Iterative Joint Detection Using Recursive Signal Cancellation

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The Random CDMA Channel

Information Theoretic CDMA Capacity

The capacity of the random CDMA channel is obtained by averaging the Shannon capacity of a fixed CDMA channel:

\[ C = E \left[ \frac{1}{N} \log \left| I + \frac{1}{\sigma^2} SS^T \right| \right] \]

- \( S \) is the matrix of spreading sequences, \( N \) the processing gain.
- Error Control Codes exploit the information theoretic capacity of the multiple access channel.
- For \( N > K \) linear filters are sufficient in approaching the information theoretic capacity of the channel.
- For \( K > N \) sophisticated joint detection methods are required to harness the channel capacity.
Linear Detection of Coded CDMA Systems

**Linear Detection**

The received filtered signal

\[ y_{MF} = S^* y = S^* S d + S^* n = R d + \hat{n} \]

is recovered with an inversion:

\[ y_{MMSE} = (R + \sigma^2 I)^{-1} y_{MF} \]

\[ y_{DEC} = R^{-1} y_{MF} \]

![Diagram of CDMA system](attachment:image.png)
Iterative Filter Implementation

Iterative Matrix Solution Methods
Consider $Mx = v$ and a splitting

$$M = S - T$$

$$Sx = Tx + v$$

leads to an iterative equation for $x$:

$$x^{(j+1)} = S^{-1} \left( Tx^{(j)} + v \right)$$

**Convergence:** Consider

$$S(x + e^{(j+1)}) = T(x + e^{(j)}) + v$$

$$Se^{(j+1)} = Te^{(j)}$$

$$e^{(j+1)} = S^{-1}Te^{(j)} = (S^{-1}T)^j e^{(0)}$$

and the solution always converges if

$$e^{(j+1)} = QA^j Q^* e^{(0)} < 1$$

$$\rho(S^{-1}T) < 1$$

where $\rho(M)$ is the spectral radius of $M$

Jacobi Iterative Receiver
The splitting matrix $S = I$ and

$$x^{(j+1)} = x^{(j)} - Mx^{(j)} + v$$

basic parallel cancellation receiver

First-Order Stationary Iterative Receiver
The splitting matrix is identical to the Jacobi method, but

$$x^{(j+1)} = x^{(j)} - \tau \left( Mx^{(j)} - v \right)$$

improved parallel cancellation with better convergence behavior

Gauss-Seidel Iterative Receiver
The splitting of the matrix $M$ is

$$M = \text{diag}(M) - L - L^T; \quad L : \text{lower triangular}$$

$$(\text{diag}(M) - L) x^{(j+1)} = L^T x^{(j)} + v$$

improved convergence achieved through serial cancellation
Iterative Filter Implementation

Iterative or Multistage Implementation uses recursive signal cancellation to achieve an approximation to a matrix filters such as the MMSE:

\[ r_k = S_d - \hat{S_d} + n \]

Capacity of Multi-Stage Filters
Multi-stage filter implementations can be characterized by their information theoretic capabilities: (shown for \( M = R \))
Iterative “Turbo” Detection of Coded CDMA Systems

The CDMA “Channel” If higher spectral efficiencies are required, a “true” joint detection method is necessary.

Signal Cancellation Just as in the case of linear multi-stage filters, an estimated signal is reconstructed and subtracted from the received signal. The iteration loop includes the error control decoder, which generates soft-estimates of the transmitted sequences.

The Turbo Principle is applied to the serial concatenation of an outer error control code with the inner CDMA channel.

Early Work on Iterative CDMA Detection


Iterative Joint Detection — A Dynamical System Interpretation

The **Turbo Principle** implies a dynamical system model with the signal variance as parameter:

\[
\sigma^2_{\text{eff}} = \frac{\sigma^2_d}{g(\sigma^2_{\text{eff}})} + f_{\text{IC}}(\sigma^2_d)
\]

Convergence of this system can be studied by examining the evolution of the values of the variances. The PDFs at the output of the filters are Gaussian according to the CLT.

Iterative “Turbo” Detection – Fundamental Observations

**CDMA Turbo Detection:** exhibits a “Turbo Cliff” and an “Error Floor”.

![Graph showing Iterative Turbo Detection](image)

- **FEC Decoder Variance** vs. **Canceler Noise Variance**
- **Interference Limitation** vs. **Noise Limitation**

**Note:** Analysis and discussion of large-scale cancellation systems:


Optimal Power Distribution Profiles

“Strong” Codes:
$J$ groups of users with partial loads $\alpha_j$.

The variance transfer function is idealized (upper-bounded) for strong codes by

$$\sigma_d^2 = g\left(\frac{\sigma_m^2}{P_j}\right) = \begin{cases} 1 & \text{if } \sigma_m^2 \geq \tau P_j \\ 0 & \text{if } \sigma_m^2 < \tau P_j \end{cases}$$

$\tau$ is the idealized SNR threshold.

Optimal Power Levels:
which maximize the slope of the load curve:

$$P_j = \frac{\sigma^2}{\tau^{1-j}} \prod_{j'=1}^{j} \frac{1}{\tau - \alpha_j} \quad 1 \leq j \leq J$$

the optimized signal-to-noise ratio is

$$\left[\frac{E_b}{N_0}\right]_{\text{opt}} = \frac{1}{2R\alpha} \left[\left(\frac{\tau}{\tau - \alpha/J}\right)^J - 1\right]$$
**Equal Power Users** $J = 1$

Using iterative turbo detection, the achievable total spectral efficiency of the system is minimized by the equal power assignment $J = 1$, the Shannon bound can no longer be achieved.

This is in marked contrast to what is known for linear filters, in particular MMSE filters:

The total spectral efficiency of a CDMA system using an MMSE filter to separate the different users is maximized by the equal power assignment.

**Observation** “Weak” codes with smoother variance transfer functions support higher system loads.

Iterative decoding with weak codes may be incomplete.

That is, weak codes in iterative systems are non-linear filters, improving the signal-to-noise ratio of each component channel.
Repetition Codes for Equal Power CDMA

Observation:
Among a large number of codes analyzed, they provide the highest spectral efficiency.

Calculation of extrinsic LLRs is simple:

\[ \lambda^{(E)}(d_i) = \sum_{i' \neq i} \lambda(y_i) = \frac{2}{\sigma^2} \sum_{i' \neq i} y_{i'} \]

The LLRs and extrinsic LLRs are exactly Gaussian distributed.

Low rate codes perform best (Rate \( R = 1/L \)):

Lemma:
The system spectral efficiency of iteratively decoded CDMA using repetition codes is maximized for \( L \to \infty \).

Using the following inequality:

\[ E \left[ 1 - \tanh \left( b^2 + b\xi \right) \right]^2 \leq \min \left\{ \frac{1}{1 + b^2}, \pi Q(b) \right\} \]

we find:

\[ \sigma_\infty^2 \leq \frac{1}{2} \left( \alpha' - 1 + \frac{\sigma^2}{L-1} + \sqrt{\left( \alpha' - 1 + \frac{\sigma^2}{L-1} \right)^2 + \frac{4\sigma^2}{L-1}} \right) \]
Conclusions

Statement 1:
Joint Detection is quasi-synonymous with iterative cancellation detection.

Statement 2:
For low system loads, iterative filter approximations may be sufficient to achieve Shannon’s capacity.

Statement 3:
The cancellation architecture is identical in both cases, they differ only in the iterative processing loop, i.e., if the error control codes are included or not.

Statement 4:
Judicious power (or rate) assignments can achieve the Shannon bound for iterative cancellation with FEC, while unequal power assignments degrade the overall capacity of linear filter receivers.