A Stochastic Approach for the Reliability Evaluation of Multi-State Systems with Dependent Components

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Abstract

A multi-state system (MSS) employs more than two discrete states to indicate different performance rates. Methods using a universal generating function (UGF) and Monte Carlo (MC) simulation are primary approaches for the reliability analysis of an MSS. However, these approaches incur a large computational overhead because the number of system states increases significantly with the number of components in an MSS. In this paper, stochastic multi-valued (SMV) models are proposed for evaluating the reliability of an MSS with dependent multi-state components (MSCs). The performance rates and their corresponding probabilities of the MSCs are simultaneously encoded in multi-valued non-Bernoulli sequences using permutations of fixed numbers of 1s and 0s. The sequences are then processed by logic gates. The effectiveness of the proposed approach is demonstrated via a comparative evaluation of a multi-state system consisting of dependent components with steady and time-varying state probabilities.

Acronyms and Abbreviations

MSS, multi-state system; MSC, multi-state component; UGF, universal generating function; MC, Monte Carlo; ESL, extended-stochastic logic; SMV, stochastic multi-valued.

Keywords: multi-state system, dependent multi-state component, stochastic multi-valued model

Nomenclature

L sequence length in number of bits

j component index for a MSS, $j = 1, 2, \dots, n$

k_j number of states for component j

g_j performance set for component j

p_j probability set for component j

$G_{j(t)}$ performance rate of component j at any instant $t \geq 0$

ϕ MSS structure function

ω system demand

Ω_ϕ general UGF composition operate

$U_{(Z)}$ UGF for MSS

1. Introduction

In conventional reliability theory, a system usually consists of components with binary states: success or failure. However, for some engineering systems such as power generation and transmission systems, the binary assumption does not accurately model their reliability behavior [1]. Hence, a multi-state system (MSS) model has been introduced to characterize the reliability of these engineering systems [2].

Markov [3, 4] and semi-Markov [5] models have been proposed for evaluating the reliability of an MSS. However, these stochastic models are only applicable for a relatively small MSS because the number of system states increases rapidly with the number of multi-state components (MSCs). In order to simplify the process, several analytical approaches, including those using multi-state binary decision diagrams (MBDDs) [6], multi-state multivalued decision diagrams (MMDDs) [7] and edge-valued multi-valued decision diagrams (EVMDDs) [8], have been introduced to analyze an MSS and to identify the critical components in the system. However, these approaches are usually applicable to an MSS with independent MSCs and a large computational overhead is encountered when analyzing complex systems in which each MSC has many states. A technique using a universal generating function (UGF) has been used to analyze a power system's reliability [9]. It was then extended to analyze the reliability of an MSS with dependent components [10]. However, the computational complexity of a UGF method is exponential in the number of components [11]. A method using fuzzy universal generating functions (FUGFs) has been presented for the performance assessment of multi-state systems to reduce the computational complexity of the UGF method [12]. A method based on inverse L_z-transform has been proposed to obtain the reliability characteristics and risk functions of multi-state power systems [13]. However, the dependency of system components is not considered in these methods.

Monte Carlo (MC) simulation has been used to evaluate the reliability of an MSS using multi-state minimal cut vectors. The dependencies and composition operators among MSCs of an MSS are simulated by MC in [14]. However, a large sample size and thus a long run time are needed to meet the accuracy requirement, due to the slow convergence typically encountered in an MC simulation.

A theory for multiple-valued logic function is proposed to analyze an MSS in [15, 16], however complex operation rules present major challenges. The advantages of stochastic computation such as computational simplicity, high speed, and fault-tolerance have been shown in [17]. Hence, a stochastic multi-value approach is proposed in [18] to model multi-valued networks. Nevertheless, the values in stochastic sequences only represent the states of MSCs, not describing the practical significance in the states.

In this paper, stochastic multi-valued (SMV) models are proposed for the analysis of a multi-state system with dependent components. The practical significance of a component's states (e.g., the processing speed of a component in an MSS) is indicated by values such as positive real numbers that are beyond the range [0, 1] in the unipolar representation of stochastic computation. Thus, an extended-stochastic logic (ESL) method [19] is utilized to expand the range of stochastic numbers to $(0,\ 2^N - 1)$, where N is the bit width of the binary values to be encoded. In the SMV models, the practical significance of a component's states and their corresponding probabilities are simultaneously encoded in the stochastic sequences to improve the evaluation efficiency. The output sequences of an SMV model encode the reliability values for an MSS with dependent MSCs. The computational complexity of a universal generating function (UGF) method increases exponentially with the number of the components' states, while the SMV models are not significantly affected by the number of components' states. In contrast to the UGF method, hence, this approach avoids the large computational complexity in evaluating the reliability of multi-state systems.

The remainder of the paper is organized as follows. Section II reviews the definitions of MSS and UGF. Section III presents SMV models to analyze an MSS with dependent MSCs. Case studies and simulation results are provided in Section IV. Finally, Section V presents conclusions and directions for future work.

2. The multi-state systems and universal generating functions

In this section, multi-state systems and universal generating functions are introduced.

2.1. Multi-state systems(MSS)

Many systems and components degrade and operate at reduced performance levels [20]. The performance of a component may vary from perfect functioning to a complete failure. Such a component is called a multi-state component (MSC), and a system consisting of MSCs is called a multi-state system (MSS). An MSS can perform tasks with various distinguished levels of performance (or states); the practical significance of a state is defined as a performance rate.

The performance rates of an MSS are determined by the performance rates of its MSCs. If n is the number of MSCs in the MSS, the performance rate $G_{j(t)}$ of component j ($1 \le j \le n$) at any instant t ($t \ge 0$) is a random variable that takes its value from a set g_j: $G_{j(t)} \in g_j$. The probabilities associated with different states of component j can be denoted by a set [21]:

$p_j = \{p_{j1}, \ldots, p_{ji}, \ldots, p_{jk_j}\}$, (1)

where k_j is the number of states of component j, p_{ji} = Pr{ $G_{j(t)}$ = g_{ji}} and $\sum_{i=1}^{k_j} p_{ji}(t) = 1$. The collection of pairs g_{ji} and p_{ji}, $i \in \{1, 2, \ldots, k_j\}$, completely determines the probability distribution (PD) of the performance of component *j*.

2.2. Universal generating functions (*UGF*)

The technique using Universal Generating Functions (UGF) is introduced in [22] for evaluating the availability of a series-parallel MSS [20, 23].

The PD of component *j* can be defined by a polynomial $u(z)$:

$$u(z) = \sum_{i=1}^{k_j} p_{ji} \,.\, z^{g_{ji}} \tag{2}$$

where g_{ji} is the performance rate of component *j* in the state i, $i \in \{1, 2, \ldots, k_j\}$, and p_{ji}= Pr{$G_{j(t)}$= g_{ji}}.

In order to obtain the PD of an MSS with an arbitrary structure function ϕ, a general composition operator Ω_ϕ is given by [10]:

$$U(z) = \Omega_\phi\{u_1(z), \ldots, u_n(z)\}$$

$$= \Omega_\phi\{\sum_{i=1}^{k_1} p_{1i}.\, z^{g_{1i}}, \ldots, \sum_{h=1}^{k_n} p_{nh}.\, z^{g_{nh}}\} \tag{3}$$

where $U(z)$ is a function to obtain the probability and performance rate for the state of an MSS. Ω_ϕ is depending on the parallel or series connections of components.

In an MSS for flow transmission (e.g., a power supply system or oil transmission system) or a task processing MSS (e.g. a control system, an information or data processing system), of which the performance is indicated by productivity, capacity or processing speed, the total performance rate of components connected in parallel is equal to the sum of the performance rates of the individual components.

When the components in a flow transmission system are connected in series, the performance rate of the system is equal to the minimum of the performance rates of the components. Assuming the components in a task processing system are connected in series with processing speeds of the two components being G_a and G_b respectively, the operation time of the system is equal to the sum of operation times of the two components, thus the processing speed of the system is equal to $\frac{1}{\frac{1}{G_a}+\frac{1}{G_b}}$.

An example is used to explain the UGF method and the structure of an MSS is shown in Fig. 1. Three MSCs constitute a series-parallel system. The transmission capacity or processing speed defines the component performance rate. The parameters of MSCs are shown in Table 1.

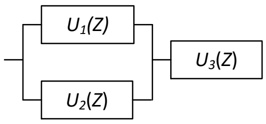


Fig. 1. The structure of an MSS.

Table 1

Parameters of MSCs in a series-parallel system [24], g_{ji} and p_{ji} are the performance rate and probability of component *j* in state i respectively.

i / g_{ji}/p_{ji}	1	2	3
g_{1i}/p_{1i}	1.5/0.8	1/0.1	0/0.1
g_{2i}/p_{2i}	2/0.7	1.5/0.2	0/0.1
g_{3i}/p_{3i}	4/0.96	0/0.04	

The PD of three MSCs can be defined as:

$u_1(z) = \sum_{i=1}^{3} p_{1i} . z^{g_{1i}}$=0.8·$z^{1.5}$+0.1·$z^1$+0.1·$z^0$

$u_2(z) = \sum_{i=1}^{3} p_{2i} . z^{g_{2i}}$=0.7·$z^2$+0.2·$z^{1.5}$+0.1·$z^0$

$u_3(z) = \sum_{i=1}^{3} p_{3i} . z^{g_{3i}}$=0.96·$z^4$+0.04·$z^0$

Assuming the MSS is a flow transmission system, the PD of entire MSS is obtained by:

$U(z) = \Omega_{\phi s}(\Omega_{\phi p}(u_1(z), u_2(z)), u_3(z))$

=0.5376· $z^{3.5}$ +0.2208· z^3 +0.0192· $z^{2.5}$ +0.0672· z^2 +0.096· $z^{1.5}$ $+ 0.0096 \cdot z^1 + 0.0496 \cdot z^0$

The PD of a task processing MSS is obtained by:

$U(z) = \Omega_{\phi s}(\Omega_{\phi p}(u_1(z), u_2(z)), u_3(z))$

=0.5376·$z^{1.87}$+0.2208·$z^{1.71}$+0.0192·$z^{1.54}$+0.0672·$z^{1.33}$+0.096·$z^{1.09}$ $+ 0.0096 \cdot z^{0.8} + 0.0496 \cdot z^0$

The availability of an MSS is provided in section III.

3. The stochastic computational model

3.1. Extended stochastic logic

In stochastic computation, signal probabilities are encoded into random binary bit streams [25]. In the binary streams, a proportional number of bits are set to “1” to indicate a probability [26]. A stochastic multiple-valued sequence is also used to indicate the probability of a multiple-valued signal. For example, the performance set $g_j = \{0, 1, 2.5\}$ and probability set $p_j = \{0.3, 0.6, 0.1\}$ of component j can be encoded into a multiple-valued stochastic sequence shown as in Fig. 2.

$$\text{" 0,1,1,1,0,1,1,1,0,2.5"} \left\{ \begin{array}{c} P_0=0.3 \\ P_1=0.6 \\ P_{2.5}=0.1 \end{array} \right.$$

Fig. 2. The stochastic encoding of an MSC using a 10-bit sequence.

Stochastic computation in the unipolar representation is limited to values in [0, 1]. However, the performance rates of an MSC or MSS may exceed this boundary. Hence, extended-stochastic logic (ESL) is used to expand the range of stochastic computation to $(0, 2^N - 1)$ for a binary value of N bits [19]. In ESL, a positive real number is encoded as the ratio of two stochastic numbers using the unipolar representation. Assuming the unipolar representation of the dividend sequence is $p_{\_U}$ and the divisor sequence is $p_{\_D}$, a positive real number x is given by: $x = \frac{p_{\_U}}{p_{\_D}}$. The ESL multiplier, divisor, adder and MIN gate are shown in Fig. 3 [19].

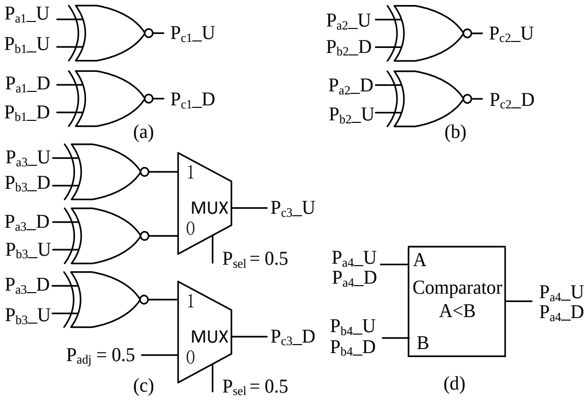


Fig. 3. (a) An ESL multiplier, (b) An ESL divisor, (c) An ESL adder, and (d) An ESL MIN gate.

3.2. *SMV models with independent components*

In the SMV models, the elements in the performance rate set g_j and corresponding probabilities of component j are simultaneously encoded in L-bit multi-value non-Bernoulli sequences. The UGF composition operators $\underset{par}{\otimes}$ / $\underset{ser}{\otimes}$ indicate the connection structure of independent components.

3.2.1. *The SMV model for parallel systems with independent components*

In a flow transmission MSS or task processing MSS containing independent parallel MSCs, the total performance of the system is the addition of the performance of each MSC. The ESL adder (Fig. 3(c)) is used to calculate the total performance of a parallel system.

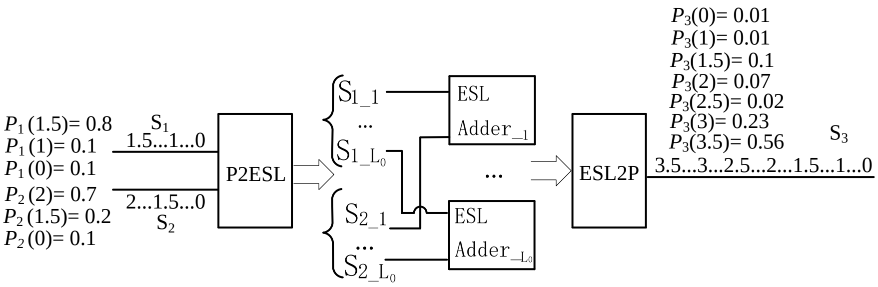


Fig. 4. The SMV model of the parallel system with independent MSCs (SMV_P).

The performance rates and probabilities of a parallel system containing two MSCs can be obtained by an SMV model of the parallel system with independent

MSCs (SMV_P) as shown in Fig. 4. The performance rates (e.g. (1.5, 1, 0) , (2, 1.5, 0)) and the corresponding probabilities (e.g. (0.8, 0.1, 0.1) , (0.7, 0.2, 0.1)) of u_1 and u_2 in Table 1 are encoded in L-bit multi-valued non-Bernoulli sequences S_1 and S_2. Since the performance rate exceeds 1, the sequences S_1 and S_2 are converted into ESL sequences. A block (M2ESL) is implemented to transform the multi-values in sequences S_1 and S_2 into L_0-bit (*e.g.* L_0=100) ESL sequences. The multiple values in the sequences S_1 and S_2 are transformed into ESL sequences S_{1_i} and S_{2_i}, the length of sequences S_{1_i} or S_{2_i} is L, ($i \in [1, L_0]$). A converter (ESL2M) in Fig. 4 is defined by transforming the output ESL sequences of ESL Adders into the multi-valued sequence S_3, including the performance rates and probabilities of the parallel subsystem.

3.2.2. SMV models for series systems with independent components

The performance of a flow transmission MSS with two independent MSCs in series connection is determined by the MSC with the least performance [24]. The SMV model of the series flow transmission system (SMV_SF) is shown in Fig. 5.

According to (6), an SMV model of series task processing system (SMV_ST) is proposed to obtain the performance of the system with two independent MSCs. The block diagram is shown in Fig. 6.

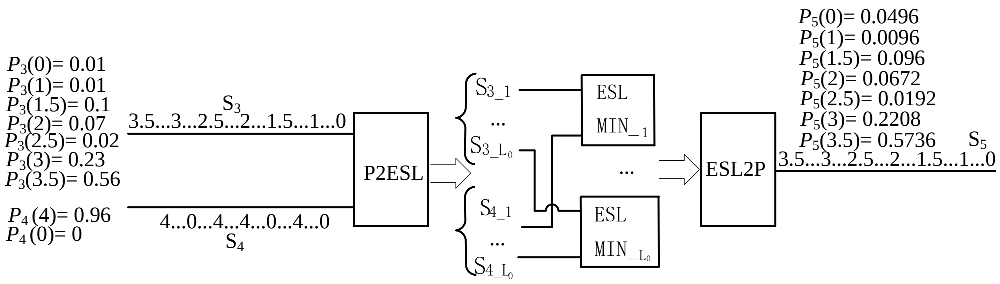


Fig. 5. The SMV model of a series flow transmission system with independent MSCs (SMV_SF).

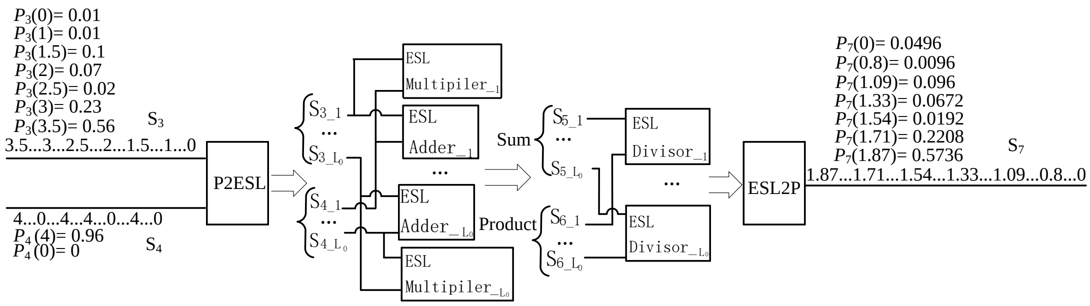


Fig. 6. The SMV model of a series task processing system with independent MSCs (SMV_ST).

The non-Bernoulli sequences S_3 and S_4 with the values (3.5, 3, 2.5, 2, 1.5, 1, 0) and (4, 0) carry the performance rates of the parallel subsystem and u_3 in Fig. 1 respectively. The probability sets P_3 and P_4 contain the probabilities with different performance rates of the parallel subsystems and u_3. The performance rates and probabilities of a series MSS can be calculated by the SMV_SF or SMV_ST according to the type of systems.

3.2.3. SMV models with dependent components

Assuming an MSS consisting of components *j* and *h*, the performance distribution of *h* depends on the state of *j*. The states of the components are distinguished by the corresponding performance rates [27], thus the performance distribution of *h* is determined by the performance rates of *j*. The structures of components *j* and *h* are shown in Fig. 7.

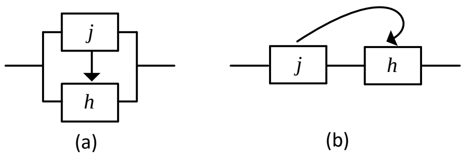


Fig. 7. (a) An MSS with two dependent parallel components, (b) An MSS with two dependent series components.

The performance set g_j and probability set p_j of component *j* are separated into *M* mutually disjoint subsets $g_j{}^l$ and $p_j{}^l$, $l \in [1, M]$. According to the dependency relationship between components *j* and *h*, the subsets of performance rates and conditional probabilities of component *h* are $\overline{g_h}^l$ and $\overline{p_h}^l$ under the condition of $g_j{}^l$

and $p_j{}^l$. The function of dependent systems is described by:

$$u_j(z) \underset{\phi}{\overset{\rightarrow}{\otimes}} \overline{u_h(z)} = \sum_{l=1}^{M} p_j{}^l \, z^{g_j{}^l} \underset{\phi}{\overset{\rightarrow}{\otimes}} \sum_{l=1}^{M} \overline{p_h}^l \, z^{\overline{g_h}^l}$$

$$= \sum_{l=1}^{M} p_j{}^l \cdot \overline{p_h}^l \cdot z^{\phi(g_j{}^l, \overline{g_h}^l)}, \qquad (7)$$

where ϕ is a composition operator determined by the MSS structure function (e.g. $\underset{par}{\otimes}$ or $\underset{ser}{\otimes}$).

In the SMV models, the performance rate subsets ($g_j{}^l$ /$\overline{g_h}^l$) and corresponding probability ($p_j{}^l$ /$\overline{p_h}^l$) of j and h are encoded in L- bit non-Bernoulli sequences S_{jl} and S_{hl}, $l \in [1, M]$. $L \times \sum_{i=1}^{k_j{}^l} p_{ji}{}^l$ bits are valid in the sequence S_{jl}, $L \times (1- \sum_{i=1}^{k_j{}^l} p_{ji}{}^l)$ bits are filled with the invalid number V, where $k_j{}^l$ is the number of elements in the subset $p_j{}^l$.

An SMV model for the parallel system with dependent MSCs j and h is presented in Fig. 8.

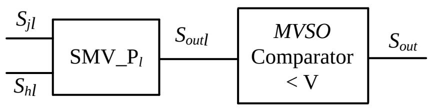


Fig. 8 An SMV model for parallel system with dependent MSCs.

The sequence S_{outl} indicates the SMV_P output of S_{jl} and S_{hl}. The L-bit sequence S_{out} represents the performance rates and probabilities of the parallel subsystem. The multiple-valued smaller operator (*MVSO*) is defined as:

$$MVSO(\, A_l < V \,) = MIN(A_l, V), \qquad (8)$$

where V is an invalid number in L-bit non-Bernoulli sequence S_{jl} and V is used to compensate the sequence S_{jl} to L bits. A number larger than the maximum performance rate of the parallel subsystem is selected as the value of V. The invalid numbers (i.e., the obtained results of invalid V) are filtered out by the *MVSO*.

An SMV model for a series subsystem with the dependent MSC j and h is shown in Fig. 9.

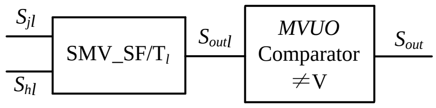


Fig. 9 An SMV model for series system with dependent MSCs.

The sequence S_{outl} is the output of the SMV_SF/T. L-bit sequence S_{out} indicates the performance rates and probabilities of the series system. The multiple-valued unequal operator (*MVUO*) is described as:

$$MVUO(\ A_m\ \neq V\ ) = \begin{cases} A_m & if A_m \neq V \\ Null & if A_m = V \end{cases}\ ,\ \mathrm{m} \in (1, L) \tag{9}$$

where an invalid number *V* in S_{jl} is used to ensure the length of the sequence to be *L* bits, and -1 can be selected as the value of *V* due to the MIN operation used in the model ESMS_F (i.e., the output of the MIN operation is -1 when the elements in the sequence and -1 are the inputs). A_m is an element in the sequence S_{outl}.The *MVUO* operator eliminates the invalid number *V* in S_{outl}.

3.2.4. SMV models with groups of dependent components

Assuming a pair of components *e* and *h* depend on the same component *j* and all the components are mutually independent of a certain state of *j*. This indicates that components of *e* and *h* are conditionally independent given the state of *j*. The structures of components *e*, *h* and *j* are presented in Fig. 10.

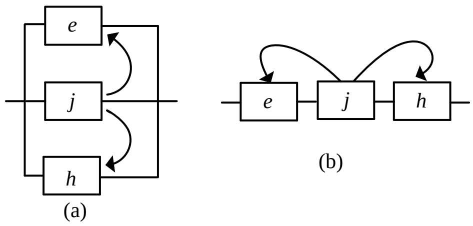


Fig. 10. (a) An MSS with group dependent parallel components, (b) An MSS with group dependent series components.

The performance set g_j and probability set $p_j$of component *j* are separated into *M* mutually disjoint subsets $g_j{}^l$ and $p_j{}^l$ as the dependency condition of components e and h, $l \in [1, M]$. The performance distributions of components e and h are described by the pairs of vectors $\overline{g_e}^l$, $\overline{p_e}^l$and $\overline{g_h}^l$, $\overline{p_h}^l$. The function of a group dependent system can be obtained as:

$$u_j(z) \underset{\phi}{\overset{\rightarrow}{\otimes}} \overline{u_e(z)} \underset{\phi}{\overset{\rightarrow}{\otimes}} \overline{u_h(z)}$$

$$=\sum_{l=1}^{M} p_j{}^l\, z^{g_j{}^l} \underset{\phi}{\overset{\rightarrow}{\otimes}} \left(\sum_{l=1}^{M} \overline{p_e}^l\, z^{\overline{g_e}^l} \underset{\phi}{\overset{\rightarrow}{\otimes}} \sum_{l=1}^{M} \overline{p_h}^l\, z^{\overline{g_h}^l}\right)$$

$$=\sum_{l=1}^{M} p_j{}^l \cdot \left(\overline{p_h}^l \cdot \overline{p_h}^l\right) \cdot z^{\phi\left(g_j{}^l,\ \phi\left(\overline{g_e}^l,\ \overline{g_h}^l\right)\right)}, \tag{10}$$

where the function ϕ indicates a composition operator $\underset{par}{\otimes}$ or $\underset{ser}{\otimes}$ in accordance with the type of components' connection.

In the SMV models, the performance rate subsets ($g_j{}^l$ /$\overline{g_e}^l/\overline{g_h}^l$) and corresponding probability ($p_j{}^l$ /$\overline{p_e}^l/\overline{p_h}^l$) of j, e and h are encoded in L-bit non-Bernoulli sequences S_{jl}, $S_{el}$and S_{hl}, $l \in [1, M]$. $L\times\sum_{i=1}^{k_j{}^l} p_{ji}{}^l$ bits are valid in S_{jl}, $L\times(1- \sum_{i=1}^{k_j{}^l} p_{ji}{}^l)$ bits are filled with the invalid number V, and $k_j{}^l$ is the number of elements in subset $p_j{}^l$.

An SMV model for a parallel/series system with a group of dependent MSCs j, e and h is presented in Fig. 11. S_{outl} is an output sequence of the stochastic logic unit (SLU). The SLU is an SMV_P model (Fig. 4) and the invalid value filter (IVF) is an *MVSO* operator, when the system is a parallel system. If the system is a series system, the SLU is an SMV_SF (Fig. 5) or SMV_ST (Fig. 6) depending on if the type of the system is a flow transmission system or a task processing system, the IVF is a *MVUO* operator. The performance rates and probabilities of the parallel/series system are obtained by the L-bit sequence S_{out}.

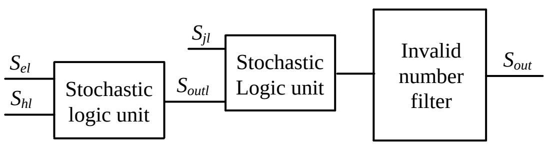


Fig. 11. An SMV model for a parallel/series system with a group of dependent MSCs.

3.2.5. Availability assessment of an MSS

The state of an MSS depends on the relation between the MSS performance and demand [24]. The relation between system performance and demand ω is determined by the system adequacy index (r_i), which takes the form:

$$r_i = g_i - \omega, \quad (11)$$

where *i* denotes an acceptable state if and only if $r_i \geq 0$.

The availability of an MSS is the probability of the system staying in the subset of acceptable states. It is defined as the probability that the MSS performance rate is greater than the demand ω [24].

$$A(\omega) = \delta_A(U(z), \omega) = \delta_A(\sum_{i=1}^{K} p_i z^{g_i}, \omega) = \sum_{i=1}^{K} p_i . \alpha_i \quad (12)$$

where $\alpha_i = \begin{cases} 1, r_i \geq 0 \\ 0, r_i < 0 \end{cases}$, and *K* is the number of system states.

For the example in Fig. 1, assuming the demand is 1.5, the MSS availability is calculated as:

$$A(1.5) = \delta_A(U(z), 1.5) = \sum_{i=1}^{7} p_i . \alpha_i = 0.9408$$

Assume that the reliability requirement for an MSS is set to 0.9, then $\mathrm{A}(1.5) = 0.9408 > 0.9$ meets the system availability.

According to (12), the SMV model to obtain the availability of an MSS is presented in Fig. 12. L-bit multi-value non-Bernoulli sequence S_{out} indicates the performance rates and probabilities of the system. S_{result} is an L-bit binary non-Bernoulli sequence representing the availability of the MSS by the probability of 1's. The multiple-valued equal or larger (MVEL) operator is defined by:

$$MVEL(A_m \geq \omega) = \begin{cases} 1 & if A_m \geq \omega \\ 0 & if A_m < \omega \end{cases}, \text{and } m \in (1, L), \quad (13)$$

where A_m is an element in the sequence S_{out}, ω is the demand of the system, and L is the length of the sequences.

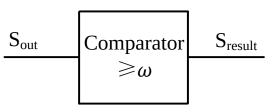


Fig. 12. An SMV model to obtain the availability of an MSS

3.2.6. Random fluctuations

The simulation output values are probabilistic rather than deterministic due to the random fluctuation as an inherent feature of stochastic computation [28]. It follows approximately a Gaussian distribution. This fluctuation can be analyzed quantitatively by investigating the mean and variance of the output distribution. The accuracy can be improved by increasing the length of stochastic sequences. The detailed discussion is presented in the next section.

4. Case studies

In this section, several case studies are presented to validate the accuracy and efficiency of the SMV models. A non-repairable series-parallel MSS (Fig. 13) consisting of dependent MSCs with steady and time-vary probabilities is considered first. The structure presented is interpreted as a flow transmission MSS or a task processing MSS. The proposed SMV approach is compared to the UGF [10, 27] and MC methods [14]. Simulations are run on a computer with a 3.4-GHz Intel microprocessor with 16GB memory.

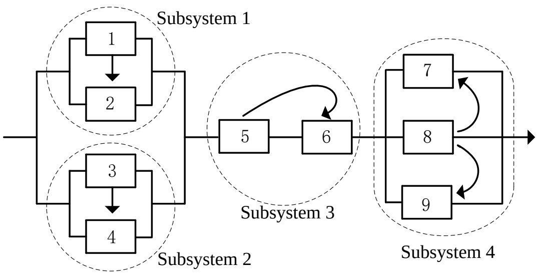


Fig. 13. The structure of a non-repairable series-parallel MSS [10].

In subsystems 1 and 2 , MSC_2 and MSC_4 depend on the states of MSC_1 and MSC_3 respectively; the state of MSC_6 is determined by the state of MSC_5 in subsystem 3; MSC_7 and MSC_9 both depend on the state of MSC_8 in subsystem 4. The states of MSCs are distinguished by the corresponding performance rates. The performance distribution of an affected component is determined by the performance rate of the influencing component.

4.1. *Example 1*

In this example, a system of components with steady state probabilities is presented to validate the SMV models. The performance rates and corresponding probabilities of the influencing and affected components are presented in Tables 2 and 3 respectively.

Table 2
Unconditional performance distributions of components [10], $g_{ji}$and p_{ji} indicate the performance rate and probability of component *j* in state *i*.

g_{ji}/p_{ji} \ i	1	2	3	4
g_{1i}/p_{1i}	2/0.5	1.5/0.2	1/0.2	0/0.1
g_{3i}/p_{3i}	2/0.6	1.5/0.2	1/0.1	0/0.1
g_{5i}/p_{5i}	5/0.6	4/0.2	2/0.15	0/0.05
g_{8i}/p_{8i}	1.5/0.7	1/0.2	0/0.1	

Table 3
Conditional performance distributions of components [10], $g_{ji}$and p_{ji} indicate the performance rate and probability of component *j* in state *i*.

g_{ji}/p_{ji}	condition \ i	1	2	3	4
g_{2i}/p_{2i}	$0 \leq g_{1i} \leq 1$	2.5/0.8	2/0.1	1/0.05	0/0.05
	$1 < g_{1i} \leq 1.5$	2/0.7	1.5/0.1	1/0.15	0/0.05
	$g_{1i} > 1.5$	1.5/0.5	1/0.4	0/0.1	
g_{4i}/p_{4i}	$0 \leq g_{3i} \leq 1$	2/0.85	1.5/0.1	0/0.05	
	$1 < g_{3i} \leq 1.5$	1.5/0.65	1/0.3	0/0.05	
	$g_{3i} > 1.5$	1/0.9	0/0.1		
g_{6i}/p_{6i}	$0 \leq g_{5i} \leq 4$	4/0.8	2/0.15	0/0.05	
	$g_{5i} > 4$	5/0.25	2.5/0.7	0/0.05	
g_{7i}/p_{7i}	$0 \leq g_{8i} \leq 1$	2/0.8	1/0.1	0/0.1	
	$g_{8i} > 1$	1.5/0.75	1/0.2	0/0.05	
g_{9i}/p_{9i}	$g_{8i} = 0$	2/0.8	1.5/0.1	1/0.05	0/0.05
	$g_{8i} > 0$	2/0.2	1.5/0.6	1/0.1	0/0.1

In Tables 2 and 3, g_{ji} is the performance rate (or level) of the component *j* in the state i, p_{ji} denotes the probability of j in the state i, and the condition refers to the dependence condition of the components.

4.1.1. The SMV model for the system

An SMV model of the non-repairable series-parallel MSS is presented in Fig. 14.

The SMV_SF or SMV_ST is used to evaluate the reliability of the system in accordance with the type of the system.

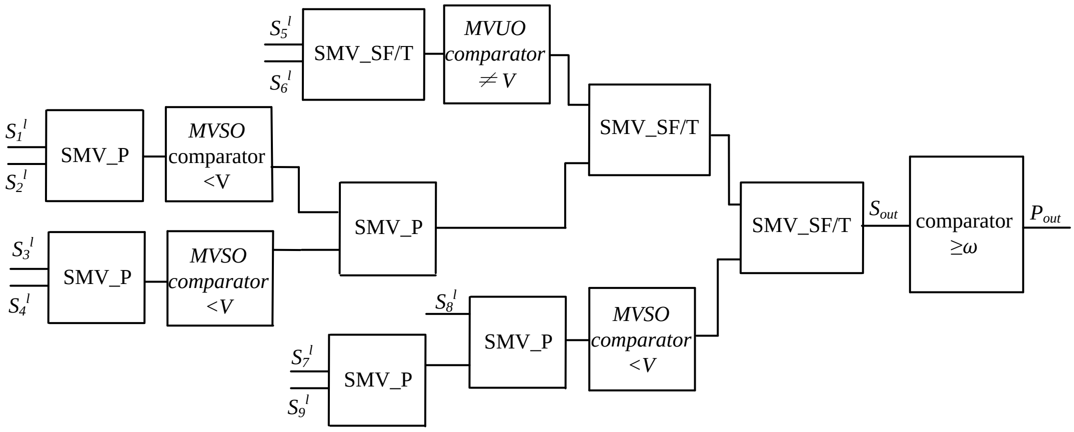


Fig. 14. The SMV model of the flow transmission system.

In Fig. 14, S_i^l ($i \in [1, 9]$, $l \in \{1, 2, 3\}$) is a non-Bernoulli sequence encoding the performance rates and corresponding probabilities of component i. Due to the parallel connection and the dependence relation between MSC_1 and MSC_2 in subsystem 1, the SMV model for a parallel system with dependent MSCs (Fig. 8) is used to evaluate the reliability of the subsystem. The models for subsystems 1 and 2 are the same owing to the same structure. The SMV_P (Fig. 4) is used to deal with the parallel system consisting of only subsystems 1 and 2. The SMV model for a series system with dependent MSCs (Fig. 9) is used to analyze the serial subsystem 3. The SMV model for a parallel system with a group of dependent MSCs (Fig. 11) is used to assess the reliability of the subsystem 4. S_{out} is the output sequence encoding the performance rates and probabilities of the flow transmission system. ω is the demand performance of the system. P_{out} is the availability of the system when the system meets the demand performance ω.

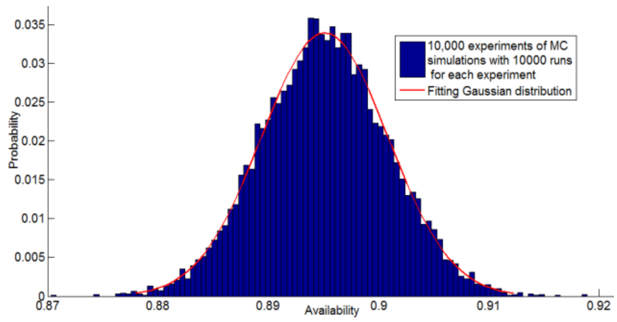
4.1.2. Simulation results

The PD for each performance rate of a flow transmission system using the UGF [10, 27], MC [14] and the proposed SMV approaches are shown in Table 4, in which the probabilities of different states of the system are obtained. 1000-bit non-Bernoulli sequences are used in the SMV experiments. The results of MC simulations were obtained from 1000 runs. Error is defined as the ratio of the difference between the simulation result by SMV or MC and the accurate result by UGF over the accurate one. According to the simulation result, the SMV approach produces more accurate results than the MC method.

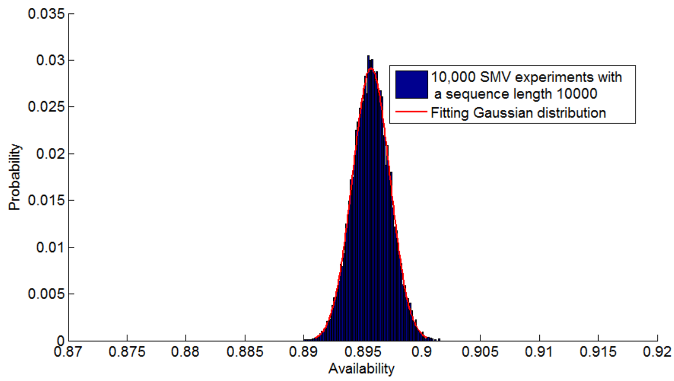
Table 4
The PD of a flow transmission system with different performance rate using UGF, MC, and SMV methods.

Performance rate	UGF [10, 27]	SMV approach		MC [14]	
	probability	probability	Error	probability	Error
5	0.098	0.0978	0.2%	0.0971	0.71%
4.5	0.0018	0.0018	0%	0.0019	5.56%
4	0.004	0.0041	2.5%	0.0043	7.5%
3.5	0.1819	0.1821	0.11%	0.1825	2.2%
3	0.4173	0.4170	0.07%	0.4165	0.19%
2.5	0.0343	0.0345	0.58%	0.0348	1.46%
2	0.0096	0.0096	0%	0.0097	1.04%
1.5	0.1643	0.1645	0.12%	0.1639	0.24%
1	0.0538	0.0537	0.19%	0.0539	0.19%
0	0.035	0.0349	0.29%	0.0354	1.14%

Assuming that the required demand of the flow transmission system is defined as $\omega = 2$, the comparison of the simulation results for the distribution of the system availability is shown in Fig. 15. The results are obtained by 10,000 SMV experiments with a 1000-bit non-Bernoulli sequence and 10,000 MC simulations with 1000 runs in each simulation. According to the Central Limit Theorem, the distribution of a large number of samples follows a Gaussian distribution. Fig. 15 shows that a Gaussian distribution with the mean and variance obtained from the simulations fits the distribution of the data very well using MC and SMV methods. The difference between the mean value of MC simulation and SMV is negligible (given by the mean value of 0.8951 for MC and 0.8957 for SMV model). In contrast, the standard deviation σ of the SMV model and MC simulation results are significantly different. The standard deviation is 0.0058 for MC simulation (as shown in Fig. 15(a)) and 0.0016 for the SMV model (as shown in Fig. 15(b)). This indicates that the SMV approach achieves more accurate evaluation results compared to MC for the same number of simulation runs.



(a)



(b)

Fig. 15. Comparison of the simulation results for the distribution of the system availability (demand performance rate of the system: ω =2). (a) 10,000 MC [14] simulations with 1000 runs in each simulation of the flow transmission system; (b) 10,000 SMV experiments with a sequence length of 1000 bits for the availability of the flow transmission system.

100 experiments are run using SMV and MC methods to obtain the mean value of the system availability and average run time. The SMV and MC simulation results and the computation results with the UGF method are shown in Table 5. Because the use of non-Bernoulli sequences reduces the random fluctuation in stochastic computation, the SMV approach produces more accurate results and is more efficient than MC simulation, as indicated by the results of system availability and run time in

Table 5. Also shown in Table 5 is that the SMV approach using different sequence lengths leads to different accuracies of the system availability and different run time. It can be seen that the SMV approach with a reasonable sequence length (e.g. 5k bits) produces a very accurate result with a shorter run time compared to the UGF method.

Table 5
Run time and error of MC simulation [14] and SMV, compared to the UGF method [10, 27], for the availability of the flow transmission system in Fig. 13. *N*: the number of simulation runs for MC; *L*: the sequence length for SMV models; the definition of error is the same as in Table 4.

UGF [10, 27]	Availability	0.8962			
	Average run time for UGF (s)	0.4102			
	N/L	**1k**	**5k**	**10k**	**100k**
MC simulation [14]	Availability	0.8941	0.8947	0.8955	0.8960
	Error	0.2343%	0.1673%	0.0781%	0.0223%
	Average run time for MC method (s)	0.2988	1.593	3.468	39.582
SMV approach	Availability	0.8956	0.8959	0.8960	0.8961
	Error	0.0669%	0.0334%	0.0223%	0.0112%
	Average run time for SMV approach (s)	0.0354	0.1642	0.3697	3.731

4.2. Example 2

In this example, a system consisting of multi-state components with time-varying probabilities is presented to validate the SMV models. The structure of this system is the same as the system in Fig. 13. The performance rates and corresponding time-varying probabilities of the influencing and affected components are presented in Tables 6 and 7 respectively. The probabilities in each state of components 1 and 3 are the same as the probabilities of components 8 and 5 respectively. In Tables 6 and 7, $p_{ij}(t)$ is the performance probability of component *j* in state *i* at time *t*, and it meets the condition $\sum_{i=1}^{k_j} p_{ji}(t) = 1$, where k_j is the number of states of component *j*. All components' performance probabilities in the highest state at the beginning of system operation (t=0) are set to 1, because a system is considered to start with a perfect performance.

Table 6

Unconditional performance distributions of components with time-varying probabilities: g_{ji} denotes the performance rate of component *j* in state *i*, *t* is the number of months, 0≤ *t* ≤ 100.

$g_{ji}/p_{ji}(t)$ \ i	$g_{1i}/p_{1i}(t)$	$g_{3i}/p_{3i}(t)$	g_{5i}	g_{8i}
1	8/ 1-0.0072*t*	9/1-0.0079*t*	8	7
2	7/ 0.0005*t*	8/0.0006*t*	7	6
3	6/0.0004*t*	7.5/ 0.0004*t*	6.5	5
4	5.5/ 0.0007*t*	7/0.0003*t*	6	4.5
5	5/ 0.0003*t*	6.5/ 0.0006*t*	5.5	4
6	4/ 0.0003*t*	6/ 0.0009*t*	5	2.5
7	3/ 0.0011*t*	4/ 0.0012*t*	4	1.5
8	2/ 0.001*t*	3/ 0.0015*t*	2.5	1
9	1/ 0.0009*t*	1/ 0.0007*t*	1	0.5
10	0/ 0.002*t*	0/0.0017*t*	0	0

Table 7

Conditional performance distributions of components with time-varying probabilities: g_{ji} denotes the performance rate of component *j* in state *i*, *t* is the number of months, 0≤ *t* ≤ 100.

$g_{ji}/p_{ji}(t)$	condition \ i	1	2	3	4	5	6
$g_{2i}/p_{2i}(t)$	$0 \leq g_{1i} \leq 2$	7/1-0.0073*t*	6/0.0016*t*	5/0.0006*t*	4/0.0019*t*	2/0.0014*t*	0/0.0018*t*
	$2 < g_{1i} \leq 5$	6/1-0.0084*t*	5.5/0.0006*t*	5/0.0006*t*	3/0.0018*t*	1/0.0022*t*	0/0.0032*t*
	$g_{1i} > 5$	5/1-0.0087*t*	4.5/0.003*t*	4/0.0018*t*	0/0.0039*t*		
$g_{4i}/p_{4i}(t)$	$0 \leq g_{3i} \leq 3$	5/1-0.0077*t*	4/0.001*t*	3/0.0017*t*	2/ 0.0013*t*	1/0.0012*t*	0/0.0025*t*
	$3 < g_{3i} \leq 7$	5/1-0.0086*t*	3/0.0022*t*	2.5/0.0003*t*	2/0.0023*t*	1/0.0011*t*	0/0.0027*t*
	$g_{3i} > 7$	4.5/1-0.0078*t*	4/0.0028*t*	3/0.0008*t*	2.5/0.0007*t*	0/0.0035*t*	
$g_{6i}/p_{6i}(t)$	$0 \leq g_{5i} \leq 4$	6/1-0.0094*t*	4/0.0013*t*	3/0.0015*t*	2.5/0.0011*t*	1.5/0.0014*t*	0/0.0041*t*
	$g_{5i} > 4$	5/ 1-0.0063*t*	3.5/0.0005*t*	3/ 0.001*t*	2.5/0.0005*t*	2/0.0028*t*	0/0.0015*t*
$g_{7i}/p_{7i}(t)$	$0 \leq g_{8i} \leq 5$	7/1-0.0096*t*	6/0.0014*t*	4/0.0021*t*	3/0.0025*t*	1/ 0.002*t*	0/0.0016*t*
	$g_{8i} > 5$	4.5/1-0.007*t*	4/ 0.0026*t*	3/0.0014*t*	2.5/0.0016*t*	0.5/0.0012*t*	0/0.0002*t*
$g_{9i}/p_{9i}(t)$	$g_{8i} = 0$	5/1-0.0076*t*	4.5/0.0023*t*	4/0.0012*t*	3/0.0005*t*	1.5/0.0021*t*	0/0.0015*t*
	$g_{8i} > 0$	4/1-0.006*t*	3.5/0.0004*t*	3/0.0005*t*	2/0.0011*t*	0.5/0.0009*t*	0/0.0031*t*

In a system consisting of MSCs with time-varying probabilities, similar to [31], the time *T* is divided into *M* equal time intervals, i.e., $\Delta T = T/M$. *M* is determined by a tradeoff between accuracy and efficiency. The discretization can provide a relatively accurate estimate of the continuous probability of a component with a reasonable *M*.

For example 2, the simulation results obtained by using the SMV, UGF and MC methods for the system reliability at time *t* are presented in Fig. 16 (with demand performance rate of the system: ω =1). As shown in Fig. 16, the simulation results using the SMV approach with a sequence length 10k bits is more accurate than the MC method with 10k runs. The error of simulation results in time *t* with a sequence length 10k bits is less than 0.0849% compared to the UGF method (the definition of error is the same as in Table 4). However, the runtime of the SMV approach is only 0.4436s, compared to the calculation time of the UGF method, 34.1623s. Hence, the SMV approach is more efficient than the UGF method for evaluating the reliability of complex systems consisting of MSCs with time-varying probabilities, and it produces a very accurate simulation result of the system reliability.

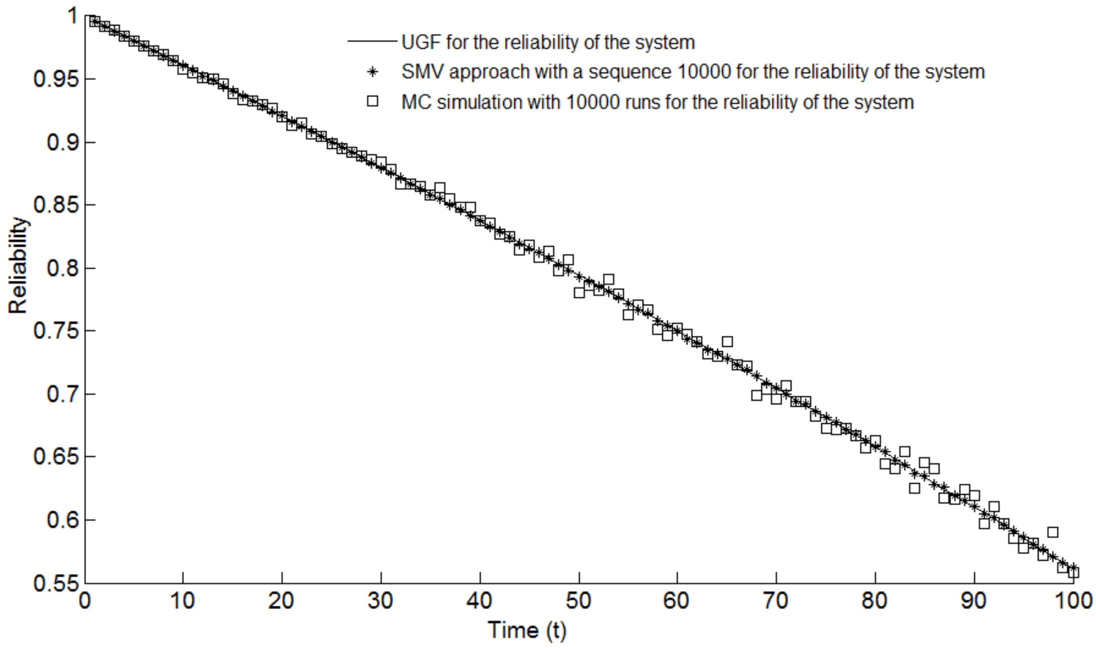


Fig. 16. The simulation results obtained by using the SMV, UGF and MC methods for the system reliability at time *t* (demand performance rate of the system: ω =1, 0≤ *t* ≤100).

4.3.Discussion of SMV approach

The SMV model simulations of the flow transmission system indicate that the result of every single experiment fluctuates around the expected mean value. The fluctuation can be analyzed quantitatively [17]. Assuming that experiment results x is a random variable with the same mean u and variance σ^2, the fluctuation is measured by the standard deviation [29],

$$e = |x - u| \approx \frac{\sigma}{\sqrt{n}} \tag{14}$$

where n is the number of experiments. The result indicates that the fluctuation is proportional to σ. (14) applies to both the SMV and MC methods. Due to the faster convergence of SMV approach, the standard deviation σ of SMV model is smaller than that of the MC simulation, hence the SMV approach is more accurate than MC simulation for the same value of n.

In stochastic computation, the result follows approximately a Gaussian distribution for a large number of runs [17, 30]. A parameter Z_c determines the confidence interval of the simulation results with a sequence length L, and the error in the simulation result is given by [31]:

$$E = \frac{Z_C}{u}\sqrt{\frac{\sigma^2}{L}} = \frac{Z_C}{u} \cdot \frac{\sigma}{\sqrt{L}} \tag{15}$$

where E is defined as the ratio of the disparity between an approximate value and the accurate one over the accurate result, u and σ are the accurate mean and standard deviation of the distribution of the results. For a confidence level of 95%, Z_C = 1.96. As per (15), the accuracy can be improved by increasing the length of the sequence for a given confidence level. The sequence length L can be determined by (15) for obtaining a desired evaluation accuracy.

In a mission time, the simulation results with different sequence lengths for the flow transmission system in example 1 are presented in Fig. 17, where the difference is the disparity between the approximate result obtained by SMV model and the exact values using the UGF method. As shown in Fig. 17, the difference decreases when the sequence length increases. It means that the accuracy of SMV model can be further improved by increasing the length of stochastic sequences.

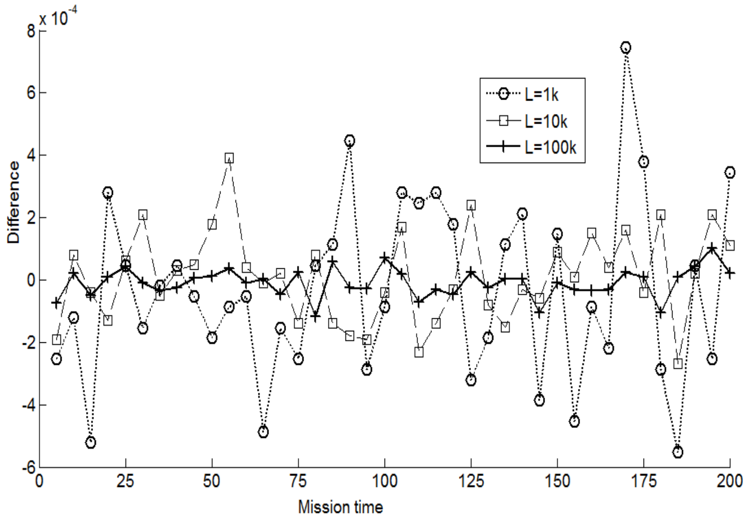


Fig. 17. The differences in the availability of the flow transmission system obtained by using the SMV model at different mission times. L is the length of the non-Bernoulli sequence. The mission time is 200 *hours* and the time interval for discretization is one *hour*.

For a parallel MSS consisting of n independent MSCs with M states, (n-1)·L ESL ADD operations and L *MVSO* operations are required; thus the computational complexity of the SMV approach with L-bit sequences is $O(n \cdot L)$. However, a complexity of $O((M+1)^n)$ is required for the UGF method [11]. Assuming the system in Fig. 13 consisting of 9 independent components with different numbers of states, the run times for SMV and UGF methods are shown in Fig. 18. The results reveal that the run time required by the SMV approach with 10k-bit sequences is shorter than the UGF method when the number of a component's states is 4 or more. The run time by the SMV approach with 100k-bit sequences or by MC simulation with 10k runs is shorter than the UGF method when the number of a component's states is 8 or more. Compared to MC simulation with 100k runs, a longer time is required by the UGF method when the number of component's states is larger than 10. The results indicate that the SMV approach is more efficient than MC simulation. The simulation results also reveal that the computation time of MC and SMV approach is not influenced by the number of component states, and the computation time of UGF method increases with an increasing number of component's states. As per (15), the reliability of the system is very accurate when the sequence length of the SMV models is 100k bits.

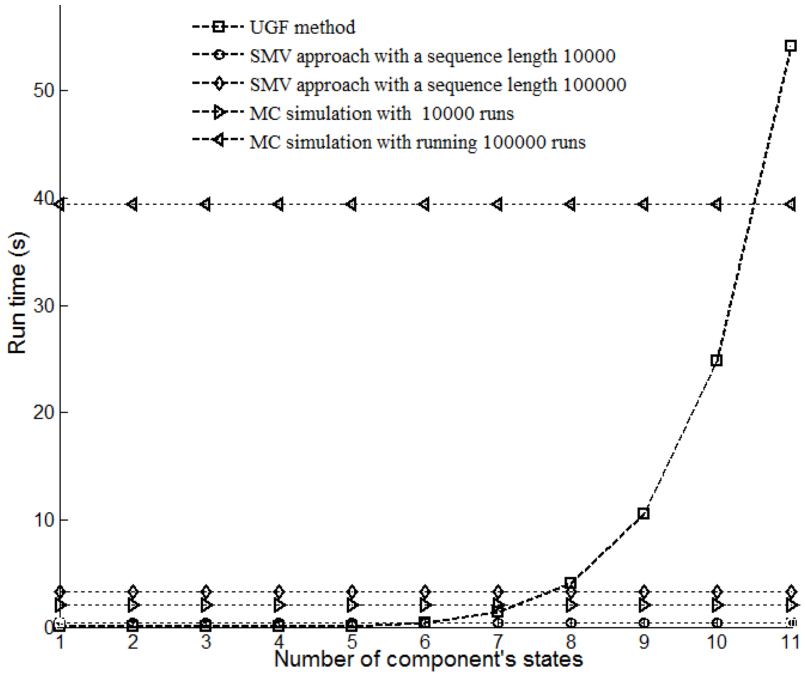


Fig. 18. The run times for the SMV and UGF methods with an increasing number of component states. The number of components *n* = 9.

For a parallel MSS consisting of n dependent MSCs with M states, assuming that the performance rate set of dependent components is separated into l mutually disjoint subsets, the complexity of SMV model with L-bit sequences is $O(n \cdot l \cdot L)$, according to the dependency relationship between components. For the same system, the complexity of the UGF method is $O(l(M+1)^n)$. The complexity of a series MSS is the same as that of a parallel MSS for the two methods. For a complex system with a large number of components and states, the SMV approach is more efficient than the UGF method, when the exact value of system reliability is not necessary.

5. Conclusion

SMV models are proposed to evaluate the availability of multi-state systems with independent and dependent multi-state components. The performance rates and corresponding probabilities of MSCs are simultaneously encoded in multi-valued non-Bernoulli sequences. The simulation results indicate that the SMV approach provides a more efficient analysis compared to Monte Carlo (MC) and UGF methods. The SMV approach is more accurate than MC simulation due to the faster convergence. As shown in the simulation results, the number of component states is not a crucial factor to influence the run time of the SMV approach; therefore, the SMV approach is suitable for the analysis of complex multi-state systems. Ongoing work includes the stochastic modeling of other types of MSS.

Acknowledgment

This research is partly supported by the Project of National Natural Science Foundation of China (Grant no. 61371024) and the Natural Sciences and Engineering Research Council (NSERC) of Canada.

References

[1] Marquez JE, Coit DW. Multi-state component criticality analysis for reliability improvement in multi-state systems. Reliab Eng Syst Saf 2007; 92:1608-1619.

[2] Barlow R, Wu A. Coherent systems with multi-state elements. Math Oper Res 1987; 3: 275–281.

[3] Lisnianski A, Elmakias D, Laredo D and Haim HB. A multi-state markov model for a short-term reliability analysis of a power generating unit. Reliab Eng Syst Saf 2012; 98: 1-6.

[4] Lisnianski A. Extended block diagram method for a multi-state system. Reliab Eng Syst Saf 2007; 92:1601-1607.

[5] Chryssaphinou O, Limnios N, and Malefaki S. Multi-state reliability systems under discrete time semi-markovian hypothesis. IEEE Trans Reliab 2011; 60(1):80-87.

[6] Amari SV, Xing L, Shrestha A, Akers J and Trivedi S. Performability analysis of multistate computing systems using multivalued decision diagrams. IEEE Trans comput 2010; 59(10):1419-1433.

[7] Shrestha A, Xing L, Dai YS. Decision diagram based methods and complexity analysis for multi-state systems. IEEE Trans Reliab 2010; 59(1): 145-161.

[8] Nagayama S, Sasao T and Butler JT. Analysis of multi-state systems with multi-state components using EVMDDs. In: 42nd International Symposium on Multiple-Valued Logic, 2012, p. 122-127.

[9] Levitin G, Lisnianski A. Optimization of imperfect preventive maintenance for multi-state systems. Reliab Eng Syst Saf 2000; 67:193-203.

[10] Levitin G. A universal generating function approach for the analysis of multi-state systems with dependent elements. Reliab Eng Syst Saf 2004; 84:285-292.

[11] Ding Y, Zuo MJ. A framework for reliability approximation of multi-state weighted k-out-of-n systems. IEEE Trans Reliab 2010; 59(2):297-308.

[12] Ding Y, Zuo MJ, Lisnianski A and Tian ZG. Fuzzy multi-state system: general definition and performance assessment. IEEE Trans Reliab 2008; 57:589-594.

[13]Lisnianski A and Ding Y. Using inverse *Lz*-transform for obtaining compact stochastic model of complex power station for short risk evaluation. Reliab Eng Syst Saf 2016; 145:19-27.
[14]Zio E, Marella M and Podofillini L. A Monte Carlo simulation approach to the availability assessment of multi-state systems with operational dependencies. Reliab Eng Syst Saf 2007; 92:871-882.
[15]Hurst SL. Multiple-valued logic-its status and its future. IEEE Trans Comput 1984; C-33(12):1160-1179.
[16]Xue JN. On multistate system analysis. IEEE Trans Reliab 1985; 34(4):pp. 329-337.
[17]Han J, Chen H, Liang J, Zhu P, Yang ZX, and Lombardi F. A stochastic computational approach for accurate and efficient reliability evaluation. IEEE Trans Comput 2013; 63(6): 1336-1350.
[18]Zhu P, Han J. Stochastic Multiple-Valued Gene Networks. IEEE Trans Biomed Circuits Syst 2014; 8:42-53.
[19]Rossello J L, Canals V, and Morro A. Probabilistic-based neural network implementation. In: The 2012 International Joint Conference on Neural Networks (IJCNN), 2012, p. 1–7.
[20]Levitin G, Lisnianski A, Ben-Haim H. Redundancy optimization for series-parallel systems. IEEE Trans Reliab 1998; 47:165-172.
[21]Ding Y, Lisnianski A. Fuzzy universal generating functions for multi-state system reliability assessment. Fuzzy Sets and Syst 2008; 159:307-324.
[22]Ushakov I. Universal generating function. Soviet J Comput System Sci 1986; 24(5): 118-129.
[23]Levitin G, lisnianski A, Ben-Haim H. Joint redundancy and maintenance optimization for multi-state series-parallel systems. Reliab Eng Syst Saf 1998; 64: 33–42.
[24]Levitin G. Universal generating function in reliability analysis and optimization. London: Berlin, Springer, 2005.
[25]Gaines BR. Stochastic Computing Systems. Advances in Information Systems Science 1969; 2:37-172.
[26]Aliee H, Zarandi HR. A fast and accurate fault tree analysis Based on stochastic logic implemented on field-programmable gate arrays. IEEE Trans Reliab 2013; 62(1):13-22.
[27]Dao CD, Zuo MJ. Reliability analysis of multi-state systems with s-dependent components. In: Annual Reliability and Maintainability Srmposium, Palm Habor, 2015, p.1-6.
[28]Li X, Qian WK, Riedel M, Bazargan K and Lilja D. A reconfigurable stochastic architecture for highly reliable computing. In: Proc Great Lakes Symp VLSI (GLVLSI), Boston, 2009, p. 315-320.
[29]Chen H, Han J. Stochastic computation models for accurate reliability evaluation of logic circuits. In: Proc Great Lakes Symp VLSI (GLVLSI), Providence, RI, USA, 2010, p.61-66.
[30]Wang R, Han J, Cockburn BF, and Elliott DG. Design, evaluation and fault-tolerance analysis of stochastic FIR filters. Microelectron Reliab 2016; 57:111–127.
[31]Zhu PC, Han J, Liu L and Lombardi F. A stochastic approach for the analysis of dynamic fault trees with spare gates under probabilistic common cause failures. IEEE Trans Reliab 2015; 64(3):1-15.