Asymptotically Optimal Power Allocation and Code Selection for Iterative Joint Detection of CDMA

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**Iterative “Turbo” Detection of Coded CDMA Systems**

The CDMA “Channel” Joint access of $K$ users with individual spreading sequences.

**Conventional Correlation Detection** Create parallel single-user channels using filtering matched to spreading sequences.

**Linear Joint Detection** Various filter techniques, most prominently MMSE filtering.

**The Turbo Principle** is applied to the serial concatenation of an outer error control code with the inner CDMA channel.

**Early Work on Iterative CDMA Detection**

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Iterative Joint Detection — A Dynamical System Interpretation

The Turbo Principle implies a dynamical system model with the signal variance as parameter:

\[ y \Lambda(d_{i}^{(1)}) \Lambda(d_{i}^{(2)}) \Lambda(d_{i}^{(K)}) \]

\[ \tanh() \tanh() \tanh() \]

\[ \sigma_{eff}^{2} \sigma_{d}^{2} \]

Convergence of this system can be studied by examining the evolution of the values of the variances. The PDFs at the output of the filters are Gaussian according to the CLT.

Iterative “Turbo” Detection – Fundamental Observations

CDMA Turbo Detection: exhibits a “Turbo Cliff” and an “Error Floor”.

\[ g(\sigma_d^2) \]

\[ f_{IC}(\sigma_{eff}^2) \]

\[ \text{Canceller Noise Variance} \]

\[ \text{FEC Decoder Variance} \]

Bit Error Rate

Rate $\frac{1}{3}$ CC ($\nu = 2$)

\[ K = 45 \]

\[ N = 15 \]

\[ \beta = 3 \]

dual-stage filter

Note: Analysis and discussion of large-scale cancellation systems:


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**Optimal Power Distribution Profiles**

**“Strong” Codes:**
J groups of users with partial loads $\alpha_j$.

The variance transfer function is idealized (upper-bounded) for strong codes by

$$\sigma^2_d = g\left(\frac{\sigma_m^2}{P_j}\right) = \begin{cases} 1 & \text{if } \sigma_m^2 \geq \tau P_j \\ 0 & \text{if } \sigma_m^2 < \tau P_j \end{cases}$$

$\tau$ is the idealized SNR threshold.

**Optimal Power Levels:**
which maximize the slope of the load curve:

$$P_j = \frac{\sigma^2}{\tau^{1-j}} \prod_{j'=1}^{j} \frac{1}{\tau - \alpha_{j'}} \quad 1 \leq j \leq J$$

the optimized signal-to-noise ratio is

$$\left[\frac{E_b}{N_0}\right]_{\text{opt}} = \frac{1}{2R\alpha} \left[ \left(\frac{\tau}{\tau - \alpha/J}\right)^J - 1 \right]$$
The average variance transfer curve of any code approaches a straight line as $J \to \infty$.

As $J \to \infty$, and $R \to 0$, the capacity of an AWGN channel is achieved:

$$\left[ \frac{E_b}{N_0} \right]_{\text{lim}} = \frac{2^{2C} - 1}{2C}; \quad C = R\alpha$$

We are working on a proof of this conjecture:

Using any code in conjunction an with optimal power allocation can achieve the AWGN channel capacity.
Equal Power Users $J = 1$

We are working on a proof of Conjecture 2:

Using iterative turbo detection, the achievable total spectral efficiency of the system is minimized by the equal power assignment $J = 1$.

This is in marked contrast to what is known for linear filters, in particular MMSE filters:

The total spectral efficiency of a CDMA system using an MMSE filter to separate the different users is maximized by the equal power assignment.

Observation “Weak” codes with smoother variance transfer functions support higher system loads.

We need to consider the resulting Binary-Input Gaussian Channel, leading to the system capacity:

$$C = \alpha RC_B \left( \frac{P}{\sigma_{\text{llr}}^2(\alpha, C)} \right)$$

- $\sigma_{\text{llr}}^2$ is the variance of the output information bit log-likelihood ratio.

Iterative decoding with weak codes may be incomplete.
Repetition Codes for Equal Power CDMA

Observation:
Among a large number of codes analyzed, they provide the highest spectral efficiency.

Calculation of extrinsic LLRs is simple:

$$\lambda^{(E)}(d_i) = \sum_{i' \neq i} \lambda(y_i) = \frac{2}{\sigma^2} \sum_{i' \neq i} y'_i$$

The LLRs and extrinsic LLRs are exactly Gaussian distributed.

Low rate codes perform best (Rate $R = 1/L$):

Lemma:

The system spectral efficiency of iteratively decoded CDMA using repetition codes is maximized for $L \to \infty$.

Using the following inequality:

$$E \left[ 1 - \tanh \left( b^2 + b \xi \right) \right]^2 \leq \min \left\{ \frac{1}{1+b^2}, \pi Q(b) \right\}$$

we find:

$$\sigma^2 \leq \frac{1}{2} \left( \alpha' - 1 + \frac{\sigma^2}{L-1} + \sqrt{\left( \alpha' - 1 + \frac{\sigma^2}{L-1} \right)^2 + \frac{4\sigma^2}{L-1}} \right)$$
Conclusions and some Speculations

- An iterative cancellation detector using “weak” FEC codes should be viewed as an iterative nonlinear filter.
- As such, using $R = 1/L$ repetition codes, the iterative filter with load $\alpha$ and equal powers has a performance lower-bounded by that of an MMSE filter with load $\alpha/(L-1)$.
- Note that both information rate loads are identical.
- For large signal-to-noise ratios, the iterative cancellation filter strictly outperforms the MMSE filter.

Conjecture 3:
For unequal power levels, the performance of the iterative filter is strictly larger that that of an MMSE filter.

Conjecture 4:
The iterative cancellation filter using repetition codes can achieve the maximum capacity of the CDMA channel using optimum power levels.