Dynamic Spectrum Leasing with Two Sellers

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***Abstract*—This paper studies dynamic spectrum leasing in a cognitive radio network. There are two spectrum sellers, who are two primary networks, each with an amount of licensed spectrum bandwidth. When a seller has some unused spectrum, it would like to lease the unused spectrum to secondary users. A coordinator helps to perform the spectrum leasing stage-by-stage. As the two sellers may have different leasing periods, there are three epochs, in which seller 1 has spectrum to lease in Epochs II and III, while seller 2 has spectrum to lease in Epochs I and II. Each seller needs to decide how much spectrum it should lease to secondary users in each stage of its leasing period, with a target at revenue maximization. It is shown that, when the two sellers both have spectrum to lease (i.e., in Epoch II), the spectrum leasing can be formulated as a non-cooperative game. Nash equilibria of the game are found in closed form. Solutions of the two sellers in their leasing periods are then derived.**

***Index Terms*—Cognitive radio, dynamic pricing, Nash equilibrium.**

I. Introduction

Cognitive radio has been considered as a promising solution to the spectrum shortage problem in the near future. In cognitive radio, spectrum access of unlicensed users (referred to as *secondary users*), which is required not to affect the communication of licensed users (referred to as *primary users*), is permitted [1]. To implement cognitive radio, two typical modes are overlay mode and underlay mode. In overlay mode, spectrum access of secondary users is permitted only when primary users are idle. Thus, secondary users are required to perform periodical spectrum sensing to detect possible transmissions of primary users. In underlay mode, secondary users can access the spectrum when primary users are transmitting. Secondary users should carefully manage their transmit power such that their generated interference to primary users is below a threshold limit.

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It is difficult to guarantee quality-of-service (QoS) of secondary users in overlay or underlay mode. In overlay mode, secondary users have to wait until primary users do not have traffic to transmit. If primary users have high traffic load, secondary users would have little chance to transmit. In underlay mode, due to the transmit power constraint, secondary users may only achieve short-range low-rate communications.

To better serve secondary users, spectrum leasing has been introduced, in which if a primary user (also called *spectrum seller*) has some unused spectrum for a certain amount of time, it leases the unused spectrum to secondary users. During the leasing period, the secondary users can use the spectrum exclusively, which guarantees their communication QoS. Furthermore, the leasing revenue can motivate the spectrum seller to use its spectrum more efficiently so as to collect more unused spectrum for leasing. Optimal spectrum leasing that maximizes the spectrum seller's revenue is an interesting topic, which is also the focus of this paper.

In the literature, spectrum leasing has been well investigated under the modes of monopoly spectrum leasing (in which there is one spectrum seller) and oligopoly spectrum leasing (in which multiple spectrum sellers exist). In monopoly spectrum leasing, e.g., the works in [2]–[4], the major target is to achieve the maximal revenue of the seller. In oligopoly spectrum leasing, e.g., the works in [5]–[12], the major target is to achieve an equilibrium in the competition among multiple spectrum sellers. In these works, spectrum leasing is performed only once, and the spectrum price is fixed for the whole spectrum leasing duration, referred to as *static spectrum leasing*. On the other hand, *dynamic spectrum leasing*, in which the spectrum price may change over time, is more appropriate for the cases that the secondary users may need spectrum at different time instants. There are limited research efforts in the literature on dynamic spectrum leasing, including the works in [13]–[15] that consider a single spectrum seller and the work in [16] that considers multiple spectrum sellers.

In this paper, we study dynamic spectrum leasing problem in a duopoly market with two sellers.[1] As the two sellers may have different leasing periods, the system has three epochs, in which seller 1 has spectrum to lease in Epochs II and III, while seller 2 has spectrum to lease in Epochs I and II. The main contributions in this paper are summarized as follows. 1) We show that, the spectrum leasing problems of the sellers in

[1]We consider two sellers (i.e., a duopoly spectrum market) for the following reasons. 1) A duopoly spectrum market is a typical and popular scenario for cognitive radio, and has been adopted by many research efforts in the literature [5]–[9]. 2) Sufficient insights can be provided by the duopoly scenario into the spectrum leasing, and our method in this paper can be extended to the scenarios with more spectrum sellers, with increased complexity in analysis and presentation. For ease of analysis and presentation, we consider a duopoly scenario.

Epoch I and Epoch III are convex optimization problems. For Epoch II, we formulate spectrum leasing of the two sellers as a non-cooperative game. We derive closed-form expressions for the Nash equilibria of the non-cooperative game. 2) The amount of spectrum that seller 1 would like to lease in Epoch III affects the non-cooperative game in Epoch II, and thus, affects the total revenues of the two sellers. By analyzing properties of seller 1's revenue in Epoch II and Epoch III, we propose a method that finds the optimal amount of spectrum that seller 1 should lease to secondary users in Epoch III.

The rest of this paper is organized as follows. In Section II, related works are reviewed. In Section III, the system model is presented, and the spectrum leasing problems for the two sellers are formulated. In Section IV, Nash equilibria of the non-cooperative game in Epoch II are derived. Section V discusses how seller 1 should distribute its spectrum to be leased in Epoch II and Epoch III. Numerical results are given in Section VI, and finally the paper is concluded in Section VII.

II. Related Works

Monopoly spectrum leasing: In the work of [2], there are a spectrum provider, a broker, and a number of secondary users. By a Stackelberg game modeling, the broker optimally decides on the number of channels it should purchase from the spectrum provider as well as the price it should use to sell the purchased spectrum to secondary users. The work in [3] also considers a broker. It is assumed that for a given spectrum price, the amount of spectrum demand from secondary users is random. The work in [4] considers the impact of spectrum leasing on primary user performance (such as possible extra interference to the primary system). An optimal solution is given for the primary user, which strikes a balance between the earned revenue and the cost.

Oligopoly spectrum leasing: Two brokers are assumed in [5]. Each broker decides on the amount of spectrum that it should purchase from spectrum providers and on the spectrum price that it should announce to secondary users, with a target at profit maximization. The work in [6] also considers two brokers, by assuming that the leased spectrum may be shared by multiple secondary users simultaneously. Therefore, interference among secondary users needs to be taken into account. The works in [7]–[9] consider a duopoly market, in which the price competition of two spectrum sellers is investigated by using game theoretical approaches. The work in [10] discusses the case with multiple sellers. By using an evolutionary game model, a solution is given to secondary users for their spectrum selection and to sellers for price setting. The work in [11] considers multiple sellers as well as one broker, in which the impact of spectrum leasing on sellers' performance (i.e., service quality degradation) is taken into account. The work in [12] considers heterogeneous secondary users, i.e., different secondary users may have different criteria on their spectrum leasing decisions.

Note that in all above works on monopoly and oligopoly spectrum leasing, the spectrum price is fixed for the whole spectrum leasing duration, and thus, the works all consider static spectrum leasing.

Dynamic pricing: In [13], dynamic pricing in monopoly spectrum leasing is performed over infinite time horizon. The spectrum price is set dynamically, with a target at long-term average revenue maximization. In [14], dynamic pricing in monopoly spectrum leasing is performed over a finite duration. The finite duration is divided into a number of stages, and the price in each stage is set up so as to maximize the overall revenue. In [15], dynamic spectrum leasing is investigated for a monopoly market. Among all channels, some are allocated as leased channels, and the others are called unleased channels. Secondary users can access the leased channels (with higher priority if primary users are also allowed to access), as well as the unleased channels with a lower priority than that of primary users. The number of leased channels is adjusted following the arrival/departure events of the primary and secondary users. In [16], dynamic spectrum leasing is investigated, considering the competition among multiple spectrum sellers. A stochastic Cournot game model is used to derive the leasing strategies (i.e., the amount of spectrum to lease) of the spectrum sellers.

In this paper, we also consider dynamic spectrum leasing. The difference of our work from those in [13]–[15] lies in that the works in [13]–[15] consider a monopoly market while we consider a duopoly market. The difference of our work from that in [16] is as follows. In [16], each spectrum seller's available spectrum for leasing is determined by the market (in other words, the spectrum seller sets the amount of spectrum that it could lease so as to maximize its profit). In our work, each spectrum seller's available spectrum is determined by traffic load of its own users. After all its users' traffic has been accommodated, the unused spectrum can be leased to secondary users.

III. System Model and Problem Formulation

Consider two spectrum sellers (seller 1 and seller 2), one coordinator, and multiple secondary users. Similar to [15], the coordinator is a centralized unit (e.g., a base station controller) which is trusted by the spectrum sellers and secondary users. It is responsible to collect information (for example, amount of spectrum to lease) from and get back to spectrum sellers, post spectrum price to secondary users, lease spectrum to secondary users, manage secondary users' access to the leased spectrum, etc. The two sellers are primary networks with a certain amount of licensed spectrum bandwidth. For each seller, when the data traffic from its own users is light, the seller may partition its spectrum bandwidth into two portions: primary portion and secondary portion. The primary portion will be assigned to the seller's own users, and the secondary portion can be leased to secondary users. In specific, consider that seller 1 and seller 2 have bandwidth Q_1 and Q_2 in their secondary portion, respectively. For each seller, the bandwidth in its secondary portion can be leased to secondary users for a duration (called *leasing period*). Consider that the two sellers' leasing periods are not identical,[2] and overlap with each other.

[2]If the two leasing periods are identical, it is a special case of the problem considered in this paper.

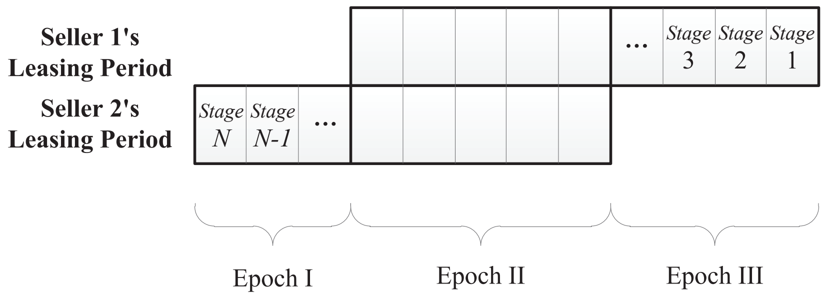

Fig. 1. Leasing periods of the two sellers.

Without loss of generality, we assume that the leasing period of seller 2 starts earlier than the leasing period of seller 1. We also assume that the leasing period of seller 2 ends earlier than that of seller 1.[3] An illustration of the two leasing periods is given in Fig. 1. Here the union of the two leasing periods contains N fixed-length stages. For presentation simplicity, the last stage of seller 1's leasing period is called stage 1, while the first stage of seller 2's leasing period is called stage N. Seller i ($i = 1, 2$) would distribute its spectrum bandwidth Q_i to be leased in the stages of its leasing period. In other words, it needs to decide on the amount of spectrum bandwidth to be leased in each stage in its leasing period, with a constraint that the total amount of leased spectrum bandwidth in the stages is bounded by Q_i. For seller i, denote the amount of spectrum bandwidth it would like to lease to secondary users in stage n as $d_{n,i}$. At the beginning of stage n, seller i should report to the coordinator the information of $d_{n,i}$.

At the beginning of stage n, after the coordinator gets the information of $d_{n,1}$ and $d_{n,2}$, it would set up a spectrum unit price (the price per unit bandwidth per stage) and lease the spectrum bandwidth $(d_{n,1} + d_{n,2})$ to secondary users. In other words, the coordinator should set up the unit price to attract $(d_{n,1} + d_{n,2})$ spectrum bandwidth demand from secondary users. Denote the price p to attract d spectrum bandwidth demand as $P(d)$, which is a function of d. Economics analysis [17], [18] has shown that price and demand typically follow a linear model, and thus, price p and spectrum bandwidth demand d satisfy the following feature:

$$p = P(d) = C_0 - C_1 \cdot d \tag{1}$$

in which C_0 and C_1 are coefficients.[4] $P(d)$ is a decreasing function of d. In addition, $d \cdot P(d)$ should be an increasing function of d (as the total revenue for more leased spectrum bandwidth should be higher), based on which we have

$$C_0 > 2C_1 (Q_1 + Q_2). \tag{2}$$

From Fig. 1, the union of the two sellers' leasing periods can be divided into three epochs: In Epoch I, only seller 2 has spectrum to lease; in Epoch II, both sellers have spectrum to lease; and in Epoch III, only seller 1 has spectrum to lease. Denote the set of stages in Epoch I, II, and III as $\mathcal{N}_{\mathrm{I}}$, $\mathcal{N}_{\mathrm{II}}$, and $\mathcal{N}_{\mathrm{III}}$, respectively. Denote the set of stages in the leasing period of seller 1 and seller 2 as $\mathcal{N}_1$ and $\mathcal{N}_2$, respectively. Thus, we have $\mathcal{N}_1 = \mathcal{N}_{\mathrm{II}} \cup \mathcal{N}_{\mathrm{III}}$ and $\mathcal{N}_2 = \mathcal{N}_{\mathrm{I}} \cup \mathcal{N}_{\mathrm{II}}$.

Seller i ($i \in \{1, 2\}$) aims at maximizing its total revenue over all the stages by deciding on $d_{n,i}, n \in \mathcal{N}_i$. Next, the spectrum leasing problem in each epoch is discussed.

A. Spectrum Leasing Problem in Epoch I

In Epoch I, only seller 2 has spectrum to lease, and it does not know when seller 1 will join the spectrum leasing market and how much spectrum bandwidth seller 1 will offer for spectrum leasing. In other words, in Epoch I, seller 2 does not know when Epoch II will start. So seller 2 assumes a monopoly market in Epoch I. At a stage in Epoch I, once an amount of spectrum is leased to secondary users, the spectrum will be used by secondary users until the last stage of seller 2's leasing period.[5]

For seller 2's spectrum leasing, it has the following two constraints for $d_{n,2}$:

$$0 \le d_{n,2} \le Q_2, \forall n \in \mathcal{N}_2,$$

$$\sum_{n \in \mathcal{N}_2} d_{n,2} \le Q_2.$$

Seller 2's collected revenue at stage n is $(C_0 - C_1 d_{n,2})\, d_{n,2}\, (n - |\mathcal{N}_{\mathrm{III}}|)$, in which $|\cdot|$ means

[3] Note that the method in this paper can be straightforwardly extended to deal with the case when the leasing period of seller 2 ends later than that of seller 1.

[4] When there are highly demanding secondary users, C_0 can be set higher and C_1 can be set lower. With less demanding secondary users, C_0 can be set lower and C_1 can be set higher, to attract more spectrum requests from secondary users.

[5] The rationale for this setting is as follows. It is possible that a secondary user may finish its transmission before the last stage of seller 2's leasing period. The secondary user's actual transmission duration depends on the user's traffic load as well as its instantaneous channel quality during the transmission. However, when a secondary user decides to lease the spectrum, it is difficult to predict the instantaneous channel quality during future transmission over the spectrum, as the instantaneous channel quality may vary dynamically during future transmission. Thus, when a secondary user decides to lease the spectrum, it is unaware when its transmission will finish. Then, at the spectrum seller's side, when some spectrum is leased to a secondary user, the seller assumes that the leased spectrum will be used by the secondary user until the end of the seller's leasing period. If the secondary user finishes its transmission earlier, say at stage k, then the seller will take the leased spectrum back, update its spectrum stock, and re-run our proposed scheme at stage k.

cardinality of a set. To maximize its overall revenue, seller 2 should solve the following optimization problem[6]:

Problem 1:

$$\begin{array}{ll} \max\limits_{\{d_{n,2}|n\in\mathcal{N}_2\}} & \sum\limits_{n\in\mathcal{N}_2} \left(C_0 - C_1 d_{n,2}\right) d_{n,2} \left(n - |\mathcal{N}_{\mathrm{III}}|\right) \\ \text{s.t.} & \sum\limits_{n\in\mathcal{N}_2} d_{n,2} \le Q_2 \\ & d_{n,2} \ge 0, \forall n \in \mathcal{N}_2. \end{array} \tag{3}$$

Problem 1 is a convex optimization problem, because 1) its objective function is a concave function with respect to the vector of variables $\{d_{n,2}|n \in \mathcal{N}_2\}$, and 2) its feasible region is a convex set. Thus, the global optimal solution of Problem 1 can be achieved by existing numerical optimization methods.

B. Spectrum leasing Problem in Epoch II

At Epoch II's first stage (denoted as stage l), seller 1 has available spectrum bandwidth Q_1, while we denote the remaining spectrum bandwidth of seller 2 as Q_2^{II} (in other words, spectrum bandwidth with amount $(Q_2 - Q_2^{\mathrm{II}})$ has been leased out by seller 2 in Epoch I). At the beginning of stage l, each seller does not know the presence of the other seller, and thus, assumes a monopoly spectrum leasing. So each seller reports to the coordinator the amount of spectrum bandwidth it would like to lease to secondary users in the stage. In specific, seller 1 has the following constraints for $d_{n,1}$:

$$0 \le d_{n,1} \le Q_1, \forall n \in \mathcal{N}_1,$$

$$\sum_{n\in\mathcal{N}_1} d_{n,1} \le Q_1.$$

So seller 1 solves the following convex optimization problem:

$$\begin{array}{ll} \max\limits_{\{d_{n,1}|n\in\mathcal{N}_1\}} & \sum\limits_{n\in\mathcal{N}_1} \left(C_0 - C_1 d_{n,1}\right) d_{n,1} n \\ \text{s.t.} & \sum\limits_{n\in\mathcal{N}_1} d_{n,1} \le Q_1 \\ & d_{n,1} \ge 0, \forall n \in \mathcal{N}_1, \end{array} \tag{4}$$

and reports to the coordinator the values of $d_{l,1}$ ($d_{l,1}$ is from the optimal solution of the above problem) and $|\mathcal{N}_1|$ (the leasing duration for the $d_{l,1}$ spectrum bandwidth). On the other hand, seller 2 has the following constraints for $d_{n,2}$:

$$0 \le d_{n,2} \le Q_2^{\mathrm{II}}, \forall n \in \mathcal{N}_2 \backslash \mathcal{N}_{\mathrm{I}},$$

$$\sum_{n\in\mathcal{N}_2\backslash\mathcal{N}_{\mathrm{I}}} d_{n,2} \le Q_2^{\mathrm{II}}.$$

So seller 2 solves the following convex optimization problem:

$$\begin{array}{ll} \max\limits_{\{d_{n,2}|n\in\mathcal{N}_2\backslash\mathcal{N}_{\mathrm{I}}\}} & \sum\limits_{n\in\mathcal{N}_2\backslash\mathcal{N}_{\mathrm{I}}} \left(C_0 - C_1 d_{n,2}\right) d_{n,2} \left(n - |\mathcal{N}_{\mathrm{III}}|\right) \\ \text{s.t.} & \sum\limits_{n\in\mathcal{N}_2\backslash\mathcal{N}_{\mathrm{I}}} d_{n,2} \le Q_2^{\mathrm{II}} \\ & d_{n,2} \ge 0, \forall n \in \mathcal{N}_2\backslash\mathcal{N}_{\mathrm{I}}, \end{array} \tag{5}$$

and reports to the coordinator the values of $d_{l,2}$ ($d_{l,2}$ is from the optimal solution of the above problem) and $(|\mathcal{N}_2| - |\mathcal{N}_{\mathrm{I}}|)$ (which is the length of seller 2's remaining leasing period, and is mathematically equal to $|\mathcal{N}_2 \backslash \mathcal{N}_{\mathrm{I}}|$ or $|\mathcal{N}_{\mathrm{II}}|$), in which $|\mathcal{N}_{\mathrm{I}}|$ stands for the length of preceding stages. Then the coordinator feeds back to the two sellers by telling 1) that now two sellers have spectrum to lease, 2) how much spectrum bandwidth each seller offers in this stage, and 3) how long the leasing period is for each seller. From $d_{l,1}$ and $|\mathcal{N}_1|$ in the feedback information, seller 2 can find out the available stock of seller 1, by searching the value of Q_1 (using bisection search) that makes $d_{l,1}$ be in the optimal solution of the problem in (4). Similarly, seller 1 can also find out the available stock of seller 2. Based on stock information of the other seller, each seller adjusts the amount of offered spectrum bandwidth ($d_{l,1}$ or $d_{l,2}$) and resubmits to the coordinator, and the coordinator decides on a unit price based on (1) with total spectrum demand $(d_{l,1} + d_{l,2})$. In each subsequent stage (say stage n) in Epoch II, by knowing the existence of the other seller, each seller reports to the coordinator the amount of offered spectrum bandwidth ($d_{n,1}$ or $d_{n,2}$), and the coordinator decides on a unit price based on (1) with total spectrum demand $(d_{n,1} + d_{n,2})$.

In every stage in Epoch II, once an amount of spectrum bandwidth of a seller is leased to secondary users, the spectrum will be used by secondary users until the last stage of the corresponding seller's leasing period.

A decision that seller 1 should make in Epoch II is the amount Q_1^{III} of spectrum bandwidth it reserves for Epoch III, where $Q_1^{\mathrm{III}} \in [0, Q_1]$. In other words, seller 1 would like to lease spectrum bandwidth $(Q_1 - Q_1^{\mathrm{III}})$ in Epoch II.

In Epoch II, the announced unit price at each stage (say stage n) depends on the sum of $d_{n,1}$ and $d_{n,2}$. Thus, there is a non-cooperative game between the two sellers. In this game, the two players are seller 1 and seller 2, and the strategy of seller 1 and seller 2 are $\mathcal{S}_1 \triangleq \{d_{n,1}|n \in \mathcal{N}_{\mathrm{II}}\}$ and $\mathcal{S}_2 \triangleq \{d_{n,2}|n \in \mathcal{N}_{\mathrm{II}}\}$, respectively. The payoff function of seller 1 and seller 2 can be expressed as

$$R_1\left(\mathcal{S}_1, \mathcal{S}_2\right) \triangleq \sum_{n\in\mathcal{N}_{\mathrm{II}}} \left(C_0 - C_1\left(d_{n,1} + d_{n,2}\right)\right) d_{n,1} n$$

and

$$R_2\left(\mathcal{S}_1, \mathcal{S}_2\right) \triangleq \sum_{n\in\mathcal{N}_{\mathrm{II}}} \left(C_0 - C_1\left(d_{n,1} + d_{n,2}\right)\right) d_{n,2} \left(n - |\mathcal{N}_{\mathrm{III}}|\right),$$

respectively. Define the feasible region of seller 1's strategy as

$$\mathcal{F}_1(y) = \left\{ \{d_{n,1}|n \in \mathcal{N}_{\mathrm{II}}\} \Big| \sum_{n\in\mathcal{N}_{\mathrm{II}}} d_{n,1} \le y, 0 \le d_{n,1} \le y \right\},$$

which can be written as a simple form

$$\mathcal{F}_1(y) = \left\{ \{d_{n,1}|n \in \mathcal{N}_{\mathrm{II}}\} \Big| \sum_{n\in\mathcal{N}_{\mathrm{II}}} d_{n,1} \le y, d_{n,1} \ge 0 \right\},$$

when seller 1 would like to lease to secondary users spectrum bandwidth amount y in Epoch II, and define the feasible region of seller 2's strategy as

$$\mathcal{F}_2(z) = \left\{ \{d_{n,2}|n \in \mathcal{N}_{\mathrm{II}}\} \Big| \sum_{n\in\mathcal{N}_{\mathrm{II}}} d_{n,2} \le z, d_{n,2} \ge 0 \right\}$$

[6] In Epoch I, seller 2 does not know the value of $|\mathcal{N}_{\mathrm{III}}|$. However, it knows the value of $(n - |\mathcal{N}_{\mathrm{III}}|)$ (the length from stage n until the end of seller 2's leasing period). Thus, in Problem 1, we use notation $(n - |\mathcal{N}_{\mathrm{III}}|)$, for consistence of the formulated spectrum leasing problems in the three epochs.

when seller 2 would like to lease to secondary users spectrum bandwidth amount z in Epoch II. The objective of seller 1 is to solve the following optimization problem

Problem 2:

$$\begin{array}{ll} \max_{\mathcal{S}_1} & R_1\left(\mathcal{S}_1, \mathcal{S}_2\right) \\ \text{s.t.} & \mathcal{S}_1 \in \mathcal{F}_1\left(Q_1 - Q_1^{\mathrm{III}}\right), \end{array} \tag{6}$$

and the objective of seller 2 is to solve the following optimization problem

Problem 3:

$$\begin{array}{ll} \max_{\mathcal{S}_2} & R_2\left(\mathcal{S}_1, \mathcal{S}_2\right) \\ \text{s.t.} & \mathcal{S}_2 \in \mathcal{F}_2\left(Q_2^{\mathrm{II}}\right). \end{array} \tag{7}$$

For the non-cooperative game of the two sellers, a Nash equilibrium defines a strategy pair $(\mathcal{S}_1, \mathcal{S}_2)$ that a seller cannot earn more revenue by deviating from its strategy while keeping the other seller's strategy unchanged. In other words, a Nash equilibrium should be a joint optimal solution of Problem 2 and Problem 3.

C. Spectrum Leasing Problem in Epoch III

In Epoch III, only seller 1 is active in the spectrum market, and thus, monopoly spectrum leasing is performed. Once an amount of spectrum bandwidth is leased to secondary users, the spectrum will be used by secondary users until the end of Epoch III.

Seller 1 has the following constraints for $d_{n,1}$ in its spectrum leasing in Epoch III:

$$0 \le d_{n,1} \le Q_1^{\mathrm{III}}, \forall n \in \mathcal{N}_{\mathrm{III}},$$
$$\sum_{n \in \mathcal{N}_{\mathrm{III}}} d_{n,1} \le Q_1^{\mathrm{III}}.$$

To maximize its revenue in Epoch III, seller 1 solves the following optimization problem.

Problem 4:

$$\begin{array}{lll} V\left(Q_1^{\mathrm{III}}\right) \triangleq & \max_{\{d_{n,1} | n \in \mathcal{N}_{\mathrm{III}}\}} & \sum_{n \in \mathcal{N}_{\mathrm{III}}} \left(C_0 - C_1 d_{n,1}\right) d_{n,1} n \\ & \text{s.t.} & \sum_{n \in \mathcal{N}_{\mathrm{III}}} d_{n,1} \le Q_1^{\mathrm{III}} \\ & & d_{n,1} \ge 0, \forall n \in \mathcal{N}_{\mathrm{III}}. \end{array} \tag{8}$$

Similar to Problem 1, Problem 4 is also a convex optimization problem, and thus, can be solved by existing numerical optimization methods.

D. Overall Leasing Strategy of the Sellers

In Epoch I, since seller 2 does not know when Epoch II will start, it assumes a monopoly market, and solves Problem 1 to get its leasing strategy in Epoch I.

At the beginning of Epoch II, some of seller 2's spectrum has been leased to secondary users in preceding stages. In other words, seller 2 could not "go back" to revise its leasing strategy in Epoch I. Thus, seller 2's leasing strategy decision for Epoch II is separate from that in Epoch I. In specific, in Epoch II, seller 2 follows a Nash equilibrium of the non-cooperative game.

For seller 1 at the beginning of Epoch II, it knows that both sellers will be present in Epoch II and it will be the only seller in Epoch III. Thus, seller 1 can use the amount of spectrum it reserves for Epoch III, i.e., Q_1^{III}, as an input for Epoch II's non-cooperative game, and find the optimal value of Q_1^{III} that maximizes its overall revenue. Thus, seller 1's leasing strategies in Epoch II and III are jointly determined, as follows.

From the perspective of seller 1, it can adjust Q_1^{III}. For a specific Q_1^{III}, the two sellers need to follow a Nash equilibrium in the non-cooperative game in Epoch II. Thus, the strategy of seller 1 can be written as Q_1^{III} and $\mathcal{S}_1$, while the strategy of seller 2 can be written as $\mathcal{S}_2$.

When seller 1 reserves spectrum bandwidth Q_1^{III} for Epoch III, it means that seller 1 would like to lease spectrum bandwidth $(Q_1 - Q_1^{\mathrm{III}})$ in Epoch II. Accordingly, we denote the revenue of seller 1 in Epoch II as $U(Q_1 - Q_1^{\mathrm{III}})$, a function of $(Q_1 - Q_1^{\mathrm{III}})$. Then for seller 1 to maximize its overall revenue, the following optimization problem should be solved

Problem 5:

$$\begin{array}{ll} \max_{Q_1^{\mathrm{III}}} & U\left(Q_1 - Q_1^{\mathrm{III}}\right) + V\left(Q_1^{\mathrm{III}}\right) \\ \text{s.t.} & 0 \le Q_1^{\mathrm{III}} \le Q_1 \end{array} \tag{9}$$

in which $V\left(Q_1^{\mathrm{III}}\right)$ is defined in (8).

In the following, in Section IV we find out Nash equilibria in Epoch II for a specific Q_1^{III}, and in Section V we select the optimal value of Q_1^{III} for seller 1.

IV. Nash Equilibria in the Non-Cooperative Game in Epoch II with Given Q_1^{III}

A. Existence and Uniqueness of Nash Equilibrium in the Non-Cooperative Game in Epoch II

Recall that a Nash equilibrium of the non-cooperative game in Epoch II should be a joint optimal solution of Problem 2 and Problem 3. Since the objective functions of Problem 2 and Problem 3 are continuous and concave, and the feasible regions of the two sellers' strategies are convex, closed, bounded, and uncoupled[7], there exists at least one Nash equilibrium point for the non-cooperative game in Epoch II according to Theorem 1 of [19].

For uniqueness of Nash equilibrium, we have the following theorem.

Theorem 1: When $|\mathcal{N}_{\mathrm{III}}| \le 12$, there is only one Nash equilibrium for the non-cooperative game in Epoch II.

Proof:

Define the vectorized strategy of seller 1 and seller 2 in Epoch II as $\boldsymbol{x}_1 = [d_{|\mathcal{N}_{\mathrm{III}}|+|\mathcal{N}_{\mathrm{II}}|,1}, d_{|\mathcal{N}_{\mathrm{III}}|+(|\mathcal{N}_{\mathrm{II}}|-1),1}, \ldots, d_{|\mathcal{N}_{\mathrm{III}}|+1,1}]^T$ and $\boldsymbol{x}_2 = \left[d_{|\mathcal{N}_{\mathrm{III}}|+|\mathcal{N}_{\mathrm{II}}|,2}, d_{|\mathcal{N}_{\mathrm{III}}|+(|\mathcal{N}_{\mathrm{II}}|-1),2}, \ldots, d_{|\mathcal{N}_{\mathrm{III}}|+1,2}\right]^T$, respectively, in which $[\cdot]^T$ means transpose operation. The payoff function of seller 1 and seller 2 can be rewritten as $R_1\left(\mathcal{S}_1, \mathcal{S}_2\right) = R_1\left(\boldsymbol{x}_1, \boldsymbol{x}_2\right)$ and $R_2\left(\mathcal{S}_1, \mathcal{S}_2\right) = R_2\left(\boldsymbol{x}_1, \boldsymbol{x}_2\right)$, respectively. Denote $\boldsymbol{x} = \left(\boldsymbol{x}_1^T, \boldsymbol{x}_2^T\right)^T$ and define

$$\sigma(\boldsymbol{x}) = R_1\left(\boldsymbol{x}_1, \boldsymbol{x}_2\right) + R_2\left(\boldsymbol{x}_1, \boldsymbol{x}_2\right). \tag{10}$$

[7]When the two feasible regions are independent from each other, we say that the two feasible regions are uncoupled.

Then the pseudo-gradient of $\sigma(\boldsymbol{x})$ can be given as

$$\boldsymbol{k}(\boldsymbol{x}) = \begin{bmatrix} \nabla_1 R_1(\boldsymbol{x}_1, \boldsymbol{x}_2) \\ \nabla_2 R_2(\boldsymbol{x}_1, \boldsymbol{x}_2) \end{bmatrix} \tag{11}$$

where $|\mathcal{N}_{\mathrm{II}}| \times 1$ matrix $\nabla_1 R_1(\boldsymbol{x}_1, \boldsymbol{x}_2)$ is the gradient of $R_1(\boldsymbol{x}_1, \boldsymbol{x}_2)$ with respect to vector $\boldsymbol{x}_1$, and $|\mathcal{N}_{\mathrm{II}}| \times 1$ matrix $\nabla_2 R_2(\boldsymbol{x}_1, \boldsymbol{x}_2)$ is the gradient of $R_2(\boldsymbol{x}_1, \boldsymbol{x}_2)$ with respect to vector $\boldsymbol{x}_2$. According to Theorem 2 and Theorem 6 of [19], Nash equilibrium of the non-cooperative game in Epoch II is unique if the $2|\mathcal{N}_{\mathrm{II}}| \times 2|\mathcal{N}_{\mathrm{II}}|$ symmetric matrix $\boldsymbol{L}(\boldsymbol{x}) = -\left[\boldsymbol{K}(\boldsymbol{x}) + \boldsymbol{K}^T(\boldsymbol{x})\right]$ is positive definite, where $\boldsymbol{K}(\boldsymbol{x})$ is the Jacobian matrix of $\boldsymbol{k}(\boldsymbol{x})$ with respect to $\boldsymbol{x}$. After some math manipulation, the matrix $\boldsymbol{L}(\boldsymbol{x})$ can be written as the following form

$$\boldsymbol{L}(\boldsymbol{x}) = \begin{bmatrix} \boldsymbol{L}_{11}(\boldsymbol{x}) & \boldsymbol{L}_{12}(\boldsymbol{x}) \\ \boldsymbol{L}_{21}(\boldsymbol{x}) & \boldsymbol{L}_{22}(\boldsymbol{x}) \end{bmatrix} \tag{12}$$

where $\boldsymbol{L}_{11}(\boldsymbol{x}) = \mathrm{Diag}\big(4C_1(|\mathcal{N}_{\mathrm{III}}| + |\mathcal{N}_{\mathrm{II}}|), 4C_1(|\mathcal{N}_{\mathrm{III}}| + |\mathcal{N}_{\mathrm{II}}| - 1), ..., 4C_1(|\mathcal{N}_{\mathrm{III}}| + 1)\big)$, $\boldsymbol{L}_{12}(\boldsymbol{x}) = \boldsymbol{L}_{21}(\boldsymbol{x}) = \mathrm{Diag}\big(C_1(|\mathcal{N}_{\mathrm{III}}| + 2|\mathcal{N}_{\mathrm{II}}|), C_1(|\mathcal{N}_{\mathrm{III}}| + 2|\mathcal{N}_{\mathrm{II}}| - 2), ..., C_1(|\mathcal{N}_{\mathrm{III}}| + 2)\big)$, and $\boldsymbol{L}_{22}(\boldsymbol{x}) = \mathrm{Diag}\big(4C_1|\mathcal{N}_{\mathrm{II}}|, 4C_1(|\mathcal{N}_{\mathrm{II}}| - 1), ..., 4C_1\big)$. Here $\mathrm{Diag}(\cdots)$ means a diagonal matrix with all diagonal elements listed in $(\cdots)$. The matrix $\boldsymbol{L}(\boldsymbol{x})$ can be guaranteed to be positive definite, if the leading principal minors are all positive [20], i.e., the determinant of $m \times m$ upper-left submatrix of $\boldsymbol{L}(\boldsymbol{x})$ is larger than 0 for $m = 1, 2, ..., 2|\mathcal{N}_{\mathrm{II}}|$. Since there is

$$\mathrm{Det}\left(\begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{D} \end{bmatrix}\right) = \mathrm{Det}(\boldsymbol{A})\,\mathrm{Det}\left(\boldsymbol{D} - \boldsymbol{C}\boldsymbol{A}^{-1}\boldsymbol{B}\right)$$

when matrix $\boldsymbol{A}$ is invertible [21], the determinant of $m \times m$ upper-left submatrix of $\boldsymbol{L}(\boldsymbol{x})$ is larger than 0 for $m = 1, 2, ..., 2|\mathcal{N}_{\mathrm{II}}|$ when the following inequalities hold

$$\begin{aligned} &12\left(|\mathcal{N}_{\mathrm{II}}| - k\right)^2 + 12|\mathcal{N}_{\mathrm{III}}|\left(|\mathcal{N}_{\mathrm{II}}| - k\right) - |\mathcal{N}_{\mathrm{III}}|^2 > 0, \\ &\qquad\qquad \forall k = 0, 1, ..., \left(|\mathcal{N}_{\mathrm{II}}| - 1\right), \end{aligned} \tag{13}$$

i.e., when

$$\frac{\left(|\mathcal{N}_{\mathrm{II}}| - k\right)}{|\mathcal{N}_{\mathrm{III}}|} > \left(-\frac{1}{2} + \frac{1}{\sqrt{3}}\right), \forall k \in 0, 1, ..., \left(|\mathcal{N}_{\mathrm{II}}| - 1\right). \tag{14}$$

The inequalities in (14) hold if

$$|\mathcal{N}_{\mathrm{III}}| < \frac{1}{-\frac{1}{2} + \frac{1}{\sqrt{3}}} = 12.9282. \tag{15}$$

This completes the proof. ■

From the proof, it can be seen that the derivation of value 12 largely depends on the fact that there are two sellers in the spectrum market.

As the number of stages in Epoch III is normally limited, it is very likely that the value of $|\mathcal{N}_{\mathrm{III}}|$ is bounded by 12, and thus, Nash equilibrium of the non-cooperative game in Epoch II is unique. Nevertheless, in next subsection, we show how to find Nash equilibria in the non-cooperative game in Epoch II without constraint $|\mathcal{N}_{\mathrm{III}}| \leq 12$ (i.e., Nash equilibrium may or may not be unique).

B. Finding Nash Equilibria in the Non-Cooperative Game in Epoch II

As aforementioned, a Nash equilibrium of the non-cooperative game in Epoch II is a joint optimal solution of Problem 2 and Problem 3. As both Problem 2 and Problem 3 are convex problems and satisfy the Slater's condition, KKT condition is a sufficient and necessary condition for optimal solution for each problem [22]–[24].

For the ease of presentation, we denote $Q_1^{\mathrm{II}_c} = Q_1 - Q_1^{\mathrm{III}}$ as the spectrum bandwidth amount that seller 1 would like to lease to secondary users in Epoch II. For Problem 2, the KKT condition is

$$2C_1 n d_{n,1} - \left(C_0 - C_1 d_{n,2}\right) n + \lambda - \mu_n = 0,\ \forall n \in \mathcal{N}_{\mathrm{II}} \tag{16a}$$
$$\lambda\left(\sum_{n \in \mathcal{N}_{\mathrm{II}}} d_{n,1} - Q_1^{\mathrm{II}_c}\right) = 0 \tag{16b}$$
$$\mu_n d_{n,1} = 0,\ \forall n \in \mathcal{N}_{\mathrm{II}} \tag{16c}$$
$$\sum_{n \in \mathcal{N}_{\mathrm{II}}} d_{n,1} \leq Q_1^{\mathrm{II}_c} \tag{16d}$$
$$d_{n,1} \geq 0, \forall n \in \mathcal{N}_{\mathrm{II}} \tag{16e}$$
$$\lambda \geq 0; \mu_n \geq 0,\ \forall n \in \mathcal{N}_{\mathrm{II}} \tag{16f}$$

where λ and μ_n are Lagrange multipliers associated with the constraints $\sum_{n \in \mathcal{N}_{\mathrm{II}}} d_{n,1} \leq Q_1^{\mathrm{II}_c}$ and $d_{n,1} \geq 0$, respectively.

For Problem 3, the KKT condition is

$$\begin{aligned} 2C_1\left(n - |\mathcal{N}_{\mathrm{III}}|\right) d_{n,2} - \left(C_0 - C_1 d_{n,1}\right)\left(n - |\mathcal{N}_{\mathrm{III}}|\right)& \\ + \zeta - \nu_n = 0,\ \forall n \in \mathcal{N}_{\mathrm{II}}& \end{aligned} \tag{17a}$$
$$\zeta\left(\sum_{n \in \mathcal{N}_{\mathrm{II}}} d_{n,2} - Q_2^{\mathrm{II}}\right) = 0 \tag{17b}$$
$$\nu_n d_{n,2} = 0,\ \forall n \in \mathcal{N}_{\mathrm{II}} \tag{17c}$$
$$\sum_{n \in \mathcal{N}_{\mathrm{II}}} d_{n,2} \leq Q_2^{\mathrm{II}} \tag{17d}$$
$$d_{n,2} \geq 0, \forall n \in \mathcal{N}_{\mathrm{II}} \tag{17e}$$
$$\zeta \geq 0; \nu_n \geq 0,\ \forall n \in \mathcal{N}_{\mathrm{II}} \tag{17f}$$

where ζ and ν_n are Lagrange multipliers associated with the constraints $\sum_{n \in \mathcal{N}_{\mathrm{II}}} d_{n,2} \leq Q_2^{\mathrm{II}}$ and $d_{n,2} \geq 0$, respectively.

To get Nash equilibrium of the non-cooperative game in Epoch II, the equations (16) and (17) should be solved jointly.

We have two properties for the joint optimal solution:

- **Property 1:** Equality should hold in (16d) and (17d) (in other words, we have $\sum_{n \in \mathcal{N}_{\mathrm{II}}} d_{n,1} = Q_1^{\mathrm{II}_c}$ and $\sum_{n \in \mathcal{N}_{\mathrm{II}}} d_{n,2} = Q_2^{\mathrm{II}}$).
- **Property 2:** If $d_{n,1} > 0$ ($n \in \mathcal{N}_{\mathrm{II}}$), then we have $\mu_n = 0$; if $d_{n,2} > 0$, then we have $\nu_n = 0$.

Property 1 is due to the facts that the objective function of Problem 2 is a monotonically increasing function of $d_{n,1}$ ($n \in \mathcal{N}_{\mathrm{II}}$) and that the objective function of Problem 3 is a monotonically increasing function of $d_{n,2}$ ($n \in \mathcal{N}_{\mathrm{II}}$). Property 2 can be obtained directly from the equalities (16c) and (17c).

Next, we try to find the expressions of $d_{n,1}$ and $d_{n,2}$ by solving (16) and (17).

From the equalities (16a) and (17a), $d_{n,1}$ and $d_{n,2}$ for $n \in \mathcal{N}_{\mathrm{II}}$ can be expressed as

$$d_{n,1} = \frac{(C_0 - C_1 d_{n,2})\, n - \lambda + \mu_n}{2C_1 n}, \tag{18}$$

$$d_{n,2} = \frac{(C_0 - C_1 d_{n,1})\,(n - |\mathcal{N}_{\mathrm{III}}|) - \zeta + \nu_n}{2C_1\,(n - |\mathcal{N}_{\mathrm{III}}|)}, \tag{19}$$

from which we have

$$d_{n,1} = \frac{2\,(C_0 n - \lambda + \mu_n)}{3C_1 n} - \frac{C_0\,(n - |\mathcal{N}_{\mathrm{III}}|) - \zeta + \nu_n}{3C_1\,(n - |\mathcal{N}_{\mathrm{III}}|)}, \tag{20}$$

$$d_{n,2} = -\frac{C_0 n - \lambda + \mu_n}{3C_1 n} + \frac{2\,(C_0\,(n - |\mathcal{N}_{\mathrm{III}}|) - \zeta + \nu_n)}{3C_1\,(n - |\mathcal{N}_{\mathrm{III}}|)}. \tag{21}$$

Define $\mathcal{Z}_1 = \{n | d_{n,1} > 0, d_{n,2} > 0, n \in \mathcal{N}_{\mathrm{II}}\}$, $\mathcal{Z}_2 = \{n | d_{n,1} > 0, d_{n,2} = 0, n \in \mathcal{N}_{\mathrm{II}}\}$, $\mathcal{Z}_3 = \{n | d_{n,1} = 0, d_{n,2} > 0, n \in \mathcal{N}_{\mathrm{II}}\}$ and $\mathcal{Z}_4 = \{n | d_{n,1} = 0, d_{n,2} = 0, n \in \mathcal{N}_{\mathrm{II}}\}$. Then $\{\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \mathcal{Z}_4\}$ constitutes a decomposition of the set $\mathcal{N}_{\mathrm{II}}$, which means that $\mathcal{Z}_1 \bigcup \mathcal{Z}_2 \bigcup \mathcal{Z}_3 \bigcup \mathcal{Z}_4 = \mathcal{N}_{\mathrm{II}}$ and $\mathcal{Z}_i \bigcap \mathcal{Z}_j = \emptyset$ ($\emptyset$ being a null set) for $i \neq j$ and $i, j \in \{1, 2, 3, 4\}$. Totally there are $2^{2|\mathcal{N}_{\mathrm{II}}|}$ decompositions.

Next we find out the expressions of $d_{n,1}$ and $d_{n,2}$ for a specific decomposition $\{\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \mathcal{Z}_4\}$.

From Property 1, we have

$$\sum_{n \in \mathcal{Z}_1} d_{n,1} + \sum_{n \in \mathcal{Z}_2} d_{n,1} = Q_1^{\mathrm{II}_c},$$

$$\sum_{n \in \mathcal{Z}_1} d_{n,2} + \sum_{n \in \mathcal{Z}_3} d_{n,2} = Q_2^{\mathrm{II}}.$$

In the two equations, substituting the expressions of $d_{n,1}$ and $d_{n,2}$ in (20) and (21) for $n \in \mathcal{Z}_1$, substituting the expressions of $d_{n,1}$ and $d_{n,2}$ in (18) and (19) for $n \in \mathcal{Z}_2$ and $n \in \mathcal{Z}_3$, and using Property 2, we have the following equations:

$$\begin{aligned} -A_{11}\lambda + A_{12}\zeta &= Q_1^{\mathrm{II}_c} - \sum_{n \in \mathcal{Z}_1} \tfrac{C_0}{3C_1} - \sum_{n \in \mathcal{Z}_2} \tfrac{C_0}{2C_1}, \\ A_{21}\lambda - A_{22}\zeta &= Q_2^{\mathrm{II}} - \sum_{n \in \mathcal{Z}_1} \tfrac{C_0}{3C_1} - \sum_{n \in \mathcal{Z}_3} \tfrac{C_0}{2C_1} \end{aligned} \tag{22}$$

where

$$A_{11} = \sum_{n \in \mathcal{Z}_1} \frac{2}{3C_1 n} + \sum_{n \in \mathcal{Z}_2} \frac{1}{2C_1 n}, \tag{23}$$

$$A_{12} = \sum_{n \in \mathcal{Z}_1} \frac{1}{3C_1\,(n - |\mathcal{N}_{\mathrm{III}}|)}, \tag{24}$$

$$A_{21} = \sum_{n \in \mathcal{Z}_1} \frac{1}{3C_1 n}, \tag{25}$$

$$A_{22} = \sum_{n \in \mathcal{Z}_1} \frac{2}{3C_1\,(n - |\mathcal{N}_{\mathrm{III}}|)} + \sum_{n \in \mathcal{Z}_3} \frac{1}{2C_1\,(n - |\mathcal{N}_{\mathrm{III}}|)}. \tag{26}$$

Note that A_{11}, A_{12}, A_{21} and A_{22} are all nonnegative. According to the equations in (22), the Lagrange multipliers λ and ζ can be expressed as

$$\begin{aligned} \lambda = \; & -\tfrac{A_{22}}{A_{11}A_{22} - A_{21}A_{12}} \left(Q_1^{\mathrm{II}_c} - \sum_{n \in \mathcal{Z}_1} \tfrac{C_0}{3C_1} - \sum_{n \in \mathcal{Z}_2} \tfrac{C_0}{2C_1} \right) \\ & -\tfrac{A_{12}}{A_{11}A_{22} - A_{21}A_{12}} \left(Q_2^{\mathrm{II}} - \sum_{n \in \mathcal{Z}_1} \tfrac{C_0}{3C_1} - \sum_{n \in \mathcal{Z}_3} \tfrac{C_0}{2C_1} \right), \end{aligned} \tag{27}$$

$$\begin{aligned} \zeta = \; & -\tfrac{A_{21}}{A_{11}A_{22} - A_{21}A_{12}} \left(Q_1^{\mathrm{II}_c} - \sum_{n \in \mathcal{Z}_1} \tfrac{C_0}{3C_1} - \sum_{n \in \mathcal{Z}_2} \tfrac{C_0}{2C_1} \right) \\ & -\tfrac{A_{11}}{A_{11}A_{22} - A_{21}A_{12}} \left(Q_2^{\mathrm{II}} - \sum_{n \in \mathcal{Z}_1} \tfrac{C_0}{3C_1} - \sum_{n \in \mathcal{Z}_3} \tfrac{C_0}{2C_1} \right). \end{aligned} \tag{28}$$

With the aid of Property 2 and using equations (18), (19), (20), and (21), the closed-form expressions of $d_{n,1}$ and $d_{n,2}$ for $n \in \mathcal{N}_{\mathrm{II}}$ are given as follows:

$$d_{n,1} = \begin{cases} \frac{2(C_0 n - \lambda)}{3C_1 n} - \frac{C_0(n - |\mathcal{N}_{\mathrm{III}}|) - \zeta}{3C_1(n - |\mathcal{N}_{\mathrm{III}}|)} & \text{if } n \in \mathcal{Z}_1 \\ \frac{C_0 n - \lambda}{2C_1 n} & \text{if } n \in \mathcal{Z}_2 \\ 0 & \text{if } n \in \mathcal{Z}_3 \bigcup \mathcal{Z}_4 \end{cases} \tag{29}$$

$$d_{n,2} = \begin{cases} -\frac{C_0 n - \lambda}{3C_1 n} + \frac{2(C_0(n - |\mathcal{N}_{\mathrm{III}}|) - \zeta)}{3C_1(n - |\mathcal{N}_{\mathrm{III}}|)} & \text{if } n \in \mathcal{Z}_1 \\ \frac{C_0(n - |\mathcal{N}_{\mathrm{III}}|) - \zeta}{2C_1(n - |\mathcal{N}_{\mathrm{III}}|)} & \text{if } n \in \mathcal{Z}_3 \\ 0 & \text{if } n \in \mathcal{Z}_2 \bigcup \mathcal{Z}_4 \end{cases} \tag{30}$$

where λ and ζ are given in (27) and (28), respectively.

By now, given the decomposition $\{\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \mathcal{Z}_4\}$, expressions of $d_{n,1}$ and $d_{n,2}$ for $n \in \mathcal{N}_{\mathrm{II}}$ are derived. To guarantee that every equality or inequality in (16) and (17) is satisfied, a feasibility check is further required, which is given as follows:

1) λ and ζ, which can be calculated from (27) and (28), are non-negative.
2) $d_{n,1}$ and $d_{n,2}$, which are calculated from (29) and (30), are non-negative for $n \in \mathcal{N}_{\mathrm{II}}$.
3) μ_n and ν_n, which can be calculated from (18) and (19) given the obtained $d_{n,1}$, $d_{n,2}$, λ and ζ, are non-negative for $n \in \mathcal{N}_{\mathrm{II}}$.

If the above feasibility check passes, the decomposition $\{\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \mathcal{Z}_4\}$ is said to be *feasible*, and the derived $d_{n,1}$ and $d_{n,2}$ expressions in (29) and (30) for $n \in \mathcal{N}_{\mathrm{II}}$ given the decomposition $\{\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \mathcal{Z}_4\}$ form a Nash equilibrium of the non-cooperative game in Epoch II.

For the set $\mathcal{N}_{\mathrm{II}}$, there are $2^{2|\mathcal{N}_{\mathrm{II}}|}$ possible decompositions. To find all Nash equilibria of the game in Epoch II, an exhaustive search of all $2^{2|\mathcal{N}_{\mathrm{II}}|}$ decompositions is required. As the number of stages in Epoch II is normally very limited, and the calculations in checking feasibility of each decomposition are simple, an exhaustive search of all $2^{2|\mathcal{N}_{\mathrm{II}}|}$ decompositions is considered to be acceptable. In addition, the following theorem is helpful in reducing the complexity in the exhaustive search.

Theorem 2: For a feasible decomposition, if there exists a stage (say stage n) in $\mathcal{Z}_4$ (i.e., $d_{n,1} = d_{n,2} = 0$), then all stages with a lower index in Epoch II should belong to $\mathcal{Z}_4$.

Proof: We use the proof by contradiction. In the Nash equilibrium of the decomposition, suppose there is $n\dagger$ satisfying $n\dagger < n, n\dagger \in \mathcal{N}_{\mathrm{II}} \backslash \mathcal{Z}_4$. We first assume that $n\dagger \in \mathcal{Z}_2$, which indicates that $d_{n\dagger,1} > 0, d_{n\dagger,2} = 0$. Then the total revenue collected over stage n and stage $n\dagger$ by seller 1 is $(C_0 - C_1 d_{n\dagger,1})\, d_{n\dagger,1} n\dagger$. By interchanging seller 1's offered spectrum bandwidth amounts in stage n and stage $n\dagger$, the total revenue that seller 1 collects in stages n and $n\dagger$ becomes $(C_0 - C_1 d_{n\dagger,1})\, d_{n\dagger,1} n$, which is larger than $(C_0 - C_1 d_{n\dagger,1})\, d_{n\dagger,1} n\dagger$ since $n\dagger < n$. This contradicts the definition of Nash equilibrium.

Similarly, $n\dagger \in \mathcal{Z}_1$ or $n\dagger \in \mathcal{Z}_3$ also leads to a contradiction.

This completes the proof. ■

Remark: Theorem 2 shows that in a feasible decomposition, if $\mathcal{Z}_4$ is not empty, then it contains consecutive stages until the end of Epoch II. Therefore, in the exhaustive search of all possible decompositions, we can skip those decompositions in which $\mathcal{Z}_4$ contains non-consecutive stages or does not last until the end of Epoch II. Thus, the number of decompositions that we should check reduces from $2^{2|\mathcal{N}_{\rm II}|}$ to $\sum_{i=0}^{|\mathcal{N}_{\rm II}|} 3^i$. The term $\sum_{i=0}^{|\mathcal{N}_{\rm II}|} 3^i$ is calculated as follows. We only need to check decompositions in which $\mathcal{Z}_4$ contains consecutive stages until the end of Epoch II. When $\mathcal{Z}_4$ contains the last i ($i \in \{0,1,2,...,|\mathcal{N}_{\rm II}|\}$) stages in Epoch II, there are $3^{|\mathcal{N}_{\rm II}|-i}$ decompositions since each of the first ($|\mathcal{N}_{\rm II}| - i$) stages in Epoch II can be in $\mathcal{Z}_1$, $\mathcal{Z}_2$, or $\mathcal{Z}_3$. Thus, the number of decompositions that we should check is $\sum_{i=0}^{|\mathcal{N}_{\rm II}|} 3^{|\mathcal{N}_{\rm II}|-i}$, which is mathematically equal to $\sum_{i=0}^{|\mathcal{N}_{\rm II}|} 3^i$.

So far all Nash equilibria of the non-cooperative game in Epoch II have been found. If there exists only one unique Nash equilibrium (e.g., when $|\mathcal{N}_{\rm III}| \le 12$), then both sellers follow the unique Nash equilibrium. If there are two or more Nash equilibria, the two sellers need to select one Nash equilibrium to follow. Here the two sellers agree to follow the *max-min Nash equilibrium*, defined as the Nash equilibrium that maximizes the minimum unit-bandwidth revenue of the two sellers. Here for seller 1, its unit-bandwidth revenue is the ratio of its total revenue in Epoch II to $Q_1^{\mathrm{II}_c}$; for seller 2, its unit-bandwidth revenue is the ratio of its total revenue in Epoch II to $Q_2^{\rm II}$.

V. Total Revenue Maximization for seller 1

In the previous section, we have found the strategies of the two sellers in Epoch II with a specific $Q_1^{\rm III}$ (the bandwidth that seller 1 reserves for Epoch III). Now, we try to solve Problem 5, i.e., find out the optimal value of $Q_1^{\rm III}$ that maximizes seller 1's total revenue. A method by exhaustive search could be: 1) for each possible value of $Q_1^{\rm III}$, search all possible Nash equilibria, find the max-min Nash equilibrium, and calculate the revenue that seller 1 can earn during its leasing period with the max-min Nash equilibrium; 2) for different $Q_1^{\rm III}$, compare the revenue values that seller 1 can earn during its leasing period, and select the optimal $Q_1^{\rm III}$ that makes seller 1 earn the most revenue. However, the complexity of the exhaustive search method is huge, due to the infinite number of values of $Q_1^{\rm III} \in [0, Q_1]$. Thus, we target at an approximation method to select $Q_1^{\rm III}$.

When $Q_1^{\rm III} = x$, $U(Q_1 - x)$ and $V(x)$ given in (8) are the revenue of seller 1 in Epoch II and Epoch III, respectively. To select x (i.e., $Q_1^{\rm III}$), we need to evaluate how $V(x)$ and $U(Q_1 - x)$ change when x varies.

Lemma 1: The function $V(x)$ is an increasing and concave function with x.

Proof: The proof follows a similar procedure to the proof of Lemma 6 of [25]. ■

Now we evaluate function $U(Q_1 - x)$ when x varies. To evaluate $U(Q_1 - x)$ for a specific decomposition $\{\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \mathcal{Z}_4\}$, we need to know $d_{n,1}$ and $d_{n,2}$ ($n \in \mathcal{N}_{\rm II}$) in the Nash equilibrium corresponding to the decomposition. Therefore, next we show how $d_{n,1}$ and $d_{n,2}$ change when x varies.

Consider a decomposition $\{\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \mathcal{Z}_4\}$. Consider two $Q_1^{\mathrm{II}_c}$ values (recalling that $Q_1^{\mathrm{II}_c} = Q_1 - Q_1^{\rm III}$): $Q^\dagger$ and $Q^\ddagger$, with $Q^\dagger \le Q^\ddagger$. Assume the decomposition $\{\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \mathcal{Z}_4\}$ is feasible for both $Q_1^{\mathrm{II}_c}$ values. For the decomposition, denote the corresponding Nash equilibrium when $Q_1^{\mathrm{II}_c} = Q^\dagger$ as

$$\left(\mathcal{S}_1^\dagger, \mathcal{S}_2^\dagger\right) \triangleq \left(\left\{d_{n,1}^\dagger | n \in \mathcal{N}_{\rm II}\right\}, \left\{d_{n,2}^\dagger | n \in \mathcal{N}_{\rm II}\right\}\right),$$

and the corresponding Nash equilibrium when $Q_1^{\mathrm{II}_c} = Q^\ddagger$ as

$$\left(\mathcal{S}_1^\ddagger, \mathcal{S}_2^\ddagger\right) \triangleq \left(\left\{d_{n,1}^\ddagger | n \in \mathcal{N}_{\rm II}\right\}, \left\{d_{n,2}^\ddagger | n \in \mathcal{N}_{\rm II}\right\}\right).$$

Then the following lemmas can be expected.

Lemma 2: For seller 1, $d_{n,1}^\dagger \le d_{n,1}^\ddagger$ for $n \in \mathcal{Z}_2$, and $d_{n,1}^\dagger = d_{n,1}^\ddagger = 0$ for $n \in \mathcal{Z}_3 \bigcup \mathcal{Z}_4$.

Proof: By the definitions of set $\mathcal{Z}_3$ and $\mathcal{Z}_4$, seller 1 does not offer spectrum bandwidth to be leased in stages in $\mathcal{Z}_3$ and $\mathcal{Z}_4$, and thus, $d_{n,1}^\dagger = d_{n,1}^\ddagger = 0$ for $n \in \mathcal{Z}_3 \bigcup \mathcal{Z}_4$.

From (23), (25), and the fact that $\mathcal{Z}_1 \cup \mathcal{Z}_2 \ne \emptyset$, we have $A_{11} > A_{21} \ge 0$. From (24), (26), and the fact that $\mathcal{Z}_1 \cup \mathcal{Z}_3 \ne \emptyset$, we have $A_{22} > A_{12} \ge 0$. Thus, we have $A_{11}A_{22} - A_{21}A_{12} > 0$.

For $n \in \mathcal{Z}_2$, with the aid of (29) and (27), we have

$$\begin{aligned} d_{n,1}^\dagger - d_{n,1}^\ddagger &= \tfrac{C_0 A_{22}}{2C_1 n (A_{11}A_{22} - A_{21}A_{12})} \left(Q^\dagger - Q^\ddagger\right) \\ &\le 0 \end{aligned} \tag{31}$$

in which the inequality comes from the fact that $A_{22} \ge 0$, $Q^\dagger \le Q^\ddagger$, and $(A_{11}A_{22} - A_{21}A_{12}) > 0$.

This completes the proof. ■

Lemma 3: For seller 2, $d_{n,2}^\dagger \le d_{n,2}^\ddagger$ for $n \in \mathcal{Z}_3$, and $d_{n,2}^\dagger = d_{n,2}^\ddagger = 0$ for $n \in \mathcal{Z}_2 \bigcup \mathcal{Z}_4$.

Proof: The proof is similar to the proof for Lemma 2, and thus, is omitted here. ■

Theorem 3: If a decomposition $\{\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \mathcal{Z}_4\}$ is feasible when $Q_1^{\rm III} = x \in \mathcal{I}$ where $\mathcal{I} \subseteq [0, Q_1]$ is an interval, then when the Nash equilibrium corresponding to the decomposition is followed by the two sellers in Epoch II, seller 1's revenue $U(Q_1 - x)$ in Epoch II can be written as $U(Q_1 - x) = G(x) - H(x)$ where $G(x)$ and $H(x)$ are monotonically increasing functions with respect to $x \in \mathcal{I}$.

Proof: Suppose the Nash equilibrium corresponding to the decomposition $\{\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \mathcal{Z}_4\}$ is $(\{d_{n,1} | n \in \mathcal{N}_{\rm II}\},$

$\{d_{n,2}|n \in \mathcal{N}_{\text{II}}\})$. Then $U(Q_1 - x)$ can be written as

$$\begin{aligned}
&U(Q_1 - x) \\
&= \sum_{n\in\mathcal{N}_{\text{II}}} \left(C_0 - C_1\left(d_{n,1} + d_{n,2}\right)\right) d_{n,1}n \\
&\overset{(a)}{=} \sum_{n\in\mathcal{Z}_1} \left(C_0 - C_1\left(d_{n,1} + d_{n,2}\right)\right) d_{n,1}n \\
&\quad + \sum_{n\in\mathcal{Z}_2} \left(C_0 - C_1 d_{n,1}\right) d_{n,1}n \\
&\overset{(b)}{=} \sum_{n\in\mathcal{Z}_1} \left(\tfrac{C_0}{3} + \tfrac{\zeta}{3(n-|\mathcal{N}_{\text{III}}|)} + \tfrac{\lambda}{3n}\right)\left(\tfrac{\zeta}{3C_1(n-|\mathcal{N}_{\text{III}}|)} - \tfrac{2\lambda}{3C_1 n}\right. \\
&\quad \left.+ \tfrac{C_0}{3C_1}\right) n + \sum_{n\in\mathcal{Z}_2} \left(C_0 - C_1 d_{n,1}\right) d_{n,1}n \\
&= \sum_{n\in\mathcal{Z}_1} \left(\tfrac{\zeta^2}{9C_1(n-|\mathcal{N}_{\text{III}}|)^2} + \tfrac{2C_0\zeta}{9C_1(n-|\mathcal{N}_{\text{III}}|)} + \tfrac{C_0^2}{9C_1}\right) n \\
&\quad - \sum_{n\in\mathcal{Z}_1} \left(\tfrac{\zeta\lambda}{9C_1 n(n-|\mathcal{N}_{\text{III}}|)} + \tfrac{2\lambda^2}{9C_1 n^2} + \tfrac{C_0\lambda}{9C_1 n}\right) n \\
&\quad + \sum_{n\in\mathcal{Z}_2} \left(C_0 - C_1 d_{n,1}\right) d_{n,1}n
\end{aligned} \tag{32}$$

where (a) holds since $d_{n,1} = 0$ for $n \in \mathcal{Z}_3 \bigcup \mathcal{Z}_4$ and $d_{n,2} = 0$ for $n \in \mathcal{Z}_2$, and (b) can be obtained by substituting $d_{n,1}$ and $d_{n,2}$ according to (29) and (30).

As the decomposition $\{\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \mathcal{Z}_4\}$ is feasible, λ and ζ are non-negative. Additionally, from (27) and (28), it can be seen that λ and ζ are monotonically decreasing with $Q_1^{\text{II}_c}$, i.e., $(Q_1 - x)$. So in the expression (32), both the term

$$\sum_{n\in\mathcal{Z}_1} \left(\frac{\zeta^2}{9C_1\left(n - |\mathcal{N}_{\text{III}}|\right)^2} + \frac{2C_0\zeta}{9C_1\left(n - |\mathcal{N}_{\text{III}}|\right)} + \frac{C_0^2}{9C_1}\right) n$$

and the term

$$\sum_{n\in\mathcal{Z}_1} \left(\frac{\zeta\lambda}{9C_1 n\left(n - |\mathcal{N}_{\text{III}}|\right)} + \frac{2\lambda^2}{9C_1 n^2} + \frac{C_0\lambda}{9C_1 n}\right) n$$

are monotonically decreasing with $Q_1^{\text{II}_c}$, and thus, are monotonically increasing with x (as $Q_1^{\text{II}_c} = Q_1 - x$). It can be also checked that the term $\sum_{n\in\mathcal{Z}_2} \left(C_0 - C_1 d_{n,1}\right) d_{n,1}n$ in (32) is a monotonically increasing function with respect to $Q_1^{\text{II}_c}$ (since the function $\left(C_0 - C_1 y\right) y$ is monotonically increasing with y and $d_{n,1}$ grows with $Q_1^{\text{II}_c}$ [from Lemma 2]), and thus, is a monotonically decreasing function with respect to x.

Define

$$G(x) = \sum_{n\in\mathcal{Z}_1} \left(\frac{\zeta^2}{9C_1\left(n - |\mathcal{N}_{\text{III}}|\right)^2} + \frac{2C_0\zeta}{9C_1\left(n - |\mathcal{N}_{\text{III}}|\right)} + \frac{C_0^2}{9C_1}\right) n \tag{33}$$

and

$$\begin{aligned}
H(x) = &\sum_{n\in\mathcal{Z}_1} \left(\frac{\zeta\lambda}{9C_1 n\left(n - |\mathcal{N}_{\text{III}}|\right)} + \frac{2\lambda^2}{9C_1 n^2} + \frac{C_0\lambda}{9C_1 n}\right) n \\
&- \sum_{n\in\mathcal{Z}_2} \left(C_0 - C_1 d_{n,1}\right) d_{n,1}n.
\end{aligned} \tag{34}$$

It can be seen that $U(Q_1 - x) = G(x) - H(x)$, and both $G(x)$ and $H(x)$ monotonically increase with x.

This completes the proof. ■

In Lemma 2, Lemma 3, and Theorem 3, it is assumed that the decomposition $\{\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \mathcal{Z}_4\}$ is feasible for $x = Q_1 - Q^\dagger$, $x = Q_1 - Q^\ddagger$ or $x \in \mathcal{I}$. The next theorem will answer the following question: If a decomposition is feasible for a specific value of x, will it continue to be feasible if x increases or decreases?

Theorem 4: Assume a decomposition $\{\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \mathcal{Z}_4\}$ is feasible for $x = x_0 \in [0, Q_1]$. If x increases from x_0, then there exists a point denoted $x_1 \in [x_0, Q_1]$ such that the decomposition is always feasible in interval $[x_0, x_1]$, and is always infeasible in interval $(x_1, Q_1]$. If x decreases from x_0, then there exists a point denoted $x_2 \in [0, x_0]$ such that the decomposition is always feasible in interval $[x_2, x_0]$, and is always infeasible in interval $[0, x_2)$.

Proof: Here we only prove the case when x increases, as the case when x decreases can be proved similarly.

For an x (i.e., Q_1^{III}) value, the feasibility of decomposition $\{\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \mathcal{Z}_4\}$ is checked as follows: calculate λ and ζ based on (27) and (28), calculate $d_{n,1}$ and $d_{n,2}$ based on (29), (30), and the calculated λ and ζ values, and calculate μ_n and ν_n based on (18), (19), and the calculated $d_{n,1}$, $d_{n,2}$, λ and ζ values. If all the values of λ, ζ, $d_{n,1}$, $d_{n,2}$, μ_n, and ν_n ($n \in \mathcal{N}_{\text{II}}$) are non-negative, then the decomposition $\{\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \mathcal{Z}_4\}$ is feasible; otherwise, it is infeasible.

Expressions (27) and (28) show that λ and ζ are linear functions of x (i.e., Q_1^{III}).

Expressions (29) and (30) show that $d_{n,1}$ and $d_{n,2}$ are linear functions of λ and ζ, and thus, are linear functions of x.

Expressions (18) and (19) show that μ_n and ν_n are linear functions of λ, ζ, $d_{n,1}$, and $d_{n,2}$, and thus, are linear functions of x.

Overall, λ, ζ, $d_{n,1}$, $d_{n,2}$, μ_n, and ν_n ($n \in \mathcal{N}_{\text{II}}$) are all linear functions of x (i.e., Q_1^{III}).

When $x = x_0$, as the decomposition $\{\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \mathcal{Z}_4\}$ is feasible, all the λ, ζ, $d_{n,1}$, $d_{n,2}$, μ_n, and ν_n ($n \in \mathcal{N}_{\text{II}}$) are non-negative. When x increases from x_0, values of λ, ζ, $d_{n,1}$, $d_{n,2}$, μ_n, and ν_n linearly change accordingly. If at one point, say $x = x_1$, one of λ, ζ, $d_{n,1}$, $d_{n,2}$, μ_n, and ν_n decreases to value 0, then we can see that for $x \in [x_0, x_1]$, the decomposition $\{\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \mathcal{Z}_4\}$ is always feasible, and for $x \in (x_1, Q_1]$, the decomposition is always infeasible.[8]

This completes the proof. ■

Remark: Theorem 4 shows that if a decomposition $\{\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \mathcal{Z}_4\}$ is feasible for $x = x_0$, then there exists an interval of x containing x_0 such that the decomposition is feasible inside the interval, and infeasible outside the interval.

Based on Lemma 1, Theorem 3, and Theorem 4, we propose that seller 1 uses the following Algorithm 1 to select x (i.e., Q_1^{III}).

[8]As an extreme case, if λ, ζ, $d_{n,1}$, $d_{n,2}$, μ_n, and ν_n all keep non-negative when x increases from x_0 to Q_1, then we have $x_1 = Q_1$.

Algorithm 1 Searching procedure for x (i.e., Q_1^{III}).

1: Set $x^* = 0$, and $R^* = 0$.
2: Set $x^\dagger = 0$
3: For $x = x^\dagger$, find out all feasible Nash equilibria, and pick up the max-min Nash equilibrium and corresponding decomposition $\{\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \mathcal{Z}_4\}$.
4: Find (using bisection search) a point denoted x_1 such that the decomposition picked in Step 3 is feasible for $x \in [x^\dagger, x_1]$, and infeasible for $x \in (x_1, Q_1]$.
5: Set $x^\ddagger = x_1$.
6: For complexity reduction, approximately seller 1 considers that the decomposition picked in Step 3 always leads to the *max-min Nash equilibrium* of the non-cooperative game with any $x \in [x^\dagger, x^\ddagger]$. In other words, for any $x \in [x^\dagger, x^\ddagger]$, both sellers always follow the Nash equilibrium corresponding to the decomposition picked in Step 3. Then the revenue of seller 1 can be written as $U(Q_1 - x) + V(x)$. Here $U(Q_1 - x)$ is the difference of two monotonically increasing functions of x (from Theorem 3), while $V(x)$ is an increasing function of x (from Lemma 1). Thus, $U(Q_1 - x) + V(x)$ can be viewed as the difference of two monotonically increasing functions of $x \in [x^\dagger, x^\ddagger]$. To maximize the difference of two monotonically increasing functions, a polyblock method can be used to find the global optimal solution (please refer to [26]–[28] for details). Denote the optimal point as $\hat{x}$ and the corresponding revenue $U(Q_1 - \hat{x}) + V(\hat{x})$ of seller 1 as $\hat{R}$.
7: If $\hat{R} > R^*$, then set $x^* = \hat{x}$ and $R^* = \hat{R}$.
8: If $x^\ddagger = Q_1$, then terminate the algorithm, and output x^*.
9: Set $x^\dagger = x^\ddagger$, and proceed to Step 3.

In the algorithm, x^* denotes the optimal selection of seller 1 for x, and R^* denotes the corresponding overall revenue of seller 1. For $x = x^\dagger = 0$, in Step 3 we first select the max-min Nash equilibrium and corresponding decomposition $\{\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \mathcal{Z}_4\}$. In Steps 4 and 5, we find the interval of x, denoted $[x^\dagger, x^\ddagger]$, such that the previously picked decomposition is feasible inside the interval and infeasible when $x > x^\ddagger$. In Step 6, we approximately consider that for $x \in [x^\dagger, x^\ddagger]$, both sellers always follow the Nash equilibrium corresponding to the decomposition picked in Step 3. Then for $x \in [x^\dagger, x^\ddagger]$, seller 1's revenue $U(Q_1 - x) + V(x)$ can be shown as the difference of two monotonically increasing functions of x. A polyblock algorithm can be used to find the global optimal value of $x \in [x^\dagger, x^\ddagger]$, denoted $\hat{x}$, such that the overall revenue of seller 1 is maximized. Then the $\hat{x}$ is a candidate for seller 1's selection of x (Step 7). Since interval $[x^\dagger, x^\ddagger]$ has been dealt with in Step 6, we proceed to the next interval starting from $x^\ddagger$ in Step 9, to repeat the procedure and find other candidates for seller 1's selection of x. Among all the candidates, the one that has the maximal overall revenue of seller 1 is eventually selected by seller 1.

Overall, the strategies of the two sellers are as follows. In Epoch I, seller 2 derives its optimal strategy by solving Problem 1. At the beginning of Epoch II, seller 1 uses Algorithm 1 to find the value of x, denoted x^*. Then in the non-cooperative game in Epoch II with $Q_1^{\mathrm{III}} = x^*$, both sellers follow the max-min Nash equilibrium. In Epoch III, seller 1 can derive its optimal strategy by solving Problem 4 with $Q_1^{\mathrm{III}} = x^*$.

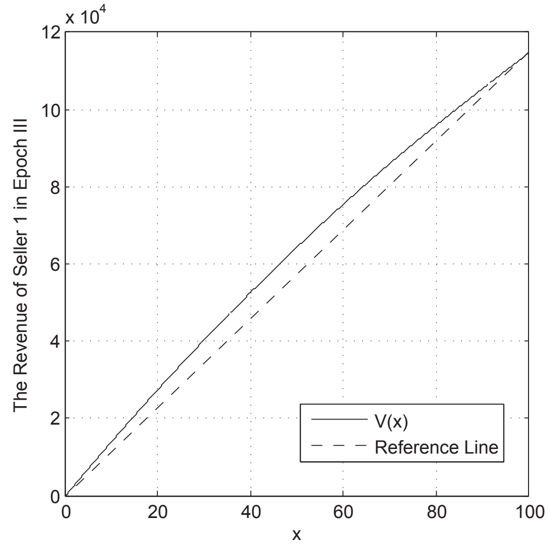

Fig. 2. $V(x)$ versus x (i.e., Q_1^{III}).

VI. Numerical Results

A. Verification of the Analysis

We use numerical results by Matlab to verify the theoretical analysis in this paper. Since the spectrum leasing problem in Epoch I and Epoch III are both convex optimization problems, here we focus on Epoch II, and the number of stages in Epoch III is fixed as $|\mathcal{N}_{\mathrm{III}}| = 3$. At the beginning of Epoch II, seller 1 has spectrum bandwidth with amount $Q_1 = 100$, while seller 2 has available spectrum bandwidth with amount $Q_2^{\mathrm{II}} = 60$. Here the unit of Q_1 and Q_2^{II} is MHz, which can also be approximately transformed to sub-carriers if the system is based on orthogonal frequency division multiplexing (OFDM).

We take $C_0 = 480$ and $C_1 = 1$, which satisfies the requirement in (2). Other configurations of C_0 and C_1 that satisfy the requirement in (2) can also be adopted.

1) Effectiveness of Theorem 2: In this subsection, the effectiveness of Theorem 2 in complexity reduction is verified. Table I lists the number of all possible decompositions and the number of decompositions that should be checked for feasibility with the aid of Theorem 2. It is clear that using Theorem 2 can significantly reduce the number of decompositions that should be checked.

2) Verification of Lemma 1: In this subsection, Lemma 1 is verified. Fig. 2 plots the function $V(x)$ (the revenue of seller 1 in Epoch III) as x (i.e., Q_1^{III}) grows from 0 to 100. From Fig. 2, it can be seen that the function $V(x)$ is an increasing and concave function with respect to x, which is consistent with Lemma 1. Note that the reference line in Fig. 2 is a straight line connecting points $(0, V(0))$ and $(100, V(100))$, which helps to observe the concavity of function $V(x)$.

TABLE I
THE NUMBER OF DECOMPOSITIONS WITHOUT AND WITH THE AID OF THEOREM 2.

$\|\mathcal{N}_{\mathrm{II}}\|$	2	4	6	8	10	15	20
Total number of decompositions	16	256	4096	6.6×10^5	1.0×10^6	1.1×10^9	1.1×10^{12}
Checked decompositions (with Theorem 2)	13	121	1093	9841	8.9×10^4	2.2×10^7	5.2×10^9

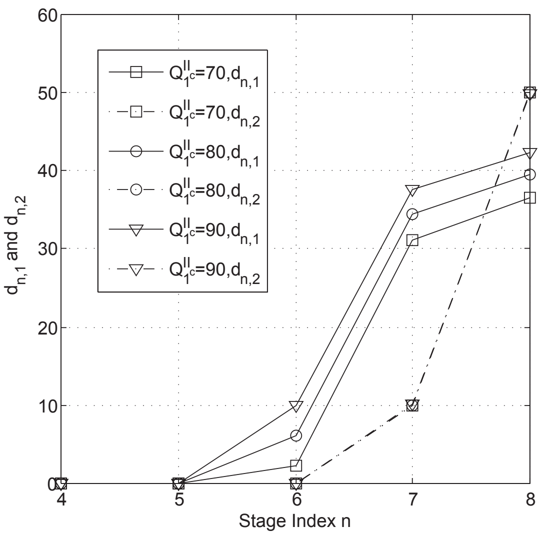

Fig. 3. $d_{n,1}$ and $d_{n,2}$ versus n for $\mathcal{Z}_1 = \{7,8\}$, $\mathcal{Z}_2 = \{6\}$, $\mathcal{Z}_3 = \emptyset$, $\mathcal{Z}_4 = \{4,5\}$, and $Q_1^{\mathrm{II}_c} = 70, 80, 90$.

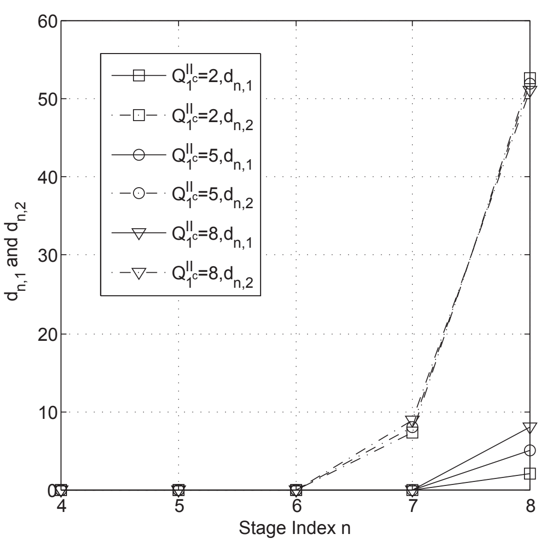

Fig. 4. $d_{n,1}$ and $d_{n,2}$ versus n for $\mathcal{Z}_1 = \{8\}$, $\mathcal{Z}_2 = \emptyset$, $\mathcal{Z}_3 = \{7\}$, $\mathcal{Z}_4 = \{4,5,6\}$, and $Q_1^{\mathrm{II}_c} = 2, 5, 8$.

TABLE II
THE DECOMPOSITIONS USED WHEN VERIFYING THEOREM 3

	$\mathcal{Z}_1$	$\mathcal{Z}_2$	$\mathcal{Z}_3$	$\mathcal{Z}_4$
1st decomposition	$\{7,8\}$	$\{6\}$	$\emptyset$	$\{4,5\}$
2nd decomposition	$\{7,8\}$	$\emptyset$	$\emptyset$	$\{4,5,6\}$

3) Verification of Lemma 2 and Lemma 3: In this subsection, Lemma 2 and Lemma 3 are verified. Consider $\mathcal{N}_{\mathrm{II}} = \{4,5,6,7,8\}$. Fig. 3 plots $d_{n,1}$ and $d_{n,2}$ versus the stage index n for a feasible decomposition in which $\mathcal{Z}_1 = \{7,8\}$, $\mathcal{Z}_2 = \{6\}$, $\mathcal{Z}_3 = \emptyset$, and $\mathcal{Z}_4 = \{4,5\}$ when $Q_1^{\mathrm{II}_c}$ is set to be 70, 80, and 90. Fig. 4 plots $d_{n,1}$ and $d_{n,2}$ versus the stage index for a feasible decomposition in which $\mathcal{Z}_1 = \{8\}$, $\mathcal{Z}_2 = \emptyset$, $\mathcal{Z}_3 = \{7\}$, and $\mathcal{Z}_4 = \{4,5,6\}$ when $Q_1^{\mathrm{II}_c}$ is set to be 2, 5, and 8. From Fig. 3 and Fig. 4, it can be seen that, when $Q_1^{\mathrm{II}_c}$ changes, $d_{n,1}$ and $d_{n,2}$ vary in the same way as Lemma 2 and Lemma 3 describe.

4) Verification of Theorem 3: In this subsection, the characteristic of $U(Q_1 - x)$ described in Theorem 3 is verified. Still consider $\mathcal{N}_{\mathrm{II}} = \{4,5,6,7,8\}$. Two decompositions are investigated, which are listed in Table II. Consider two intervals of x: $[0,30]$ and $[40,70]$, in which the two decompositions are feasible, respectively. Fig. 5 and Fig. 6 plot the function $U(Q_1 - x)$ as well as $G(x)$ and $H(x)$ (from Theorem 3) for the two decompositions over the two corresponding intervals, respectively. It can be seen that both the functions $G(x)$ and $H(x)$ are monotonically increasing for each decomposition in the corresponding interval of x, which is consistent with Theorem 3.

B. Performance of Algorithm 1

1) Computation complexity of Algorithm 1: Computation complexity of Algorithm 1 largely depends on the number of iterations (i.e., how many times are Steps 3–7 repeated?). Table III shows the iteration number for different settings of $|\mathcal{N}_{\mathrm{II}}|$ and $|Q_1^{\mathrm{II}_c}|$. It can be seen that in all the cases, the number of iterations is 2 or 3, which shows that Algorithm 1 is efficient in reducing computation complexity.

2) Suboptimality of Algorithm 1: In Algorithm 1, to search optimal x (i.e., Q_1^{III}), an approximation is used to reduce the computation complexity, as follows. For the non-cooperative game with $x^\dagger$, in Step 3 of Algorithm 1, we first find the max-min Nash equilibrium and the corresponding decomposition $\{\mathcal{Z}_1, \mathcal{Z}_2, \mathcal{Z}_3, \mathcal{Z}_4\}$. We then find an interval of x, denoted $[x^\dagger, x^\ddagger]$, such that the decomposition picked in Step 3 is a feasible decomposition of the non-cooperative game with any $x \in [x^\dagger, x^\ddagger]$. Then we approximately consider that the decomposition, which leads to the max-min Nash equilibrium

TABLE III
NUMBER OF ITERATIONS IN ALGORITHM 1

	$\lvert Q_1^{\mathrm{II}_c}\rvert = 20$	$\lvert Q_1^{\mathrm{II}_c}\rvert = 40$	$\lvert Q_1^{\mathrm{II}_c}\rvert = 60$	$\lvert Q_1^{\mathrm{II}_c}\rvert = 80$	$\lvert Q_1^{\mathrm{II}_c}\rvert = 100$
$\lvert\mathcal{N}_{\mathrm{II}}\rvert = 4$	2	2	2	3	3
$\lvert\mathcal{N}_{\mathrm{II}}\rvert = 6$	2	2	3	3	3
$\lvert\mathcal{N}_{\mathrm{II}}\rvert = 8$	2	3	3	3	3

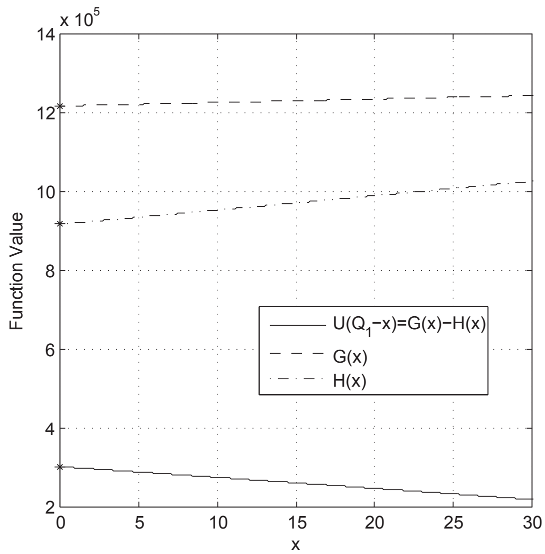

Fig. 5. Functions $U(Q_1 - x)$, $G(x)$ and $H(x)$ ($x \in [0, 30]$) for the first decomposition in Table II.

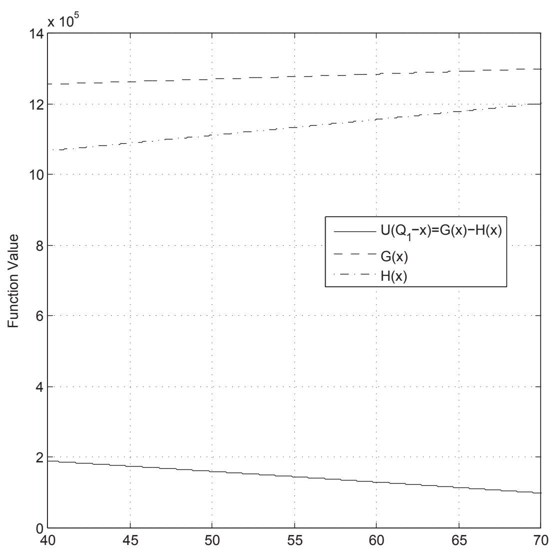

Fig. 6. Functions $U(Q_1 - x)$, $G(x)$ and $H(x)$ ($x \in [40, 70]$) for the second decomposition in Table II.

of the non-cooperative game with $x^\dagger$, also leads to the *max-min Nash equilibrium* of the non-cooperative game with any $x \in [x^\dagger, x^\ddagger]$. In other words, for any $x \in [x^\dagger, x^\ddagger]$, both sellers always follow the Nash equilibrium corresponding to the decomposition picked in Step 3. Due to this approximation, Algorithm 1 in general is suboptimal in finding the optimal x.

From Theorem 1, if $|\mathcal{N}_{\mathrm{III}}| \leq 12$, then the non-cooperative game in Epoch II always has a unique Nash equilibrium, which means that the decomposition picked in Step 3 of Algorithm 1 is the only feasible decomposition for any $x \in [x^\dagger, x^\ddagger]$. In other words, the decomposition indeed always leads to the *max-min Nash equilibrium* of the non-cooperative game with any $x \in [x^\dagger, x^\ddagger]$. Thus, Algorithm 1 is optimal if $|\mathcal{N}_{\mathrm{III}}| \leq 12$.

We have tried a large number of scenarios with $|\mathcal{N}_{\mathrm{III}}| \in \{13, 14, ..., 30\}$, and have found that the associated non-cooperative games in Epoch II all have unique Nash equilibrium, which means our Algorithm 1 is optimal in those considered scenarios. However, we find that it is hard to prove theoretically that Nash equilibrium of the non-cooperative game in Epoch II is unique or not unique when $|\mathcal{N}_{\mathrm{III}}| \geq 13$. This is an interesting problem, and we leave it for future investigation.

C. Comparison with a Cooperative Scheme

Now we compare with a cooperative scheme. The difference of the cooperative scheme from our proposed scheme is as follows. When the two sellers know the existence of each other (i.e., at the beginning of Epoch II), the two sellers cooperate to jointly maximize the total revenue of them over Epoch II and III, by solving the following optimization problem.

$$\begin{aligned}
\max_{\substack{\{d_{n,1}|n\in\mathcal{N}_1\},\\ \{d_{n,2}|n\in\mathcal{N}_{\mathrm{II}}\}}} \quad & \sum_{n\in\mathcal{N}_{\mathrm{II}}} \left(C_0 - C_1(d_{n,1}+d_{n,2})\right) d_{n,1} n \\
& + \sum_{n\in\mathcal{N}_{\mathrm{III}}} \left(C_0 - C_1 d_{n,1}\right) d_{n,1} n \\
& + \sum_{n\in\mathcal{N}_{\mathrm{II}}} \left(C_0 - C_1(d_{n,1}+d_{n,2})\right) d_{n,2} (n - |\mathcal{N}_{\mathrm{III}}|) \\
\text{s.t.} \quad & \sum_{n\in\mathcal{N}_{\mathrm{II}}\cup\mathcal{N}_{\mathrm{III}}} d_{n,1} \leq Q_1 \\
& \sum_{n\in\mathcal{N}_{\mathrm{II}}} d_{n,2} \leq Q_2^{\mathrm{II}} \\
& d_{n,1} \geq 0, \forall n \in \mathcal{N}_{\mathrm{II}} \cup \mathcal{N}_{\mathrm{III}} \\
& d_{n,2} \geq 0, \forall n \in \mathcal{N}_{\mathrm{II}}.
\end{aligned} \tag{35}$$

For performance comparison, the simulation is set up as follows. Since the cooperative scheme and our proposed scheme perform the same in Epoch I, we set $\mathcal{N}_{\mathrm{I}} = \emptyset$. And $\mathcal{N}_{\mathrm{II}} = \{6, 5, 4, 3\}$, $\mathcal{N}_{\mathrm{III}} = \{2, 1\}$. We fix the sum of Q_1 and Q_2 to be 200, and consider three configurations of (Q_1, Q_2): $(50, 150)$, $(100, 100)$, and $(150, 50)$. Fig. 7 shows the achieved

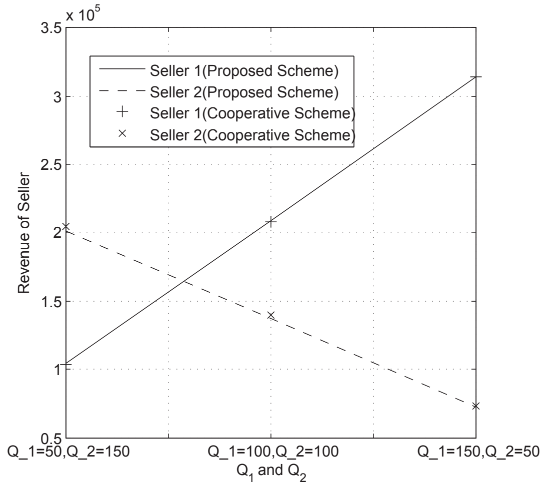

Fig. 7. The revenue of the two sellers in our proposed scheme and the cooperative scheme.

revenue of the two sellers in our proposed scheme and the cooperative scheme. It can be seen that each seller's revenue in our proposed non-cooperative scheme is very close to that in the cooperative scheme, thus verifying the efficiency of our proposed scheme.

D. Performance with Random Leasing Periods

We consider that each seller has alternating ON and OFF states. Here an ON and OFF state mean that the seller has and does not have spectrum to lease, respectively. When the state of the seller changes from OFF to ON, we call it an arrival, and when the state of the seller changes from ON to OFF, we call it a departure. Thus, for the seller, the duration from an arrival to the following departure is its leasing period. For seller $i \in \{1, 2\}$, the ON duration and OFF duration are exponential distributed with mean value being $1/\mu_i$ and $1/\lambda_i$, respectively. Here λ_i and μ_i are called the arrival and departure rate, respectively. The time duration of one stage is set as 1. We vary μ_1, μ_2, λ_1, and λ_2. For seller 1 and seller 2, when an ON state begins, the amount of available spectrum to lease is $Q_1 = 100$ and $Q_2 = 60$, respectively.

Following the random arrival and departure processes of the two sellers, we have a number of zero-seller, one-seller, and two-seller epochs.

- A zero-seller epoch means a number of consecutive stages in which neither seller has spectrum to lease.
- A one-seller epoch means a number of consecutive stages in which only one seller has spectrum to lease. The seller can get its leasing strategy by solving an optimization problem similar to Problem 1 in Section III-A.
- A two-seller epoch means a number of consecutive stages in which both sellers have spectrum to lease. If a two-seller epoch is followed by a one-seller epoch, the leasing strategy in the two-seller epoch and the following one-seller epoch can be obtained similar to our treatment for Epoch II and Epoch III in Section IV and V. If a two-seller epoch is followed by a zero-seller epoch, the leasing strategy in the two-seller epoch can be obtained similar to our treatment for our Epoch II in Section IV with $Q_1^{\text{III}} = 0$.

Without loss of generality, the average revenue of seller 1 per leased stage (i.e., the total revenue of seller 1 divided by the number of total stages in which seller 1 has spectrum to lease) is plotted in Fig. 8, for varying μ_1, μ_2, λ_1, and λ_2. We have the following observations.

- When μ_1 increases, seller 1's average revenue decreases. This is because for a higher μ_1, seller 1's leasing period has less duration. A shorter leasing period reduces the flexibility in seller 1's spectrum leasing, thus reducing seller 1's average revenue.
- When μ_2 increases, seller 1's average revenue increases. This is because higher μ_2 leads to shorter leasing period of seller 2, which means less competition of seller 2 to seller 1. Thus, seller 1's average revenue increases.
- When λ_1 increases, seller 1's average revenue almost keeps the same. The reason is as follows. When λ_1 increases, seller 1's average OFF state duration (expressed as $1/\lambda_1$) decreases. However, when seller 1 is ON, its chance to overlap with seller 2's leasing period is not affected by seller 1's average OFF duration. In other words, seller 1's average OFF duration length does not affect the competition that seller 1 receives from seller 2 when seller 1 is ON. Thus, seller 1's average revenue almost keeps the same.
- When λ_2 increases, seller 1's average revenue decreases. The reason is as follows. When λ_2 increases, seller 2's average OFF state duration (expressed as $1/\lambda_2$) decreases. Thus, when seller 1 is ON, its chance to overlap with seller 2's leasing period is higher. In other words, seller 1 receives more competition from seller 2, and thus, its average revenue decreases.

VII. Conclusions

In this paper, we investigate spectrum leasing with two sellers, in which seller 1 leases spectrum in Epoch II and Epoch III, and seller 2 leases spectrum in Epoch I and Epoch II, as shown in Fig. 1. In Epoch I, only seller 2 has spectrum to lease, and its strategy is derived by solving a convex problem. In Epoch II, since the two sellers both have spectrum to lease, competition between the two sellers exists. Thus, the spectrum leasing in Epoch II is formulated as a non-cooperative game. Nash equilibria of the game are derived in closed form by jointly solving two optimization problems. By analyzing the choice of seller 1 for Epoch III, seller 1's strategy in Epochs II and III and seller 2's strategy in Epoch II are developed.

In this paper, we consider a duopoly market. When there are three or more sellers, similarly the union of their leasing periods can be divided into a number of epochs. For an epoch with one seller, the seller can derive its leasing strategy by solving a convex optimization problem similar to Problem 1 in this paper. For an epoch with two or more sellers, a game model can be set up, and all sellers follow the Nash equilibrium that maximizes the minimum unit-bandwidth revenue of the sellers. If a seller's leasing period also continues

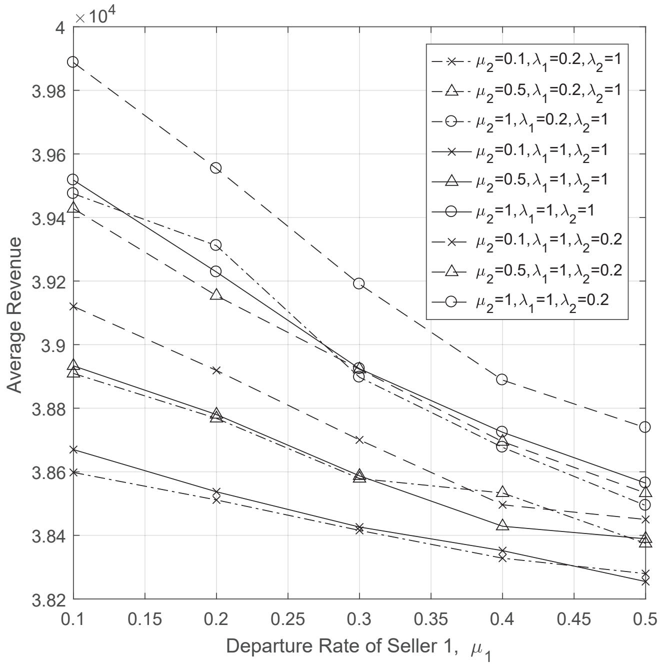

Fig. 8. Average revenue of seller 1.

into subsequent epochs, the seller's leasing strategies over the multiple epochs can be derived jointly. In other words, similar to the method in Section V in this paper, the seller can use the amount of the spectrum reserved for later epochs as an input to the game model, and find the optimal amount of the reserved spectrum that maximizes its overall revenue.

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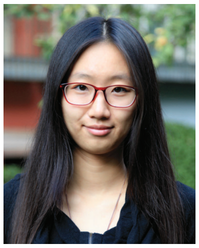

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