Abstract—As Moore’s law is coming to an end, research on technologies alternative to the complementary metal-oxide semiconductor (CMOS) has been extensively pursued over the last few decades. Many emerging nanotechnologies assemble circuits based on majority logic. It is generally known that majority logic is more expressive and hardware efficient than Boolean logic, however majority logic presents unique challenges at many levels such as arithmetic design and synthesis. With the rediscovery of majority logic as a computational primitive in the post-CMOS era, this paper briefly reviews recent studies from various perspectives to highlight accomplishments and open problems across many domains of majority logic design.

Index Terms—Majority logic, logic synthesis, approximate computing, nanotechnology, reliability.

I. INTRODUCTION

MOORE’S law predicts that the number of transistors on a microchip will double approximately every eighteen months, as observed over the past 50 years for the development of integrated circuits. However, as the feature size of complementary metal-oxide semiconductor (CMOS) transistors continues to scale down, fundamental physical limits lead to many severe challenges, such as increased leakage currents, doping fluctuations, and expensive lithography [1]. In the post-CMOS era, new nanometer-scale technologies are being developed to improve upon or replace traditional CMOS technology to achieve a faster switching speed, higher integration density, and lower power dissipation.

A majority gate (or voter, MV) performs a voter function with an odd number of inputs, i.e., it outputs true (or false) if more than half of the inputs are true (or false). A 3-input MV implements the logic function, \( F = M(A, B, C) = AB + BC + AC \), where \( A, B, \) and \( C \) are the inputs. Majority logic has been widely used to design fault-tolerant architectures, such as the \( N \)-modular redundancy for reliable system design. Unlike CMOS technology that relies on Boolean logic, a number of novel emerging technologies function on the principles of the majority or minority logic [2], such as quantum-dot cellular automata (QCA) [3], nanomagnetic logic (NML) [4], and spin-wave technology [5]. Majority logic is straightforward and thus efficient for implementations on a specific nanotechnology. It also benefits from its capability to realize functions using circuits with fewer gates, as shown in Fig. 1. In this paper, we review the use of majority logic for synthesis and arithmetic computation in different nanoscale technologies.

The remainder of this paper is organized as follows. Section II introduces majority logic-based nanometer-scale technologies. Majority logic synthesis methods are reviewed in Section III. Section IV discusses majority logic-based approximate arithmetic designs. Finally, Section V concludes this paper with a discussion of challenges and prospects.

Fig. 1: The full adders at the gate level using majority gates and inverters. It performs the addition of \( A + B + C_{in} \) and produces a carry output, \( C_{out} \), and a sum, \( S \). M3: three input MV; M5: five input MV. The circuit characteristics are evaluated by the number of utilized MVs, the number of utilized inverters (INVs), and the critical path delay (D) from theoretical level.

II. MAJORITY LOGIC-BASED EMERGING TECHNOLOGIES

This section introduces several beyond-CMOS technologies using majority logic as the design primitive.

A. Quantum-dot Cellular Automata (QCA)

Instead of transistors, the basic element in QCA is a cell composed of four single tunnel structures with four quantum dots on the corners and two electrons [3], [11], [12]. The electrons are positioned on two different diagonals with a +1 or -1 polarization state for the logic value ‘1’ or ‘0’, respectively.
TABLE I: Majority Logic-based Nanotechnologies

<table>
<thead>
<tr>
<th>Technology</th>
<th>Quantum-dot Cellular Automata</th>
<th>Nanomagnet Logic</th>
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<th>Memristive Logic</th>
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<tbody>
<tr>
<td>Basic Elements</td>
<td>QCA cells with four quantum dots and two electrons</td>
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<td>Information Propagation</td>
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<td>Electric currents to transport spin angular momentum</td>
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<td>The charge passing through memristors</td>
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<tr>
<td>Features</td>
<td>High operating frequency Low power High circuit density</td>
<td>Low operating frequency Low power Non-volatility High circuit density</td>
<td>High endurance Low power Non-volatility High circuit density</td>
<td>High operating frequency Low power Non-volatility High circuit density</td>
<td>High operating frequency Low power Non-volatility High circuit density</td>
</tr>
</tbody>
</table>

Due to Coulomb repulsion, the polarization spreads from one cell to its neighbor cells without moving electrons between cells. A 3-input MV consists of four QCA cells that operate by electromagnetic interactions, as shown in Table I.

B. Nanomagnet Logic (NML)

NML uses the two stable magnetization states of nanoscale magnets as logic values ‘0’ and ‘1’ [13], [14]. The magnets are placed close to each other, information is propagated using the magnetic interaction among neighboring magnets. An MV in NML is made of three input magnets surrounding a central element [4], as shown in Table I.

C. Spin-based Technology

Spin-based technology for MV implementation includes spin-wave technology [15], all spin logic [16], and spin-transfer torque technology [8]. Consider the spin-transfer torque technology as an example; a spin torque MV can be realized by magnetic tunnel junctions (MTJs). As one of the emerging spintronic technologies, an MTJ is made of two ferromagnetic layers separated by a thin insulating tunnel barrier: one free layer with a changeable magnetization and the other with a fixed magnetization. The resultant low or high resistance of the junction encodes a bit of information. An all-magnetic MV based on a spin-transfer torque MTJ (STT-MTJ), referred to as STTMAJ, can realize Boolean and non-Boolean functions without using transistors, as shown in Table I.

D. Plasmonic Technology

The plasmonic technology utilizes the propagation of interfacing surface-plasmon-polariton (SPP) waves between a dielectric and a metal. It localizes the electromagnetic energy in dimensions, thus overcoming the diffraction limitation of photonics. The wave nature of the computation has motivated the studies on implementing MVs with plasmons. In [9], the plasmonic-based logic utilizes the phase of the SPP wave as logic variables, which follow the majority roles. A three-input plasmonic MV layout is presented in Table I.

E. Memristive Logic

Memristive logic uses the resistance states of memristors as basic building blocks for logic operations. A memristor is a non-volatile memory element that can change its resistance state depending on the amount of charge that has passed through it with very low power consumption. Memristive logic has been recently exploited for in-memory computing by making logic gates executable in an array configuration [17]. The functionally complete Majority+NOT logic family is preferable for implementation [18]. An MV can be implemented in a Resistive Random Access Memory (ReRAM) array by utilizing V–R logic or R–V logic. A ReRAM-based R–V MV is presented in Table I.

Table I summarizes the basics for those aforementioned majority logic-based nanotechnologies. There are also some alternative technologies relying on minority logic, such as single electron tunneling [19], [20] and tunneling phase logic [21]. As complementary to majority logic, minority logic shares similar rules for circuit implementation.

III. MAJORiTY LOGIC SYNTHESIS

In synthesis, a logic function is extracted from a high-level description to an optimized gate-level representation [22]. Various synthesis methods are based on logic primitives for circuit design using different technology or implementation platforms. For example, the use of NOR or NAND gates are common for CMOS technology [23]. Field programmable gate arrays (FPGAs) are built on reconfigurable lookup tables (LUTs) [24], [25]. For majority logic-based nanotechnologies, it is necessary to describe the logic functions by using primitives of MVs and inverters.

By synthesizing circuits using logic primitives of majority and NOT gates (such as the Majority-Invert-Graphs, or MIGs), and AND and NOT gates (such as the AND-OR-Invert-Graphs, or AOIGs), it has been shown that the majority logic (together with NOT) can achieve up to a 33% reduction in logic depth [26]. However, a direct transformation from Boolean logic to majority logic may likely not lead to an optimal gate-level implementation. Therefore, majority logic synthesis plays an important role to obtain an optimized netlist for the circuit to
be realized. Moreover, majority logic synthesis is applicable to CMOS-based applications such as FPGA-based technology mapping over the LUT structure [27].

This section briefly reviews majority logic synthesis methods for obtaining optimized circuit implementations.

A. Function-oriented Majority Logic Synthesis

1) Synthesis in Majority-of-3 (maj-3) Forms

The research in majority logic synthesis has been focused on three-input majority logic functions for QCA. Based on the geometric interpretation of Boolean functions, thirteen standard functions have been synthesized manually in [6], [28]. An automated method has been proposed in [29], but it cannot optimize the number of MVs and majority levels. Further improvement based on this approach has been studied in [30] to obtain the optimized majority logic function by introducing a metric to evaluate a majority network. However, these approaches cannot handle functions with more than three inputs. Various algorithms have been developed in the preprocessing and decomposition of circuits, and in converting each node into Majority-of-3 (maj-3) expressions [31]–[33].

Alternatively, a logic representation structure referred to as Majority-Invert-Graphs (MIG) has been recently proposed for optimizing majority logic expressions based on Boolean logic representation [26]. An MIG describes the maj-3 network through a ternary tree format by using majority nodes and regular/complemented edges. It is derived from an AOIG, by fixing one input as ‘0’ or ‘1’ to realize an AND or OR operation. A dedicated Boolean algebra is introduced to derive transformation rules for the optimization of the MIG representations. To further optimize both the number of MVs and majority levels, there are some studies on MIG-based optimization methods by utilizing the error masking property of majority functions [34], functional hashing [35], node-merging using Boolean satisfiability (SAT) [27], [36] or Boolean resubstitution [37], [38]. Fig. 2 (a)-(b) presents the optimized logic networks based on AOIG and MIG for a 4-bit adder, respectively. Moreover, there are some studies on logic synthesis based on an extended logic representation of MIG, referred to as XOR-Majority graph (XMG) [39], given exclusive-OR (XOR), MV, and inverters as primitives. Fig. 2 (c) shows the optimized logic network based on XMG for a 4-bit adder.

2) Synthesis in Majority-of-n (maj-n) Forms

When using MVs with a larger fan-in in majority logic synthesis can reduce the majority levels and the number of required MVs. Synthesis approaches for maj-3 networks can be generalized to exploit Majority-of-n (maj-n) networks [40]. However, it does not result in optimized circuits for maj-n networks. Therefore, there are some efforts on logic synthesis for maj-n networks. A heuristic-based majority logic synthesis method has been presented by using a library based on 3-input and 5-input MVs in [33], [41] has proposed a synthesis flow for a maj-n network by converting each look-up table to an MV without decomposition. This approach has the drawback of using massive inversions without optimization. Chu et al. have presented a synthesis method to represent Boolean functions in maj-5 forms to optimize the number of MVs, followed by majority levels [42]. An improved approach has been studied in [43], which synthesizes networks composed of maj-3 and maj-5 by encoding majority logic functions into linear optimization models.

The efficiency of physical circuit implementations of MVs with different input sizes relies on the underlying technology; therefore, efficient decomposition of maj-n to maj-3 has been extensively explored. A sorting network with $O(n \log_2 n)$ [44] and the median selection algorithm with linear complexity [45] provide effective approaches to generate maj-3 networks with the asymptotic bound and the quasi-linear bound, respectively. To obtain a minimum MIG for small values of $n$, Testa et al. map monotone maj-n functions to maj-3 functions using binary decision diagrams (BDDs) and Shannon’s expansion with the quadratic bound [46]. Chattopadhyay et al. further develop a constructive and linear decomposition approach [47].

B. Technology-aware Majority Logic Synthesis

The function-oriented majority logic synthesis is developed based on conventional metrics, i.e., the number of MVs and the majority levels. These approaches are technology-independent, so without considering physical implementations on a specific technology. Direct mapping into a circuit using basic building blocks may lead to inefficiency. Several optimization algorithms for multilevel majority logic synthesis have been developed based on the MIG by considering technology-aware metrics [22].

Since QCA has a relatively expensive implementation of inverters, logic synthesis algorithms to minimize or eliminate inversion have been developed based on the MIG [48], [49]. Testa et al. have rewritten the MIGs to an inversion-free network by moving all inverters to the primary inputs and also restricting the fan-out of each MIG node to a maximum value [49]. Moreover, an SAT-based method developed on MIGs has been proposed by considering constraints in fan-out
and implementation depth [50]. Torres et al. have presented an energy-aware model for logic synthesis for QCA based on an estimate of the energy dissipation [51]. The majority logic synthesis approach in [52] minimizes the number of wire-crossings in single-layer QCA circuits by analyzing the impact of input ordering on wire-crossing. [53] has proposed a cross-aware logic synthesis technique to exploit the inherent bit-level parallelism of ReRAM crossbar arrays.

IV. MAJORITY LOGIC-BASED APPROXIMATE ARITHMETIC CIRCUIT DESIGNS

Approximate computing improves the efficiency of circuits and systems with a controlled loss of accuracy [54]. Although approximate designs based on Boolean logic can be utilized for majority logic-based nanotechnologies, a naive mapping does not fully leverage the properties of majority logic [55]. Hence, dedicated designs have been developed for majority logic-based arithmetic functions and, more recently, approximate circuits.

This section discusses majority logic-based designs for approximate arithmetic circuits, including adders, subtractors, compressors, and multipliers. Also, error-tolerant applications based on the reviewed designs are introduced.

A. MAJORITY LOGIC-BASED APPROXIMATE ADDERS AND SUBTRACTORS

A full adder (FA) performs the addition \( A + B + C_{in} \) and produces a carry output, \( C_{out} = M(A, B, C_{in}) \), and a sum, \( S = M(C_{out}, M(A, B, C_{in}), C_{in}) \) [56], [57]. A full subtractor (FS) performs the subtraction denoted by \( A - B - B_{in} \) and then produces one borrow output, \( B_{out} = M(\overline{A}, B, B_{in}) \), and one difference, \( D = M(B_{out}, M(A, B, B_{in}), \overline{B}_{in}) \) [58].

1) Approximate Full Adders and Subtractors

Approximate FAs (AFAs) and approximate FSs (AFSs) have been developed based on the simplification of the truth table and logic functions. Various AFA designs [59], [60] are variants of the design in [61], denoted by AFA1, which accurately generates \( C_{out} \) and approximately computes \( S \). Therefore, errors do not propagate to the more significant bits in a multi-bit adder. The AFA design in [62], denoted by AFA2, approximately generates \( C_{out} \) and \( S \); however, the errors introduced to \( C_{out} \) and \( S \) compensate each other. Labrado et al. presented an AFS design [61], which accurately computes \( B_{out} \) and generates an erroneous \( D \) in two input cases. As shown in Table II, compared with their exact counterparts, AFA and AFS designs save two MVs, one inverter and one unit of delay. AFA1 introduces a one-bit error when the exact result is 0, which is not included when computing MRED. Therefore, AFA1 and AFA2 share similar results in hardware and error metrics, except for the MRED.

2) Approximate Multi-bit Adders and Subtractors

Approximate 2-bit adders (A2As) have been proposed by cascading two AFAs [62], logic simplification based on the truth table [64], and shortening the carry chain [65]. Table II gives the results in hardware and error metrics of these A2A designs: (1) Zhang et al. have assembled A2As based on AFA1 and AFA2 [62]. The use of two different types of AFAs leads to more accurate A2A designs, denoted by A2A1 (whose lower significant bit is computed by AFA1) and A2A2 (whose lower significant bit is computed by AFA2). (2) The A2A design based on the truth table [64] and denoted by A2A3, requires two more MVs than AFA-based A2As. However, it benefits from higher accuracy. (3) To shorten the critical path of multi-bit adders, two A2A designs with no carry out bit [65] and denoted by A2A4 and A2A5, have been proposed for multi-bit adders.

By discarding some of the primary input bits and the carry in bit, Chu et al. have used two approximate 4-bit adders (A4As) to cut off the carry chain and reduce the hardware when building large adders [66]. Approximate 4-bit subtractors (A4Ss) have been designed for constructing unsigned restoring array dividers [68]: A4S1 correctly generates the borrow output and A4S2 decreases the accuracy in the borrow propagation to alleviate the issue of a long delay. The comparison results in Table II indicate that A4S1 with \( NMED = 0.1250 \) and A4S2 \( NMED = 0.1821 \) save five and seven MVs, respectively. They are respectively preferred for the computation of the most and less significant bits. Approximate multi-bit designs can be assembled by cascading the aforementioned designs [64], i.e., implementing the least significant bits (LSBs) by approximate circuits, whereas computing the most significant bits (MSBs) by exact counterparts.

B. MAJORITY LOGIC-BASED APPROXIMATE MULTIPLIERS

For an unsigned multiplier, let \( A = a_{n-1}a_{n-2}...a_0 \) be the multiplicand and \( B = b_{n-1}b_{n-2}...b_0 \) be the multiplier. Let \( pp_{ij} \) denote the partial product (PP) bit in the \( i \)th row and the \( j \)th column, where \( 0 \leq i, j \leq n - 1 \). The PPs are first generated as \( pp_{ij} = M(a_j, b_i) \) [63]. For the signed multiplication, the Booth algorithm is widely utilized to reduce the number of PPs by recoding the multiplier [69]. A radix-2^k Booth multiplier groups the multiplier bits into sets of \( k+1 \) adjacent bits. Then, the multiplicand is weighted by \( \pm 2^{k-1}, \pm 2^{k-2}, ..., 0 \) to generate PPs. The higher radix the Booth algorithm is, the fewer PPs are generated, thereby reducing the complexity of PP accumulation.

For both unsigned and signed multipliers, the PPs are then accumulated by using different circuit structures, such as a carry save adder array [70], the Wallace tree [71], or the Dadda tree [72]. In general, FAs, half adders, and 4:2 compressors are used

| Table II: Hardware and Error Metrics of ML-based Approximate Adders and Subtractors |
|---------------------|-----|-----|-----|-----|-----|-----|-----|
| Designs | MV | INV | D | MRED | NMED | RMSE | ER |
| One-bit adders | Exact [58] | 3 | 2 | 1 | 0.0416 | 0.083 | 0.25 | 0.25 |
| AFA1 [61] | 1 | 1 | 0.1875 | 0.083 | 0.25 | 0.25 |
| AFA2 [62] | 1 | 1 | 0.1875 | 0.083 | 0.25 | 0.25 |
| One-bit subtractors | Exact [58] | 3 | 2 | 2 | - | - | - | - |
| AFS [61] | 1 | 1 | 0.0416 | 0.083 | 0.25 | 0.25 |
| Two-bit adders | Exact [65] | 6 | 4 | 3 | - | - | - | - |
| A2A1 [62] | 2 | 2 | 0.1977 | 0.0893 | 1 | 0.4375 |
| A2A2 [65] | 2 | 2 | 0.1977 | 0.0893 | 1 | 0.4375 |
| A2A3 [64] | 4 | 2 | 0.1977 | 0.0893 | 1 | 0.4375 |
| A2A4 [65] | 6 | 4 | 0.1429 | 1.3693 | 0.0563 |
| A2A5 [65] | 6 | 6 | 0.1429 | 1.3693 | 0.0563 |
| Four-bit subtractors | Exact [58] | 7 | 6 | 5 | 0.6160 | 0.1250 | 2.6458 | 0.8125 |
| A4S1 [66] | 6 | 6 | 0.6160 | 0.1250 | 2.6458 | 0.8125 |
| A4S2 [66] | 6 | 6 | 0.6160 | 0.1250 | 2.6458 | 0.8125 |

*The error metrics including the normalized mean error distance (NMED), the mean relative error distance (MRED), the root-mean-square error (RMSE), and the error rate (ER) [67] are utilized to assess the error characteristics of approximate designs.
TABLE III: Hardware and Error Metrics of ML-based Approximate 4:2 Compressors

<table>
<thead>
<tr>
<th>Approximate Designs Based on AFAs</th>
<th>MV</th>
<th>INVD</th>
<th>MRED</th>
<th>NMED</th>
<th>RMSE</th>
<th>ER (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AC1 [64]</td>
<td>2</td>
<td>2/0.1625</td>
<td>0.075</td>
<td>0.0124</td>
<td>37.5</td>
<td></td>
</tr>
<tr>
<td>AC2 [64]</td>
<td>2</td>
<td>2/0.1955</td>
<td>0.075</td>
<td>0.0124</td>
<td>37.5</td>
<td></td>
</tr>
<tr>
<td>AC5 [77]</td>
<td>2</td>
<td>2/0.2052</td>
<td>0.075</td>
<td>0.0124</td>
<td>37.5</td>
<td></td>
</tr>
<tr>
<td>AC4 [76]</td>
<td>2</td>
<td>2/0.2344</td>
<td>0.1</td>
<td>0.0706</td>
<td>43.75</td>
<td></td>
</tr>
<tr>
<td>AC5 [77]</td>
<td>1</td>
<td>2/0.2804</td>
<td>0.1</td>
<td>0.0701</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>AC6 [64]</td>
<td>4</td>
<td>2/0.7539</td>
<td>0.0875</td>
<td>0.0707</td>
<td>40.63</td>
<td></td>
</tr>
<tr>
<td>AC7 [78]</td>
<td>1</td>
<td>2/0.2031</td>
<td>0.125</td>
<td>0.0707</td>
<td>50</td>
<td></td>
</tr>
</tbody>
</table>

1) Approximate Partial Product Generation

For unsigned array multipliers, the authors in [64] have proposed a majority logic-based approximate $2 \times 2$ multiplier with a 3-bit output and a compensation bit, where each bit is generated by one MV. The compensation bit is useful to compensate for the accuracy loss in larger multipliers. The more compensation bit is disregarded, the less accurate the result is.

For radix-4 Booth multipliers, approximate PP generators with single-sided or double-sided errors have been developed based on different implementation of encoding schemes [73], [74]. Compared with the radix-4 Booth algorithm, radix-8 and radix-16 Booth algorithms suffer from a long carry propagation to generate odd multiples of the multiplicand. Therefore, approximate recoding adders (ARAs) without carry propagation have been considered for computing odd multiples [75]. For calculating 3A, 5A, and 7A, 2-bit, 3-bit, and 4-bit ARAs have been designed to compute the sum of $A$ and $2A$, the sum of $A$ and $4A$, and the sum of $-A$ and $8A$, respectively.

2) Approximate Partial Product Accumulation

For the PP compression, AFAs and approximate 4:2 compressors (ACs) have been used in Wallace and Dadda trees [64]. A 4:2 compressor takes five inputs: four primary inputs and one carry input from the lower bit accumulation. It produces three outputs: two primary outputs ($\text{Carry} \text{ and } \text{Sum}$) and one carry output ($\text{C}_{\text{out}}$). It is commonly implemented by cascading two FAs. 

A comprehensive comparison of ACs is provided in Table III. The existing ACs are designed using three criteria: using AFAs, changing truth tables and using output reduction: (1) AFAs can be used to replace the exact counterparts [64], AC1 and AC2 are designed by cascading two FA1s and two FA2s, respectively; AC3 uses AFA1 for the lower part computation and AFA2 for the higher part computation. AC3 produces the smallest MRED. For higher accuracy, only the less significant FA is substituted by an AFA [79]. (2) The entries in a truth table can be changed by introducing errors to reduce circuit complexity [76], [77]. Compared with AC4 [76], although with an increase of 11.1% in MRED and 14.2% in ER, AC5 [77] improves the accuracy in RMSE by 11.3% with a reduction of one MV, two inverters, and one delay unit. (3) To further decrease the hardware complexity, two ACs are designed by reducing the number of outputs [64], [78]. These designs use two output bits (rather than three) for the result, so $\text{C}_{\text{out}}$ is omitted. Different from AC6 [64], AC7 [78] uses only four input operands and one MV.

Therefore, among the AC designs with five input operands and three outputs, AC3 [64] and AC5 [77] are superior in terms of accuracy and hardware. The AC6 [64] benefits from its characteristics of the fewer number of outputs, which can decrease the complexity of PP reduction and compression design. The AC7 [78] with only two outputs is specially designed for four input operands.

Instead of using 4:2 compressors, there are some designs of approximate compressors with an arbitrary number of inputs for PP reduction [66], [80], [81]. More interestingly, an algorithm of constructing a heuristic MIG for exact and approximate compressors using maj-$n$ logic networks has been developed in [81]. Moreover, the low power truncation technique [78] and complementary strategies for the errors based on a probabilistic analysis [74] can be utilized to achieve a superior trade-off between accuracy and hardware efficiency.

C. Applications

Majority logic-based approximate arithmetic units have been applied in error tolerant applications.

Case studies of image processing include addition, filtering, multiplication, sharpening, smoothing and compression. The output quality is commonly evaluated by the peak signal-to-noise ratio (PSNR) and the structural similarity (SSIM). The greater the PSNR and SSIM values are, the less distortion occurs. The use of approximate 8-bit adders for addition in image filtering approximately results in a PSNR value of 44 dB and an SSIM value of 0.95, with a reduction of 40% in area-and-delay product (ADP) [65]. The approximate $8 \times 8$ multiplier in [78] provides a PSNR value of around 40 dB for image multiplication, with a saving of approximately 70% in ADP.

NN applications include classification using a multilayer perceptron and joint face detection and alignment using a multi-task convolution NN (MTCNN). Two often employed measurements are the true positive rate (TPR) and the false positive rate (FPR). For the joint face detection and alignment using the Face Detection Data Set and Benchmark dataset, approximate $8 \times 8$ multipliers in inference reduce the number of multiply-and-accumulate units by 5.1%, with a saving of approximately 18.9% in ADP and an accuracy loss of 7.4% in TPR, compared to the use of exact multipliers [74].

V. Conclusions, Challenges and Prospects

Emerging nano-meter technologies have shown great potentials as alternatives to conventional CMOS technology for their high density and low energy.

For those majority logic-based technologies, including QCA, NML, memristive logic, spin-based technology, and plasmonic technology, logic synthesis by leveraging the characteristics of majority logic plays a crucial role in circuit design. However, the costs and performances of circuits constructed by MVs with different inputs and inverters highly depend on a physical implementation. For example, the number of utilized gates and the wire crossings can be reduced when realizing a function in maj-$n$ networks, but this may be at the cost of an increased gate area. Therefore, to obtain an optimized circuit implementation, an automatic logic synthesis approach with a comprehensive consideration from functional, physical, and geometric levels is worthy of further studies.
Approximate circuits provide opportunities for majority logic-based nanotechnologies to improve hardware efficiency. Designs of majority logic-based approximate arithmetic circuits are briefly reviewed, including adders, subtractors, compressors, and multipliers. Compared to addition, subtraction, and multiplication, division requires more complex computing structures. Majority logic-based approximate 4-bit subtractors have been developed for unsigned restoring array dividers in [68]. However, more efficient designs could be obtained by exploiting the unique structures of various types of dividers. Moreover, there is a lack of research on common computational blocks such as those for fast Fourier transform and multiply–accumulate units.

Finally, most current approximate arithmetic designs are developed independently from the physical characteristics of nanotechnologies. Compared with conventional Boolean logic, the utilization of majority logic as a primitive can improve the reliability of computational circuits [82]–[84]. However, defects may appear in the manufacturing process and modify the correct logic function [85], [86]. The impact of different types of defects on the effectiveness of approximate arithmetic circuits awaits further investigation. It will also be interesting to see if approximate computing can provide effective mitigation strategies for the reliability issues that have been long-lasting and plaguing nanotechnologies.

ACKNOWLEDGMENT

This work was supported by NSERC (RES0048688, RES0051374 and RES0054326), and NSFC (62022041 and X. S. Hu, “On-chip clocking for nanomagnet logic devices,” IEEE TNANO, vol. 9, no. 3, pp. 348–351, 2010.


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