Optimal Resource Allocation for Wireless Powered Sensors: A Perspective From Age of Information

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***Abstract*—We investigate a wireless powered sensor network, in which multiple sensors generate data and send their data to a base station (BS) periodically. Each sensor first harvests energy from the BS via wireless power transfer and then uses its available energy to transmit to the BS its data. We target minimal average age of information, by optimizing the energy harvesting time and the bandwidth allocation during the sensors' transmissions. The research problem is hard to solve, as some notations in the problem do not have a closed-form expression. To optimally solve the problem, we first show that there is a one-to-one mapping from the energy harvesting time to the bandwidth allocation. We also develop a method to obtain the bandwidth allocation vector corresponding to each value of the energy harvesting time. Then we get the optimal energy harvesting time by investigating and comparing different sub-regions of energy harvesting time. Numerical results show optimality of our solution and its performance gain over a benchmark scheme based on the traditional threshold-based method.**

***Index Terms*—Wireless power transfer, energy harvesting, age of information.**

I. Introduction

In recent years, the Internet-of-things (IoT) has been growing quickly, which is very helpful in many applications including environmental monitoring, connected and automated vehicles, smart home, connected health, etc. To facilitate various IoT applications, wireless sensors are usually deployed to monitor the environments and then upload the collected data to a base station (BS) or a fusion center [1]. To power the wireless sensors, energy harvesting from ambient sources or from wireless power transfer (WPT) sources is a good candidate solution, and has been studied extensively in the literature [2]–[4].

Many IoT applications, such as connected vehicles and connected health, are delay sensitive [1]. When a destination receives some information, freshness of the information is of importance. Age of information (AoI) is a metric for information freshness, and has drawn a lot of attention recently [5]–[7]. Consider a source node and a destination node. At a moment (say τ_0), some information (a burst of packets) is generated at the source node; and at moment τ_1, the destination node finishes receiving the information. Then, AoI at instant $t \in [\tau_0, \tau_1]$ is defined as $t - \tau_0$, which is actually the elapsed time duration since generation of the information. For delivery of the information, the end-to-end delay is $\tau_1 - \tau_0$, which is actually AoI at the moment when the whole information is delivered. It can be seen that, compared to the end-to-end delay metric, AoI can measure the freshness of each packet (or each bit) in the target information. This AoI feature fits well with delay-sensitive applications, in which freshness of each packet is a major concern.

Recently, the AoI metric has been widely used in energy-harvesting wireless sensor networks (WSNs), for system performance analysis and/or optimization of sensing and transmission policies. The works in [8]–[12] consider WSNs that harvest energy from ambient environments (such as solar, wind, etc.). The works in [8] and [9] consider a non-fading channel between a sensor and its sink. Online sensing and transmission policies are designed for the cases with finite, infinite, or one-unit battery capacity. The work in [10] takes into account instantaneous channel state information of a Rayleigh fading channel when designing an online transmission policy, while the work in [11] develops an online sensing policy considering data erasure due to the wireless fading channel. In [12], the work is focused on a sensor-sink pair aided by an energy-harvesting-powered relay node. Both offline and online transmission policies are developed. The works in [13]–[15] consider WSNs powered by WPT. In [13], a slave node harvests energy from a master node that uses WPT, When the harvested energy at the slave node is more than a threshold, it uploads its information to the master node. Closed-form AoI in the information transmission process is derived. In [14], an external power station is applied to send RF signals to a wireless sensor. When the sensor is fully charged, its uses all its harvested energy to transmit a fixed amount of data. Taking link outages into consideration, AoI of the system is derived in closed form, and is then minimized by finding the optimal battery capacity. The work in [15] also considers a sensor-sink pair using an external power station. A metric termed *urgency-aware AoI (U-AoI)* is defined, and then derived in closed form. The U-AoI of the system is minimized by finding the optimal waiting time before each sensing.

In this letter, we consider a WSN powered by WPT. The major difference between our paper and works [13]–[15] lies in that we consider transmissions of multiple sensors to a sink while works [13]–[15] consider only one sensor-sink pair.

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In specific, we investigate a system with multiple wireless sensors that have periodical data to transmit to a BS. Each sensor first gets energy harvested through WPT from the BS for a while and then uploads its data to the BS. In such a framework, when to stop energy harvesting and how to allocate the bandwidth resources for data uploading are of importance for minimization of the AoI of the system. An optimization problem is formulated, which is hard to solve because some notations in the problem formulation do not have a closed-form expression. To optimally solve the formulated problem, which optimizes over variables of energy harvesting time and bandwidth allocation, we first transform it to an equivalent problem that optimizes over only the energy harvesting time. Then we find the optimal solution of the transformed problem, which is also the optimal solution of the original problem formulation.

II. System Model and Problem Formulation

Consider a wireless sensor system with one BS and N sensors. The N sensors constitute set $\mathcal{N} \triangleq \{1,2,...,N\}$. The N sensors are used to monitor diverse physical quantities related to an application scenario and then upload their sensed data to the BS.[1] Each sensor has periodical data generation, i.e., it generates a burst of information after every T duration. For sensor i ($\in \mathcal{N}$), the amount of data it generates each time is D_i nats. The data generation moments of the N sensors are different. Without loss of generality, within time interval $[0,T]$, the data generation moments of the N sensors are s_1, s_2, ..., s_N, respectively, with $0 \le s_1 \le s_2 \le ... \le s_{N-1} \le s_N \le T$.

None of the N sensors have stable power supply. Thus, they are all wireless-powered by the radio frequency signals from the BS. Energy harvesting and data transmission are performed alternatively. Specifically, time is divided into frames, and each frame has duration T. In each frame, a time duration of $t_0 \ge 0$ is firstly used for wireless power transfer from the BS to all sensors, and then a time duration of $t_1 \ge 0$ is used for data transmission, in which sensor i ($i \in \mathcal{N}$) will upload D_i nats to the BS. Consider the time frame from moment 0 to moment T. Suppose the ending moment of wireless charging, i.e., t_0, lies between s_n and s_{n+1}. Thus, at the frame (and each subsequent frame), sensors $1,2,...,n$ upload data generated in the current frame, while sensors $n+1,n+2,...,N$ upload data generated in the preceding frame. An illustration of the charging and data transmission process over multiple frames is given in Fig. 1.

Denote the channel gain from the BS to sensor i as g_i and the channel gain from sensor i to the BS as h_i. Both g_i and h_i keep unchanged for $i \in \mathcal{N}$ within the duration of T. This setup is reasonable when the sensors make observations frequently or the channels are slow-fading.

The BS has transmit power p_T. The energy conversion efficiency of the sensors' energy harvesters is η. Then within duration t_0, the energy harvested by sensor i can be written as $E_i(t_0) = p_T \eta g_i t_0, \forall i \in \mathcal{N}$. For data transmission of these N sensors, a total bandwidth w_T is available. Denote $w_i \ge 0$ as the allocated bandwidth for sensor i. Then we have $\sum_{i=1}^{N} w_i = w_T$. Given that the available energy of sensor i is $E_i(t_0)$, to upload D_i nats within time duration t_1, the transmit power is $\frac{E_i(t_0)}{t_1}$. Then according to the Shannon capacity formula, we have

$$D_i = t_1 w_i \log\left(1 + \frac{E_i(t_0)h_i}{t_1 w_i \sigma^2}\right), \forall i \in \mathcal{N}, \tag{1}$$

in which σ^2 is noise power spectrum density. According to (1), t_1 can be denoted as a function of t_0 and Ξ_i, expressed as $t_1(t_0,\Xi_i)$, where Ξ_i represents information related to sensor i (such as D_i, w_i, h_i, g_i). Note that $t_1(t_0,\Xi_i)$ has no closed-form expression. Since all the sensors use the same duration, i.e., t_1, for transmission to the BS, $t_1(t_0,\Xi_i)$ should be the same for all sensors. Thus, we have $t_1(t_0,\Xi_i) = t_1(t_0), \forall i \in \mathcal{N}$.

In a frame, the total time for wireless charging and data transmission is $t_T = t_0 + t_1(t_0)$, which is actually a function of t_0 (written as $t_T(t_0)$). Since wireless charging and data transmission should happen within a frame, we have $t_T(t_0) = t_0 + t_1(t_0) \le T$.

Consider a frame with duration from moment 0 to moment T. For sensor i, $i = 1,2,...,n$, its generated data in this frame are uploaded to the BS in this frame. Thus, the associated AoI at time $t(t \in [s_i, t_T(t_0)])$ can be written as $t - s_i$ and the sensor's accumulated AoI in the frame is $\Theta_i = \int_{s_i}^{t_T(t_0)}(t - s_i)dt = \frac{1}{2}\left(t_T(t_0) - s_i\right)^2$.

For sensor i, $i = n+1, n+2, ..., N$, its AoI has two parts:

- Data generated in the previous frame are transmitted in the current frame. The associated AoI in the current frame, i.e., at time $t(t \in [0, t_T(t_0)])$, can be written as $t - s_i + T$. The corresponding accumulated AoI in the current frame is $\Phi_i = \int_0^{t_T(t_0)}(t - s_i + T)dt = \frac{1}{2}\left(\left(t_T(t_0) + T - s_i\right)^2 - \left(T - s_i\right)^2\right)$.
- Data generated in the current frame will be transmitted in the next frame. The associated AoI in the current frame, i.e., at time $t(t \in [s_i, T])$ can be written as $t - s_i$. The corresponding accumulated AoI in the current frame is $\Psi_i = \int_{s_i}^{T}(t - s_i)dt = \frac{1}{2}\left(T - s_i\right)^2$.

Overall, in the current frame, the average AoI of the system is expressed as $\tilde{\Delta} = \frac{1}{T}(\sum_{i=1}^{n}\Theta_i + \sum_{i=n+1}^{N}[\Phi_i + \Psi_i])$.

In this letter, our target is to minimize the average AoI $\tilde{\Delta}$. To achieve this goal, we need to optimize t_0 and w_i, $\forall i \in \mathcal{N}$. Specifically, the following optimization problem is formulated.

Problem 1:

$$\min_{t_0, n, \{w_i | i \in \mathcal{N}\}} \tilde{\Delta}$$

$$\text{s.t.} \quad 0 < t_0 < T; \quad t_T(t_0) \le T; \tag{2a}$$

$$s_n \le t_0 \le s_{n+1}; \quad n \in \{0\} \bigcup \mathcal{N}; \tag{2b}$$

$$t_1(t_0,\Xi_i) = t_1(t_0), \forall i \in \mathcal{N}; \tag{2c}$$

$$\sum_{i=1}^{N} w_i = w_T; \quad w_i \ge 0, \forall i \in \mathcal{N} \tag{2d}$$

where s_0 and s_{N+1} are defined as 0 and T, respectively, for the ease of presentation.

[1] A typical application is health monitoring network, in which multiple sensors are implemented separately on the body of one person to collect the blood pressure, respiratory rate, heart rate, moving speed, etc., for the purpose of monitoring the person's health status. The sensors may also perform fall detection as well as activity classification for older persons.

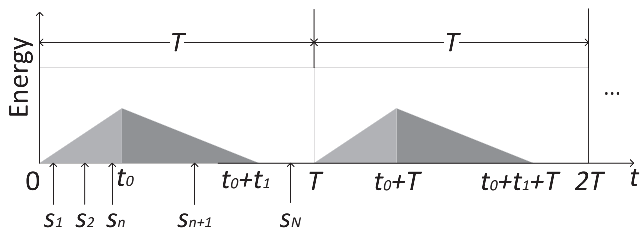

Fig. 1: Illustration of time frames.

III. Optimal Solution of Problem 1

In Problem 1, the functions $t_1(t_0, \Xi_i)$ and $t_1(t_0)$ do not have a closed-form expression, which brings a big challenge to solve Problem 1. Next, we will develop a method to get the optimal solution of Problem 1.

A. Problem Transformation

In Problem 1, the variables to be optimized are t_0, n, and $\{w_i | i \in \mathcal{N}\}$. We first try to transform Problem 1 to another problem that optimizes over t_0 and n only. We achieve this by representing w_i as a function of t_0.

Define function $z_i(D_i, t_0)$ such that

$$D_i = z_i(D_i, t_0) \log\left(1 + \frac{p_T \eta g_i h_i t_0}{z_i(D_i, t_0)\sigma^2}\right), \forall i \in \mathcal{N}. \tag{3}$$

Then from (1), it can be seen that $t_1(t_0) w_i = z_i(D_i, t_0), \forall i \in \mathcal{N}$. Therefore, combining the fact in (2d), we have

$$t_1(t_0) = \sum_{i=1}^{N} \frac{t_1(t_0) w_i}{w_T} = \sum_{i=1}^{N} \frac{1}{w_T} z_i(D_i, t_0). \tag{4}$$

In addition, from $t_1(t_0) w_i = z_i(D_i, t_0)$, we have

$$w_i = \frac{z_i(D_i, t_0)}{t_1(t_0)} \overset{\text{from (4)}}{=} \frac{w_T z_i(D_i, t_0)}{\sum\limits_{i=1}^{N} z_i(D_i, t_0)}, \forall i \in \mathcal{N}. \tag{5}$$

To get the expression of w_i in (5), we need to find the expression of $z_i(D_i, t_0)$ for $i \in \mathcal{N}$. The function $z_i(D_i, t_0)$ is actually the inverse function of $f_i(x, t_0) \triangleq x \log\left(1 + \frac{p_T \eta g_i h_i t_0}{x \sigma^2}\right)$ with x for $i \in \mathcal{N}$. By checking the first-order derivative, it can be proved that $f_i(x, t_0)$ is monotonically increasing with x when $x > 0$. Hence the inverse function of $f_i(x, t_0)$, i.e., $z_i(D_i, t_0)$, is also a monotonically increasing function. On the other hand, if $z_i(D_i, t_0)$ goes to infinity, we have

$$\lim_{z_i(D_i, t_0) \to \infty} z_i(D_i, t_0) \log\left(1 + \frac{p_T \eta g_i h_i t_0}{z_i(D_i, t_0)\sigma^2}\right) = \frac{p_T \eta g_i h_i t_0}{\sigma^2} \tag{6}$$

which, together with (3), imposes a lower bound on t_0, expressed as

$$t_0 > \frac{D_i \sigma^2}{p_T \eta g_i h_i}, \forall i \in \mathcal{N}. \tag{7}$$

For the ease of following presentation, we define $t_0^{\min} \triangleq \max\limits_{i \in \mathcal{N}} \left\{ \frac{D_i \sigma^2}{p_T \eta g_i h_i} \right\}$. Therefore, for a given $t_0 \in (t_0^{\min}, T)$, the value of $z_i(D_i, t_0)$ can be searched by a bisection search method such that (3) holds. With the value of $z_i(D_i, t_0)$, we can further get the value of w_i by using (5). In other words, for a given $t_0 \in (t_0^{\min}, T)$, we can find the values of $w_1, w_2, ..., w_N$. Thus, for Problem 1, we only need to optimize over variables t_0 and n.

The objective function of Problem 1 is re-written as

$$\begin{aligned} \tilde{\Delta} &= \frac{1}{2T}\Bigg(\sum_{i=1}^{n} \left(t_T^2(t_0) - 2 t_T(t_0) s_i + s_i^2\right) \\ &+ \sum_{i=n+1}^{N} \left[t_T^2(t_0) + 2 t_T(t_0)(T - s_i) + (T - s_i)^2\right] \Bigg) \\ &= \frac{1}{2T} Y(t_T, n) \end{aligned}$$

with

$$\begin{aligned} Y(t_T, n) \triangleq \quad & N t_T^2(t_0) - 2 t_T(t_0) \sum_{i=1}^{N} s_i + 2(N-n) T t_T(t_0) \\ & + \sum_{i=1}^{n} s_i^2 + \sum_{i=n+1}^{N} (T - s_i)^2. \end{aligned}$$

Thus, Problem 1 is equivalent to the following optimization problem

Problem 2:

$$\begin{aligned} \min_{t_0, n} \quad & Y(t_T, n) \\ \text{s.t.} \quad & t_0^{\min} < t_0 < T; \quad t_T(t_0) \le T, & \text{(8a)} \\ & s_n \le t_0 \le s_{n+1}; \quad n \in \{0\} \bigcup \mathcal{N}. & \text{(8b)} \end{aligned}$$

B. Optimal Solution of the Transformed Problem

In Problem 2, $t_T(t_0)$ appears in both the objective function and constraints. Thus, it is necessary to characterize $t_T(t_0)$. Recalling that $t_T(t_0) = t_0 + t_1(t_0)$ and $t_1(t_0) = \frac{1}{w_T} \sum_{i=1}^{N} z_i(D_i, t_0)$ from (4), we will first investigate a feature of $z_i(D_i, t_0)$ in Lemma 1.

Lemma 1: With D_i given, $z_i(D_i, t_0)$ is decreasing and convex with t_0 for $i \in \mathcal{N}$.

Proof: We first prove the monotonicity of $z_i(D_i, t_0)$ with t_0. For $t_0^{\dagger} < t_0^{\ddagger}$, according to (3), we have

$$D_i = f_i(z_i(D_i, t_0^{\dagger}), t_0^{\dagger}) = f_i(z_i(D_i, t_0^{\ddagger}), t_0^{\ddagger}), \forall i \in \mathcal{N}. \tag{9}$$

Suppose $z_i(D_i, t_0^{\dagger}) \le z_i(D_i, t_0^{\ddagger})$. It can be also checked that the function $f_i(x, t_0)$ is increasing with both x and t_0. Hence we have $f_i(z_i(D_i, t_0^{\dagger}), t_0^{\dagger}) < f_i(z_i(D_i, t_0^{\ddagger}), t_0^{\ddagger})$, which contradicts the fact in (9). Therefore, we should have $z_i(D_i, t_0^{\dagger}) > z_i(D_i, t_0^{\ddagger})$ for $i \in \mathcal{N}$, i.e., $z_i(D_i, t_0)$ is decreasing with t_0.

Next we turn to prove the convexity of $z_i(D_i, t_0)$ with t_0 for $i \in \mathcal{N}$. Consider $t_0^{\dagger}$ and $t_0^{\ddagger}$ such that $t_0^{\dagger} \ne t_0^{\ddagger}$. Note that the function $f_i(x, t_0)$ is a concave function with $(x, t_0)^T$ (here $(\cdot)^T$ means transpose operation), which can be proved by checking that the Hessian matrix of $-f_i(x, t_0)$ is semi-definite. Hence for $\forall \alpha \in [0, 1]$, we have

$$\begin{aligned} D_i \quad &= \alpha D_i + (1-\alpha) D_i \\ &= \alpha f_i\left(z_i(D_i, t_0^{\dagger}), t_0^{\dagger}\right) + (1-\alpha) f_i\left(z_n(D_i, t_0^{\ddagger}), t_0^{\ddagger}\right) \\ &\le f_i\big(\alpha z_i(D_i, t_0^{\dagger}) + (1-\alpha) z_i(D_i, t_0^{\ddagger}), \alpha t_0^{\dagger} + (1-\alpha) t_0^{\ddagger}\big). \end{aligned} \tag{10}$$

The inequality in (10) indicates that $\alpha z_i(D_i, t_0^{\dagger}) + (1 - \alpha) z_i(D_i, t_0^{\ddagger})$ is larger than the x' value such that $D_i =$

$f(x', \alpha t_0^\dagger + (1-\alpha)t_0^\ddagger)$, i.e., $x' = z_i(D_i, \alpha t_0^\dagger + (1-\alpha)t_0^\ddagger)$. Considering the increasing monotonicity of $f(x, t_0)$ with x and t_0, we have

$$\alpha z_i(D_i, t_0^\dagger) + (1-\alpha) z_i(D_i, t_0^\ddagger) \geq z_i\left(D_i, \alpha t_0^\dagger + (1-\alpha)t_0^\ddagger\right),$$

which proves function $z_i(D_i, t_0)$ is convex with t_0 ($i \in \mathcal{N}$). ■

Remark: $t_T(t_0) = t_0 + t_1(t_0)$ and $t_1(t_0) = \frac{1}{w_T}\sum_{i=1}^N z_i(D_i, t_0)$. Thus, from Lemma 1, we can see that $t_T(t_0)$ is also a convex function with t_0.

Since $t_T(t_0)$ is a convex function with t_0, when t_0 varies from 0 to T, the minimal value of $t_T(t_0)$, denoted as $t_T^\S$, can be found by using a golden-section search method (which is able to return the minimum value of a one-dimensional convex function), and we denote the corresponding t_0 value as $t_0^\S$.

Next we turn to solve Problem 2. Recall that the feasible region of t_0 is $(t_0^{\min}, T)$. Suppose $t_0^{\min}$ satisfies $s_{k-1} < t_0^{\min} < s_k$. Then the feasible region of t_0 consists of the following $N-k+2$ sub-regions: $(t_0^{\min}, s_k]$, $[s_k, s_{k+1}]$, $[s_{k+1}, s_{k+2}]$, ..., $[s_{N-1}, s_N]$, $[s_N, T)$. To solve Problem 2, we can find the optimal objective function value over each sub-region, and get $N-k+2$ objective function values, and then we pick up the minimal value.

Now we show how to get the optimal objective function value of Problem 2 over a sub-region of t_0, denoted as $[t_0^{\mathrm{L}}, t_0^{\mathrm{R}}]$ (here superscript L and R stand for "left" and "right"). We first get the feasible region of t_T, denoted as $[t_T^{\mathrm{L}}, t_T^{\mathrm{R}}]$ when t_0 is within the sub-region. Recall that $t_T(t_0)$ is a convex function with t_0.

- If $t_0^\S \in [t_0^{\mathrm{L}}, t_0^{\mathrm{R}}]$, then $t_T^{\mathrm{L}} = t_T^\S$, and $t_T^{\mathrm{R}} = \max\{t_T(t_0^{\mathrm{L}}), t_T(t_0^{\mathrm{R}})\}$;
- Otherwise, $t_T^{\mathrm{L}} = \min\{t_T(t_0^{\mathrm{L}}), t_T(t_0^{\mathrm{R}})\}$, and $t_T^{\mathrm{R}} = \max\{t_T(t_0^{\mathrm{L}}), t_T(t_0^{\mathrm{R}})\}$.

Since $Y(t_T, n)$ (objective function of Problem 2) is a quadratic function of t_T, it is easy to get the minimal value of $Y(t_T, n)$ in closed form for $t_T \in [t_T^{\mathrm{L}}, t_T^{\mathrm{R}}]$.

The above gives how to find the optimal solution of Problem 2, which is also optimal solution of Problem 1, since the two problems are equivalent.

Complexity Analysis: We can see that to find the optimal t_0 value, we need to first find the value of $t_0^\S$, and then solve Problem 2 for $N-k+2$ sub-regions of t_0. The value of $t_0^\S$ is found by using a golden-section search, the complexity of which is $O(\log(1/\epsilon_2))$, where ϵ_2 is the error tolerance for golden-section search. In each iteration of the golden-section search with a specific t_0 value, we need to calculate $z_i(D_i, t_0)$ for $i \in \mathcal{N}$ by using a bisection search, the complexity of which is $O(N\log(1/\epsilon_1))$, with ϵ_1 the error tolerance for the bisection search. Thus, the complexity in getting the value of $t_0^\S$ is $O\left(N\log(1/\epsilon_1)\log(1/\epsilon_2)\right)$. For each sub-region of t_0, optimal solution of Problem 2 can be found in closed form, with complexity $O(1)$. Thus, complexity of solving Problem 2 over all $N-k+2$ sub-regions of t_0 is $O(N)$. Further, after the optimal t_0 value for Problem 2 is found, we can get the corresponding optimal w_i values ($i \in \mathcal{N}$) by using a bisection method, with complexity $O(N\log(1/\epsilon_1))$. Overall, the complexity in solving Problem 1 is $O\left(N\log(1/\epsilon_1)\log(1/\epsilon_2) + N + N\log(1/\epsilon_1)\right)$.

IV. Numerical Results

Numerical results are presented next to show the performance of our proposed method. Unless otherwise specified, we have the following system parameter setting. $\mathcal{N} = \{1, 2, 3, 4\}$. The carrier frequency is 500MHz, $\sigma^2 = -140$dBm, p_T=1W, w_T = 1MHz, and $\eta = 0.5$. Both g_i and h_i $i \in \mathcal{N}$ experience free-space attenuation with path loss component being 3 at a distance of 50m, as well as Rayleigh fading with mean being 1. The length of one frame is $T = 0.1$s. The amount of data generated in each frame is $D_i = 800$ nats for $i \in \mathcal{N}$.

In Fig. 2, average AoI is plotted for each t_0 value within $(t_0^{\min}, T)$, with $s_1 = 0.1T, s_2 = 0.4T, s_3 = 0.6T, s_4 = 0.8T$. As analyzed in the preceding section, the feasible region of t_0 consists of multiple sub-regions. Fig. 2 also shows our analytically obtained optimal t_0 value. It can be seen that the analytically obtained optimal t_0 value indeed minimizes the average AoI, thus verifying the optimality of our solution.

From our discussion in Sections II and III, for a sensor (say sensor i), if its data generation moment s_i is after t_0, then its generated data in the current frame will be transmitted in the next frame, leading to longer AoI. Thus, it may look intuitive that AoI can be shortened if all the s_i's are shifted to the beginning portion of a frame. To check this, we design the following simulation. For N sensors, consider $(0, T_{\max})$ as the window for all $s_1, s_2, ...s_N$. Without loss of generality, we consider that $s_1, s_2, ..., s_N$ are evenly placed between $\frac{T_{\max}}{N}$ and $\frac{(N-1)T_{\max}}{N}$, i.e., $s_1 = \frac{T_{\max}}{N}, s_N = \frac{(N-1)T_{\max}}{N}$. Thus, a smaller $T_{\max}$ means that the s_i's are closer to the beginning moment of each frame. Fig. 3 shows the minimal average AoI obtained by our proposed solution for different N and $T_{\max}$ values. We have a counter-intuitive observation: a smaller $T_{\max}$ does not guarantee smaller AoI. The reason is as follows. Consider a specific $T_{\max}$. Suppose we can find two sensors, say sensor j and sensor k, with s_j smaller than the optimal t_0 and s_k larger than the optimal t_0. So sensor j's generated data in a frame can be transmitted in the current frame, while sensor k's data need to be transmitted in the next frame. When $T_{\max}$ decreases, indeed it is likely that s_k may be smaller than the optimal t_0, and thus sensor k's generated data in a frame may be transmitted in the current frame, which reduces sensor k's AoI. However, with a smaller $T_{\max}$, sensor j's s_j moves towards the beginning moment of the frame, which increases sensor j's AoI. Thus, overall, with a smaller $T_{\max}$, the average AoI of the system may decrease or increase.

Now we compare our proposed solution with a benchmark scheme. The benchmark scheme borrows the idea of threshold-based transmission in [13], and thus, is referred to as *threshold-based scheme*. In specific, when the harvested energy level of the sensors are all more than a threshold, the sensors stop energy harvesting and start data transmissions. During transmissions, the bandwidth allocation to the sensors is optimized so as to achieve the minimal AoI. Fig. 4 shows average AoI of our proposed solution and the threshold-based scheme when the threshold varies. It can be seen that when the threshold increases, the average AoI of the threshold-based scheme fluctuates. It can also be seen that our proposed solution outperforms the threshold-based scheme as our solution has a lower average AoI.

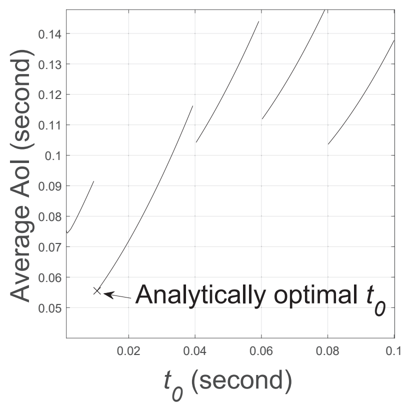

Fig. 2: Average AoI versus t_0.

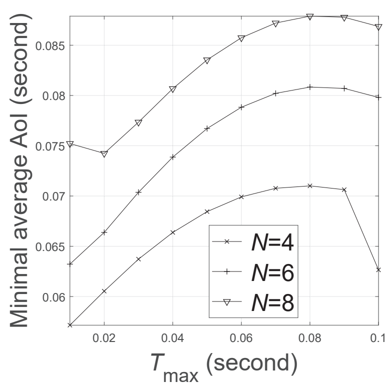

Fig. 3: Minimal average AoI versus $T_{\max}$.

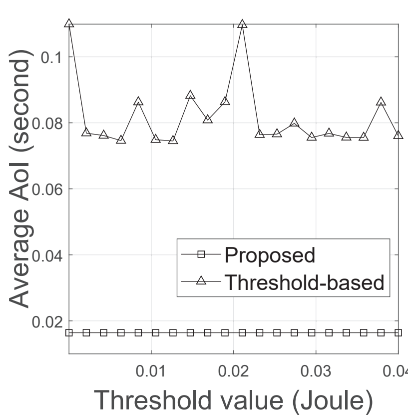

Fig. 4: Average AoI of our proposed solution and the threshold-based scheme.

V. Conclusion

We investigate energy harvesting and bandwidth allocation in a WPT-based WSN. Our target is minimal AoI by optimizing the energy harvesting time t_0 and the bandwidth allocation for data transmissions from the sensors. To solve the research problem, we first discover a one-to-one mapping from t_0 to the bandwidth allocation, i.e., for each t_0 value, there is a corresponding bandwidth allocation vector for the sensors. We also develop a method to find the corresponding bandwidth allocation vector. Based on this finding, our research problem is transformed to another problem that minimizes average AoI by optimizing t_0. Closed-form solution of the transformed problem is obtained for each sub-region of t_0, and the best solution among all t_0's sub-regions is the optimal solution of our research problem. Our proposed solution has complexity linear to the number of sensors, and thus, is appropriate for a low-complexity WSN.

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