Optimal Resource Allocation in Wireless Powered Relay Networks with Nonlinear Energy Harvesters

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***Abstract*—This paper investigates a multiple-relay decode-and-forward (DF) network with simultaneous wireless information and power transfer (SWIPT). The relays use the power splitting (PS) technique to receive radio frequency (RF) energy and information simultaneously from the source. To characterize nonlinearity of energy harvesters at the relays, two existing models are adopted, which are the logistic function model and the constant-linear-constant (CLC) model. We target end-to-end throughput maximization by optimizing the power and bandwidth assignment for every source-relay-destination link and the PS ratio at every relay node. The formulated problems are demonstrated to be nonconvex. Through a series of analysis and transformations, we find global optimal solutions for the formulated problems. Numerical results verify the effectiveness of our proposed methods.**

***Index Terms*—Simultaneous wireless information and power transfer (SWIPT), multiple relays, throughput maximization, nonlinear energy harvesting model.**

I. Introduction

Simultaneous wireless information and power transfer (SWIPT) is a promising technical solution for energy-constrained wireless networks, which enables a transmitter to send energy and information simultaneously to a receiver via the radio frequency (RF) signals [1], [2]. To realize SWIPT, the receiver may use the power splitting (PS) technique, in which the received signal is split into two portions, which are used for energy harvesting (EH) and for information decoding (ID), respectively [3]. By adjusting the PS ratio between EH and ID, the rate of information transmission and the rate of energy harvesting can be balanced [4], [5].

A special application of SWIPT lies in relay networks, in which a relay node with no battery extracts both energy and information from the source's signal through SWIPT, and then forwards the received signal (in amplify-and-forward (AF) mode or decode-and-forward (DF) mode) to the destination by using the harvested energy. In such a SWIPT-powered relay network, it is not necessary to equip relay nodes with external battery supply [4]–[11]. For SWIPT-powered relay networks, ergodic capacity analysis and/or outage probability analysis are performed in [4] for DF relay networks, in [5] for AF relay networks, in [6] for AF relay networks with multiple-antenna relay and co-channel interference, and in [7] for AF networks with relay selection from multiple randomly distributed relays. PS ratio optimization is performed in [8] that maximizes the outage capacity and in [9]–[11] that maximize end-to-end throughput. In [8], PS ratio is optimized in a DF relay network. In [9], multiple antennas are assumed at an AF relay, and PS ratio and antenna selection strategy are optimized jointly. In [10], beamforming vector and PS ratio are optimized with multiple antennas implemented at the source node, AF relay, and destination node. In [11], PS ratio is optimized over multiple channels separately.

For all these works [4]–[11] for SWIPT-powered relay networks, a linear model is assumed for the energy harvester at a relay node, which means that the output power of the energy harvesting circuit grows linearly with the power of input RF signal. However, measurements [12] show that output power of a practical energy harvester is non-linear with the input RF signal power. In the literature, two models to characterize the non-linearity are shown to match well with the measurement data, as follows. A *logistic function model* is used in [12], which models the energy harvester output as a logistic function of the input RF signal power. In another model in [13], the energy harvester output is modelled as a linear function of the input RF signal power when the input RF signal power is between two thresholds, referred to as the *constant-linear-constant (CLC) model*. There is limited work in the literature for performance analysis in SWIPT-powered relay networks with a nonlinear energy harvester. The work in [14] investigates the system secrecy outage probability for a SWIPT-powered single-relay system under the logistic function model.

In this paper, we investigate throughput maximization in a SWIPT-powered multiple-relay network considering non-linear energy harvesters at the relays. Multiple DF relays with the PS technique are used. End-to-end throughput is targeted to be maximized by optimizing the transmit power and bandwidth on every source-relay-destination link and the PS ratio at every relay node. Two optimization problems are formulated for the logistic function model and the CLC model, respectively. Both problems are shown to be non-convex. For the case with the logistic function model, with a series of transformations, the original optimization problem is transformed to a standard monotonic optimization problem, whose global optimal solution can be achieved by using a polyblock algorithm. For the case with the CLC model, we first transform the optimization problem, and then derive a semi-closed-form optimal solution for the transformed problem. The major differences between this paper and [14] are as follows. 1) The work [14] investigates closed-form expressions of

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system secrecy outage probability, while we target end-to-end throughput maximization. 2) The work [14] considers logistic function model, while we consider both logistic function model and the CLC model. 3) The work [14] considers a single relay, while we consider multiple relays. 4) The work [14] focuses on performance analysis, but does not analytically find the optimal system configuration. We focus on analytically deriving the optimal configuration of the system.

II. System Model and Problem Formulation

Consider a two-hop multiple-relay network, in which a source node denoted as S would like to send its information to its destination denoted as D. A direct link from S to D does not exist due to physical obstacles [5]. Thus, S uses N relays denoted as $R_1, R_2, ..., R_N$ to help forward its signals. The relays have no external power supply, and thus, they use the PS technique to harvest energy and decode information from S. Denote the link $S \rightarrow R_n \rightarrow D$ as link n, for $n \in \mathcal{N} \triangleq \{1, 2, ..., N\}$. The channel gain from S to R_n is h_n, and the channel gain from R_n to D is g_n. In the system, all the channel gains keep stable in one fading block, and are randomly and independently distributed over fading blocks following a distribution function. Denote σ^2 as the power spectrum density of noise at each node.

S has total transmit power denoted as p_T and total bandwidth denoted as w_T. It assigns bandwidth w_n and transmit power p_n for its transmission to relay R_n. Thus, we have $\sum_{n=1}^{N} w_n = w_T$, and $\sum_{n=1}^{N} p_n = p_T$.

Time is divided into frames, and the length of each frame is equal to a fading block duration denoted as T. One frame is equally divided into two phases, each with time duration $\frac{T}{2}$. Consider a target frame. In the first phase, relay R_n ($n \in \mathcal{N}$) uses the PS technique for SWIPT: a fraction β_n ($0 \leq \beta_n \leq 1$) of the received signal's power expressed as $p_T h_n$ is used for energy harvesting, and the remaining $(1 - \beta_n)$ of the received signal power is for information decoding. To boost the energy harvester, the power of received energy $p_T h_n \beta_n$ should be not less than a threshold, denoted as E_{th}, which requires $\beta_n \geq \frac{E_{\text{th}}}{p_T h_n} \triangleq \beta_n^{\min}$ for $n \in \mathcal{N}$. In the second phase of the target frame, R_n forwards to the destination D with transmit power q_n over its assigned bandwidth w_n. Hence the transmit power is expressed as $q_n = \frac{\frac{T}{2}\phi(p_T h_n \beta_n)}{\frac{T}{2}} = \phi(p_T h_n \beta_n)$. Here $\phi(\cdot)$ is the output power of the energy harvester as a function of the input RF signal power. For the nth link, the throughput from S to R_n is $\frac{w_n}{2} \log\left(1 + \frac{p_n h_n (1-\beta_n)}{\sigma^2 w_n}\right)$, and the throughput from R_n to D is $\frac{w_n}{2} \log\left(1 + \frac{\phi(p_T h_n \beta_n) g_n}{\sigma^2 w_n}\right)$. As DF relaying mode is used, the throughput on link n can be written as $\frac{1}{2} \min\{w_n \log(1 + \frac{p_n h_n (1-\beta_n)}{\sigma^2 w_n}), w_n \log(1 + \frac{\phi(p_T h_n \beta_n) g_n}{\sigma^2 w_n})\}$ according to [15], and the end-to-end throughput from S to D over all relays can be written as

$$\begin{aligned} & C(\{\beta_n\}, \{p_n\}, \{w_n\}) \\ & \triangleq \frac{1}{2} \sum_{n=1}^{N} \min \left\{ w_n \log\left(1 + \frac{p_n h_n (1-\beta_n)}{\sigma^2 w_n}\right), \right. \\ & \left. w_n \log\left(1 + \frac{\phi(p_T h_n \beta_n) g_n}{\sigma^2 w_n}\right) \right\}. \end{aligned} \tag{1}$$

For an energy harvester, experimental measurements [12] show that, when the input RF power is above a threshold, output power of the energy harvester grows with the input RF power, but the growth rate becomes smaller and smaller until the output power saturates. To approximate this feature, two models are introduced in the literature. **1) Logistic function model**: In this model, $\phi(x)$ is written as $\phi^L(x) = \left(\frac{M}{1+e^{-a(x-b)}} - \frac{M}{1+e^{ab}}\right) / \left(1 - \frac{1}{1+e^{ab}}\right)$, where M represents the maximal power that the energy harvester can harvest, and a and b are nonlinearity parameters. This model is used in [12], [16]. **2) CLC model**: In this model, $\phi(x)$ is written as

$$\phi^C(x) = \begin{cases} 0, \text{when } x < x_L, \\ c(x - x_L), \text{when } x_L \leq x \leq x_U, \\ c(x_U - x_L), \text{when } x > x_U \end{cases} \tag{2}$$

where x_L and x_U are two thresholds, and c is a parameter to characterize the growth rate of $\phi^C(x)$ with x between the two thresholds. This model is used in [13].

We target maximal end-to-end throughput by optimizing the transmit power and bandwidth on every source-relay-destination link and the PS ratio at every relay node. Accordingly, the following optimization problem is formulated:

Problem 1:

$$\max_{\{\beta_n\},\{p_n\},\{w_n\}} \quad C(\{\beta_n\}, \{p_n\}, \{w_n\})$$

$$\text{s.t.} \quad \beta_n^{\min} \leq \beta_n \leq 1, \forall n \in \mathcal{N}, \tag{3a}$$

$$p_n \geq 0, w_n \geq 0, \forall n \in \mathcal{N}, \tag{3b}$$

$$\sum_{n=1}^{N} w_n = w_T; \sum_{n=1}^{N} p_n = p_T. \tag{3c}$$

Considering the non-convexity of $\phi^L(x)$ and $\phi^C(x)$, it can be checked that Problem 1 is a non-convex optimization problem, whose optimal solution is hard to find. In Section III, we will address this challenge, as follows. In Problem 1, the optimization variables are $\{\beta_n\}, \{p_n\}, \{w_n\}$. For the logistic function model or CLC model, we first convert Problem 1 to an equivalent problem with optimization variables $\{\beta_n\}, \{w_n\}$. The equivalent problem is further converted to another equivalent problem with optimization variables $\{\beta_n\}$. After solving the last equivalent problem, we get optimal solution of Problem 1.

III. Optimal Solution

A. The Case under the Logistic Function Model

In this subsection, the logistic function model is adopted to characterize the energy harvesters, i.e., $\phi^L(x)$ defined in Section II is adopted.

For Problem 1, the objective function is expressed in (1). In the expression, the term $w_n \log\left(1 + \frac{p_n h_n (1-\beta_n)}{\sigma^2 w_n}\right)$ is a decreasing function with β_n, and the term $w_n \log\left(1 + \frac{\phi^L(p_T h_n \beta_n) g_n}{\sigma^2 w_n}\right)$ is an increasing function with β_n considering that $\phi^L(x)$ is an increasing function of x. Hence, when the objective function of Problem 1 achieves its maximal, the above two terms $w_n \log\left(1 + \frac{p_n h_n (1-\beta_n)}{\sigma^2 w_n}\right)$ and $w_n \log\left(1 + \frac{\phi^L(p_T h_n \beta_n) g_n}{\sigma^2 w_n}\right)$ should be equal, i.e., we should have

$$p_n h_n (1 - \beta_n) = \phi^L(p_T h_n \beta_n) g_n, \tag{4}$$

which further indicates $p_n = \frac{\phi^L(p_T h_n \beta_n) g_n}{h_n(1-\beta_n)}$. Together with the fact that $p_n \le p_T$, we have an upper bound of β_n, denoted as $\hat{\beta_n} \in (0,1)$, such that $\frac{\phi^L(p_T h_n \hat{\beta_n})}{(1-\hat{\beta_n})} = \frac{p_T h_n}{g_n}$. Together with constraint (3a), we know that β_n should be bounded as $\beta_n^{\min} \le \beta_n \le \hat{\beta_n}, \forall n \in \mathcal{N}$.

By replacing p_n with $\frac{\phi^L(p_T h_n \beta_n) g_n}{h_n(1-\beta_n)}$ and by relaxing the two constraints in (3c) as $\sum_{n=1}^{N} w_n \le w_T$ and $\sum_{n=1}^{N} p_n \le p_T$, respectively,[1] Problem 1 is equivalent to the following problem

Problem 2:

$$\max_{\{\beta_n\},\{w_n\}} \ \frac{1}{2}\sum_{n=1}^{N} w_n \log\left(1+\frac{\phi^L(p_T h_n \beta_n) g_n}{\sigma^2 w_n}\right)$$
$$\text{s.t.} \quad \beta_n^{\min} \le \beta_n \le \hat{\beta_n}, \forall n \in \mathcal{N}, \tag{5a}$$
$$w_n \ge 0, \forall n \in \mathcal{N}, \tag{5b}$$
$$\sum_{n=1}^{N} w_n \le w_T, \tag{5c}$$
$$\sum_{n=1}^{N} \frac{\phi^L(p_T h_n \beta_n) g_n}{h_n(1-\beta_n)} \le p_T. \tag{5d}$$

Problem 2 is non-convex. Here we first investigate Problem 2 for given $\beta_1, \beta_2, ..., \beta_N$. With given $\beta_1, \beta_2, ..., \beta_N$, the objective function of Problem 2 is concave with $\boldsymbol{w} \triangleq (w_1, w_2, ..., w_N)^T$ (where $(\cdot)^T$ denotes the transpose operation), as $\partial^2\{w_n \log\left(1+\frac{\phi^L(p_T h_n \beta_n) g_n}{\sigma^2 w_n}\right)\}/\partial(w_n)^2 = -\frac{\left(\phi^L(p_T h_n \beta_n) g_n\right)^2}{w_n(w_n\sigma^2+\phi^L(p_T h_n \beta_n) g_n)^2} \le 0$. Further, all the constraints of Problem 2 are linear with $\boldsymbol{w}$. Thus, Problem 2 with given $\beta_1, \beta_2, ..., \beta_N$ is a convex optimization problem with $\boldsymbol{w}$. In addition, Problem 2 with given $\beta_1, \beta_2, ..., \beta_N$ satisfies the Slater condition [17]. In this case, KKT condition is a sufficient and necessary condition for the optimal $w_n, \forall n \in \mathcal{N}$ [17], which can be given as follows

$$\ln\left(1+\frac{\phi^L(p_T h_n \beta_n) g_n}{\sigma^2 w_n}\right) - \frac{\phi^L(p_T h_n \beta_n) g_n}{\phi^L(p_T h_n \beta_n) g_n + w_n \sigma^2} + \mu_n - \nu = 0, \tag{6a}$$
$$\mu_n w_n = 0, \forall n \in \mathcal{N}; \nu\left(w_T - \sum_{n\in\mathcal{N}} w_n\right) = 0, \tag{6b}$$
$$\mu_n \ge 0, \forall n \in \mathcal{N}; \nu \ge 0, \tag{6c}$$
$$\text{Constraints}(5b),(5c), \tag{6d}$$

where μ_n and ν are Lagrange multipliers associated with the constraints (5b) and (5c), respectively. For $n \in \mathcal{A} \triangleq \{n'|w_{n'} > 0\}$, it can be checked that $\mu_n = 0$ from (6b). Then from (6a) we know that $f(\frac{\phi^L(p_T h_n \beta_n) g_n}{\sigma^2 w_n}) = \nu$ with $f(x) \triangleq \ln(1+x) - x/(1+x)$. Thus, we have $\frac{\phi^L(p_T h_n \beta_n) g_n}{\sigma^2 w_n} = \gamma \triangleq f^{-1}(\nu)$. Note that to be physically meaningful, both ν and $f^{-1}(\nu)$ should be less than infinity. For $n \in \mathcal{N} \setminus \mathcal{A} = \{n'|w_{n'} = 0\}$, from (6a) we know that $\frac{\phi^L(p_T h_n \beta_n) g_n}{\sigma^2 w_n} = f^{-1}(\nu - \mu_n) \le f^{-1}(\nu) < \infty$, which implies that we should have $\phi^L(p_T h_n \beta_n) g_n = 0$. As an overall result, for $n \in \mathcal{N}$, we have $w_n = \frac{\phi^L(p_T h_n \beta_n) g_n}{\sigma^2 \gamma}$. The above discussion also indicates that constraint (5b) would be non-binding unless $\phi^L(p_T h_n \beta_n) g_n = 0$. For constraint (5c), it is binding due to the following two reasons: 1) The objective function of Problem 2 is increasing with w_n since the partial derivative of the objective function with respect to w_n is nonnegative. 2) The left-hand function of constraint (5c) is increasing with w_n for $n \in \mathcal{N}$. Hence, we have $\sum_{n\in\mathcal{N}} w_n = w_T$ at optimality. Accordingly, γ can be derived as $\gamma = \frac{1}{w_T \sigma^2}\sum_{n=1}^{N} \phi^L(p_T h_n \beta_n) g_n$. With this derived γ value, optimal w_n ($n \in \mathcal{N}$) for Problem 2 with given $\beta_1, \beta_2, ..., \beta_N$ is a closed-form expression of $\beta_1, \beta_2, ..., \beta_N$, given as $w_n = \frac{\phi^L(p_T h_n \beta_n) g_n}{\sigma^2 \gamma}$. Substituting these optimal closed-form expressions of $w_1, w_2, ..., w_N$ into Problem 2 and with some mathematical manipulations, Problem 2 transforms to the following equivalent optimization problem

Problem 3:

$$\max_{\{\beta_n\}} \ \sum_{n=1}^{N} \phi^L(p_T h_n \beta_n) g_n$$
$$\text{s.t.} \quad \beta_n^{\min} \le \beta_n \le \hat{\beta_n}, \forall n \in \mathcal{N}, \tag{7a}$$
$$\sum_{n=1}^{N} \frac{\phi^L(p_T h_n \beta_n) g_n}{h_n(1-\beta_n)} \le p_T. \tag{7b}$$

Recall that $\phi^L(x)$ defined in Section II is an increasing function. Thus, both the objective function of Problem 3 and the left-hand side function of (7b) are increasing functions with the vector $\boldsymbol{\beta} \triangleq (\beta_1, \beta_2, ..., \beta_N)^T$. Hence Problem 3 falls into the standard form of *monotonic optimization problem*, i.e.,

$$\max_{\boldsymbol{y}} \ f(\boldsymbol{y}) \quad \text{s.t.} \quad \boldsymbol{y}_L \le \boldsymbol{y} \le \boldsymbol{y}_U; \ \ g(\boldsymbol{y}) \le 0,$$

where variable $\boldsymbol{y}$ is a vector, $\boldsymbol{y}_L$ and $\boldsymbol{y}_U$ represent lower and upper bounds of $\boldsymbol{y}$, respectively, and $f(\boldsymbol{y})$ and $g(\boldsymbol{y})$ are monotonically increasing functions with $\boldsymbol{y}$. For a standard monotonic optimization problem, there is a polyblock algorithm to achieve the ε-optimal solution ($\varepsilon > 0$ is a predefined parameter) [18, p. 2316], i.e., the gap between the achieved objective function by the polyblock algorithm and the globally maximal objective function is bounded by ε. Due to space limit, detailed procedure of the polyblock algorithm is omitted.

B. The Case under the CLC Model

In this subsection, the CLC model is adopted for the energy harvester, i.e., $\phi^C(x)$ defined in (2) is adopted. Looking into the expression of $\phi^C(x)$, to guarantee positive harvested energy, we should have $p_T h_n \beta_n \ge x_L$, i.e., $\beta_n \ge \frac{x_L}{p_T h_n} \triangleq \beta_n^L, \forall n \in \mathcal{N}$. On the other hand, when input RF power at an energy harvester is more than x_U, the energy harvester will become saturated. Thus, there is no need to set the input RF power to be larger than x_U, i.e., we have $p_T h_n \beta_n \le x_U$, which implies $\beta_n \le \frac{x_U}{p_T h_n}, \forall n \in \mathcal{N}$.

When β_n is bounded in $[\beta_n^L, \frac{x_U}{p_T h_n}]$, then we have $\phi^C(x) = c(x - x_L)$. Similar to the discussion for (4), when the objective function of Problem 1 achieves its maximal, we should have $p_n h_n(1-\beta_n) = \phi^C(p_T h_n \beta_n) g_n = c\left(p_T h_n \beta_n - x_L\right) g_n$, which indicates $p_n = \frac{c(p_T h_n \beta_n - x_L) g_n}{h_n(1-\beta_n)}$. As $p_n \le p_T$, we

[1]The relaxation does not affect the equivalence of Problems 1 and 2.

know that β_n is upper bounded by $\tilde{\beta_n}$, in which $\tilde{\beta_n}$ satisfies $\frac{c(p_T h_n \tilde{\beta_n} - x_L)}{(1-\tilde{\beta_n})} = \frac{p_T h_n}{g_n}, \forall n \in \mathcal{N}$. Thus, overall, we have $\max\{\beta_n^L, \beta_n^{\min}\} \le \beta_n \le \min\{\tilde{\beta_n}, \frac{x_U}{p_T h_n}, 1\}$.

By replacing p_n with $\frac{c(p_T h_n \beta_n - x_L) g_n}{h_n(1-\beta_n)}$, Problem 1 under the CLC model is equivalent to the following problem

Problem 4:

$$\max_{\{\beta_n\},\{w_n\}} \ \frac{1}{2}\sum_{n=1}^{N} w_n \log\left(1 + \frac{c\,(p_T h_n \beta_n - x_L)\,g_n}{\sigma^2 w_n}\right)$$

$$\text{s.t.} \quad \max\{\beta_n^L, \beta_n^{\min}\} \le \beta_n \le \min\{\tilde{\beta_n}, \frac{x_U}{p_T h_n}, 1\}, \quad \forall n \in \mathcal{N}, \tag{8a}$$

$$w_n \ge 0, \forall n \in \mathcal{N}; \sum_{n=1}^{N} w_n \le w_T, \tag{8b}$$

$$\sum_{n=1}^{N} \frac{c\,(p_T h_n \beta_n - x_L)\,g_n}{h_n(1-\beta_n)} \le p_T. \tag{8c}$$

Similar to the transformation from Problem 2 to Problem 3, Problem 4 can be transformed to

Problem 5:

$$\max_{\{\beta_n\}} \ \sum_{n=1}^{N} c\,(p_T h_n \beta_n - x_L)\,g_n$$

$$\text{s.t.} \quad \max\{\beta_n^L, \beta_n^{\min}\} \le \beta_n \le \min\{\tilde{\beta_n}, \frac{x_U}{p_T h_n}, 1\}, \forall n \in \mathcal{N}, \tag{9a}$$

$$\sum_{n=1}^{N} \frac{c\,(p_T h_n \beta_n - x_L)\,g_n}{h_n(1-\beta_n)} \le p_T. \tag{9b}$$

For Problem 5, its objective function is a linear function with β_n for $n \in \mathcal{N}$, and the left-hand side function in (9b) is convex with β_n when $\frac{p_T h_n}{x_L} > 1$.[2] Therefore, Problem 5 is a convex optimization problem. It can be checked that Problem 5 satisfies Slater's condition. Thus, the KKT condition of Problem 5 can serve as a sufficient and necessary condition of its optimal solution [17], which can be written as

$$c p_T h_n g_n - \Xi \frac{c g_n}{h_n} \frac{(p_T h_n - x_L)}{(\beta_n - 1)^2} + \Gamma_n - \Delta_n = 0, \tag{10a}$$

$$\Gamma_n\left(\beta_n - \max\{\beta_n^L, \beta_n^{\min}\}\right) = 0, \forall n \in \mathcal{N}, \tag{10b}$$

$$\Delta_n\left(\beta_n - \min\{\tilde{\beta_n}, \frac{x_U}{p_T h_n}, 1\}\right) = 0, \forall n \in \mathcal{N}, \tag{10c}$$

$$\Xi\left(p_T - \sum_{n=1}^{N} \frac{c\,(p_T h_n \beta_n - x_L)\,g_n}{h_n(1-\beta_n)}\right) = 0, \tag{10d}$$

$$\Gamma_n \ge 0, \Delta_n \ge 0, \forall n \in \mathcal{N}, \tag{10e}$$

$$\Xi \ge 0, \tag{10f}$$

$$\text{Constraints}(9a), (9b), \tag{10g}$$

where Γ_n, Δ_n, and Ξ are Lagrange multipliers associated with left-hand side of (9a), right-hand side of (9a), and (9b), respectively.

[2]If $\frac{p_T h_n}{x_L} \le 1$, then relay R_n has no chance to work, as output power of its energy harvester is always zero.

According to (10b) and (10c), when $\max\{\beta_n^L, \beta_n^{\min}\} < \beta_n < \min\{\tilde{\beta_n}, \frac{x_U}{p_T h_n}, 1\}$, we have $\Gamma_n = \Delta_n = 0$. Hence, according to (10a), we have

$$\beta_n = 1 - \sqrt{\frac{\Xi\,(p_T h_n - x_L)}{p_T h_n^2}}. \tag{11}$$

For a given Ξ, if the calculated β_n in (11) is larger than its upper bound $\min\{\tilde{\beta_n}, \frac{x_U}{p_T h_n}, 1\}$, then we should have $\beta_n = \min\{\tilde{\beta_n}, \frac{x_U}{p_T h_n}, 1\}$ according to (10a) and (10c). Similarly, if the calculated β_n in (11) is smaller than its lower bound $\max\{\beta_n^L, \beta_n^{\min}\}$, then we should have $\beta_n = \max\{\beta_n^L, \beta_n^{\min}\}$ according to (10a) and (10b). Overall, β_n ($\forall n \in \mathcal{N}$) can be expressed as a function of Ξ as

$$\beta_n(\Xi) = \left[1 - \sqrt{\frac{\Xi\,(p_T h_n - x_L)}{p_T h_n^2}}\right]\Bigg|_{\max\{\beta_n^L, \beta_n^{\min}\}}^{\min\{\tilde{\beta_n}, \frac{x_U}{p_T h_n}, 1\}}, \tag{12}$$

with operation $[x]|_a^b \triangleq \max\{a, \min\{x, b\}\}$.[3] The $\beta_n(\Xi)$ in (12) is a non-increasing function with Ξ for $n \in \mathcal{N}$.

On the other hand, both the left-hand side function of (9b) and the objective function of Problem 5 are increasing functions with β_n for $n \in \mathcal{N}$. So it is better to set β_n as large as possible, which indicates that the optimal solution of Problem 5 happens when the constraint (9b) becomes active,[4] i.e.,

$$\sum_{n=1}^{N} \frac{c\,(p_T h_n \beta_n(\Xi) - x_L)\,g_n}{h_n(1-\beta_n(\Xi))} = p_T. \tag{13}$$

The left-hand side function of constraint (13) is increasing with $\beta_n(\cdot)$, and $\beta_n(\Xi)$ is monotonic with Ξ. Thus, the left-hand side function of constraint (13) is also monotonic with Ξ. Hence we can use a bisection search method to get the value of Ξ that satisfies (13). With the value of Ξ, the optimal β_n for Problem 5 can be obtained by using (12). Such an optimal solution of β_n is in semi-closed-form, as expression of $\beta_n(\Xi)$ in (12) is in closed-form and the value of Ξ needs to be obtained by using a bisection search.

IV. Numerical Results

In this section, numerical results are presented to verify the effectiveness of our proposed methods. Similar to [12], the system parameters are set as follows. The carrier frequency is 915MHz, $\sigma^2 = -95$dBm, $E_{\min} = 1\mu$W, both g_n and h_n are subject to Rician distribution with a Rician factor being 3dB, the path loss exponent is 3.6, the antenna gain on every link is 10dB, and $w_T = 2$MHz. There are four relay nodes. As SWIPT can only be applied for short-range harvesting links and communication links, we consider two short-range location setups: the coordinates (in unit of meter) of source, destination, and the four relays are $(-5, 0)$, $(10, 0)$, $(0, 10)$, $(0, 5)$, $(0, -5)$, and $(0, -10)$ in the 1st setup, and are $(-15, 0)$, $(25, 0)$, $(0, 10)$, $(0, 5)$, $(0, -5)$, and $(0, -10)$ in the 2nd setup. For the energy harvester, by utilizing the curve fitting tool on

[3]Accordingly, for the KKT condition in (10), it is hard to say whether constraint (9a) is binding or not, which depends on whether the truncation operations in (12) are active.

[4]Accordingly, for the KKT condition in (10), the constraint (9b) is binding.

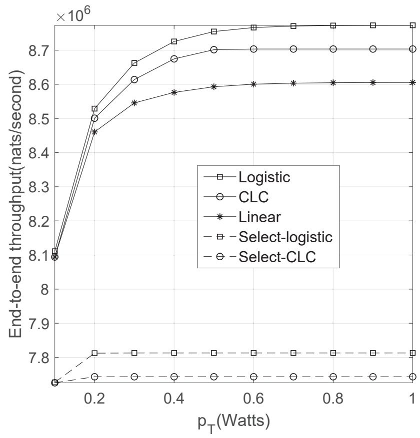

Fig. 1: End-to-end throughput vs. p_T in the 1st location setup.

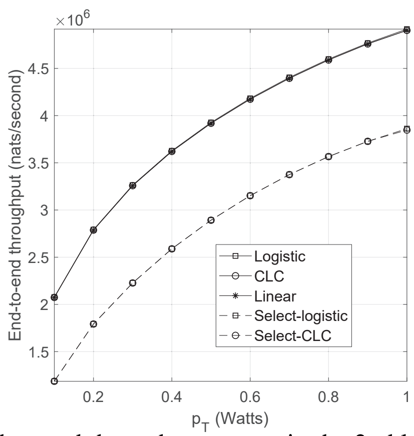

Fig. 2: End-to-end throughput vs. p_T in the 2nd location setup.

the measured data in [12], it is calculated that $M = 2.3\times10^{-2}$, $a = 170$, and $b = 1.398\times10^{-2}$ for the logistic function model, and $c = 0.7833$, $x_L = 0$, and $x_U = 3 \times 10^{-2}$ for CLC model. When running the polyblock algorithm, ε is set as 1×10^{-2}. As a comparison, for each of the logistic function model and CLC model, a relay selection method in [7] is also simulated, in which the source uses all transmit power and bandwidth to transmit to one selected relay, and then the selected relay forwards to the destination. In addition, the optimal allocation of transmit power, bandwidth, and PS ratios under the linear energy harvesting model is also simulated for comparison.

Fig. 1 shows the end-to-end throughput versus p_T in the 1st location setup. Our proposed methods under the logistic function model and CLC model outperform other methods. For the two selection schemes, their throughput do not increase when p_T increases from 0.2 to 1, for the following reason. The selected best relay intends to have a very good first-hop channel gain, and thus, its harvested energy has a very large chance to get saturated when $p_T \geq 0.2$.

Fig. 2 shows the end-to-end throughput versus p_T in the 2nd location setup. The throughput in our methods and in the linear model are almost the same. This is because with larger link distances in the 2nd location setup, the energy harvesters have a low chance to get saturated (one major difference among the two nonlinear energy harvesting models and the linear energy harvesting model lies in whether or not to model saturation and/or how to model saturation). Also due to the low chance for energy harvesters to get saturated, the throughput with the two selection schemes increase with p_T.

V. Conclusion

A two-hop multiple-relay DF network with SWIPT under the PS technique is investigated, taking into account the nonlinearity of energy harvesters. The logistic function model and CLC model are adopted to characterize the nonlinearity. Transmit power and bandwidth on every source-relay-destination link and the PS ratio at every relay node are optimized so as to maximize the end-to-end throughout. For each nonlinear model, the formulated problem is non-convex. With a series of analysis and transformations, we find the global optimal solution of each formulated optimization problem. Our optimal solution with the CLC model is in a semi-closed-form.

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