Coverage Analysis of Cooperative NOMA in Millimeter Wave Networks

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***Abstract*—This letter analyzes the coverage probability of a cooperative non-orthogonal multiple access (NOMA) millimeter wave (mmWave) network consisting of source (S) and destination (D) pair without a direct link. A cooperative relay (R), which is selected from a set of active users, helps D to receive its data. The transmission from S to R is based on NOMA, by which R can receive its own data and D's data simultaneously. Two relay selection schemes are developed. For each scheme, the favorable users that meet a rate threshold are selected to be in the decoding set. Then from the decoding set, a relay is selected that is nearest to S (in the 1st selection scheme), or nearest to D (in the 2nd selection scheme). For both schemes, we characterize the spatial density (location dependent) of decoding sets and derive the coverage probability. Both schemes outperform orthogonal multiple access. We also derive the coverage of randomly picking a relay (i.e., no selection) to quantify the benefits of relay selection in mmWave NOMA.**

***Index Terms*—Cooperative communications, millimeter wave (mmWave), non-orthogonal multiple access (NOMA), relay, 5G.**

I. Introduction

The wireless networks will experience huge data traffic increase with a compound annual growth rate being 46% from 2018 to 2022. To meet this growth, fifth generation (5G) wireless may utilize non-orthogonal multiple access (NOMA) [1]. It is a paradigm shift from traditional orthogonal multiple access (OMA). NOMA permits a single resource block to be utilized simultaneously by two or more users. Although this allows for inter-user interference, spectral efficiency is enhanced. NOMA may be implemented via power domain multiplexing and successive interference cancellation (SIC) decoding [1], [2]. Compared to OMA, NOMA offers higher sum rates, lower outage and improved fairness [3].

Another 5G candidate is millimeter wave (mmWave), which is above the sub-6 GHz spectrum [4]. Since the marrying of NOMA and mmWave facilitates higher spectral efficiency and an abundance of spectrum, their coverage and rate performance with beam misalignment [5] and outage performance with random beamforming [6] have been analyzed. On the other hand, compared to sub-6 GHz communications, mmWave communications are more likely to experience very weak link or even no link due to the much higher path losses and blockage losses in mmWave, which largely shrinks the mmWave coverage area [7]. In general, the coverage can be improved by the use of relays [8]–[10]. Again due to the large chance for a source-destination pair to experience very weak link or no link, densely deployed relays may be needed to guarantee coverage. Thus, it may be costly to use dedicated relays. A feasible solution is to employ existing users as potential relays. As an incentive, a user also gets some service rate if it provides relaying for another user (destination). This can be achieved by using NOMA, i.e., the source sends to the relay both the destination's signal and the relay's signal by using superposition coding, and then the relay forwards the destination's signal to the destination. This is referred to as *cooperative mmWave NOMA*, which enhances both spectral efficiency and coverage in mmWave communications. Although sub-6 GHz cooperative NOMA has been studied extensively (see [11], [12] and references therein), the mmWave version has remained un-analyzed thus far. This letter fills this gap.

The critical challenges of mmWave propagation include high path losses (diffraction and penetration losses up to 40 dB), atmospheric absorption, blockages and beam alignment issues [4]. Thus, some users may have no signal reception (e.g., cell-edge users). We thus consider such a user, D, which has no coverage from the source S. In this network, relay R is selected from a pool of active user nodes (e.g., cell users near the base station). So S-D link is non-existent while S-R and R-D links are good. The selected user of the pool will act as R. The R-to-D channel is analogous to a direct device-to-device channel [13].

Therefore, we consider a fading block for communications from S to D. As relaying is needed, the fading block is divided into two equal-duration time slots. In the first time slot, S transmits to R only (as the S-D link does not exist). Via the power principle of NOMA, the transmit signal will be a weighted signal consisting of data signals of both R and D. At the end of the first time slot, R will apply SIC demodulation and recover both types of data. In the second time slot, R will transmit the decoded data to D. The rate requirement of D is R_D. For the selected relay R, a "bonus" rate R_R is promised.

For this simple yet prototypical network, we characterize the ability of destination (D) to achieve a sufficient signal-to-noise ratio (SNR) and the ability of both relay (R) and D to meet the rate requirements.

Comparisons to OMA: Recall that S has separate data symbols to transmit to both R and D. For this, OMA would require a total of three time slots: one time slot to transmit R's data, and two time slots to transmit D's data via R. In contrast, when NOMA is used, superimposed data of R and D can be transmitted in the first time slot, and R can transmit D's data in the next slot, decreasing the required number of time slots from three to two, which improves spectral efficiency.

II. System Model

Consider a source (denoted as S), a destination (denoted as D), as well as a set of users that are distributed in a circular disc $\mathcal{D}$ of radius $\mathcal{R}$ as a homogeneous Poisson point process (PPP) Φ with a density λ. We assume no direct link from S to D, and a relay R is selected from active users that are receiving data from S. Here, we only consider relays that are

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in line of sight (LOS) from S and D because non line of sight path losses exceed 20 dB or more over LOS links [7]. A link of length d becomes LOS with probability of $\mathrm{e}^{-\beta d}$ where $\beta = \frac{\mu \mathbb{E}[W]}{\pi}$ is a blockage parameter in which μ means the blockage density, and $\mathbb{E}[W]$ represents the average perimeter of the blockage objects [4]. $\mathbb{E}[\cdot]$ and $\mathbb{P}(\cdot)$ represent expectation and probability, respectively.

In this case, S simultaneously transmits x_R and x_D, intended for R and D, respectively, using the principle of NOMA with power scaling factors a_R and a_D where $a_R^2 + a_D^2 = 1$. Without loss of generality, we set $a_R < a_D$, and thus, R decodes x_D first.[1] In the first time-slot, S transmits the superimposed symbols to the selected relay (relay selection in Section III). The received signal at R can be written as

$$y_R = h_{SR}\left(a_R x_R + a_D x_D\right) \sqrt{P_S G_S G_R \Psi}\, d_{SR}^{-\alpha/2} + w_R \quad (1)$$

where h_{SR} is small scale fading in $S-R$ link, P_S is the total transmit power at S, G_S and G_R are the directional antenna gains at S and R, respectively, $\Psi = \frac{c}{4\pi f}$ is a path loss at 1 meter distance and c and f are the speed of light in free space and the operating mmWave frequency, d_{SR} is the distance between S and R, α is the path loss exponent, and w_R is the additive white Gaussian noise (AWGN) with power N_0 at R. Now, R first decodes x_D by treating the x_R term as interference, which will result in the signal to interference-plus-noise ratio (SINR) of

$$\gamma_{R,D} = \frac{P_S \Psi G_S G_R |h_{SR}|^2 d_{SR}^{-\alpha} a_D^2}{P_S \Psi G_S G_R |h_{SR}|^2 d_{SR}^{-\alpha} a_R^2 + N_0}. \quad (2)$$

Next, with error free decoding, R removes the x_D term from y_R and decodes its own symbol x_R. The SNR at R to decode x_R is thus $\gamma_{R,R} = P_S \Psi G_S G_R |h_{SR}|^2 d_{SR}^{-\alpha} a_R^2 / N_0$. Next, R forwards x_D to D in the second time slot. The received signal at D is $y_D = h_{RD} x_D \sqrt{P_R G_R G_D \Psi}\, d_{RD}^{-\alpha/2} + w_D$, where h_{RD} is small scale fading in $R-D$ link, P_R is the transmit power of R, G_D is the directional antenna gain at D, d_{RD} is the distance between R and D, and w_D is the AWGN at D. Now, the received SNR at D is given by

$$\gamma_D = \frac{P_R \Psi G_R G_D |h_{RD}|^2 d_{RD}^{-\alpha}}{N_0}. \quad (3)$$

Similar to [4], [5], we assume that h_{SR} and h_{RD} are Nakagami-m distributed for analytical tractability. Thus, channel power gains $|h_{SR}|^2$ and $|h_{RD}|^2$ are independent Gamma random variables (r.v.) with shape parameter $m \geq 1$ and rate parameter $\nu > 0$.

III. Coverage Analysis

Coverage is the probability that both R and D meet their individual data rate requirements R_R and R_D, respectively. Due to the half-duplex relaying, SINR or SNR thresholds of $\tau_R = 2^{2R_R} - 1$ and $\tau_D = 2^{2R_D} - 1$ must be met while decoding the data of R and D, respectively. Therefore, coverage depends on two events: (1) a relay can successfully decode its own message and the message for D in the first time slot, and (2) in the second time slot, D successfully decodes its message from R. The channel coherence time is assumed to be at least one fading block.

[1] Our analysis can be straightforwardly extended to the case with $a_R > a_D$.

Since multiple nodes are capable of decoding and forwarding data to D, selecting the best among them improves coverage. Therefore, we analyze several relay selection strategies and compare their coverage performances.

1) Selection Scheme 1 ($\mathcal{S}_1$): We consider source S to be at origin, and D is at $(L, 0)$ in polar coordinate system and an arbitrary relay is located at (r, θ). With this, the location dependent distance between an arbitrary relay to D is $d_{RD} = \sqrt{r^2 - 2rL\cos(\theta) + L^2} \triangleq \rho$, and we have $d_{SR} = r$. This selection scheme works as follows: first, a set of nodes that can decode S's message to D and successfully transmit it to D is selected. This set is called the decoding set, which is an inhomogeneous PPP $\hat{\Phi}_1$ because it is a subset of Φ and the selection of its members is influenced by path loss and blockage, i.e., selection is not random. The density function of the inhomogeneous PPP $\hat{\Phi}_1$, $\hat{\lambda}_1(r,\theta)$, is described in Lemma 1. Second, from the decoding set, a relay is selected, which is closest to S (recall that any node in the decoding set can guarantee the rate requirement of D, thus, picking a relay closest to S can maximize the chance of meeting the rate requirement of R).

Lemma 1. *The density function of the decoding set $\hat{\Phi}_1$ is characterized by $\hat{\lambda}_1(r,\theta) = \lambda \mathrm{e}^{-\beta(r+\rho)} \mathrm{e}^{-\nu(\xi_D r^\alpha + \zeta_D \rho^\alpha)} \sum_{n=0}^{m-1} \sum_{k=0}^{m-1} \frac{\nu^{n+k}}{n!k!} \xi_D^n \zeta_D^k\, r^{n\alpha}\, \rho^{k\alpha}$, where $\xi_D = \frac{\tau_D N_0}{P_S \Psi G_S G_R (a_D^2 - a_R^2 \tau_D)}$ and $\zeta_D = \frac{\tau_D N_0}{P_R \Psi G_R G_D}$.*

Proof. A relay in Φ is retained in $\hat{\Phi}_1$ if it can decode the message for D in the first time slot and can successfully deliver this message to D in the second time slot. Also, since both $S-R$ and $R-D$ links need to be in LOS condition, their LOS probabilities are given by $\mathrm{e}^{-\beta r}$ and $\mathrm{e}^{-\beta\rho}$, respectively. Let P_1 be the probability that a relay meets the above criteria, which can be written mathematically as

$$\begin{aligned} P_1 &= \mathrm{e}^{-\beta(r+\rho)}\, \mathbb{P}\left(\gamma_{R,D} \geq \tau_D\, ,\, \gamma_D \geq \tau_D\right) \\ &\overset{(a)}{=} \mathrm{e}^{-\beta(r+\rho)}\, \mathbb{P}\left(\gamma_{R,D} \geq \tau_D\right) \mathbb{P}\left(\gamma_D \geq \tau_D\right) \\ &\overset{(b)}{=} \mathrm{e}^{-\beta(r+\rho)}\, \mathbb{P}\left(|h_{SR}|^2 \geq \xi_D r^\alpha\right)\, \mathbb{P}\left(|h_{RD}|^2 \geq \zeta_D \rho^\alpha\right) \\ &\overset{(c)}{=} \mathrm{e}^{-\beta(r+\rho)}\, \mathrm{e}^{-\nu(\xi_D r^\alpha + \zeta_D \rho^\alpha)} \\ &\quad \times \sum_{n=0}^{m-1} \sum_{k=0}^{m-1} \frac{\nu^{n+k}}{n!\,k!} \xi_D^n \zeta_D^k\, r^{n\alpha}\, \rho^{k\alpha}, \end{aligned} \quad (4)$$

where (a) is due to the independence of two events, (b) is obtained using (2) and (3), and (c) is obtained using the complementary cumulative distribution function (CCDF) of Gamma r.v. with an integer-valued shape parameter m. We also assume $a_D^2 > a_R^2 \tau_D$, as otherwise the coverage probability will be automatically zero. This assumption is widely used in the analysis of NOMA networks [11], [12]. Now, by thinning Φ with P_1, we obtain the density function shown in Lemma 1. ■

The average size of the decoding set $\hat{\Phi}_1$ is given by

$$\hat{\Lambda}_{\mathcal{D},1} = \int_{\theta=0}^{2\pi} \int_{r=0}^{\mathcal{R}} \hat{\lambda}_1(r,\theta) r dr d\theta. \quad (5)$$

To derive coverage probability, distribution of the distance of the selected relay in $\hat{\Phi}_1$ from S is required, which is given in Lemma 2.

Lemma 2. *The probability that a relay in $\hat{\Phi}_1$ located at distance y is closest to S is given by*

$$P_{\text{nearest}}^1 = \exp\left(-\int_{\theta=0}^{2\pi}\int_{r=0}^{y} \hat{\lambda}_1(r,\theta) r dr d\theta\right). \quad (6)$$

Proof. Let y be the distance from S to the nearest relay in $\hat{\Phi}_1$. This means no relay is located within $\mathcal{C}(0,y)$ ($\mathcal{C}(0,y)$ being a circle centered at origin S and with radius y), which corresponds to the void probability of the PPP $\hat{\Phi}_1$. This probability is given by

$$\begin{aligned} P_{\text{nearest}}^1 &= \mathbb{P}\left(r > y\right) \\ &= \mathbb{P}\left\{\text{no decoding relays in } \mathcal{C}(0,y)\right\} \\ &= \exp\left(-\hat{\Lambda}_{\mathcal{D},1}([0,y))\right) \end{aligned} \quad (7)$$

where $\hat{\Lambda}_{\mathcal{D},1}([0,y))$ is the mean number of decoding relays in $\mathcal{C}(0,y)$, which is obtained from (5) by replacing $\mathcal{R}$ by y. Now by substituting the expression of $\hat{\Lambda}_{\mathcal{D},1}([0,y))$ in (7), we obtain (6). ■

Theorem 1. *The coverage probability of the selection scheme $\mathcal{S}_1$ is given by*

$$\begin{aligned} P_{\text{cov}}^{\mathcal{S}_1} = &\frac{\left(1-\mathrm{e}^{-\hat{\Lambda}_{\mathcal{D},1}}\right)}{A}\sum_{j=0}^{m-1}\sum_{k=0}^{m-1}\sum_{n=0}^{m-1}\frac{\nu^{j+k+n}}{j!\,k!\,n!}\xi_R^j\xi_D^n\zeta_D^k \\ &\times\int_{\theta=0}^{2\pi}\int_{r=0}^{\mathcal{R}}\mathrm{e}^{-\beta(r+\rho)}\mathrm{e}^{-\nu((\xi_D+\xi_R)r^\alpha+\zeta_D\rho^\alpha)}r^{\alpha(j+n)} \\ &\times\rho^{\alpha k}\exp\Bigg(-\lambda\sum_{n=0}^{m-1}\sum_{k=0}^{m-1}\frac{\nu^{n+k}}{n!\,k!}\xi_D^n\zeta_D^k\int_{\omega=0}^{2\pi}\int_{z=0}^{r} \\ &\times z^{\alpha n}\rho_z^{\alpha k}\mathrm{e}^{-\beta(z+\rho_z)}\mathrm{e}^{-\nu(\xi_D z^\alpha+\zeta_D\rho_z^\alpha)}zdzd\omega\Bigg)rdrd\theta, \end{aligned} \quad (8)$$

where $\rho_z = \sqrt{z^2 - 2zL\cos(\omega) + L^2}$ and A is given by

$$\begin{aligned} A = &\sum_{k=0}^{m-1}\sum_{n=0}^{m-1}\frac{\nu^{k+n}}{k!\,n!}\xi_D^n\zeta_D^k\int_{\theta=0}^{2\pi}\int_{r=0}^{\mathcal{R}}\mathrm{e}^{-\beta(r+\rho)}\mathrm{e}^{-\nu((\xi_D)r^\alpha+\zeta_D\rho^\alpha)} \\ &\times r^{\alpha n}\rho^{\alpha k}\exp\left(-\int_{\omega=0}^{2\pi}\int_{z=0}^{r}\hat{\lambda}_1(z,\omega)zdzd\omega\right)rdrd\theta. \end{aligned} \quad (9)$$

Proof. Here, coverage is the probability that the selected relay from $\hat{\Phi}_1$ meets the SNR threshold τ_R. Mathematically,

$$\begin{aligned} P_{\text{cov}}^{\mathcal{S}_1} &= \mathbb{P}\left(\gamma_{R,R}\geq\tau_R\right) = \mathbb{P}\left(|h_{SR}|^2\geq\frac{\tau_R N_0 r^\alpha}{P_S\Psi G_S G_R}\right) \\ &= \mathbb{E}\left[\exp\left(-\nu\xi_R r^\alpha\right)\sum_{j=0}^{m-1}\frac{\nu^j}{j!}\left(\xi_R r^\alpha\right)^j\right] \end{aligned} \quad (10)$$

where $\xi_R = \frac{\tau_R N_0}{P_S\Psi G_S G_R}$. In (10), the expectation needs to be taken over the PPP $\hat{\Phi}_1$. Since the selected relay must be from $\hat{\Phi}_1$ and is closest to S, it also needs to satisfy the probabilities given in (4) and (6). Therefore, the overall coverage can be obtained by jointly averaging (10) with these probabilities over the area of the disc $\mathcal{D}$ and normalizing with the joint probability of P_1 and P_{nearest}^1. Now using the probabilities in (4) and (6), expression (10) can be written as follows:

$$\begin{aligned} P_{\text{cov}}^{\mathcal{S}_1} = &\frac{\left(1-\mathrm{e}^{-\hat{\Lambda}_{\mathcal{D},1}}\right)}{A}\int_{\theta=0}^{2\pi}\int_{r=0}^{\mathcal{R}}\Bigg[\exp\left(-\nu\xi_R r^\alpha\right) \\ &\times\sum_{j=0}^{m-1}\frac{\nu^j}{j!}\left(\xi_R r^\alpha\right)^j\mathrm{e}^{-\beta(r+\rho)}\mathrm{e}^{-\nu(\xi_D r^\alpha+\zeta_D\rho^\alpha)} \\ &\times\sum_{n=0}^{m-1}\sum_{k=0}^{m-1}\frac{\nu^{n+k}}{n!\,k!}\xi_D^n\zeta_D^k r^{n\alpha}\rho^{k\alpha} \\ &\times\exp\left(-\int_{\omega=0}^{2\pi}\int_{z=0}^{r}\hat{\lambda}_1(z,\omega)zdzd\omega\right)\Bigg]rdrd\theta, \end{aligned} \quad (11)$$

where the term $\left(1-\mathrm{e}^{-\hat{\Lambda}_{\mathcal{D},1}}\right)$ in the numerator is due to the coverage probability being zero if no relay is present in the decoding set and A is the normalization factor given in (9). Now with some mathematical manipulation of (11), we obtain (8). The two-fold integrals in (8) can be computed in MATLAB using trapezoidal method and do not incur significant computation complexity. ■

2) Selection Scheme 2 ($\mathcal{S}_2$): To make the analysis tractable, the origin is shifted to the location of D so that S is located at (L,π) in polar coordinates. An arbitrary relay is located at (l,ϕ) and its distance from S is given by $d_{SR} = \sqrt{l^2 + 2lL\cos(\phi) + L^2} \triangleq \delta$, and we have $d_{RD} = l$. This displacement of origin does not affect the performance when radius $\mathcal{R}$ of user disc $\mathcal{D}$ is much larger than L, and is used just to aid the tractability of the analysis. Here, a decoding set is formed again. In this case, the decoding set includes the nodes that can meet the SINR requirements of R and D in the first hop. This means, the conditions $\gamma_{R,D}\geq\tau_D$ and $\gamma_{R,R}\geq\tau_R$ are met by the decoding set of relays which forms an inhomogeneous PPP, denoted by $\hat{\Phi}_2$ with a density function $\hat{\lambda}_2(l,\phi)$ given in Lemma 3. Next, an LOS relay is selected from $\hat{\Phi}_2$ that is closest to D. This is equivalent to selecting a relay that provides the maximum chance to meet the rate requirement of D in the second hop.

Lemma 3. *The density function of the decoding set $\hat{\Phi}_2$ is given by $\hat{\lambda}_2(l,\phi) = \lambda\,\mathrm{e}^{-\beta\delta}\mathrm{e}^{-\nu\eta\delta^\alpha}\sum_{n=0}^{m-1}\frac{\nu^n}{n!}\eta^n\delta^{n\alpha}$, where $\eta = \xi_D$ if $\frac{\tau_D}{1+\tau_D} < a_D^2 \leq \frac{\tau_D(1+\tau_R)}{\tau_R(1+\tau_D)}$ and $\eta = \xi_R$ if $\frac{\tau_D(1+\tau_R)}{\tau_R(1+\tau_D)} \leq a_D^2 < 1$.*

Proof. Let P_2 be the probability that a relay in Φ is in LOS from S with corresponding LOS probability of $\mathrm{e}^{-\beta\delta}$ and is retained in $\hat{\Phi}_2$, which can be written mathematically as

$$\begin{aligned} P_2 &= \mathrm{e}^{-\beta\delta}\,\mathbb{P}\left(\gamma_{R,D}\geq\tau_D\,,\,\gamma_{R,R}\geq\tau_R\right) \\ &= \mathrm{e}^{-\beta\delta}\,\mathbb{P}\left(|h_{SR}|^2\geq\xi_D\delta^\alpha\,,\,|h_{SR}|^2\geq\xi_R\delta^\alpha\right) \\ &= \mathrm{e}^{-\beta\delta}\,\mathbb{P}\left(|h_{SR}|^2\geq\delta^\alpha\,\max\{\xi_D,\xi_R\}\right). \end{aligned} \quad (12)$$

Here, closed form expression of P_2 can be derived. Since we have $a_D^2 > a_R^2\tau_D$, a_D must satisfy $a_D^2 > \frac{\tau_D}{1+\tau_D}$, and the coverage is zero for $0 \leq a_D^2 \leq \frac{\tau_D}{1+\tau_D}$. It can be shown by simple mathematical manipulations that $\xi_D > \xi_R$ if a_D satisfies $\frac{\tau_D}{1+\tau_D} < a_D^2 \leq \frac{\tau_D(1+\tau_R)}{\tau_R(1+\tau_D)}$, and $\xi_D < \xi_R$ if a_D satisfies $\frac{\tau_D(1+\tau_R)}{\tau_R(1+\tau_D)} \leq a_D^2 < 1$. Then, using the CCDF of Gamma r.v., P_2 can be written as $P_2 = \mathrm{e}^{-\beta\delta}\exp\left(-\nu\eta\delta^\alpha\right)\sum_{n=0}^{m-1}\frac{\nu^n}{n!}\eta^n\delta^{n\alpha}$. ∎

Then, the average number of relays in $\hat{\Phi}_2$ is given by

$$\hat{\Lambda}_{\mathcal{D},2} = \lambda\sum_{n=0}^{m-1}\frac{(\nu\eta)^n}{n!}\int_{\phi=0}^{2\pi}\int_{l=0}^{\mathcal{R}}\mathrm{e}^{-\beta\delta-\nu\eta\delta^\alpha}\delta^{n\alpha}l dl d\phi. \quad (13)$$

In Theorem 2, we provide the coverage probability of this selection scheme.

Theorem 2. *The coverage probability of the selection scheme $\mathcal{S}_2$ is given by*

$$P_{\mathrm{cov}}^{\mathcal{S}_2} = \frac{B_1}{B_2}\sum_{j=0}^{m-1}\sum_{n=0}^{m-1}\frac{\nu^{j+n}}{j!\,n!}\zeta_D^j\eta^n\int_{\phi=0}^{2\pi}\int_{l=0}^{\mathcal{R}}l^{\alpha j}\delta^{\alpha n} \times \mathrm{e}^{-\nu(\zeta_D l^\alpha+\eta\delta^\alpha)}\mathrm{e}^{-\beta(l+\delta)}\exp\Bigg(-\lambda\sum_{n=0}^{m-1}\frac{(\nu\eta)^n}{n!} \times\int_{\omega=0}^{2\pi}\int_{z=0}^{l}\mathrm{e}^{-\beta(z+\delta_z)}\mathrm{e}^{-\nu\eta\delta_z^\alpha}\delta_z^{\alpha n}z dz d\omega\Bigg)l dl d\phi, \quad (14)$$

where $\delta_z = \sqrt{z^2+2zL\cos(\omega)+L^2}$ and B_1 and B_2 are given by

$$B_1 = 1-\exp\left(-\int_{\phi=0}^{2\pi}\int_{l=0}^{\mathcal{R}}\mathrm{e}^{-\beta l}\hat{\lambda}_2(l,\phi)l dl d\phi\right),$$

$$B_2 = \sum_{n=0}^{m-1}\frac{(\nu\eta)^n}{n!}\int_{\phi=0}^{2\pi}\int_{l=0}^{\mathcal{R}}\mathrm{e}^{-\nu(\eta\delta^\alpha)}\mathrm{e}^{-\beta(l+\delta)}l^{\alpha j}\delta^{\alpha n} \times\exp\left(-\int_{\omega=0}^{2\pi}\int_{z=0}^{l}\mathrm{e}^{-\beta z}\hat{\lambda}_2(z,\omega)z dz d\omega\right)l dl d\phi.$$

Proof. Here, to achieve the coverage, the selected relay from $\hat{\Phi}_2$ must be in LOS from D and needs to meet $\gamma_D \geq \tau_D$. Therefore, we have

$$\begin{aligned}
P_{\mathrm{cov}}^{\mathcal{S}_2} &= \mathrm{e}^{-\beta l}\,\mathbb{P}\left(\gamma_D \geq \tau_D\right) = \mathrm{e}^{-\beta l}\,\mathbb{P}\left(|h_{RD}|^2 \geq \frac{\tau_D N_0 l^\alpha}{P_R\Psi G_S G_R}\right)\\
&= \mathbb{E}\left[\mathrm{e}^{-\beta l}\exp\left(-\nu\zeta_D l^\alpha\right)\sum_{j=0}^{m-1}\frac{\nu^j}{j!}\left(\zeta_D l^\alpha\right)^j\right]\\
&= \frac{B_1}{B_2}\int_{\phi=0}^{2\pi}\int_{l=0}^{\mathcal{R}}\Bigg[\mathrm{e}^{-\beta l}\exp\left(-\nu\zeta_D l^\alpha\right)\sum_{j=0}^{m-1}\frac{\nu^j}{j!}\left(\zeta_D l^\alpha\right)^j\\
&\quad\times\mathrm{e}^{-\beta\delta}\exp\left(-\nu\eta\delta^\alpha\right)\sum_{n=0}^{m-1}\frac{\nu^n}{n!}\eta^n\delta^{n\alpha}\\
&\quad\times\exp\left(-\int_{\omega=0}^{2\pi}\int_{z=0}^{l}\mathrm{e}^{-\beta z}\hat{\lambda}_2(z,\omega)z dz d\omega\right)\Bigg]l dl d\phi
\end{aligned} \quad (15)$$

where B_1 is due to the fact that coverage becomes zero if there is no LOS relay in the decoding set and B_2 is the normalization factor. Here, we follow the similar technique as in (11), i.e., by taking the average over the disc $\mathcal{D}$ of product of probabilities of a relay being in the decoding set and also being closest to D and LOS to D. Now with some mathematical manipulations of (15), we obtain (14). ∎

3) Selection Scheme 3 ($\mathcal{S}_3$): Since the selection schemes $\mathcal{S}_1$ and $\mathcal{S}_2$ require knowledge of distance or location information, the immediate question is what the performance loss is if such information is not available. To answer it, we consider random relay selection from the decoding set. Coverage probability of it is given in Theorem 3.

Theorem 3. *When a relay is selected at random from $\hat{\Phi}_2$, the coverage probability is given by*

$$P_{\mathrm{cov}}^{\mathcal{S}_3} = \frac{\lambda\left(1-\mathrm{e}^{-\hat{\Lambda}_{\mathcal{D},2}}\right)}{\hat{\Lambda}_{\mathcal{D},2}}\sum_{j=0}^{m-1}\sum_{n=0}^{m-1}\frac{\nu^{n+j}}{n!\,j!}\eta^n\zeta_D^j\int_{\phi=0}^{2\pi}\int_{l=0}^{\mathcal{R}} \times l^{\alpha j+1}\delta^{\alpha n}\mathrm{e}^{-\beta(l+\delta)}\mathrm{e}^{-\nu(\eta\delta^\alpha+\zeta_D l^\alpha)}dl d\phi. \quad (16)$$

Proof. The proof follows from the proof of Theorem 2 where the coverage is the probability that a relay meets the SNR threshold at D and comes from the decoding set $\hat{\Phi}_2$. Therefore the averaging is done only over the probability P_2, that is

$$P_{\mathrm{cov}}^{\mathcal{S}_3} = \frac{\lambda\left(1-\mathrm{e}^{-\hat{\Lambda}_{\mathcal{D},2}}\right)}{\hat{\Lambda}_{\mathcal{D},2}}\int_{\phi=0}^{2\pi}\int_{l=0}^{\mathcal{R}}\Bigg[\mathrm{e}^{-\beta l}\exp\left(-\nu\zeta_D l^\alpha\right)\sum_{j=0}^{m-1} \times\frac{\nu^j}{j!}\left(\zeta_D l^\alpha\right)^j\mathrm{e}^{-\beta\delta}\exp\left(-\nu\eta\delta^\alpha\right)\sum_{n=0}^{m-1}\frac{\nu^n}{n!}\eta^n\delta^{n\alpha}\Bigg]l dl d\phi,$$

where the term $\left(1-\mathrm{e}^{-\hat{\Lambda}_{\mathcal{D},2}}\right)$ is due to the coverage being zero if no relay is present in $\hat{\Phi}_2$. Then, with some mathematical manipulations, we obtain (16). ∎

Optimal power allocation: For each of the relay selection schemes, we can use a one-dimensional search to find the optimal power allocation (i.e., optimal a_D and a_R), which maximizes the coverage probability.

Implementation: Recall that the potential relays are other active users. When serving those potential relays in their own assigned resource blocks, the channel gain (main lobe gain) information from the source to those relays can be obtained. Based on the information, the decoding set can be determined, and a relay can be selected from it. Then the source aligns its beam to the selected relay, and the relay aligns its beam to the destination.

IV. Numerical Results

Here we verify the derived analytical results via 10^5 Monte-Carlo simulations per run. We set the parameters: $G_S = G_R = G_D = 18$ dBi, $\alpha = 2, \mathcal{R} = 1000$ m, $L = 200$ m, $\lambda = 5\times10^{-5}$/m^2, $\mu = 2\times10^{-4}$, $\mathbb{E}[W] = 60$ m, unless otherwise specified. The optimal values of the power allocation factors obtained by one-dimensional search are used in each relay

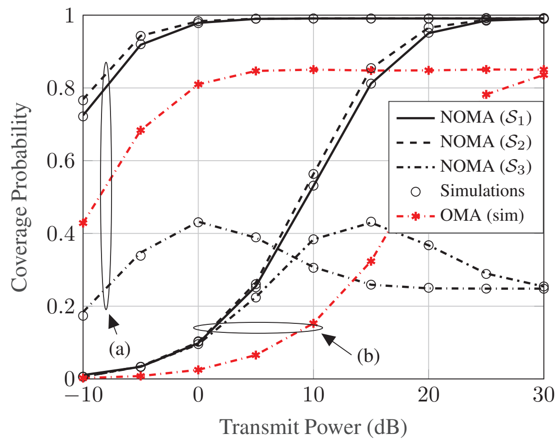

Fig. 1: Coverage probability vs transmit power for NOMA using $\mathcal{S}_1, \mathcal{S}_2$ and $\mathcal{S}_3$ and OMA, where the curves (a) are for $\{R_D, R_R\} = \{1, 3\}$ bps/Hz, and (b) for $\{R_D, R_R\} = \{1.9, 6\}$ bps/Hz.

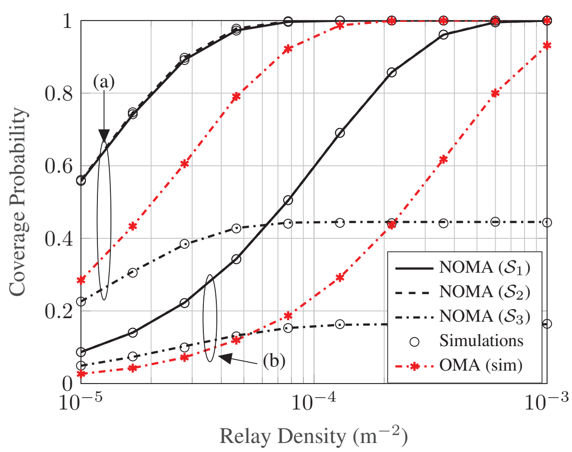

Fig. 2: Coverage probability vs relay density for rate thresholds $\{R_D, R_R\} = \{1, 3\}$ bps/Hz with $P_S = P_R = 0$ dB. The curves (a) are for $\mu = 2 \times 10^{-4}$/m^2, and (b) are for $\mu = 5 \times 10^{-4}$/m^2.

selection scheme. For comparison, we also simulate OMA transmission with same rate requirements at R and D that uses best-worst relay selection criteria [14].

The two-fold integrals in the analytical results can be computed using the trapezoidal method and do not incur significant computational complexity. For instance, to compute the integral in (8), we use an angular resolution of one degree and radial resolution of 10 meters resulting the total number of intervals to be 360×100 = 36,000 and the inner integrals are computed using integral2 MATLAB function.

In Fig. 1, we plot coverage expressions (8), (14) and (16) for NOMA and OMA. This figure shows that $\mathcal{S}_2$ scheme (i.e., closest-to-destination relay selection) provides the best coverage followed closely by $\mathcal{S}_1$ scheme (i.e., closest-to-source relay selection). However, with random relay selection, the coverage probability decreases after an initial increase when transmit power increases. The reason is that for large transmit powers, the decoding set has relay nodes farther away from D. If such a node is picked randomly, then the resulting $R-D$ link is less likely to be LOS, reducing coverage at D.

To study the effect of relay density on the coverage, Fig. 2 plots the coverage probabilities (8), (14) and (16) versus the relay density for two values of blockage density $\mu = \{2 \times 10^{-4}/\mathrm{m}^2, 5 \times 10^{-4}/\mathrm{m}^2\}$. Here, the coverage for $\mathcal{S}_1$, $\mathcal{S}_2$ and OMA improve with increasing relay density λ. Note also that, these coverage curves shift to lower values when the blockage density μ is increased. This suggests that if μ increases, λ should be increased to maintain the same coverage. For a random relay, coverage flattens for higher λ because its selection is independent of density as long as at least one relay exists in the decoding set.

V. Conclusion

This letter derives coverage probability of three mmWave cooperative NOMA relay selection schemes. Both closest-to-source and closest-to-destination relay selection perform closely and both outperform OMA. However, coverage due to a random relay may be worse than that of OMA depending on transmit power level and relay density.

This work considers a destination with no direct link from the source. If a direct link exists, the system may select to perform direct transmission or cooperative transmission by using a relay, which can be investigated in future works.

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