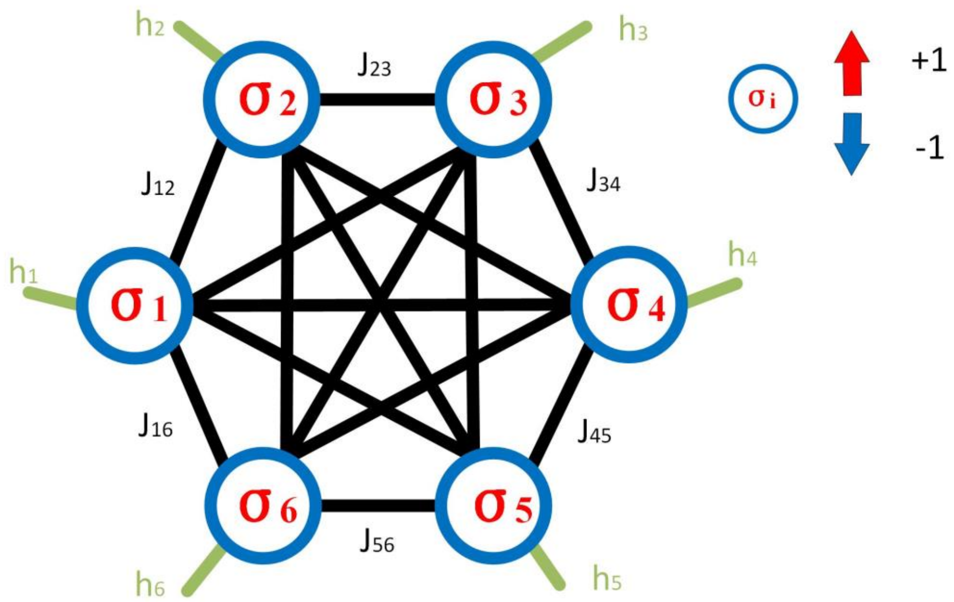
The Ising Model



A fully connected Ising model with 6 spins

The Hamiltonian of an *N*-spin Ising model [1]:

$$H(\sigma_1, \ldots, \sigma_N) = -\sum_{i,j} J_{ij}\sigma_i\sigma_j - \sum_i h_i\sigma_i$$

σ_i (σ_j): the state of the ith (jth) spin (upward +1 and downward -1)

J_{ij}: the interaction value between the ith spin and jth spin

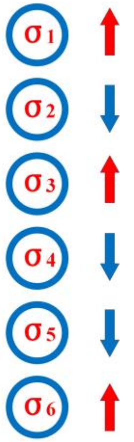
h_i: the external field of the ith spin

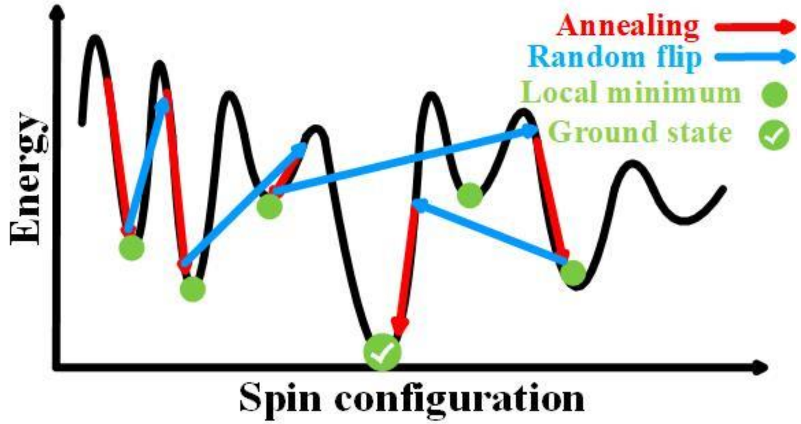
Solving CO Problems via an Ising Machine

A CO problem → Mapping to Ising model → Finding ground state of the Ising energy → Getting the optimal solution

Annealing process

Initial configuration





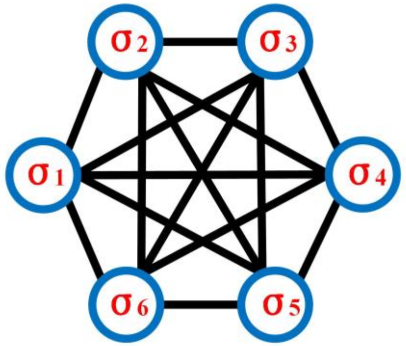
Optimal solution

The limitation of the conventional annealing algorithms:

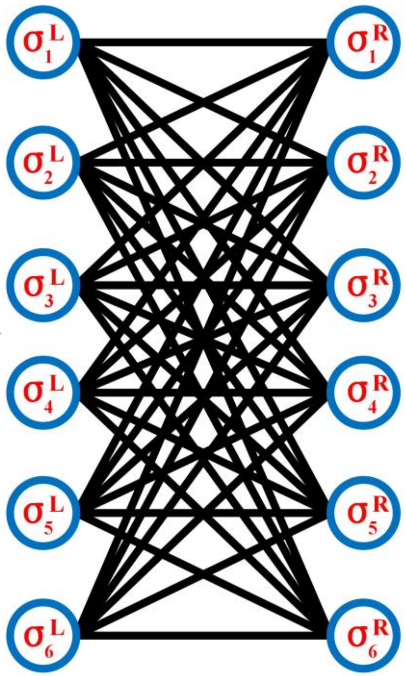
Only one spin can be updated per iteration for the fully connected Ising model.
It decreases the annealing speed.

How to achieve parallel spin update?

Parallel Annealing



The conventional structure



The two-layer spin structure [1]

- Each spin is connected with all the other spins in the other layer but has no interaction with the spins in the same layer.
- The interactions between σ_i^R and σ_i^L are called self-interactions (ω_i).
- All spins in the right and left layers are updated simultaneously per iteration.
- The Hamiltonian (H_P) [1]:

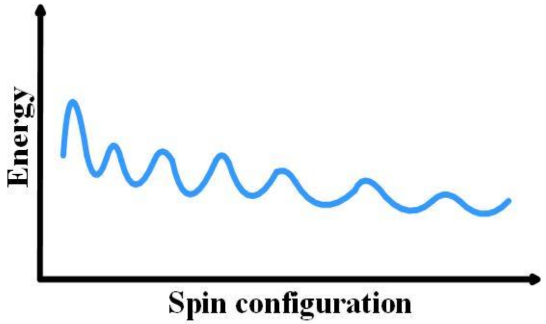
$$H_P = -\sum_{i,j} J_{ij}\sigma_i^L\sigma_j^R - \frac{1}{2}\sum_i h_i\left(\sigma_i^L + \sigma_i^R\right) + \omega_i \sum_i \left(1 - \sigma_i^L\sigma_i^R\right)$$

Is it efficient for solving constrained combinatorial optimization problems?

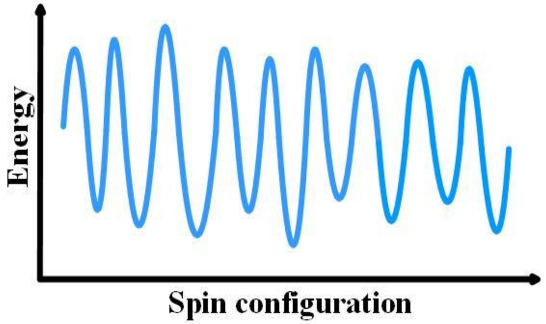
[1] T. Okuyama, T. Sonobe, K. Kawarabayashi, M. Yamaoka, “Binary optimization by momentum annealing," Physical Review E, vol. 100, no. 1, p. 012111, 2019.

Max-cut Problems & Traveling Salesman Problems

Max-cut Problems



Traveling Salesman Problems (TSPs)



Energy landscape of solving CO problems via Ising machine

$$H_{Max-cut} = \frac{1}{2}\sum_{i}\sum_{j} W_{ij} - \frac{1}{2}\sum_{i}\sum_{j} W_{ij}\sigma_i\sigma_j$$

$$H_{TSP} = \frac{A}{4}\sum_{k\neq l}\sum_{i} W_{kl}\sigma_{ik}\sigma_{(i+1)l} + \frac{A}{2}\sum_{k\neq l}\sum_{i} W_{kl}\sigma_{ik} + \frac{B}{4}\sum_{i}\sum_{k}\sum_{l}\sigma_{ik}\sigma_{il} + \frac{(n-2)B}{2}\sum_{i}\sum_{k}\sigma_{ik} + \frac{C}{4}\sum_{i}\sum_{k}\sum_{j}\sigma_{ik}\sigma_{jk} + \frac{(n-2)C}{2}\sum_{i}\sum_{k}\sigma_{ik} + \frac{A}{4}\sum_{k\neq l}\sum_{i} W_{kl} + (\frac{n^3}{4} - n^2 + n)(B + C)$$

Constant

Interaction

External field

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Temperature Function

- For conventional parallel annealing algorithm [1]:

$$T_s = \frac{1}{\beta_0 ln(1+s)}$$

T_s approaches a value that makes P_i to be low but not zero.

Spins can flip at the end of annealing.

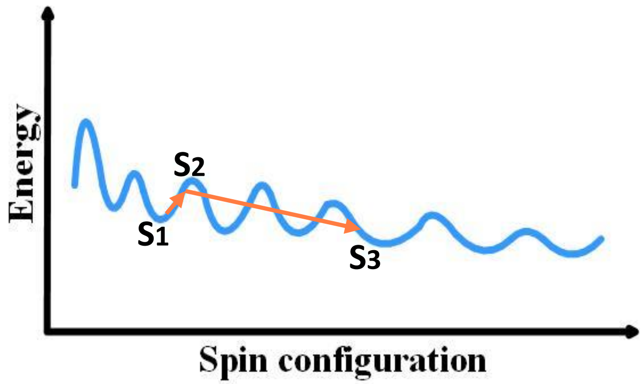
- For the improved parallel annealing algorithm:

$$T_s = T_{init} r^{s-1}$$

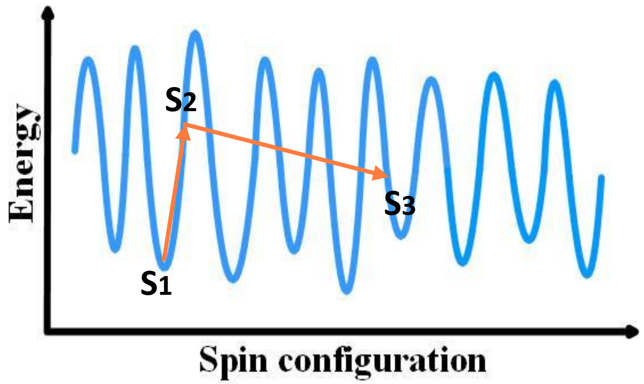
T_s approaches a value that makes P_i to be zero.

Spins can not flip at the end of annealing.

Max-cut Problems



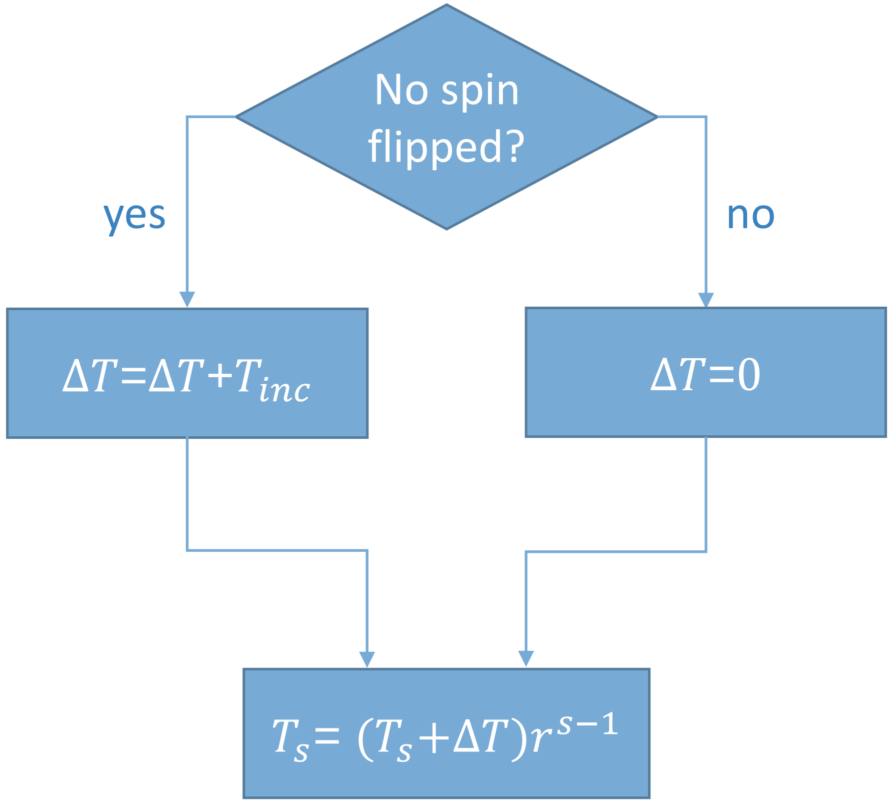
TSPs



[1] T. Okuyama, T. Sonobe, K. Kawarabayashi, M. Yamaoka, “Binary optimization by momentum annealing," Physical Review E, vol. 100, no. 1, p. 012111, 2019.

Dynamic Offset

- The Ising model is stuck in a local minimum and hard to escape when T_s is low.
- A dynamic offset ΔT is introduced.
  - T_s increases by ΔT when there is no flip among the spins.
  - ΔT resets to zero when the configuration of spins changes.
- The Ising model can escape from a local minimum quickly and have a higher probability to find a better solution.

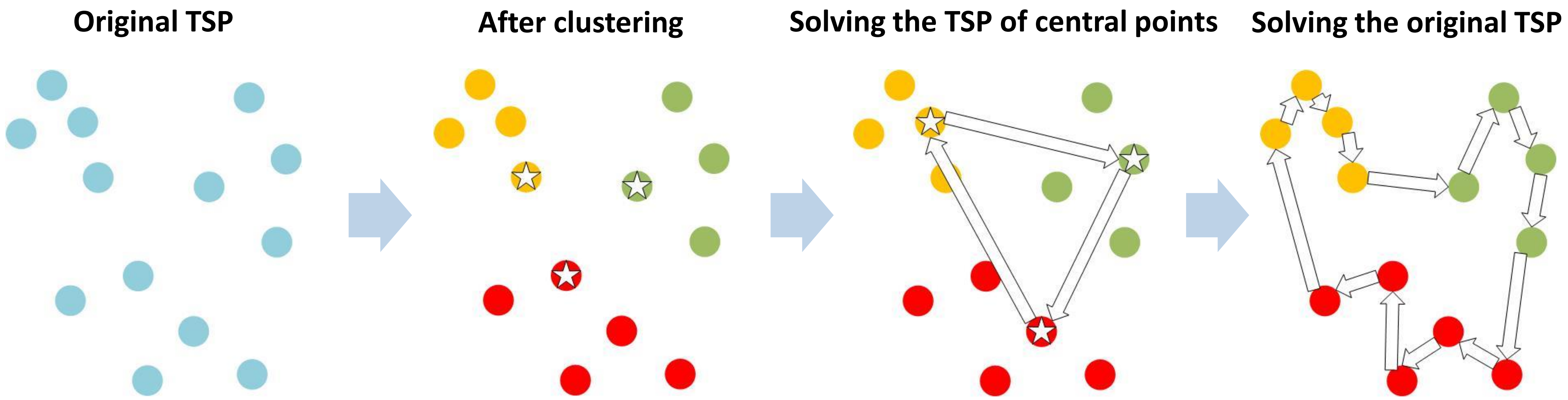


Improved Parallel Annealing for TSPs

1. Initialize spin configuration, temperature, and dynamic offset
2. **For** *s = 1* to *iteration* do
   (update left layer if *s* is odd, right layer if *s* is even)
3. Update temperature value using an exponential temperature function
4. Calculate energy variation and spin-flip probability of each spin
5. Determine new state of each spin
6. Update dynamic offset
7. **End for**

The solution quality decreases when the number of cities in the TSP is large.

A Clustering Approach [2]



[2] A. Dan, R. Shimizu, T. Nishikawa, S. Bian, and T. Sato, “Clustering approach for solving traveling salesman problems via Ising model based solver," the 57th ACM/IEEE Design Automation Conference (DAC), pp. 1-6, 2020.

The K-medoids Clustering

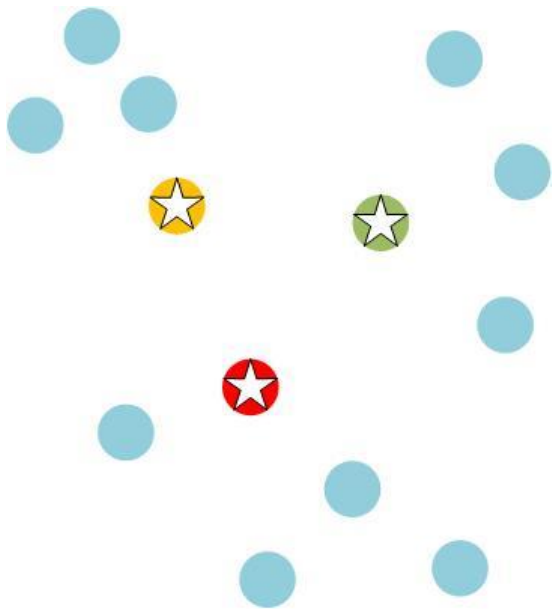
Step 1

1. **For** *i* = *1* to *M*
2. $D_i = \sum_{j=1}^{M} W_{ij}$
3. **End for**
4. Choose *k* cities with the first *k* smallest *D* as the central points

M: the number of cities for clustering

W: the distance matrix

k: the number of groups

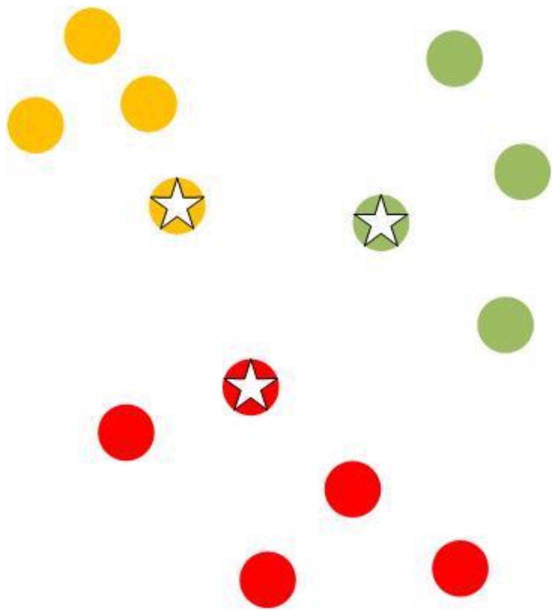


Find the 3 central points

The K-medoids Clustering

Step 2

1. **For** *i* = *1* to *M*
2. Assign the *i*th city to the closest central point
3. **End for**

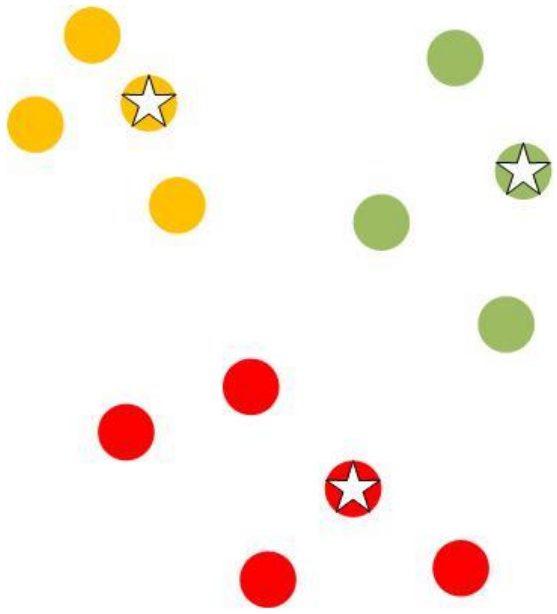


Assign all points into groups

The K-medoids Clustering

Step 3

1. **For** *each group*
2. **For** *each cities in the group*
3. $d_i = \sum_j W_{ij}$
4. **End for**
5. Choose the city with the smallest *d* as the new central points of the current group
6. **End for**



Update new central points

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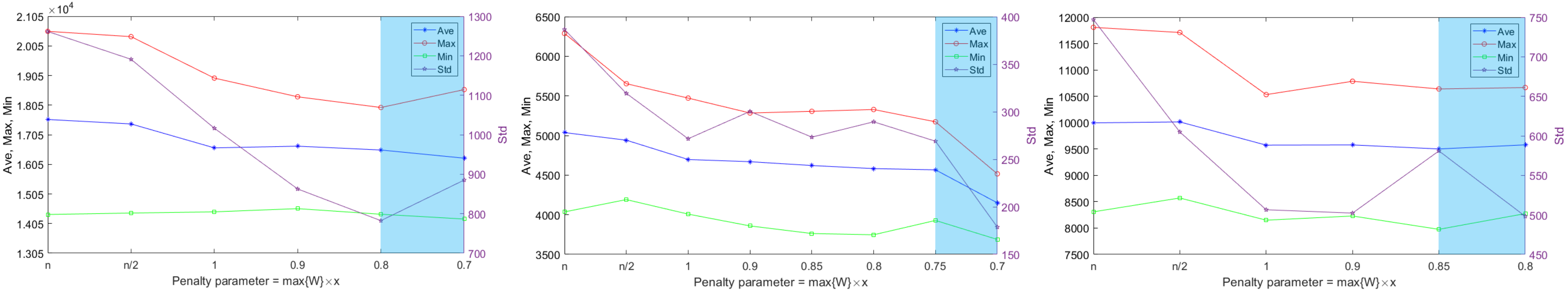
Penalty Parameter Setting



(a) (b) (c)

The effect of the penalty parameters *B* and *C* (*B=C*) on the quality of solutions: (a) for *12* cities from *gr431*, (b) for *14* cities from *burma14*, and (c) for *16* cities from *ulysses16*. The blue shadow area indicates the results do not meet constraints.

- The found distance is shorter when the penalty parameter is smaller.
- Too small penalty parameter values produce results that do not meet constraints.

$$B = C = 1 \times \max\{\boldsymbol{W}\}$$

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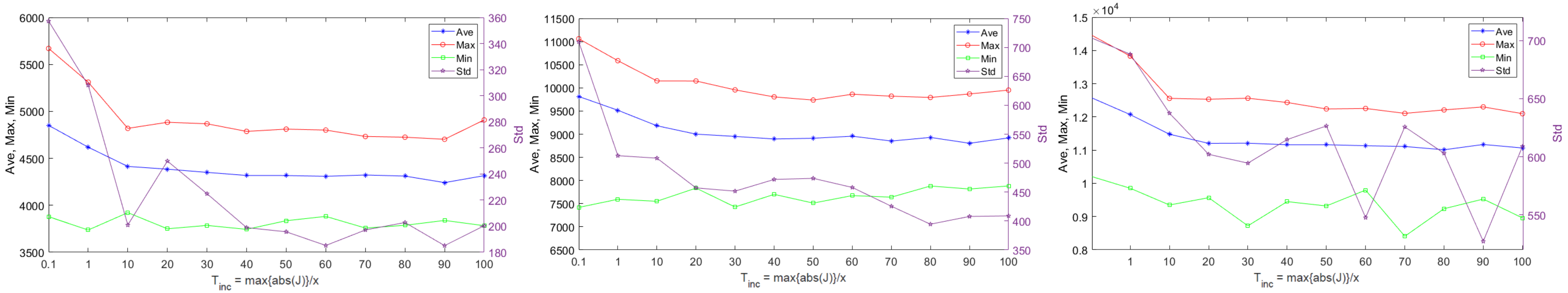
Dynamic offset setting



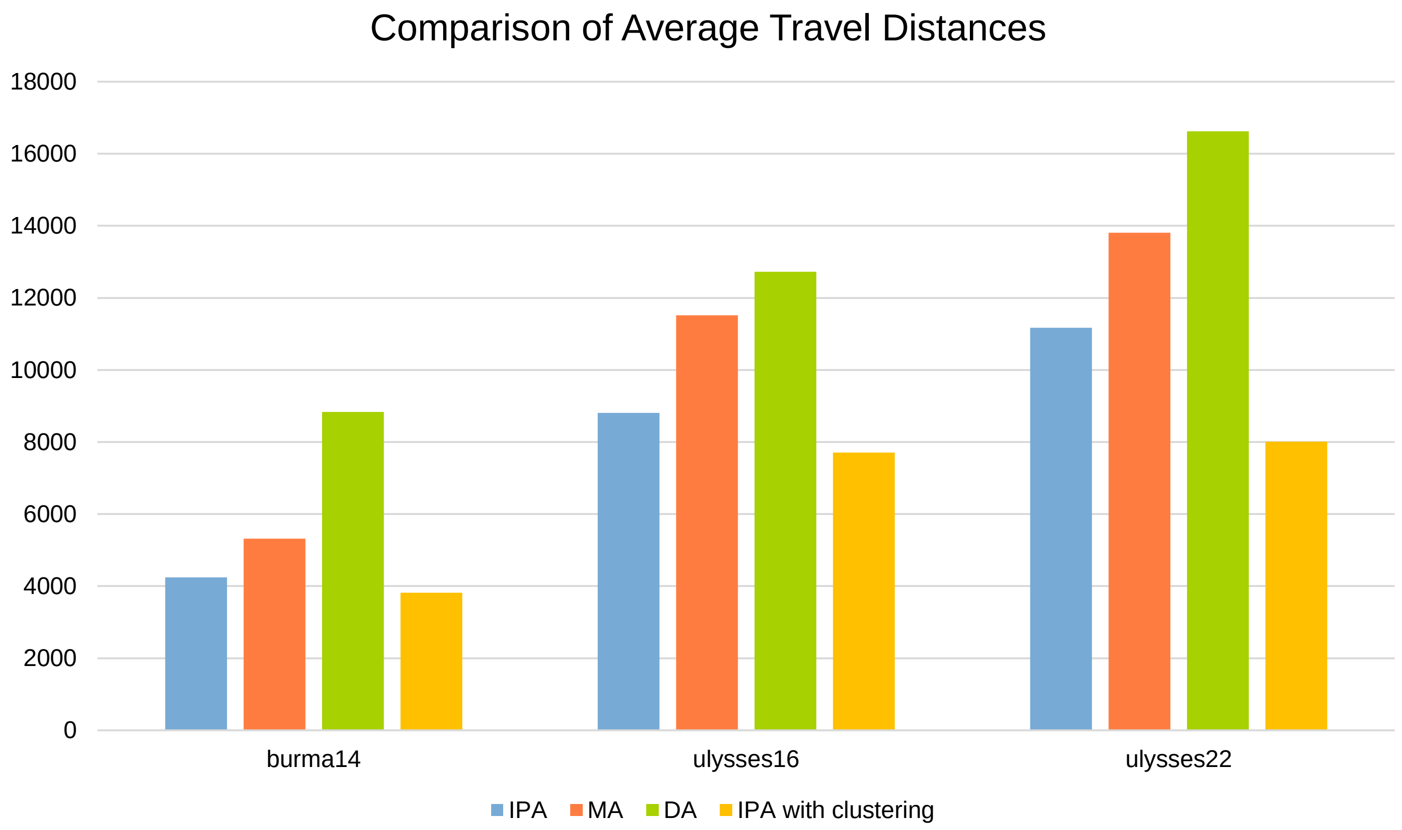
(a) (b) (c)

The effect of T_{inc} on the quality of solutions: (a) for the benchmark *burma14*, (b) for the benchmark *ulysses16*, and (c) for the benchmark *ulysses22*.

- The solution quality tends to be stable when $\frac{\max\{\text{abs}(\mathbf{J})\}}{90} \leq T_{inc} \leq \frac{\max\{\text{abs}(\mathbf{J})\}}{10}$.
- The traveling distance increases when $T_{inc} < \frac{\max\{\text{abs}(\mathbf{J})\}}{90}$ for *burma14*.

$$T_{inc} = \frac{\max\{\text{abs}(\mathbf{J})\}}{90}$$

Evaluation



- Parallel annealing algorithms, including the improved parallel annealing (IPA) and momentum annealing (MA) [1], have better performance than the conventional annealing algorithm, i. e., digital annealing (DA) [3].
- The IPA can find a shorter travel distance than MA.
- The found distance is shorter after using the clustering approach.

[1] T. Okuyama, T. Sonobe, K. Kawarabayashi, M. Yamaoka, “Binary optimization by momentum annealing," Physical Review E, vol. 100, no. 1, p. 012111, 2019.
[3] S. Tsukamoto, M. Takatsu, S. Matsubara, and H. Tamura, “An accelerator architecture for combinatorial optimization problems," Fujitsu Sci. Tech. J, vol. 53, no. 5, pp. 8-13, 2017.

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Conclusion

- Parallel spin update improves the speed of annealing as all spins can be updated in every iteration.
- The improved parallel annealing algorithm achieves higher solution quality after applying an exponential temperature function, a dynamic offset, and a clustering approach.
- The runtime of IPA is *44.4*× faster than the DA and *19.9*× faster than the MA.
- Parallel annealing algorithms show potential in the development of energy-efficient systems.

Acknowledgement

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Thank you!

Questions?

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