Abstract—MIMO wireless downlinks using transmit zero-forcing beamforming (TZFBF) with $M_t$ transmit antennas can serve up to $K=M_T$ receivers, each equipped with one antenna. To maximize the sum rate, waterfiling can be used. It is shown that unlike classical waterfilling, TZFBF waterfilling cannot simply drop the poorer spatial modes during optimization. Instead, receive antenna selection (RAS) must be incorporated and achieving the maximum sum rate requires an exhaustive search over $2^{M_r |C_k|} = 2^{2O} - 1$ iterations to find the optimal subset $S$ of active receivers where $|S| = 1, \ldots, M_T$. In principle, a separate RAS algorithm can be used in conjunction with waterfilling to reduce the exponential complexity $O(2^{2O})$ of the exhaustive search to linear complexity $O(M_T)$. We develop optimization algorithms that emulate classical waterfilling by progressively reducing the effects of poor spatial modes in $M_T$ iterations. They do so by performing RAS jointly during the waterfilling process at little additional complexity. By avoiding a separate RAS process, complexity is thus further reduced. For the typical case where $K > M_T$, we propose a 2-phase framework that helps reduce the overall complexity while meeting the TZFBF dimensional constraints. Numerical results over different channel conditions are given.

Keywords- Multi-user, downlink beam-forming, antenna selection.

I. INTRODUCTION

For the downlink of a wireless base station equipped with multiple antennas where coordination is feasible among the transmit chains but not among the mobile terminals, simultaneous transmissions to multiple users are possible when channel state information is available at the transmitter. The optimum scheme for maximizing the downlink sum rate is accomplished via dirty paper coding [1 - 3]. However, dirty paper coding has very high computational complexity. A reduced complexity sub-optimal alternative is beamforming where each user’s stream is coded independently and multiplied by a beamforming weight vector for transmission via multiple antennas. Despite its reduced complexity, beamforming has been shown to achieve a large fraction of the capacity promised in multi-user systems when the number of users is large [4], [5]. However, the determination of optimal weight vectors is a tedious non-convex optimization problem. A sub-optimal beamforming technique is zero-forcing beamforming (ZFBF) where the weight vectors are chosen to enforce zero co-channel interference (CCI) among all users. When channel state information is available at the transmitter (CSIT), transmit zero-forcing beamforming (TZFBF) can be implemented so that data for each user is encoded individually and spatial multiplexing is achieved. When channel conditions are poor, the TZFBF sum rate will drop significantly. This is because TZFBF pre-coding involves channel-inversion, which requires significant extra power to boost poor spatial modes, resulting in low power efficiency. Given that cellular base stations are generally sited at higher ground for coverage, poor channel conditions due to fading correlation can occur at the cell edges. It has been shown that the spread of arriving signal spreads of 2° and 5° for angle of departures of 50° and 20° respectively. To help remedy this situation, proper power allocation over the spatial modes is required. Waterfilling can be done since CSIT is available. It will be shown however that using the classical waterfilling method for TZFBF does not help remove the effects of poor spatial modes. Instead, the sum rate optimization process must also incorporate a receive antenna selection (RAS) process.

In this paper, we develop efficient RAS algorithms that are executed jointly during the waterfilling process with little additional complexity introduced to the original waterfilling process. By avoiding a separate RAS process, complexity is thus further reduced. To meet the TZFBF pre-coding constraint, a base station with $M_t$ transmit antennas can serve up to $K=M_T$ receivers, each equipped with one antenna. To account for the typical case where $K>M_T$, we propose a 2-phase framework that helps reduce the overall complexity while meeting the TZFBF dimensional constraints.

II. SYSTEM MODEL

We focus on the multi-user downlink of a base station (BS) serving a group of $K$ geographically distributed users via spatial multiplexing. The BS has $M_T$ transmit antennas while each user has one or more antennas, each coupled with a receiver. The algorithms developed here are applicable to users with one or more antennas. For ease of reference however, we will assume that each user terminal has one antenna, i.e., there are $M_R=K$ receive antennas and $H$ is the $M_R \times M_T$ channel matrix. Channel state information at the transmitter (CSIT) is assumed available, e.g., via time-division duplex (TDD). Assuming $M_R\leq M_T$, $M_R$ data streams $d = \{d_1, \ldots, d_{M_R}\}$ are precoded at the transmitter to form the $(M_T \times 1)$ transmit vector $s$. TZFBF pre-coding uses the right inverse of $H$ (see (1)) so that $M_R$ spatial channels are created and each data stream arrives decoupled without co-channel interference (CCI) at each user after passing through the channel.
y = \sqrt{E_s} / M_f^r H_s n + n = \sqrt{E_s} / M_f^r H H^H d + n = \sqrt{E_s} / M_f^r d + n \quad (1)

This paper assumes a quasi-static, flat fading Rayleigh channel that is constant over several transmission blocks. \( H \) is complex with entries \(-CN(0,1)\) and \( n \) is \((M_R \times 1)\) with covariance \( \mathcal{E}\{n n^H\} = N_0 I_{M_0}\). All \( M_R \) data streams \( d_i \) are i.i.d. \(-CN(0,1)\) and uncorrelated, i.e., \( R_{dd} = \mathcal{E}\{d d^H\} = \text{diag}(\gamma_1, \ldots, \gamma_{M_R})\). \( E_s \) is the total average transmit energy per symbol and \( E_p / M_f^r \) is the average energy transmitted by each antenna per channel use. To constrain the total transmit power, \( R_{pp} = \mathcal{E}\{s s^H\} \) must satisfy \( \text{tr}(R_{pp}) = M_f^r \). As \( H \) may be rectangular and not of full rank, a Moore-Penrose pseudo-inverse \( H^+ \) must be used and \( H^+ = H^H (HH^H)^{-1} \) in this context. Hence full row rank for \( H \) is needed and this imposes the constraint \( M_S \leq M_f^r \). The full row rank assumption is reasonable in a cellular environment. However, it is common to find \( K > M_f^r \) users and a subset \( S \) must then be drawn from the pool of \( K \) users with cardinality \(|S| \leq M_f^r\) so that pre-coding with \( H^+ \) can proceed, where \( H_S \subset H \) is the \((S \times M_f^r)\) subset channel matrix and \(|S| \leq M_f^r\). For ease of reference, we define the best subset containing \( M_f^r \) users as \( S^* \), i.e., \(|S^*| = M_f^r\), and the resulting \((M_f^r \times M_f^r)\) channel sub-matrix as \( H^*_S \subset H \).

III. TZFBF SUM RATE OPTIMIZATION APPROACHES

In this section, the key approaches that lead to efficient TZFBF sum rate optimization algorithms (of which RAS is an important component) will be developed. The first approach works when the TZFBF pre-coding constraint is met, i.e., when \( M_S \leq M_f^r \). A simpler second approach is developed under the 2-phase framework to cater for \( M_f^r > M_f^r \). In effect, the algorithms arising from these approaches emulate the classical waterfilling process but are sub-optimal.

A. Waterfilling for TZFBF

To serve the largest possible number of users at any instant, we assume that the best subset of \( M_f^r \) users, \( S^* \), has been drawn from the pool of \( K > M_f^r \) users. The resulting \((M_f^r \times M_f^r)\) channel sub-matrix is \( H_S \subset H \) allowing TZFBF pre-coding with \( H^+ \) and the channel rate \( C(H_S) \) of a fixed MIMO channel specified by \( H \), is given by (8), (9)

\[
C(H_S) = \max_{R_{pp}} \left( \log_2 \det \left( I + A H_R H^H \right) \right) \text{bps/Hz} \\
= \max_{R_{pp}} \left( \sum_{i=1}^{M_S} \log_2 (1 + A \gamma_i) \right) \quad \text{where} \\
H_R = \text{diag}(\gamma_1, \ldots, \gamma_{M_S}), \quad \gamma_i = \mathcal{E}\{|d|_i^2\}, \quad A = E_s / M_f^r N_0 \text{ and } M_S = \text{rank}(H). \quad (2)
\]

The power constraint \( \text{tr}(R_{pp}) = M_f^r \) and over the \( \gamma_i \geq 0 \) power values allocated for the subset \( S^* \) of chosen users. The power constraint can be expanded as

\[
\text{tr}(R_{pp}) = \text{tr}(H_R R_{dd} H^H) = \text{tr}(R_{dd} (H H^H)^{-1}) \\
= \sum_{i=1}^{M_S} \langle \gamma_i / b_i \rangle = M_f^r \\
\text{where}\quad \langle 1 / b_i \rangle = \left( [H H^H]^{-1} \right)_{ii}. \quad (3)
\]

The objective for maximization in (2) is concave in the variables \( \gamma_i \) with \( i = 1, \ldots, M_S \), and can be maximized using Lagrangian methods and meeting the Kuhn-Tucker conditions.

The energy allocation policy is then

\[
\gamma_i = (\mu_i b_i - A^{-1})_+, \quad i = 1, \ldots, M_S \quad \text{and} \quad \mu_i = M_f^r \left( 1 + N_0 / E_s \sum_{i=1}^{M_S} (b_i)_[+] \right)^{-1}, \quad (4)
\]

where \( x_+ = \max(0, x) \), subjected to the power constraint in (3) and \( \mu_i \) is the water-level. Hence the energy allocation policy for TZFBF has the form \( \gamma_i = (\mu_i b_i - c_i)_+ \), subjected to \( \Sigma\{\gamma_i / b_i\} = M_f^r \) instead of the classical form \( \gamma_i = (\mu_i - c_i)_+ \), subjected to \( \Sigma\{\gamma_i\} = M_f^r \). However, relying on the (4), power allocation policy as in (5) does not help optimize the TZFBF sum rate because the spatial mode gains \( \lambda_i \) are not explicit in the channel rate expression (2). Instead, they are captured within the \( 1 / b_i \) values as

\[
\{1 / b_i\} = \left( [H H^H]^{-1} \right)_{ii} = \left( [U \Sigma U^H]^{-1} \right)_{ii} \\
= \left\{ \sum_{j=1}^{M_S} u_{ij} u_{ij}^T \lambda_j \right\}_i, \quad i = 1, \ldots, M_S, \quad (7)
\]

where \( u_{ij} \) are elements of the matrix \( U \) from the channel SVD. Equation (7) shows that each \( 1 / b_i \) value is influenced by the entire set of channel mode gains \( \{\lambda_j\} \ ; j = 1, \ldots, M_S \). Hence, poor spatial mode gains will result in large magnitudes across all \( 1 / b_i \) values. When subjected to (3), low \( \gamma_i \) values will arise and result in low channel rates. Due to these reasons, using the (4), \( \mu_i \) is a good policy alone as in classical waterfilling does not help remove the effect of poor spatial modes. To change the channel mode gains \( \lambda_j \), consider the following. Let \( H_S \) represent a channel matrix that has a row vector removed from \( H \). Let \( \sigma_{\max}(H_S) \geq \sigma_2(H_S) \geq \ldots \geq \sigma_{\min}(H_S) \) and \( \sigma_{\max}(H) \geq \sigma_2(H) \geq \ldots \geq \sigma_{\min}(H) \) be their singular values respectively. Noting that \( M_S \leq M_f^r \), then

\[
\sigma_{\max}(H_S) \geq \sigma_{\max}(H) \geq \sigma_2(H_S) \geq \ldots \geq \sigma_{\min}(H_S) \geq \sigma_{\min}(H) \quad \text{where} \quad S \subseteq S^*, \quad (8)
\]

Hence the extremal singular values of \( H \) lie between those of \( H_S \). In addition, the lower singular values of \( H \) are increased. If these increases outweigh the decreases in the higher singular values, a better set of \( 1 / b_i \) values may result and the channel rate may then be increased. Eliminating a row vector in \( H \), is equivalent to dropping a receive antenna. The new values of \( \sigma_{\max}(H_S) \) and \( \sigma_{\min}(H_S) \) will depend on the row vector that was eliminated from \( H \), and a judicious choice must be made. This is tantamount to receive antenna selection where a simple exact solution is lacking, as pointed out by e.g., [11], [12]. The only mechanism for optimum selection of antennas is an exhaustive search of all possible combinations. The number of iterations needed for an exhaustive search has exponential complexity order \( O(2^{M_f^r}) \) and is \( \Sigma_f(1, C_{pp}) = 2^{M_f^r} - 1 \), where \( S \subseteq S^* \) is the active receiver set with \( 1 \leq |S| \leq M_f^r \) and the resulting channel sub-matrix \( H_S \subset H \). At this juncture, it is clear that receive antenna selection (RAS) is required within the TZFBF sum rate optimization process in order to remove the causes of poor spatial modes. Waterfilling must be performed on each antenna combination to evaluate the channel rate and the subset with the maximum rate is chosen

\[
C(H^*_S)_{\text{TZFBF}} = \max_{S \subseteq S^*, \{1 / b_i\}} \left( \sum_{i=1}^{M_S} \log_2 (1 + A \gamma_i) \right) \quad (9)
\]

where \( H^*_S \subset H \ ; \quad M_f^r = \text{rank}(H_S) = |S| \).

One approach to help reduce the high computational complexity associated with an exhaustive search is to find a...
way of emulating the convenient process available in classical waterfilling where the poorest spatial mode is dropped at each turn. Since RAS is involved, this could be realized by using sub-optimal antenna selection algorithms (examples in [13] or [14]) in tandem with waterfilling. This forms a 2-step process where a RAS algorithm is first invoked to choose an active user subset and waterfilling is then performed to evaluate the rate of that subset. Unlike in classical waterfilling, the stopping criterion is not clear and all $M_T$ row vectors of $H$, must be examined. However, this helps reduce the search complexity from exponential order $O(2^{M_T})$ to linear $O(M_T^2)$. Antenna selection algorithms can generally be classified into two categories, viz., incremental selection or decremental selection. The former chooses the next best antenna whereas the latter drops the next worst antenna. In this way, both algorithm types require $M_T$ iterations in the 2-step process discussed above. The computational complexity for this 2-step process depends largely on the chosen antenna selection algorithm. Within the TZFBF context, the upper bound computational complexity of the algorithms in [13] is $M_T^3$ for the incremental algorithm and $M_T^2(M_T-1)$ for the decremental algorithm. When waterfilling is included in each iteration, the computational complexity is $O(M_T^3 \cdot 2^s)$ and $O(M_T^3 \cdot 3^s)$ for $M_T \geq 4$. The computation complexity order for the incremental algorithm in [14] is $O(M_T^3) + \delta T$ and is around $O(M_T^3 k)$ for $M_T \geq 4$ when waterfilling is included in $M_T$ iterations. Note that the computational complexity of waterfilling is dominated by the evaluation of $(HH)^{1/2}$ and is upper bounded by $O(M_T^3 k)$. In the next section, we develop approaches to help reduce the computation complexity incurred by the RAS processing segment.

B. First Approach: Joint Waterfilling & Antenna Selection

The motivation starts from (3) where a $\gamma$ value will be low when its associated $1/b$ value is high. If users associated with high $1/b$ values could be removed from a set of users, then applying the same power constraint will mean higher values for the remaining $\gamma$ and (2) will increase. Dropping the “undesired” users corresponds to removing undesired rows of the channel matrix $H$, and an approach different from those in [13] and [14] is as follows. First, (4) can be expressed as

$$\{1/b; i = 1,...,M'_T\} = ([H,H]^H)^{-1}_i = \{A_{i1}\Delta_1^{-1},...,A_{ik}\Delta_k^{-1}\},$$

where $A_{ik}$ are the cofactors of the diagonal elements $h_{ij}$ in $H,H^H$ and $\Delta$ is the determinant of $H,H^H$. E.g., $A_{ik}$ is found after eliminating row $k$ and column $k$ in $H,H^H$. This corresponds to eliminating row $k$ in $H$, to give a sub-matrix $H$. Since $H,H^H$ is positive definite Hermitian, the Inclusion Principle [10] applies and $\det(H,H^H) = \Delta$. $H$ can be greater than $\det(H,H^H) = \Delta$, when the lower eigen-values of $H,H^H$, transit from $\lambda_i(H,H^H) < 1$ to $\lambda_i(H,H^H) \geq 1$ after the row elimination in $H$. From (10), we see that $\max(1/b)$ is associated with $\max(A_{ik}) = A_{i\text{max}}$, the largest cofactor. This means that eliminating row $m$ and column $m$ of $H,H^H$, will result in a $H,H^H$ that possesses the largest determinant ($\Delta_{i\text{max}} = A_{i\text{max}}$). This corresponds to removing row $m$ in $H$, to give $H$. At high SNR, the resulting one-antenna-reduced channel $H$ approaches the largest possible capacity among all such channels. A similar situation exists for TZFBF and this is shown in (12) where $\Delta_{iE}N_o$ dominates under high SNR. Equation (12) is derived as follows. Let $\gamma = (\mu_i b_i - c)_i$, represent the computation of $\gamma_i$ after removing row $k$ and then applying the $(\gamma_i = \max(0, \gamma_i))$ policy. As usual, the computation of the $1/b$ values and water level $\mu_i$ is done first before executing $(\gamma_i)_i$. The TZFBF rate expression (9) can be re-written as

$$C(H) = \sum_{i=1}^{M'_T} \log_2 \left(1 + \frac{\Delta_i E_s / N_o + \sum_{j=1}^{M'_T} (A_{ij})}{(A_{ij})^H M_R'} \right)$$

where $(A_{ij})$, mean cofactors associated with $H$. Considering only those row eliminations that result in non-negative power allocations, i.e., $\gamma_i \geq 0, \forall i$, (11) may be re-written as

$$C(H) = \sum_{i=1}^{M'_T} \log_2 \left(\frac{M_R' \log_2 \left(\Delta_i E_s / N_o + \sum_{j=1}^{M'_T} (A_{ij})\right)}{(A_{ij})^H M_R'} \right)$$

(12)

The impact of this approach on $(A_{ij})$ is evaluated next. Using [10] with $Z_{ij} = H,H^H$, $\prod_{i=1}^{M'_T-1} \lambda_i (Z_{ij}) \leq \det(Z_{ij}) \prod_{i=1}^{M'_T-1} \lambda_{1+i}(Z_{ij})$.

$$\Delta_i / \lambda_{\min}(Z_{ij}) \leq \lambda_{\text{max}}(Z_{ij}) \leq \lambda_{\text{max}}(Z_{ij}) \leq \lambda_{\min}(Z_{ij}) \leq \lambda_{\text{max}}(Z_{ij}) \leq \lambda_{\min}(Z_{ij})$$

(13)

where $(Z_{ij})_{M'_T\times1}$ is a $(M'_T \times 1)$ principal sub-matrix of $H,H^H$ and $(A_{ij})$ is the set of cofactors associated with the diagonal elements of $H,H^H$. Hence, a larger $\Delta_i$ value will generally result in larger $(A_{ij})$ values and helps maximize Term I in (12). However, there is no guarantee that this process will lead to the global maximum. Consider two search paths where $H_1$ and $H_2$ are sub-matrices arising from eliminating rows #1 and #2 in $H$, respectively. Assuming that $\Delta_{i1} > \Delta_{i2}$, the bounds given by (14) [10]

$$\lambda_{\text{min}}(Z_{ij}) \leq \lambda_{\text{min}}(Z_{ij}) \leq \lambda_{\text{max}}(Z_{ij}) \leq \lambda_{\text{max}}(Z_{ij}) \leq \lambda_{\text{min}}(Z_{ij})$$

(14)

are not tight enough to guarantee that $\max(A_{ij}) > \max(A_{ij})_i$ when applying (13). For example, there is a possibility that

$$\Delta_{i1} / \lambda_{\min}(H_1,H_1^H) < \Delta_{i2} / \lambda_{\min}(H_2,H_2^H)$$

(15)

Hence, the proposed one-antenna-reduction process is sub-optimal. A second reason for sub-optimality is that the rejected cases associated with negative power allocations could have contained a search path leading to the global maximum. A third reason for sub-optimality is that the chosen $(A_{ij})$, may not be the set that minimizes the product in Term II. We note however that Term II is less significant than Term I for all SNR values and channel conditions. This can be seen by applying the inequality of arithmetic- and geometric- means (AM-GM inequality), which results in

$$(M_R' \log_2 \sum_{i=1}^{M'_T} A_{ii} - M_R' \log_2 M_R') \geq \log_2 \prod_{i=1}^{M'_T} A_{ii},$$

(16)

with equality only when $(A_{ij}) = \{c\}$, where $c$ is a constant. Applying (16) to (12), we see that (Term I) > (Term II) even when the SNR is very small. This shows that considering only Term I for receive antenna selection is a reasonable approach even for very low SNR and regardless of the channel condition, which is reflected by the $(A_{ij})$ values. Since the original waterfilling process already requires computation of the $1/b$
values, the proposed receive antenna selection method does not add significant complexity. It has the effect of performing antenna selection *jointly* with the waterfilling process and removes the cause of poor spatial modes at each step. The 2-step RAS-waterfilling process reduces to a single joint process that emulates classical waterfilling but remains sub-optimal as explained above. For convenience, this decremental algorithm is referred to as JWFAS ("joint waterfilling and antenna selection") and its pseudo-code is given. From (12), JWFAS is expected to perform better at high SNR where *Term I* dominates. This is demonstrated by the numerical results which show that JWFAS performs better than the algorithms in [13] in many cases.

**C. Case of More Users Than Transmit Antennas**

Numerical results have shown that JWFAS outperforms the algorithms in [13] under a wide range of conditions with very little additional complexity. However, JWFAS is applicable only within the context of TZFBF waterfilling and the pre-coding constraint $|S| \leq M_T$ must be met. To exploit the benefits of JWFAS, we propose a two-phase framework to allow the incorporation of fast receive antenna selection methods in both phases. In Phase #1, a subset $S_r$ containing $M_r$ users is selected from the original pool of $K=M_R$ potential users. This can be done judiciously to maximize the channel rate [15] and exhaustive searches can be avoided through existing antenna selection algorithms such as [13] and [14]. In Phase #2, the TZFBF constraint is met and JWFAS can then be used for the selection of a subset $S \subseteq S_r$ where $1 \leq |S| \leq M_T$. Note that JWFAS in Phase #2 may be combined with either incremental or decremental selection algorithms from Phase #1.

**D. A Simple Second Approach**

Antenna selection methods for Phase #1 can generally be classified into incremental- and decremental- selection types. For incremental selection algorithms like Gorokhov’s Algorithm II in [13], the next best row in the original $M_R \times M_T$ channel matrix $\mathbf{H}$ is selected at each iteration to give $\mathbf{H}_r \subset \mathbf{H}$, where $\mathbf{H}_r$ is $M_T \times M_T$. Hence the selected subset of row vectors in $\mathbf{H}_r$ is *ordered*. This lends itself to a simple selection algorithm, named GA2 for convenience. GA2 simply eliminates the last row vector of $\mathbf{H}_r$ for each iteration during TZFBF waterfilling in Phase #2. A similar scheme is not possible however when decremental selection is used in Phase #1 in which the next worst row is de-selected in each iteration. This means that the resulting subset remains unordered. Hence the use of another antenna selection algorithm like JWFAS is needed in Phase #2 when decremental selection is applied in Phase #1.

**IV. NUMERICAL RESULTS**

The presence of spatial fading correlation in $\mathbf{H}$ is captured by modeling the channel as $\mathbf{H} = \mathbf{R}_r^\dagger \mathbf{H}_w \mathbf{R}_t$, where $\mathbf{H}_w$ is the i.i.d. spatially white channel and $\mathbf{R}_r$ and $\mathbf{R}_t$ are positive definite Hermitian matrices that specify the receive and transmit correlations respectively. A simple constant correlation model is used. To evaluate the effects of different angle spreads, the one-ring correlation model formulated in [16] is used. A system with $M_T=8$ is chosen to obtain insight into the algorithms’ performance. Fig.1 compares the performance of JWFAS, GA2 and GA3 (Gorokhov’s Decremental Selection Algorithm III in [13]) versus exhaustive search. The ergodic and 10%-outage rates over SNR are shown under flat Rayleigh fading conditions with zero correlation. The GA2 performance is slightly inferior to all others and will not be mentioned further. Fig.2 provides an amplified view of JWFAS and GA3 by expressing the performance as rate difference ratios defined as $(C_{\text{eh}} - C_{\text{alg}})/C_{\text{eh}}$ which is the ratio of the difference between the exhaustive search rate and an algorithm’s rate over the exhaustive search rate. The subplots are done at correlations of 0.00, 0.50 and 0.95. For the most part, JWFAS appears to have the potential to outperform GA3 and this is particularly pleasing, since JWFAS does not incur any significant additional computational load to the original TZFBF waterfilling process.

An extension of JWFAS that includes *Term II* in the RAS process is possible by comparing the rates arising from the highest and next highest $1/b_i$ values. Named ALT1, it offers better performance (see also Fig.2) at a search complexity of $O(2M_T)$. Other extensions to JWFAS are possible but not discussed here due to the lack of space. Fig.3 shows the performance versus correlation at 20dB and 5dB SNR. Fig.4 shows performance versus angle spread at 20dB SNR. As

![Figure 1: Channel Rate Performance vs. SNR](image-url)
shown, JWFAS and its derivative ALT1 perform better than GA3 over a wide range of correlation and SNR values. They are slightly inferior to GA3 when correlation exceeds 0.8 under low SNR.

V. CONCLUSION

We have shown that receive antenna selection (RAS) is needed during the transmit zero-forcing beamforming (TZFBF) sum rate maximization process. Efficient RAS algorithms that can be jointly executed during waterfilling has been developed. Computational savings are realized by avoiding separate processes for RAS and waterfilling. In fact, the RAS algorithms add insignificant loads to the original TZFBF waterfilling process. The algorithms JWFAS and its derivative ALT1 were numerically shown to outperform current RAS algorithms like those in [13] for channel conditions of practical interest. When there are more users than transmit antennas, we proposed a 2-phase framework that facilitates the incorporation of RAS algorithms. A straightforward RAS algorithm in Phase #2 is possible under this framework when incremental RAS is first applied in Phase #1.

ACKNOWLEDGMENT

The authors gratefully acknowledge the funding for this work provided by DSO National Laboratories (Singapore), Natural Sciences and Engineering Research Council (NSERC) of Canada, Alberta Informatics Circle of Research Excellence (iCORE), Rohit Sharma Professorship, and TRLabs.

REFERENCES