Efficient Data Traffic Forwarding for Infrastructure-to-Infrastructure Communications in VANETs

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***Abstract*— In this work, we consider roadside infrastructure to roadside infrastructure communications in a vehicular ad hoc network. A remote roadside unit (RSU), which does not have connection to the backbone network, needs to send its data traffic to a central RSU (which has backbone connection) by using help from passing-by vehicles. Cost is assigned to information transmission energy consumption, as well as possible violation of a soft delay bound. For each passing-by vehicle, the remote RSU needs to decide whether or not to ask for help from the vehicle, with a target at minimal rate of cost. We derive an optimal decision strategy of the remote RSU, which is shown to have a conditional pure-threshold structure, i.e., when a vehicle arrives at the remote RSU, if the queuing delay of the data traffic at the remote RSU is above a threshold, it is optimal for the remote RSU to ask for help from the vehicle, with a condition that the vehicle's speed satisfies a requirement. We also provide a method that can theoretically derive the threshold. The conditional pure-threshold structure makes our derived strategy very easy to implement with very low computation complexity.**

***Index Terms*— Delay, infrastructure-to-infrastructure (I2I) communications, vehicular ad hoc network.**

I. Introduction

A vehicular ad hoc network (VANET) can support the communications among roadside units (RSUs, the infrastructure) and vehicles. By providing safety messages, road conditions, and commercial services, VANET is essential to make a safe, intelligent, and convenient transportation system [1]–[6]. In the literature, VANET has been well investigated, mainly in two research directions: vehicle-to-infrastructure (V2I) communications and vehicle-to-vehicle (V2V) communications. It is in general assumed that all the RSUs are connected to a backbone network via wired links. However, in some cases, some RSUs may not be connected to the backbone network. For example, in remote areas, it is costly to connect all RSUs to the backbone network. Those *remote RSUs* (i.e, RSUs without backbone connection) need to send their data traffic to *central RSUs* (i.e., RSUs with backbone connection), and then the central RSUs forward the data traffic to the backbone network. This communication is referred to as roadside infrastructure-to-roadside infrastructure (I2I) communication. A cost effective method to achieve I2I communications is to use passing-by vehicles, which can carry messages from the remote RSUs and forward them to central RSUs on their path.

For such vehicle-aided I2I communications, the energy consumption of the remote RSUs is an issue. This is because those RSUs are usually deployed in remote areas, and thus, they do not have constant power supply. So the remote RSUs are often equipped with batteries, and the batteries can get recharged or renewed after a relatively long time (for example, a few months) [7]. Energy efficient data forwarding in VANETs has been investigated recently in the literature [8]–[12]. The work in [8] targets at energy consumption minimization for an RSU. A scheduling scheme is provided, which favors passing-by vehicles with higher velocity and/or shorter distance to the RSU. The work in [9] considers delivery of packets from a source to a destination by using relaying service of other nodes. A delay bound is set for each packet. If a packet cannot be delivered to the destination within the delay bound, the packet will be discarded. To maximize the packet delivery probability subject to an energy consumption constraint, it is shown that the threshold dynamic policy is optimal. The works in [10] and [11] take into account the energy for node discovery process as well as energy for information transmission, for two-hop routing and epidemic routing, respectively. Transmission policy is well designed such that packet delivery probability is maximized. The work in [12] proposes that the RSU-to-vehicle scheduling can be combined with vehicle-to-vehicle forwarding, which can largely lower the energy cost of the RSU.

Although the vehicle-aided I2I communications can tolerate a certain level of delay, a timely delivery is still preferred [13]–[15]. In general, the total delay of a data packet at a source RSU consists of two components: the queuing delay at the source RSU, and the transit delay (i.e., the time needed by a helping vehicle to travel to the destination RSU to deliver its carried data traffic). A tradeoff exists between the queuing delay and the transit delay. To minimize the transit delay, the source RSU should wait for fast vehicles, which may result in larger queuing delay. On the other hand, to minimize the queuing delay, the source RSU should pick up the first passing-by vehicle, which may lead to larger transit delay. The work in [16] considers finite-size traffic case and infinite-size traffic case. For the former case, the source file at the source RSU has a number of packets, and the time duration needed to deliver

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TABLE I

COMPARISON OF DATA TRAFFIC FORWARDING METHODS IN VANETS.

Work	Research Problem, Methodology, and Result
[8]	Problem: Minimize energy consumption of an RSU that processes requests from vehicles. Methodology: Optimization formulation and approximation. Result: A scheduler based on vehicles' locations and velocities.
[9]	Problem: Maximize the multi-hop packet delivery probability from a source to a destination subject to an energy consumption constraint. Methodology: Continuous-time Markov framework. Result: The threshold dynamic policy is optimal.
[10]	Problem: Maximize packet delivery probability in two-hop routing subject to constraints on energy consumption and relay activation rate, considering energy in information transmission and node discovery. Methodology: Fluid approximation, optimal control theory. Result: Optimal two-dimensional threshold policy in closed form for transmission and activation.
[11]	Problem: Maximize message delivery probability in epidemic routing subject to total energy consumption constraint, considering energy in information transmission and node discovery. Methodology: Continuous-time Markov framework. Result: Optimal beaconing control solution.
[12]	Problem: in RSU-to-vehicle communications, minimize RSU energy consumption by using V2V forwarding. Methodology: Integer linear programming. Result: Greedy scheduling algorithms with low complexity.
[16]	Problem: For I2I communications using vehicles as relays, minimize the delay for delivering all packets in a finite-size file, or the average packet delay for a file with infinite packets. Methodology: Markov decision process. Result: Optimal scheduling algorithms and a low-complexity sub-optimal algorithm.
[17]	Problem: For I2I communications using vehicles as relays, minimize sum of queuing delay and transit delay. Methodology: Queuing analysis. Result: Probabilistic scheduling scheme.
[18]	Problem: For I2I communications using vehicles as relays, minimize rate of weighted cost of energy consumption and queuing delay. Methodology: Traditional optimal stopping theory. Result: Optimal pure-threshold strategy.
[19]	Problem: For I2I communications (with hard delay bound) using vehicles as relays, minimize rate of weighted cost of energy consumption, queuing delay, and transit delay. Methodology: Traditional optimal stopping theory. Result: Optimal strategy without threshold structure.
This Paper	Problem: For I2I communications (with soft delay bound) using vehicles as relays, minimize rate of weighted cost of energy consumption, queuing delay, and transit delay. Methodology: New method to solve an optimal stopping problem with forced stop. Result: Optimal strategy with conditional pure-threshold structure.

all packets is minimized by using a Markov decision process. For the latter case, the source file has an infinite number of packets, and the average delay of a packet is minimized by a Markov decision process. Both queuing delay and transit delay are considered and well balanced. The work in [17] assigns each passing-by vehicle a pick-up probability, which favors faster vehicles. For a vehicle, its assigned probability is the probability that its arrival moment at the destination is earlier than its next vehicle‘s expected arrival moment at the destination.

The work in [18] considers both energy consumption and queuing delay. As a follow-up of [18], the work in [19] considers energy consumption, queuing delay, and transit delay. Cost function is assigned to energy consumption as well as packet dropping (due to delay bound violation). An optimal scheduling scheme is derived, which minimizes the rate of cost (i.e., the average cost per unit of time).

For the work in [19], a *hard delay bound* is used, which means that if an information unit at the source RSU cannot be delivered to the destination RSU within the delay bound, the information unit is considered useless and thus, is dropped at the source RSU. Different from the work in [19], here we consider a *soft delay bound*. In specific, it is desired that any information unit is delivered within the delay bound. However, when an information unit cannot be delivered within the delay bound, the information unit is considered to be partially useful and is still delivered, and a cost is charged for the delay bound violation. The rationale behind using soft delay bound in vehicle-aided I2I communications is as follows. One typical application of the source RSU is to serve as gateway for a wireless sensor network that monitors the environments (fire detection, animal tracking, etc.) in remote areas. The sensing data of the wireless sensor network are sent to the source RSU, and are subsequently delivered by the source RSU to a destination RSU with backbone connection. Then the destination RSU sends the data to the data center of the wireless sensor network. It is preferred that the sensing data are delivered within a delay bound. If the sensing data are beyond the delay bound, they still have some value (for example, for later historical studies), and will still be delivered. A similar soft delay bound model was used in [20], [21].

The contributions of this paper are summarized as follows. 1) Based on the soft delay bound model, we formulate an optimal stopping problem. In the formulated optimal stopping problem, a concept of forced stop (the definition of forced stop is given in Section II) is introduced. Due to the forced stop, the methods used in the literature to solve traditional optimal stopping problems, including the method used in [19], do not work here, and a completely new method is required. By characterizing the impact of forced stop, we develop a method to find optimal solution for the formulated problem. 2) We theoretically prove that it is optimal for the source RSU to take a *conditional pure-threshold strategy*. In specific, before a forced stop, the source RSU should transmit to a passing-by vehicle if the queuing delay is above a threshold[1], conditioned on that the total delay (queuing delay plus transit delay of the vehicle) is not more than the delay bound. The conditional pure-threshold structure can largely facilitate implementation of the strategy in a VANET. As a comparison, optimal solution in [19] does not have such a conditional pure-threshold feature, and thus, more computation is needed to make a decision in optimal solution of [19]. 3) We provide a method that quickly calculates the threshold off-line.

The following sections are organized as follows. Section II describes the considered system and formulates the research problem. Section III derives an optimal strategy of the problem, and proves that the optimal strategy has a conditional pure-threshold structure. Section IV provides an efficient method to obtain the threshold. Section V evaluates our derived strategy. Section VI concludes our paper.

Table I compares existing data traffic forwarding methods in VANETs and the method proposed in this paper, and Table II summarizes important notations used in this paper.

TABLE II
USED NOTATIONS

Symbol	Meaning
a, b	Smallest, largest possible transit delay
$\mathcal{C}$	The set of all possible stopping strategies
d	Distance from S-RSU to D-RSU
D	Soft delay bound
$F_S(x)$	Cumulative distribution function of transit delay, given in (1)
$\mathcal{F}_{n_r}$	information of arrival moments and transit delay of vehicles before Vehicle n_r
$N^\dagger(\lambda)$	Optimal stopping strategy of Problem (5)
P	Transmission power of RTS, CTS, DATA, and ACK
R	Transmission rate of DATA packets
r	Data traffic arrival rate at the S-RSU
S_n	The transit delay of the nth vehicle
T_n	The arrival moment of the nth vehicle
U_n	Total cost of using the nth vehicle (given in (2))
$V(\lambda)$	Optimal objective function of Problem (5)
$v_{\min}$	Minimal speed of vehicles
$v_{\max}$	Maximal speed of vehicles
X_n	Time interval from arrival of vehicle $n-1$ to arrival of vehicle n
$Z_n(\lambda)$	Cost function for Problem (5)
$1/\mu$	Average duration between two vehicle arrivals
κ	Communication overhead duration between S-RSU and selected vehicle
ω	Cost weight for energy consumption
β	Cost of violating soft delay bound

[1] In the sequel, “threshold” means threshold for queuing delay at the source RSU.

II. System Model and Problem Formulation

The system model is similar to that in [19], except that a soft delay bound is used here. A source RSU (S-RSU) has constant data traffic arrival rate r. The S-RSU is a remote RSU, and needs to send its data traffic to a destination RSU (D-RSU, which is a central RSU) by using the help of passing-by vehicles. The distance between the S-RSU and D-RSU is d. If a vehicle is selected to help, it takes all buffered data traffic at the S-RSU, and when the vehicle arrives at the D-RSU, it passes all the carried data traffic to the D-RSU .

At the S-RSU, denote the arrival moment of the nth ($n = 1, 2, ...$) vehicle as T_n. Without loss of generality, we set $T_0 = 0$. Similar to [16], [17], [22], the arrival process of vehicles at the S-RSU is a Poisson process with parameter μ, which means that the vehicle inter-arrival durations, denoted as $X_n = T_n - T_{n-1}$ ($n = 1, 2, ...$), are independent exponentially-distributed random variables with mean $1/\mu$. Similar to [23], the speed of the vehicles are independent random variables that are uniformly distributed between $v_{\min}$ (the minimal speed) and $v_{\max}$ (the maximal speed). As the transit delay is the duration for a vehicle to travel from the S-RSU to the D-RSU, it can be seen that the transit delay has cumulative distribution function (CDF) given as

$$F_S(x) = \begin{cases} 0 & \text{if} \quad x < a; \\ \frac{b(x-a)}{x(b-a)} & \text{if} \quad a \le x \le b; \\ 1 & \text{if} \quad x > b. \end{cases} \tag{1}$$

Here $a = d/v_{\max}$ is the smallest possible transmit delay, while $b = d/v_{\min}$ is the largest possible transit delay.

When a vehicle (say the nth vehicle) arrives, the system state is defined as the arrived vehicles' arrival moments (i.e., $T_1, T_2, ..., T_n$) and transit delay (denoted as $S_1, S_2, ..., S_n$), and the S-RSU needs to make a decision between two options, as follows.

- Option *wait*: the nth vehicle is skipped, and the S-RSU waits for later vehicles.
- Option *stop*: the S-RSU stops waiting, and passes its buffered traffic to the nth vehicle by using a four-way handshake RTS-CTS-DATA-ACK. Here RTS means request-to-send, CTS means clear-to-send, DATA carries the data traffic, and ACK means acknowledgement. Denote P as the transmission power of RTS, CTS, DATA, and ACK. Denote κ as the duration of all communication overhead (including RTS, CTS, ACK, as well as the medium access control [MAC] header of the DATA packet). Denote the transmission rate of a DATA packet as R. Similar to [20], [24], [25], we adopt a weighted cost structure for energy consumption and delay as follows. For energy consumption, we assign cost weight ω (unit of cost per Joule). Then the energy consumption cost if the S-RSU stops at the nth vehicle is expressed as $\omega P \left(T_n r/R + \kappa\right)$, in which $T_n r$ is the amount of buffered data traffic at the S-RSU. A soft delay bound D is set for each information unit in the data traffic. If one or multiple information units of the data traffic have a total delay (queuing delay plus transit delay[2]) larger than D, we say that a soft delay bound violation happens, and a fixed charge of β is set (the reason for a fixed charge for one or multiple information units with delay bound violation is given in Appendix A). Therefore, delay cost if the S-RSU stops at the nth vehicle is expressed as $\beta \mathbf{1}_{\{T_n+S_n>D\}}$, where $\mathbf{1}_{\{\cdot\}}$ is an indicator function (which is equal to 1 if the event indicated in $\{\cdot\}$ happens, and equal to 0 otherwise), and S_n is transit delay of the nth vehicle. Overall, the total cost if the S-RSU stops at the nth vehicle is given as

$$U_n = \omega P \kappa + \frac{\omega r P T_n}{R} + \beta \mathbf{1}_{\{T_n+S_n>D\}}. \tag{2}$$

Recall that for any vehicle, the transit delay is always not less than a (the smallest possible transit delay). Thus, if the queuing delay at the S-RSU is more than $(D-a)$, the total delay (queuing delay plus transit delay) will be always more than the delay bound D. Therefore, when its queuing delay is more than $(D-a)$, the S-RSU is required to stop when the next vehicle arrives, referred to as a *forced stop*.[3] The index of the vehicle that is the first arrival after moment $(D-a)$ is denoted as $C \triangleq \min\{n : T_n > D-a\}$. Thus, if the queuing delay at the S-RSU is more than $(D-a)$, it will be forced to stop at the Cth vehicle, and the moment of the forced stop is denoted as T_C. The inter-arrival duration between Vehicle C and its previous vehicle is denoted as X_C.

Denote N as index of the vehicle upon arrival of which the S-RSU stops[4]. We also use N to denote the corresponding stopping strategy. We target at the S-RSU's optimal stopping strategy with minimal rate of cost (i.e., minimal cost per unit time). Define $Y_n \triangleq \omega P \kappa + \beta \mathbf{1}_{\{T_n+S_n>D\}}$, and $\mathcal{C} \triangleq \{N : 1 \le N \le C\}$ is the set of all possible stopping strategies (i.e., due to the forced stop concept, we exclude stopping rules which will stop at a vehicle arriving after the Cth vehicle). Similar to [19], to achieve our target, equivalently we should find

$$\begin{aligned} N^* &\triangleq \arg\inf_{N\in\mathcal{C}} \frac{\mathbb{E}[U_N]}{\mathbb{E}[T_N]} \\ &= \arg\inf_{N\in\mathcal{C}} \frac{\mathbb{E}[Y_N] + \omega r P \mathbb{E}[T_N]/R}{\mathbb{E}[T_N]} \\ &= \arg\inf_{N\in\mathcal{C}} \frac{\mathbb{E}[Y_N]}{\mathbb{E}[T_N]}, \end{aligned} \tag{3}$$

where $\mathbb{E}[\cdot]$ means expectation.[5] As the vehicle inter-arrival durations are exponentially distributed (which means that the vehicle inter-arrival durations are "memoryless"), we have $\mathbb{E}[T_C] = D - a + \mathbb{E}[X_C] = D - a + 1/\mu < \infty$. Thus, $\mathbb{E}[T_N] < \infty$ for all $N \in \mathcal{C}$.

[2]Note that when the S-RSU and a vehicle are exchanging information, the vehicle is also moving. Thus, the duration of transmissions between the S-RSU and the vehicle is included in the transit delay.

[3]Reference [21] uses a concept similar to forced stop. In [21], when the target soft delay bound for a service is crossed, the system must provide the service when the next chance appears.

[4]Note that after the S-RSU stops at a vehicle and transmits its data traffic, we denote this stop moment as $T_0 = 0$, and call the next arrival vehicle as Vehicle 1 again. In other words, the formulated problem is repeated after a stop.

[5]Here we minimize $\mathbb{E}[U_N]/\mathbb{E}[T_N]$ due to the following reason. Since the optimal stopping problem is repeated after a stop, we denote the stopping time in K stops as $T_{N_1}, T_{N_2}, ..., T_{N_K}$ (which are independent and identically distributed), and the corresponding cost in the K stops as $U_{N_1}, U_{N_2}, ...U_{N_K}$ (which are independent and identically distributed), respectively. Then the average cost per unit time is given as $(U_{N_1} + U_{N_2} + ... + U_{N_K})/(T_{N_1} + T_{N_2} + ...T_{N_K})$, which converges to $\mathbb{E}[U_N]/\mathbb{E}[T_N]$ by the law of large numbers [26].

Remarks: For the formulated problem, there is a tradeoff between wait and stop. If the S-RSU waits less time and picks up a vehicle, the selected vehicle can have a larger chance to deliver all data traffic before delay bound. But it is not energy efficient, since the S-RSU needs to have information exchanges with more vehicles in a long term, thus consuming more energy. If the S-RSU waits longer time, it is energy efficient as the S-RSU needs to have information exchanges with fewer vehicles in a long term, but the chance for delay bound violation is also higher. The major challenge in solving the problem is due to the forced stop, which makes the problem different from a traditional optimal stopping problem. A traditional optimal stopping problem does not have forced stop, and thus, methods used to solve traditional optimal stopping problems, including the method used in [19], cannot be used here. In the sequel, we will develop a completely new method to solve our optimal stopping problem with forced stop.

III. An optimal stopping strategy

We have four steps in the following four subsections to derive an optimal stopping strategy for Problem (3).

A. Transformation of the original problem

Define

$$Z_n(\lambda) = Y_n - \lambda T_n = \omega P\kappa + \beta\mathbf{1}_{\{T_n+S_n>D\}} - \lambda T_n,\ \lambda > 0. \tag{4}$$

Here λ can be viewed as rate of cost.

We will first transform Problem (3) into a stopping problem that minimizes $\mathbb{E}\left[Z_N(\lambda)\right]$ [26], i.e.,

$$N^{\dagger}(\lambda) = \arg\inf_{N\in\mathcal{C}} \mathbb{E}\left[Z_N(\lambda)\right]. \tag{5}$$

Theorem 1: If i) for any particular $\lambda > 0$, Problem (5) has an optimal stopping strategy, denoted as $N^{\dagger}(\lambda)$, and ii) there exists a λ^* such that $\mathbb{E}\left[Z_{N^{\dagger}(\lambda^*)}(\lambda^*)\right] = 0$, then an optimal stopping strategy of Problem (3) is in the form of $N^{\dagger}(\lambda^*)$.

Proof: See Appendix B. ■

In the subsequent two steps in Sections III-B and III-C, we derive $N^{\dagger}(\lambda)$ for Problem (5). Then in the last step in Section III-D, we prove that there exists λ^* satisfying the above condition ii).

B. Elimination of a set of non-optimal stopping strategies

In this step, we show that, for Problem (5), a set of stopping strategies are non-optimal, and thus, can be removed from our consideration.

For any stopping strategy $N \in \mathcal{C}$, define an event: $\{T_N \le D - a, T_N + S_N > D\}$. This event means that when the S-RSU stops, it is not a forced stop, and the total delay (queuing delay plus transit delay) is more than the delay bound.

We first consider the following set of stopping strategies:

$$\mathcal{B} = \left\{N \in \mathcal{C} : \Pr\{T_N \le D - a, T_N + S_N > D\} > 0\right\},$$

in which $\Pr\{\cdot\}$ means probability of an event. Next we show that stopping strategies in $\mathcal{B}$ are strictly non-optimal for Problem (5). We can use proof by contradiction. Assume $N \in \mathcal{B}$ is optimal for Problem (5). Then based on N, we can construct a new stopping strategy, denoted as N'. The only difference of N' from N is that: if N advises a stopping such that $T_N \le D - a, T_N + S_N > D$, then N' advises that the S-RSU waits until a forced stop. It can be easily shown that $\mathbb{E}\left[Z_N(\lambda)\right] > \mathbb{E}\left[Z_{N'}(\lambda)\right]$, which contradicts the assumption that strategy N is optimal.

Hence, we need only to search for an optimal stopping strategy within the following collection of stopping strategies:

$$\mathcal{N} = \mathcal{C}\backslash\mathcal{B} = \{N \in \mathcal{C} : T_N + S_N \le D \text{ if } T_N \le D - a\}.$$

In other words, upon a vehicle arrival, if the total delay (queuing delay plus transit delay of the vehicle) is above D, and the queuing delay is less than $D - a$ (i.e., it is before the forced stop, which also means that it is still possible for the S-RSU to pick up a later vehicle that can make the total delay bounded by D), then the S-RSU should continue to wait for the next vehicle. Or equivalently, the S-RSU should skip the vehicles which arrive at the S-RSU before moment $(D-a)$ and violate the delay bound. And we re-index the not skipped vehicles as $n_r = 1, 2, \ldots$. So we have $1 \le n_r \le C_r$, where $C_r \triangleq \min\{n_r : T_{n_r} > D - a\}$ means the forced stop.[6] Denote $X_{n_r} = T_{n_r} - T_{n_r-1}$ $(n_r = 1, 2, \ldots)$. Let N_r denote the corresponding stopping time (the new index of the vehicle upon arrival of which the S-RSU stops) and stopping strategy.

Then we have the following new stopping problem

$$N_r^{\dagger}(\lambda) = \arg\inf_{N_r\in\mathcal{N}} \mathbb{E}\Big[Z_{N_r}(\lambda) = \omega P\kappa + \beta\mathbf{1}_{\{T_{N_r}+S_{N_r}>D\}} - \lambda T_{N_r}\Big]. \tag{6}$$

C. Optimal stopping strategy for Problem (6) and Problem (5)

Consider Problem (6). The concept of myopic stopping strategy is given first. Upon a vehicle arrival, the myopic stopping strategy advises the S-RSU to stop if the cost of stopping at the vehicle is not more than the expected cost of skipping the vehicle and stopping at the next vehicle.

We use $\mathcal{A}_{n_r}$ to denote the event $\{Z_{n_r}(\lambda) \le \mathbb{E}\left[Z_{n_r+1}(\lambda)|\mathcal{F}_{n_r}\right]\}$, in which $\mathcal{F}_{n_r}$ is the information up to time T_{n_r}. Here $\mathcal{F}_{n_r}$ includes the arrival moments and transit delay of all previous vehicles before Vehicle n_r. So $\mathcal{A}_{n_r}$ means that the myopic strategy advises the S-RSU to stop at Vehicle n_r. We have the following definition for a *monotone problem*.

Definition 1: Problem (6) is monotone if $\mathcal{A}_1 \subset \mathcal{A}_2 \subset \mathcal{A}_3 \subset \ldots$ almost surely (a.s.) [26].

In this definition, $\mathcal{A}_{n_r} \subset \mathcal{A}_{n_r+1} \subset \mathcal{A}_{n_r+2} \subset \ldots$ means that if the myopic strategy advises the S-RSU to stop at Vehicle n_r, then it will also advise the S-RSU to stop at any future vehicle[7] no matter what the realization of $(T_{n_r+1}, T_{n_r+2}, \ldots)$ will be (a.s.).

Now, we proceed to show that Problem (6) is a monotone problem. Since at moment T_{C_r} the S-RSU is forced to stop, we need only to consider $n_r < C_r$, i.e., $T_{n_r} = t \in [0, D-a]$.

[6] Note that C_r (in the re-indexed system with some vehicles skipped) and C (in the initial system) both mean the forced stop, corresponding to the first vehicle arrival after moment $D - a$.

[7] When we say that the myopic strategy advises the S-RSU to stop at a future vehicle, it is assumed that the S-RSU does not stop at vehicles before that future vehicle.

Then, we have

$$\begin{aligned}
Z_{n_r}(\lambda) &= \omega P\kappa - \lambda T_{n_r}, \\
\mathbb{E}\left[Z_{n_r+1}(\lambda)|\mathcal{F}_{n_r}\right] &= \mathbb{E}\left[Z_{n_r+1}(\lambda)|T_{n_r}=t\right] \\
&\overset{\text{(i)}}{=} \omega P\kappa + \beta\Pr\{n_r+1=C_r|T_{n_r}=t\} \\
&\quad - \lambda T_{n_r} - \lambda\mathbb{E}\left[X_{n_r+1}|T_{n_r}=t\right],
\end{aligned}$$

in which equality (i) uses the following two equations:

$$\mathbb{E}\left[\mathbf{1}_{\left\{T_{n_r+1}+S_{n_r+1}>D\right\}}|T_{n_r}=t\right]=\Pr\{n_r+1=C_r|T_{n_r}=t\},$$

$$T_{n_r+1} = T_{n_r} + X_{n_r+1}.$$

For $0 \le t \le D-a$, we define :

$$\begin{aligned}
&m(t,\lambda) \overset{\triangle}{=} \mathbb{E}\left[Z_{n_r+1}(\lambda)|\mathcal{F}_{n_r}\right] - Z_{n_r}(\lambda) \\
&= \beta\Pr\{n_r+1=C_r|T_{n_r}=t\} - \lambda\mathbb{E}\left[X_{n_r+1}|T_{n_r}=t\right].
\end{aligned}$$

Theorem 2: $m(t,\lambda)$ is continuous in $t\in[0,D-a]$. And if for some $t^*\in[0,D-a)$, we have $m(t^*,\lambda)\ge 0$, then $m(t,\lambda)$ is a strictly increasing function in $t\in[t^*,D-a]$.

Proof: See Appendix C. ■

For Problem (6), if the myopic strategy advises the S-RSU to stop at Vehicle n_r, based on the definition of $m(t,\lambda)$, we have $m(T_{n_r},\lambda)\ge 0$. Then from Theorem 2, we have $m(T_{n_r+1},\lambda)>0$, $m(T_{n_r+2},\lambda)>0,\ldots$, for $T_{n_r+1}<D-a, T_{n_r+2}<D-a,\ldots$. In other words, the myopic strategy also advises the S-RSU to stop at any vehicle after Vehicle n_r. Thus, Problem (6) is a monotone problem.

In general, the myopic strategy of Problem (6) advises the S-RSU to stop at the earliest possible vehicle such that the cost of stopping at the vehicle is not more than the expected cost of skipping the vehicle and stopping at the next vehicle. Thus, the myopic strategy for Problem (6) can be expressed as

$$N_r^m(\lambda) = \min\Big\{\min\left\{n_r : m(T_{n_r},\lambda)\ge 0\right\}, C_r\Big\}, \tag{7}$$

in which the superscript m stands for "myopic".

Theorem 3: The myopic stopping strategy (7) is optimal for Problem (6).

Proof: See Appendix E. ■

Considering the skipped vehicles when we transform Problem (5) to Problem (6), from Theorem 2 an optimal stopping strategy for Problem (5) is

$$N^\dagger(\lambda) = \min\Big\{\min\left\{n : T_n \ge T_{\text{th}}(\lambda), T_n+S_n\le D\right\}, C\Big\}, \tag{8}$$

in which $T_{\text{th}}(\lambda)$ is given as

$$T_{\text{th}}(\lambda) = \begin{cases} t^* & \text{if} \quad \exists t^*\in[0,D-a],\ m(t^*,\lambda)=0; \\ \infty & \text{if} \quad m(D-a,\lambda)<0. \end{cases} \tag{9}$$

D. Optimal stopping strategy for Problem (3)

Recall that $N^\dagger(\lambda)$ denotes optimal strategy of Problem (5). For Problem (5), let $V(\lambda)$ denote the optimal objective function, i.e.,

$$V(\lambda)=\inf_{N\in\mathcal{C}}\big(\mathbb{E}\left[Y_N\right]-\lambda\mathbb{E}\left[T_N\right]\big)=\mathbb{E}\left[Y_{N^\dagger(\lambda)}\right]-\lambda\mathbb{E}\left[T_{N^\dagger(\lambda)}\right].$$

Theorem 4: $V(\lambda)$ is strictly decreasing and continuous in $\lambda>0$.

Proof: See Appendix F. ■

If $\lambda\to 0$,

$$\lim_{\lambda\to 0} V(\lambda) = \lim_{\lambda\to 0}\mathbb{E}\left[Y_{N^\dagger(\lambda)}\right] \ge \omega P\kappa. \tag{10}$$

On the other hand, if $\lambda>\mu(\omega P\kappa+\beta)$, we have

$$\begin{aligned}
V(\lambda) &\overset{\text{(ii)}}{=} \omega P\kappa + \beta\Pr\{T_{N^\dagger(\lambda)}+S_{N^\dagger(\lambda)}>D\} - \lambda\mathbb{E}\left[T_{N^\dagger(\lambda)}\right] \\
&\le \omega P\kappa+\beta-\lambda\mathbb{E}\left[T_1\right] = \omega P\kappa+\beta-\frac{\lambda}{\mu}<0,
\end{aligned} \tag{11}$$

in which equality (ii) uses

$$\mathbb{E}\left[\mathbf{1}_{\left\{T_{N^\dagger(\lambda)}+S_{N^\dagger(\lambda)}>D\right\}}\right] = \Pr\{T_{N^\dagger(\lambda)}+S_{N^\dagger(\lambda)}>D\}.$$

From Theorem 4 and inequalities (10) and (11), it can be concluded that there exists one and only one $\lambda^*>0$ such that $V(\lambda^*)=0$. In other words, condition ii) of Theorem 1 is satisfied. Thus, according to Theorem 1, an optimal stopping strategy to the Problem (3) is

$$N^\dagger(\lambda^*) = \min\Big\{\min\left\{n : T_n\ge T^*, T_n+S_n\le D\right\}, C\Big\},$$

in which $T^* = T_{\text{th}}(\lambda^*)$. It can be seen that, before a forced stop, the S-RSU is optimal to transmit to a passing-by vehicle if the queuing delay is more than the threshold T^*, conditioned on that the sum of the queuing delay and the transit delay is bounded by D. In other words, the optimal stopping strategy has a conditional pure-threshold structure.

IV. Derivation of the optimal threshold T^*

To derive the optimal threshold T^*, it is intuitive to firstly obtain the threshold $T_{\text{th}}(\lambda)$ based on (9) for each $\lambda(>0)$ value (in which the solution of nonlinear equation $m(t^*,\lambda)=0$ needs to be calculated numerically), secondly obtain the optimal objective function $V(\lambda)$ of Problem (5) for each λ value, and thirdly find λ^* such that $V(\lambda^*)=0$. Although $T^*=T_{\text{th}}(\lambda^*)$ can be numerically calculated based on this intuitive method, the computational complexity is high. So next we propose to derive T^* from another perspective, to obtain T^* directly.

For Problem (3), for $t\in[0,D-a]$, we consider the following stopping strategies:

$$N(t) = \min\Big\{\min\left\{n : T_n\ge t, T_n+S_n\le D\right\}, C\Big\}$$

with corresponding objective function denoted as

$$k(t) = \frac{\omega P\kappa+\beta\Pr\{N(t)=C\}}{\mathbb{E}\left[T_{N(t)}\right]}. \tag{12}$$

Then T^* should be the value of t that minimizes $k(t)$.

Theorem 5: $k(t)$ is continuous in $t\in[0,D-a]$.

Proof: See Appendix G. ■

To minimize $k(t)$, we may investigate its derivative expressed as

$$\frac{\mathrm{d}k(t)}{\mathrm{d}t} = l(t)/\Big(\mathbb{E}\left[T_{N(t)}\right]\Big)^2,$$

where

$$l(t) = \beta \frac{\mathrm{d}\Pr\{N(t)=C\}}{\mathrm{d}t} \mathbb{E}\left[T_{N(t)}\right] - \frac{\mathrm{d}\mathbb{E}\left[T_{N(t)}\right]}{\mathrm{d}t} \left(\omega P\kappa + \beta \Pr\{N(t)=C\}\right). \quad (13)$$

We have the following theorem.

Theorem 6: The function $l(t)$ is continuous in the interval $[0, D-a]$ with $l(0) < 0$ and $l(D-a) = 0$. Moreover, if there is a $t^{\ddagger} \in [0, D-a)$ such that $l(t^{\ddagger}) \geq 0$, then $l(t) > 0$ for $t \in (t^{\ddagger}, D-a)$.

Proof: See Appendix H. ■

Theorem 6 implies that there is at most one root for $l(t) = 0$, $t \in [0, D-a)$. And if there is a root, denoted as $t^{\S}$, then $l(t) < 0$ in $t \in [0, t^{\S})$ (which means $k(t)$ is strictly decreasing in $t \in [0, t^{\S})$) and $l(t) > 0$ in $t \in (t^{\S}, D-a)$ (which means $k(t)$ is strictly increasing in $t \in (t^{\S}, D-a)$), and thus, the optimal threshold T^* should be $T^* = t^{\S}$.

Based on these conclusions, we can derive T^*, as follows. We consider the following three cases.

Case 1) If $l(D-b) \geq 0$:

Since $l(0) < 0$, and $l(t)$ is continuous in the interval $[0, D-a]$, we can see that $l(t) = 0$ ($t \in [0, D-a)$) has a unique root in $[0, D-b]$. From (40) in Appendix H, the optimal threshold T^* is the root of

$$\mu\beta t e^{-\mu(D-t)} \left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}} - \omega P\kappa\Big(1 + \mu e^{-\mu(D-b-t)} \times \left(g(D-b) - D + b - \frac{1}{\mu}\right)\Big) = 0. \quad (14)$$

The method of bisection search can be used to find the root. The corresponding minimum rate of cost is

$$k(T^*) = \frac{\omega P\kappa + \beta \Pr\{N(T^*)=C\}}{\mathbb{E}\left[T_{N(T^*)}\right]} \stackrel{\text{(iii)}}{=} \frac{\omega P\kappa + \beta e^{-\mu(D-T^*)}\left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}}}{\frac{1}{\mu} + T^* - \Big(D - b + \frac{1}{\mu} - g(D-b)\Big) e^{-\mu(D-b-T^*)}}, \quad (15)$$

in which equality (iii) comes from (34) and (35) in Appendix G, and function $g(\cdot)$ is defined in Appendix C and derived in Appendix D.

Case 2): If $l(D-b) < 0$ and $l(t)|_{t=(D-a)^-} > 0$ (which is equivalent to $\big(t - g(t) + [\beta/(\omega P\kappa)]h(t)t\big)|_{t=D-a} > 0$ from (41) in Appendix H, where function $h(\cdot)$ is defined in Appendix C and derived in Appendix D):[8]

We can see that $l(t) = 0$ ($t \in [0, D-a)$) has a unique root in $(D-b, D-a)$. From (41), the optimal threshold T^* is the root of $t - g(t) + \beta h(t)t/(\omega P\kappa) = 0$. The method of bisection search can be used to find the root. The corresponding minimum rate of cost is

$$k(T^*) = \frac{\omega P\kappa + \beta \Pr\{N(T^*)=C\}}{\mathbb{E}\left[T_{N(T^*)}\right]} \stackrel{\text{(iv)}}{=} \frac{\omega P\kappa + \beta h(T^*)}{g(T^*)}, \quad (16)$$

in which equality (iv) comes from (36) and (37) in Appendix G.

Case 3): If $l(D-b) < 0$ and $l(t)|_{t=(D-a)^-} \leq 0$ (which is equivalent to $\big(t - g(t) + [\beta/(\omega P\kappa)]h(t)t\big)|_{t=D-a} \leq 0$ from (41)):

From Theorem 6, we have $l(t) < 0$ in $[0, D-a)$. Thus, $k(t)$ is a strictly decreasing function in $[0, D-a)$. Since we want to minimize $k(t)$, the optimal threshold should be $T^* = D-a$. In other words, it is optimal to wait for a forced stop. The corresponding minimum rate of cost is

$$k(T^*) = \frac{\omega P\kappa + \beta \Pr\{N(D-a)=C\}}{\mathbb{E}\left[T_{N(D-a)}\right]} \stackrel{\text{(v)}}{=} \frac{\omega P\kappa + \beta}{g(D-a)}, \quad (17)$$

where equality (v) comes from $\Pr\{N(D-a)=C\} = 1$ and $\mathbb{E}\left[T_{N(D-a)}\right] = g(D-a)$ (which is from (37) in Appendix G).

[8]Here x^- means a value that is smaller than x but with infinitely small difference.

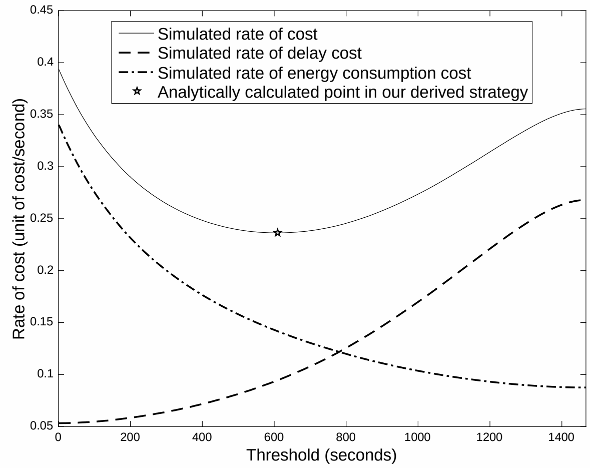

Fig. 1. Rate of cost in conditional pure-threshold strategies with different thresholds.

V. PERFORMANCE EVALUATION

We use Matlab simulation to evaluate our derived stopping strategy. The distance of the S-RSU and D-RSU is $d = 10,000$ m. The S-RSU has a data arrival rate of $r = 5$ bits/second. The soft delay bound is set to be $D = 1,800$ seconds. Vehicle arrival process at the S-RSU is a Poisson process with parameter μ. If the S-RSU decides to stop at a vehicle, the communication overhead duration is $\kappa = 938.91$ μs.[9] The transmission rate of DATA packets is $R = 11$ Mbps. The transmission power of RTS, CTS, DATA, and ACK is $P = 15.5$ dBm= 35.5 mW. The cost weight for energy consumption is $\omega = 1$ unit of cost per μJoule. We collect simulation statistics over $100,000$ simulation runs.

We first demonstrate that our stopping strategy is optimal. For this purpose, we compare our strategy with other conditional pure-threshold strategies. Here a conditional pure-threshold strategy with threshold η works as follows: before

[9]the overhead 938.91 μs is calculated based on IEEE 802.11 Standard, which includes the following: RTS preamble (192 μs), ratio of RTS size (20 bytes) to RTS transmission rate (2Mb/s), CTS preamble (192 μs), ratio of CTS size (14 bytes) to CTS transmission rate (2Mb/s), DATA preamble (192 μs), ratio of DATA MAC header size (34 bytes) to DATA transmission rate (11 Mb/s), ACK preamble (192 μs), and ratio of ACK size (14 bytes) to ACK transmission rate (11Mb/s).

forced stop, the S-RSU selects the first vehicle (say the nth vehicle) such that $T_n > \eta$ and $T_n + S_n \le D$; if the S-RSU cannot find such a vehicle, then the forced stop is decided on. We set $\beta = 500$, $\mu = 1/400$ vehicles/second, $v_{\min} = 10$ m/second, and $v_{\max} = 30$ m/second. In conditional pure-threshold strategies with threshold η varying from 0 to $(D-a)$, Fig. 1 shows the simulation results of the rate of cost, as well as the simulation results of the rate of energy consumption cost and rate of delay cost [10]. It can be seen that, when the threshold increases, the chance to have delay bound violation is larger, and thus, the delay cost is higher. On the other hand, a larger threshold means that the S-RSU has information exchanges with fewer vehicles in a long term, and thus, the energy consumption cost is lower. Fig. 1 also shows the analytically calculated threshold T^* and the corresponding analytically calculated rate of cost (i.e., $k(T^*)$ in (15)-(17) plus $\omega r P/R$, the difference of $\mathbb{E}[Y_N]/\mathbb{E}[T_N]$ from $\mathbb{E}[U_N]/\mathbb{E}[T_N]$ as shown in (3)) in our derived strategy. It is clearly shown that our derived strategy strikes an optimal balance between energy consumption cost and delay cost, and achieves the minimal rate of total cost.

We then vary the penalty cost β from 10 to $10,000$. And for each β value, we exhaustively search the simulated rate of cost in conditional pure-threshold strategies with threshold η varying from 0 to $(D-a)$. Fig. 2 shows the searched optimal threshold that achieves the (simulated) minimal rate of cost for each β value, and Fig. 3 shows the corresponding (simulated) minimal rate of cost for each β value. As a comparison, for each β value, Fig. 2 and Fig. 3 also show the analytically calculated threshold T^* in our derived strategy and corresponding analytically calculated rate of cost, respectively. It can be seen that the analytical results and exhaustively searched optimal simulation results match well. When β is small, the S-RSU waits until it is forced to stop, i.e., the optimal threshold is $(D-a) = 1,467$ seconds. This is because the penalty cost β is dominated by the benefit from delivering more traffic in a transmission. In fact, from Case 3) when deriving T^* in Section IV, we know that if $\big(t - g(t) + [\beta/(\omega P\kappa)]h(t)t\big)|_{t=D-a} \le 0$, which means

$$\beta \le \frac{\omega P\kappa\left(g(D-a) - D + a\right)}{(D-a)h(D-a)} = 36.4,$$

then the optimal threshold is $(D-a) = 1,467$ seconds. When the value of β increases, the penalty cost begins to dominate, and the optimal threshold value begins to decrease. As an extreme case, the optimal threshold becomes 0 when $\beta \to \infty$, which means that the cost of delay bound violation is too high to afford, and thus, the S-RSU should transmit to the first vehicle that meets the delay bound requirement (i.e., the sum of queuing delay and transit delay is not more than D).

Fig. 3 also shows the comparison of our derived stopping strategy with the following heuristic strategy: when the S-RSU's waiting time is less than $D-b$, it is impossible for any vehicle to violate the delay bound requirement (i.e., $T_n + S_n > D$), and thus, the S-RSU does not stop; when the S-RSU's waiting time is more than $D-b$, it is possible that a vehicle would violate the delay bound requirement, and thus, the S-RSU transmits to the next coming vehicle that satisfies the delay bound requirement. So the heuristic strategy is actually a conditional pure-threshold strategy with threshold being $(D-b)$. The simulated rate of cost in the heuristic strategy for different β values is shown in Fig. 3. It is clear that the heuristic strategy is not optimal in general.

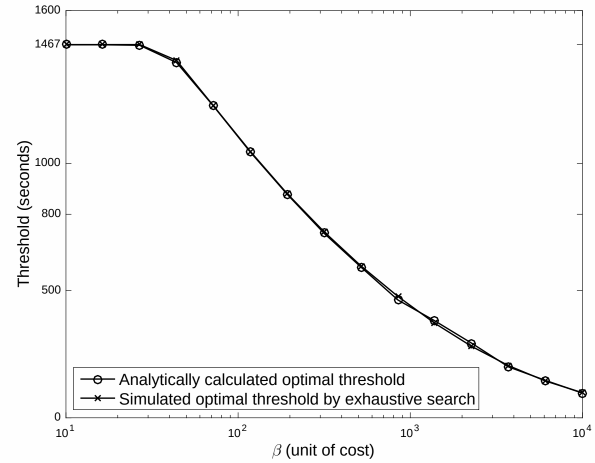

Fig. 2. The optimal threshold in conditional pure-threshold strategies.

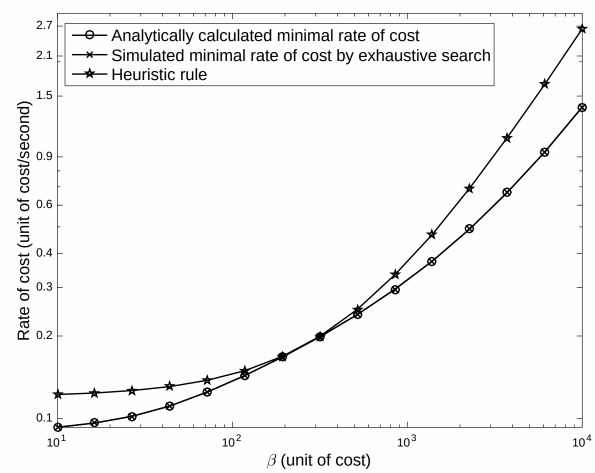

Fig. 3. The rate of cost in conditional pure-threshold strategies.

We continue to show how different arrival rates of vehicles at the S-RSU affect the optimal threshold and the rate of cost in the derived stopping strategy. We vary the arrival rate μ of vehicles from 0.0015 to 0.03 vehicles/second. For $\beta = 500$ and different μ, the analytically calculated and simulated (by exhaustive search) optimal thresholds are shown in Fig. 4, and analytical calculated and simulated minimal rates of cost are shown in Fig. 5. It can be seen that, when μ increases, the optimal threshold increases, and the minimal rate of cost decreases. This is because, when it is expected that vehicles arrive more frequently, the S-RSU can hold the traffic in its buffer for a longer time, and thus, each transmission can deliver more traffic, which leads to a smaller rate of cost.

[10]In a conditional pure-threshold strategy (including our derived strategy), a delay bound violation happens only at a forced stop. At a forced stop, the average amount of information units with delay bound violation is expressed as $r(1/\mu + \mathbb{E}[S] - a)$, in which $\mathbb{E}[S]$ is average transit delay of a vehicle. Therefore, if the rate of delay cost is expressed as R_d, the amount of information units per unit time with delay bound violation can be expressed as $(R_d/\beta)r(1/\mu+\mathbb{E}[S]-a)$, i.e., proportional to R_d. Thus, the rate of delay cost in our cost function actually can represent the performance of positive system throughput (defined as the amount of information units per unit time that can be delivered to the D-RSU within delay bound).

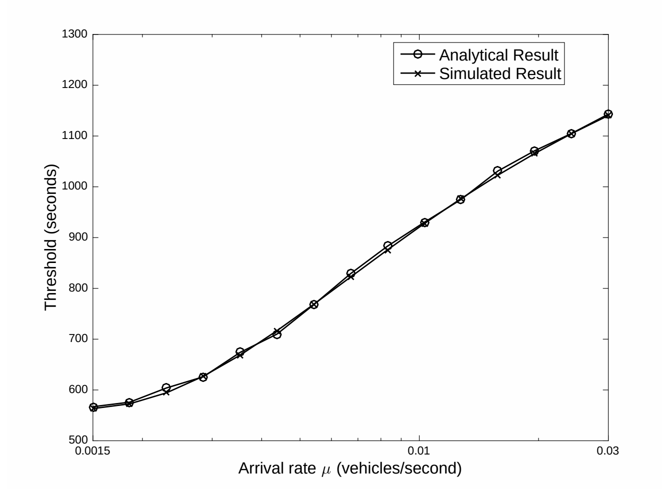

Fig. 4. The optimal threshold value for different μ.

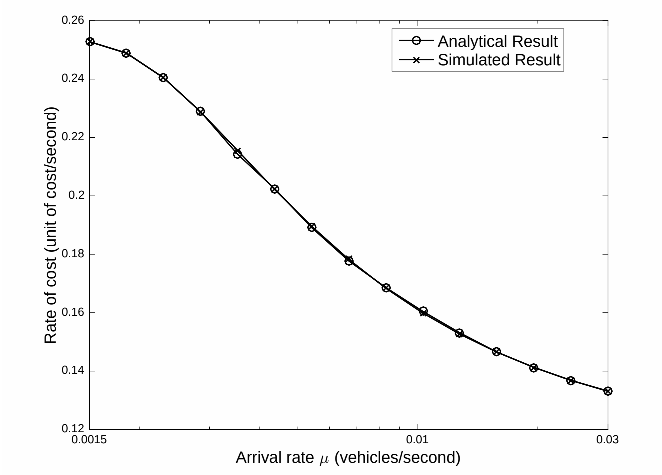

Fig. 5. The minimal rate of cost for different μ.

Next we show how the value of the soft delay bound D affects the optimal threshold and the rate of cost in the derived stopping strategy. For $\beta = 500$ and $\mu = 1/400$ vehicles/second, we vary D from 1,000 seconds to 40,000 seconds. The analytically calculated and simulated (by exhaustive search) optimal thresholds are shown in Fig. 6, and analytical calculated and simulated minimal rates of cost are shown in Fig. 7. It can be seen that, with a larger delay bound D, the S-RSU can wait more time before stop, and thus, the optimal threshold increases, and the minimal rate of cost decreases (as the S-RSU has information exchanges with fewer vehicles in a long term, thus decreasing energy consumption).

VI. Conclusion

This work studies vehicle-aided communications from a remote RSU to a central RSU. Costs are assigned to energy consumption as well as possible violation of a soft delay bound. We theoretically prove that an optimal stopping strategy has a conditional pure-threshold structure, and the threshold can be calculated offline quickly by our provided method. Upon arrival of a passing-by vehicle, if the vehicle can meet the delay bound requirement, the S-RSU only needs to compare the arrival moment of the vehicle with the threshold to make its decision. Thus, the derived stopping strategy can be implemented in a VANET easily with very low complexity.

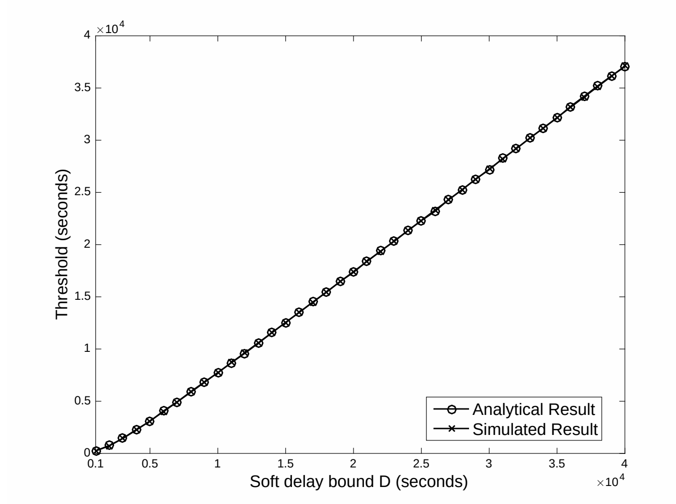

Fig. 6. The optimal threshold value for different D.

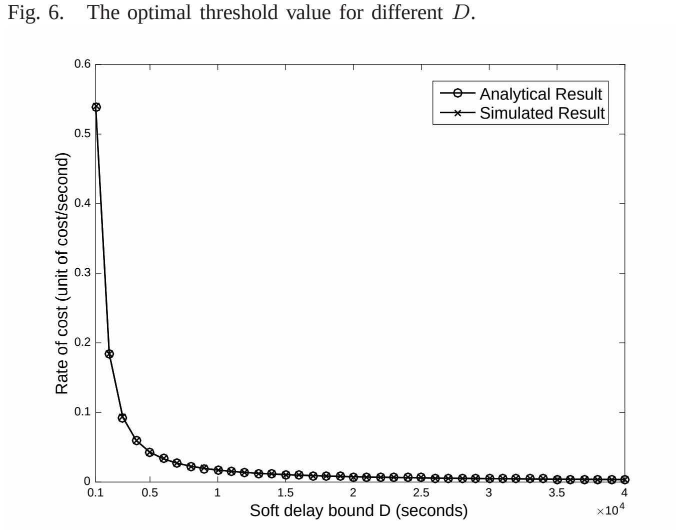

Fig. 7. The minimal rate of cost for different D.

Appendix

A. Reason to have a fixed charge for one or multiple information units with delay bound violation

As an example, we use the typical application of the S-RSU: serve as gateway for a wireless sensor network. So the data traffic at the S-RSU actually carries information of a number of "events" in the wireless sensor network, and each event is corresponding to a number of information units in the data traffic at the S-RSU.

We expect that only a very small portion of the data traffic will have delay bound violation (i.e., cannot be delivered before the delay bound). This is because, if a large portion of data traffic has delay bound violation, this means that the system is not effective to deliver the buffered data traffic at the S-RSU, and thus, a new system is needed (for example, by using cellular communications or satellite communications).

Therefore, when the S-RSU stops at a vehicle, it is very likely that the information units that have delay bound violation belong to the same event. For the same event, a single information unit with delay bound violation and multiple information units with delay bound violation have the same effect on processing of the event: both will make processing of the event at the receiver side delayed. Thus, when there is delay bound violation, we do not make the penalty charge proportional to the number of information units with delay bound violation. Rather, we use a fixed penalty charge for one or multiple information units with delay bound violation.

In addition, if the data traffic of the wireless sensor network is encrypted, an encryption segment consists of a number of information units. At the receiver side, decryption of an encryption segment can be done only after all information units in the segment are received. Then for an encryption segment, a single information unit with delay bound violation and multiple information units with delay bound violation both will make the decryption process at the receiver side delayed. Thus, it is reasonable to charge a fixed penalty when one or multiple information units have delay bound violation.

B. Proof of Theorem 1

Considering Problem (5) with λ^*, from i) we know that $N^\dagger(\lambda^*)$ is optimal stopping strategy. In other words, for any stopping strategy $N \in \mathcal{C}$, we have $\mathbb{E}\left[Z_N(\lambda^*)\right] = \mathbb{E}\left[Y_N - \lambda^* T_N\right] \geq \mathbb{E}\left[Z_{N^\dagger(\lambda^*)}(\lambda^*)\right] = 0$, in which the last equality comes from ii). Based on this, we have $\mathbb{E}\left[Y_N\right]/\mathbb{E}\left[T_N\right] \geq \lambda^*$.

Further, from (4), we have $\mathbb{E}\left[Z_{N^\dagger(\lambda^*)}(\lambda^*)\right] = \mathbb{E}\left[Y_{N^\dagger(\lambda^*)} - \lambda^* T_{N^\dagger(\lambda^*)}\right]$. As $\mathbb{E}\left[Z_{N^\dagger(\lambda^*)}(\lambda^*)\right] = 0$ which is from ii), we have $\mathbb{E}\left[Y_{N^\dagger(\lambda^*)}\right]/\mathbb{E}\left[T_{N^\dagger(\lambda^*)}\right] = \lambda^*$. Together with $\mathbb{E}\left[Y_N\right]/\mathbb{E}\left[T_N\right] \geq \lambda^*$, we can see that among all stopping strategies in $\mathcal{C}$, $N^\dagger(\lambda^*)$ minimizes $\mathbb{E}\left[Y_N\right]/\mathbb{E}\left[T_N\right]$, and thus, is an optimal stopping strategy of Problem (3), with the optimal value of the objective function of Problem (3) being λ^*.

C. Proof of Theorem 2

Since the expression of $m(t,\lambda)$ includes the following two terms: $\Pr\{n_r+1 = C_r | T_{n_r} = t\}$ and $\mathbb{E}\left[X_{n_r+1} | T_{n_r} = t\right]$ ($t \in [0, D-a]$), we need to calculate the two terms. We consider the following two cases.

- When $0 \leq t < (D-b)$: We have

$$
\begin{aligned}
&\Pr\{n_r+1 = C_r | T_{n_r} = t\} \\
\overset{\text{(vi)}}{=}&\Pr\{n_r+1 = C_r | T_{n_r+1} \geq D-b, T_{n_r} = t\} \\
&\times \Pr\{T_{n_r+1} \geq D-b | T_{n_r} = t\} \\
\overset{\text{(vii)}}{=}&h(D-b) e^{-\mu(D-b-t)} \\
=&\left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}} e^{-\mu b} e^{-\mu(D-b-t)} \\
=&e^{-\mu(D-t)} \left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}},
\end{aligned} \tag{18}
$$

  in which equality (vi) uses Total Probability Theorem and the fact $\Pr\{n_r+1 = C_r | T_{n_r+1} < D-b, T_{n_r} = t\} = 0$, and equality (vii) uses $\Pr\{n_r+1 = C_r | T_{n_r+1} \geq D-b, T_{n_r} = t\} = \Pr\{n_r+1 = C_r | T_{n_r+1} \geq D-b\}$ and $\Pr\{T_{n_r+1} \geq D-b | T_{n_r} = t\} = e^{-\mu(D-b-t)}$ (recalling that vehicle inter-arrival durations are exponentially distributed with parameter μ). Here $h(\tau) \overset{\triangle}{=} \Pr\{n_r+1 = C_r | T_{n_r+1} \geq \tau\}$, $D-b \leq \tau \leq D-a$, is derived in Appendix D.

$$
\begin{aligned}
&\mathbb{E}\left[X_{n_r+1} | T_{n_r} = t\right] \\
\overset{\text{(viii)}}{=}&\mathbb{E}\left[X_{n_r+1} | T_{n_r+1} < D-b, T_{n_r} = t\right] \\
&\times \Pr\{T_{n_r+1} < D-b | T_{n_r} = t\} \\
&+ \mathbb{E}\left[X_{n_r+1} | T_{n_r+1} \geq D-b, T_{n_r} = t\right] \\
&\times \Pr\{T_{n_r+1} \geq D-b | T_{n_r} = t\} \\
\overset{\text{(ix)}}{=}&\int_0^{D-b-t} \mu x e^{-\mu x}\,\mathrm{d}x + \big(g(D-b) - t\big) e^{-\mu(D-b-t)} \\
=&\frac{1}{\mu} - \left(D - b + \frac{1}{\mu} - g(D-b)\right) e^{-\mu(D-b-t)},
\end{aligned} \tag{19}
$$

  in which equality (viii) uses Total Probability Theorem, equality (ix) uses the fact that the vehicle inter-arrival durations are exponentially distributed with parameter μ, and $g(\tau) \overset{\triangle}{=} \mathbb{E}\left[T_{n_r+1} | T_{n_r+1} \geq \tau\right]$, $D-b \leq \tau \leq D-a$, is derived in Appendix D.
- When $(D-b) \leq t \leq (D-a)$: We have

$$
\begin{aligned}
&\Pr\{n_r+1 = C_r | T_{n_r} = t\} \\
&= h(t) = \left(\frac{D-t}{a}\right)^{\frac{\mu ab}{b-a}} e^{-\frac{\mu b(D-a-t)}{b-a}}
\end{aligned}
$$

  and $\mathbb{E}\left[X_{n_r+1} | T_{n_r} = t\right] = g(t) - t$.

Then $m(t,\lambda)$ can be expressed as

$$
\begin{aligned}
&m(t,\lambda) \\
=&\begin{cases} \beta e^{-\mu(D-t)} \left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}} - \lambda\Big(\frac{1}{\mu} - \big(D-b+\frac{1}{\mu} - g(D-b)\big) \\ \quad \times e^{-\mu(D-b-t)}\Big), \text{ if } 0 \leq t < D-b; \\ \beta h(t) - \lambda\left(g(t) - t\right), \quad \text{if } D-b \leq t \leq D-a. \end{cases}
\end{aligned} \tag{20}
$$

It can be verified from (20) that $m(t,\lambda)$ is continuous for $t \in [0, D-a]$.

Suppose $\lambda > 0$ and there is a $t^* \in [0, D-a)$ satisfying $m(t^*,\lambda) \geq 0$.

If $t^* \in [0, D-b)$, from (20), we have

$$
\begin{aligned}
m(t^*,\lambda) =&\beta e^{-\mu(D-t^*)} \left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}} - \lambda\Big(\frac{1}{\mu} - \big(D-b \\
&+ \frac{1}{\mu} - g(D-b)\big) e^{-\mu(D-b-t^*)}\Big) \geq 0.
\end{aligned}
$$

Then, we have

$$
\left.\frac{\partial m(t,\lambda)}{\partial t}\right|_{t=t^*} = \mu m(t^*,\lambda) + \lambda > 0. \tag{21}
$$

If $t^* \in [D-b, D-a)$, from (20), we have $m(t^*,\lambda) = \beta h(t^*) - \lambda\big(g(t^*) - t^*\big) \geq 0$. Then, we have

$$
\begin{aligned}
\left.\frac{\partial m(t,\lambda)}{\partial t}\right|_{t=t^*} =& \left.\left(\beta\frac{\mathrm{d}h(t)}{\mathrm{d}t} - \lambda\frac{\mathrm{d}g(t)}{\mathrm{d}t} + \lambda\right)\right|_{t=t^*} \\
\overset{\text{(x)}}{=}& \beta\mu F_S(D-t^*) h(t^*) - \lambda\mu F_S(D-t^*) \\
&\times \left(g(t^*) - t^*\right) + \lambda \\
=& \mu F_S(D-t^*)\Big(\beta h(t^*) - \lambda\left(g(t^*) - t^*\right)\Big) + \lambda \\
=& \mu F_S(D-t^*) m(t^*,\lambda) + \lambda > 0,
\end{aligned} \tag{22}
$$

where equality (x) uses the following two equations:

$$\frac{\mathrm{d}h(t)}{\mathrm{d}t} = \mu F_S(D-t)h(t),$$
$$\frac{\mathrm{d}g(t)}{\mathrm{d}t} = \mu F_S(D-t)\left(g(t)-t\right),$$

which are from Appendix D.

Thus, if for some $t^* \in [0, D-a)$, we have $m(t^*, \lambda) \geq 0$, then from (21) and (22) we have

$$\frac{\partial m(t,\lambda)}{\partial t} > 0 \quad \text{for } t \in [t^*, D-a),$$

which means $m(t,\lambda)$ is strictly increasing in $t \in [t^*, D-a]$.

D. Derivation of $h(\tau)$ and $g(\tau)$

We consider the following stopping strategy:

$$N(\tau) = \min\left\{ \min\left\{n : T_n \geq \tau, T_n + S_n \leq D\right\}, C\right\}$$

for $D-b \leq \tau \leq D-a$.

According to the definitions of $h(\tau)$ and $g(\tau)$ in Appendix C, we have

$$h(\tau) = \Pr\{N(\tau) = C\}, \qquad g(\tau) = \mathbb{E}\left[T_{N(\tau)}\right]$$

with the following boundary conditions:

$$h(D-a) = 1, \qquad g(D-a) = D-a+\frac{1}{\mu}.$$

We first derive $h(\tau)$. For $D-b < \tau \leq D-a$, consider a sufficiently small $\Delta\tau$ such that $\tau - \Delta\tau \geq D-b$. Recall that vehicles arrive at the S-RSU following a Poisson process with parameter μ. Thus, within duration $(\tau-\Delta\tau, \tau)$, the probabilities of no vehicle arrival, one vehicle arrival, and two or more vehicle arrivals are expressed as $(1-\mu\Delta\tau)$, $\mu\Delta\tau$, and $o(\Delta\tau)$ (higher order of $\Delta\tau$), respectively. Consider stopping strategy $N(\tau-\Delta\tau)$. If no vehicle arrives within duration $(\tau-\Delta\tau,\tau)$, then the S-RSU should continue to wait for the next vehicle that comes after moment τ. If one vehicle, say the nth vehicle, arrives within duration $(\tau-\Delta\tau,\tau)$, and $T_n + S_n \leq D$, then the S-RSU stops and there is no need to wait after τ. If one vehicle, say the nth vehicle, arrives within duration $(\tau-\Delta\tau,\tau)$, and $T_n+S_n > D$, then the RSU should skip this vehicle and continue to wait for the next vehicle that comes after moment τ. As a summary, we have

$$h(\tau-\Delta\tau) = (1-\mu\Delta\tau)h(\tau) + \mu\Delta\tau\left(1-F_S(D-\tau)\right)h(\tau) + o(\Delta\tau),$$

which leads to

$$\frac{h(\tau)-h(\tau-\Delta\tau)}{\Delta\tau} = \mu F_S(D-\tau)h(\tau) - \frac{o(\Delta\tau)}{\Delta\tau}.$$

Letting Δt approach zero, we have

$$\frac{\mathrm{d}h(\tau)}{\mathrm{d}\tau} = \mu F_S(D-\tau)h(\tau). \tag{23}$$

Using the initial condition $h(D-a)=1$, we obtain

$$h(\tau) = e^{\mu\int_{D-a}^{\tau} F_S(D-x)\,\mathrm{d}x} = \left(\frac{D-\tau}{a}\right)^{\frac{\mu ab}{b-a}} e^{-\frac{\mu b(D-\tau-a)}{b-a}} \tag{24}$$

for $D-b \leq \tau \leq D-a$.

Since we have

$$\frac{\mathrm{d}h(\tau)}{\mathrm{d}\tau} = \mu F_S(D-\tau)h(\tau) > 0 \quad \text{for } \tau \in [D-b, D-a)$$

and

$$\frac{\mathrm{d}h(\tau)}{\mathrm{d}\tau}\Big|_{\tau=D-a} = \mu F_S(a)h(D-a) = 0 \text{ (as } F_S(a)=0),$$

it can be concluded that $h(\tau)$ is strictly increasing in $[D-b, D-a]$.

Similar to the derivation of $h(\tau)$, for $g(\tau)$ we have

$$\begin{aligned} g(\tau-\Delta\tau) = (1-\mu\Delta\tau)g(\tau) + \mu\Delta\tau F_S(D-\tau)\tau \\ + \mu\Delta\tau(1-F_S(D-\tau))g(\tau) + o(\Delta\tau), \end{aligned}$$

which leads to

$$\frac{g(\tau)-g(\tau-\Delta\tau)}{\Delta\tau} = \mu F_S(D-\tau)\left(g(\tau)-\tau\right) - \frac{o(\Delta\tau)}{\Delta\tau}.$$

Letting $\Delta\tau$ approach zero, we have

$$\frac{\mathrm{d}g(\tau)}{\mathrm{d}\tau} = \mu F_S(D-\tau)\left(g(\tau)-\tau\right). \tag{25}$$

Using the initial condition $g(D-a) = D-a+1/\mu$, we obtain

$$\begin{aligned} g(\tau) = D - h(\tau)\Big[a - \frac{1}{\mu} + \frac{\mu b}{b-a}\left(\frac{a}{e}\right)^{\frac{\mu ab}{b-a}} \\ \times \int_a^{D-\tau}(z-a)z^{-\frac{\mu ab}{b-a}}e^{\frac{\mu bz}{b-a}}\,\mathrm{d}z\Big] \end{aligned} \tag{26}$$

for $D-b \leq \tau \leq D-a$, where $h(\tau)$ is given in (24).

E. Proof of Theorem 3

Since the S-RSU is forced to stop when $n_r = C_r$, for presentation simplicity we can set $T_{n_r} = T_{C_r}$ for $n_r > C_r$. Then the objective function of Problem (6) can be written as $Z_{n_r}(\lambda) = \omega P\kappa + \beta\mathbf{1}_{\{n_r = C_r\}} - \lambda T_{n_r}$. We have

$$\begin{aligned} \mathbb{E}\left[\inf{}_{n_r}(Z_{n_r}(\lambda))\right] &\geq \omega P\kappa + \mathbb{E}\left[\inf{}_{n_r}(-\lambda T_{n_r})\right] \\ &= \omega P\kappa - \mathbb{E}\left[\sup{}_{n_r}\lambda T_{n_r}\right] \\ &\geq \omega P\kappa - \lambda\mathbb{E}\left[T_{C_r}\right] \\ &= \omega P\kappa - \lambda\left(D-a+\frac{1}{\mu}\right) > -\infty. \end{aligned} \tag{27}$$

According to Wald's Equation, $\mathbb{E}[T_{C_r}] = \mathbb{E}[C]/\mu$. Thus, $\mathbb{E}[C_r] \leq \mathbb{E}[C] < \infty$ (since $\mathbb{E}[T_{C_r}] < \infty$), which leads to $\mathbf{1}_{\{C_r<\infty\}} = 1$ a.s.. Again, since the RSU is required to stop when $n_r = C_r$, the stopping strategies that we consider have the property $N_r(\lambda) \leq C_r$ and thus

$$\mathbb{E}\left[N_r(\lambda)\right] < \infty \quad a.s.. \tag{28}$$

According to Theorem 3.1 in [26], when the two inequalities (27) and (28) hold, there exists an optimal stopping strategy $N_r^\dagger(\lambda)$ for Problem (6), and the optimal (minimal) objective function of the problem is denoted as $V^*(\lambda) = \mathbb{E}\left[Z_{N_r^\dagger(\lambda)}\right]$.

To prove the optimality of the myopic stopping strategy $N_r^m(\lambda)$ in (7) for Problem (6), it suffices if we can show that the optimal objective function of Problem (6) is not less than the objective function of the myopic stopping strategy $N_r^m(\lambda)$, as follows.

For Problem (6), if the S-RSU is forced to stop when the vehicle index n_r is more than J, then we call this problem bounded at J. Recall that Problem (6) is monotone problem. Thus, Problem (6) bounded at J is a finite horizon monotone problem, and thus, according to Theorem 5.1 in [26], the corresponding myopic strategy for Problem (6) bounded at J, given as $N_r^{m,(J)}(\lambda) = \min\Big\{\min\left\{n_r : m(T_{n_r}, \lambda) \geq 0\right\}, C_r, J\Big\}$, is optimal, with the achieved objective function denoted as $V^{(J)}(\lambda) = \mathbb{E}\left[Z_{N_r^{m,(J)}(\lambda)}(\lambda)\right]$.

Based on $N_r^{\dagger}(\lambda)$, we define a new stopping strategy as $N_r^{[J]}(\lambda) = \min\{N_r^{\dagger}(\lambda), J\}$, for $J \geq 1$, and denote the corresponding objective function as $V^{[J]}(\lambda)$. Then we have $V^{[\infty]}(\lambda) = V^{*}(\lambda)$. Since stopping strategy $N_r^{m,(J)}(\lambda)$ is optimal for Problem (6) bounded at J, and $N_r^{[J]}(\lambda)$ is a stopping strategy for Problem (6) bounded at J, we have $V^{[J]}(\lambda) \geq V^{(J)}(\lambda)$. Then

$$
\begin{aligned}
0 \leq & V^{(J)}(\lambda) - V^{*}(\lambda) \leq \mathbb{E}\left[Z_{N_r^{[J]}(\lambda)}(\lambda)\right] - \mathbb{E}\left[Z_{N_r^{\dagger}(\lambda)}(\lambda)\right] \\
=& \mathbb{E}\left[\mathbf{1}_{\left\{N_r^{\dagger}(\lambda) > J\right\}}\left(Z_{N_r^{[J]}(\lambda)}(\lambda) - Z_{N_r^{\dagger}(\lambda)}(\lambda)\right)\right] \\
=& \mathbb{E}\left[\mathbf{1}_{\left\{N_r^{\dagger}(\lambda) > J\right\}}\left(Z_{N_r^{[J]}(\lambda)=J}(\lambda) - Z_{N_r^{\dagger}(\lambda)}(\lambda)\right)\right] \\
=& \mathbb{E}\Bigg[\mathbf{1}_{\left\{N_r^{\dagger}(\lambda) > J\right\}}\Bigg(\beta\Big(\mathbf{1}_{\{T_J + S_J > D\}} - \mathbf{1}_{\left\{T_{N_r^{\dagger}(\lambda)} + S_{N_r^{\dagger}(\lambda)} > D\right\}}\Big) \\
& \quad + \lambda(T_{N_r^{\dagger}(\lambda)} - T_J)\Bigg)\Bigg] \\
\leq & \mathbb{E}\left[\mathbf{1}_{\left\{N_r^{\dagger}(\lambda) > J\right\}}\left(2\beta + \lambda T_{N_r^{\dagger}(\lambda)}\right)\right] \\
\leq & \mathbb{E}\left[\mathbf{1}_{\left\{N_r^{\dagger}(\lambda) > J\right\}}\left(2\beta + \lambda T_{C_r}\right)\right].
\end{aligned} \tag{29}
$$

Since $\Pr\{N_r^{\dagger}(\lambda) > J\} \to 0$ as $J \to \infty$, then we have $\mathbb{E}\left[\mathbf{1}_{\left\{N_r^{\dagger}(\lambda) > J\right\}}\left(2\beta + \lambda T_{C_r}\right)\right] \to 0$ as $J \to \infty$. Then from (29) we have

$$
\begin{aligned}
V^{*}(\lambda) &= \lim_{J\to\infty} V^{(J)}(\lambda) = \lim_{J\to\infty} \mathbb{E}\left[Z_{N_r^{m,(J)}(\lambda)}(\lambda)\right] \\
&= \lim \inf_{J\to\infty} \mathbb{E}\left[Z_{N_r^{m,(J)}(\lambda)}(\lambda)\right] \\
&\overset{\text{(xi)}}{\geq} \mathbb{E}\left[\lim \inf_{J\to\infty} Z_{N_r^{m,(J)}(\lambda)}(\lambda)\right] \\
&\overset{\text{(xii)}}{=} \mathbb{E}\left[Z_{N_r^{m}(\lambda)}(\lambda)\right],
\end{aligned} \tag{30}
$$

in which inequality (xi) follows from (27) by applying Fatou's lemma [27], and equality (xii) follows from the fact that $N_r^{m,(J)}(\lambda)$ is an increasing sequence of stopping strategies converging to $N_r^{m}(\lambda)$. Because of (28), $N_r^{m}(\lambda)$ is a fixed integer from some J on a.s.. Thus, we have $\lim\inf_{J\to\infty} Z_{N_r^{m,(J)}(\lambda)}(\lambda) = Z_{N_r^{m}(\lambda)}(\lambda)$ a.s..

Inequality (30) means that the achieved objective function in the optimal stopping strategy for Problem (6) is not less than the achieved objective function of the myopic stopping strategy $N_r^{m}(\lambda)$. So the myopic stopping strategy is optimal for Problem (6).

F. Proof of Theorem 4

Consider positive λ_2 and λ_1 satisfying $\lambda_2 > \lambda_1$. We have

$$
\begin{aligned}
V(\lambda_1) &= \mathbb{E}\left[Y_{N^{\dagger}(\lambda_1)}\right] - \lambda_1 \mathbb{E}\left[T_{N^{\dagger}(\lambda_1)}\right] \\
&> \mathbb{E}\left[Y_{N^{\dagger}(\lambda_1)}\right] - \lambda_2 \mathbb{E}\left[T_{N^{\dagger}(\lambda_1)}\right],
\end{aligned} \tag{31}
$$

in which the inequality is because $\lambda_1 < \lambda_2$.

For Problem (5) with parameter λ_2, its optimal strategy is denoted as $N^{\dagger}(\lambda_2)$. In other words, $N^{\dagger}(\lambda_2)$ minimizes the objective function of Problem (5) with parameter λ_2. Thus, we have

$$
\begin{aligned}
V(\lambda_2) &= \mathbb{E}\left[Y_{N^{\dagger}(\lambda_2)}\right] - \lambda_2 \mathbb{E}\left[T_{N^{\dagger}(\lambda_2)}\right] \\
&< \mathbb{E}\left[Y_{N^{\dagger}(\lambda_1)}\right] - \lambda_2 \mathbb{E}\left[T_{N^{\dagger}(\lambda_1)}\right].
\end{aligned} \tag{32}
$$

Combining (31) and (32), we have $V(\lambda_1) > V(\lambda_2)$. Thus, $V(\lambda)$ is strictly decreasing in $\lambda > 0$.

Next we prove that $V(\lambda)$ is uniformly continuous, i.e., for any $\epsilon > 0$, there exists a δ such that for any λ_1 and λ_2 satisfying $|\lambda_2 - \lambda_1| < \delta$, we have $|V(\lambda_2) - V(\lambda_1)| < \epsilon$. We set $\delta = \epsilon / \mathbb{E}\left[T_C\right]$. Then for any λ_1 and λ_2 satisfying $0 < \lambda_1 < \lambda_2 < \lambda_1 + \delta$, we have

$$
\begin{aligned}
& |V(\lambda_2) - V(\lambda_1)| \\
& = V(\lambda_1) - V(\lambda_2) \\
& < V(\lambda_1) - V(\lambda_1 + \delta) \\
& = V(\lambda_1) - \left(\mathbb{E}\left[Y_{N^{\dagger}(\lambda_1+\delta)}\right] - (\lambda_1 + \delta)\mathbb{E}\left[T_{N^{\dagger}(\lambda_1+\delta)}\right]\right) \\
& = V(\lambda_1) - \left(\mathbb{E}\left[Y_{N^{\dagger}(\lambda_1+\delta)}\right] - \lambda_1 \mathbb{E}\left[T_{N^{\dagger}(\lambda_1+\delta)}\right]\right) \\
& \qquad + \delta \mathbb{E}\left[T_{N^{\dagger}(\lambda_1+\delta)}\right] \\
& \overset{\text{(xiii)}}{\leq} \delta \mathbb{E}\left[T_{N^{\dagger}(\lambda_1+\delta)}\right] \overset{\text{(xiv)}}{\leq} \delta \mathbb{E}\left[T_C\right] = \epsilon,
\end{aligned}
$$

in which inequality (xiii) is because $V(\lambda_1)$ is the minimal objective function of Problem (5) with parameter λ_1, and inequality (xiv) is because $N^{\dagger}(\lambda_1 + \delta) \leq C$.

Since a uniformly continuous function is also continuous [28], it can be concluded that $V(\lambda)$ is continuous in $\lambda > 0$.

G. Proof of Theorem 5

When $0 \leq t < (D - b)$, similar to (18), we have

$$
\Pr\{N(t) = C\} = e^{-\mu(D-b-t)} h(D-b) \tag{33}
$$

$$
= e^{-\mu(D-t)} \left(\frac{b}{a}\right)^{\frac{\mu a b}{b-a}}, \tag{34}
$$

in which $h(\tau)$ is derived in Appendix D. Similar to (19), we have

$$
\mathbb{E}\left[T_{N(t)}\right] = \frac{1}{\mu} + t - \left(D - b + \frac{1}{\mu} - g(D-b)\right) e^{-\mu(D-b-t)}. \tag{35}
$$

So $\Pr\{N(t) = C\}$ and $\mathbb{E}\left[T_{N(t)}\right]$ are both continuous in $t \in [0, D-b)$, and thus, from (12), $k(t)$ is continuous in $t \in [0, D-b)$.

When $(D-b) \le t \le (D-a)$, we have

$$\Pr\{N(t)=C\} = h(t) = \left(\frac{D-t}{a}\right)^{\frac{\mu ab}{b-a}} e^{-\frac{\mu b(D-a-t)}{b-a}}, \quad (36)$$

$$\mathbb{E}\left[T_{N(t)}\right] = g(t), \quad (37)$$

where $h(\tau)$ and $g(\tau)$ are derived in Appendix D. So $\Pr\{N(t)=C\}$ and $\mathbb{E}\left[T_{N(t)}\right]$ are both continuous in $t \in [D-b, D-a]$, and thus, $k(t)$ is continuous in $t \in [D-b, D-a]$.

Moreover, around $t = D-b$ we have

$$\lim_{t<D-b,\ t\to(D-b)} \Pr\{N(t)=C\} \overset{\text{(xv)}}{=} h(D-b) \overset{\text{(xvi)}}{=} \Pr\{N(D-b)=C\},$$

in which equalities (xv) and (xvi) are from (33) and (36), respectively, and

$$\lim_{t<D-b,\ t\to(D-b)} \mathbb{E}\left[T_{N(t)}\right] = g(D-b) = \mathbb{E}\left[T_{N(D-b)}\right],$$

in which the two equalities are from (35) and (37), respectively. So $\Pr\{N(t)=C\}$ and $\mathbb{E}\left[T_{N(t)}\right]$ are both continuous at $t=D-b$, and thus, $k(t)$ is continuous at $t=D-b$.

Overall, $k(t)$ is continuous in $t \in [0, D-a]$.

H. *Proof of Theorem 6*

When $0 \le t < (D-b)$, from (34) we have

$$\frac{\mathrm{d}\Pr\{N(t)=C\}}{\mathrm{d}t} = \mu e^{-\mu(D-t)} \left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}}. \quad (38)$$

From (35), we have

$$\frac{\mathrm{d}\mathbb{E}\left[T_{N(t)}\right]}{\mathrm{d}t} = 1 + \mu e^{-\mu(D-b-t)} \left(g(D-b) - D + b - \frac{1}{\mu}\right). \quad (39)$$

Then from (13), (34), (35), (38), and (39), we have expression of $l(t)$ in (40) on top of next page.

From (40) we know that $l(t)$ is continuous in $[0, D-b)$. Taking the first-order derivative of $l(t)$, we have

$$\begin{aligned}
\frac{\mathrm{d}l(t)}{\mathrm{d}t} =& \mu\beta e^{-\mu(D-t)} \left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}} + \mu^2\beta t e^{-\mu(D-t)} \left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}} \\
&- \omega P\kappa\mu^2 e^{-\mu(D-b-t)} \left(g(D-b) - D + b - \frac{1}{\mu}\right) \\
=& \mu\Bigg\{\mu\beta t e^{-\mu(D-t)} \left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}} - \omega P\kappa\mu e^{-\mu(D-b-t)} \\
&\times \left(g(D-b) - D + b - \frac{1}{\mu}\right) - \omega P\kappa\Bigg\} \\
&+ \mu\omega P\kappa + \mu\beta e^{-\mu(D-t)} \left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}} \\
=& \mu l(t) + \mu\omega P\kappa + \mu\beta e^{-\mu(D-t)} \left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}} > \mu l(t).
\end{aligned}$$

Thus, if there is a $t^{\ddagger} \in [0, D-b)$ satisfying $l(t^{\ddagger}) \ge 0$, then we have $\frac{\mathrm{d}l(t)}{\mathrm{d}t} > 0$ for $t \in [t^{\ddagger}, D-b)$, and subsequently we have $l(t) > 0$ for $t \in (t^{\ddagger}, D-b)$.

When $(D-b) \le t \le (D-a)$, from (36), (37), (23), and (25), we have

$$\begin{aligned}
\frac{\mathrm{d}\Pr\{N(t)=C\}}{\mathrm{d}t} &= \frac{\mathrm{d}h(t)}{\mathrm{d}t} = \mu F_S(D-t)h(t), \\
\frac{\mathrm{d}\mathbb{E}\left[T_{N(t)}\right]}{\mathrm{d}t} &= \frac{\mathrm{d}g(t)}{\mathrm{d}t} = \mu F_S(D-t)\left(g(t) - t\right).
\end{aligned}$$

Then from (13), we have

$$\begin{aligned}
l(t) =& \mu\beta F_S(D-t)h(t)g(t) \\
&- \mu F_S(D-t)\left(g(t)-t\right)\left(\omega P\kappa + \beta h(t)\right) \\
=& \mu F_S(D-t)\Big(t\left(\omega P\kappa + \beta h(t)\right) - g(t)\omega P\kappa\Big) \\
=& \omega P\kappa\mu F_S(D-t)\left(t - g(t) + \frac{\beta h(t) t}{\omega P\kappa}\right) \\
=& \omega P\kappa\mu F_S(D-t)p(t), \qquad (41)
\end{aligned}$$

where $p(t) \overset{\triangle}{=} t - g(t) + \frac{\beta}{\omega P\kappa}h(t)t$. From (41), it can be seen that $l(t)$ is continuous in $t \in [D-b, D-a]$.

When $t \in [D-b, D-a]$, according to (25), we have

$$t - g(t) = \frac{-\frac{\mathrm{d}g(t)}{\mathrm{d}t}}{\mu F_S(D-t)}.$$

Then, we have

$$\begin{aligned}
p(t) =& \frac{-\frac{\mathrm{d}g(t)}{\mathrm{d}t}}{\mu F_S(D-t)} + \frac{\beta}{\omega P\kappa}h(t)t \\
=& \frac{\frac{\mathrm{d}h(t)}{\mathrm{d}t}\left(a - \frac{1}{\mu} + \frac{\mu b}{b-a}\left(\frac{a}{e}\right)^{\frac{\mu ab}{b-a}} \int_a^{D-t}(z-a)z^{-\frac{\mu ab}{b-a}} e^{\frac{\mu b z}{b-a}}\,\mathrm{d}z\right)}{\mu F_S(D-t)} \\
&- \frac{h(t)\left(\frac{\mu b}{b-a}\left(\frac{a}{e}\right)^{\frac{\mu ab}{b-a}} (D-t-a)(D-t)^{-\frac{\mu ab}{b-a}} e^{\frac{\mu b(D-t)}{b-a}}\right)}{\mu F_S(D-t)} \\
&+ \frac{\beta}{\omega P\kappa}h(t)t \\
=& h(t)q(t), \qquad (42)
\end{aligned}$$

where the second equality is from (26), the last equality is from (23) and (1), and $q(t)$ is defined as

$$\begin{aligned}
q(t) =& a - \frac{1}{\mu} + \frac{\mu b}{b-a}\left(\frac{a}{e}\right)^{\frac{\mu ab}{b-a}} \int_a^{D-t} (z-a)z^{-\frac{\mu ab}{b-a}} e^{\frac{\mu bz}{b-a}}\,\mathrm{d}z \\
&- \left(\frac{a}{e}\right)^{\frac{\mu ab}{b-a}} (D-t)^{1-\frac{\mu ab}{b-a}} e^{\frac{\mu b(D-t)}{b-a}} + \frac{\beta t}{\omega P\kappa}.
\end{aligned}$$

Taking the first-order derivative of $q(t)$, we have

$$\begin{aligned}
\frac{\mathrm{d}q(t)}{\mathrm{d}t} =& -\frac{\mu b}{b-a}\left(\frac{a}{e}\right)^{\frac{\mu ab}{b-a}} (D-t-a)(D-t)^{-\frac{\mu ab}{b-a}} e^{\frac{\mu b(D-t)}{b-a}} \\
&+ \left(1 - \frac{\mu ab}{b-a}\right)\left(\frac{a}{e}\right)^{\frac{\mu ab}{b-a}} (D-t)^{-\frac{\mu ab}{b-a}} e^{\frac{\mu b(D-t)}{b-a}} \\
&+ \frac{\mu b}{b-a}\left(\frac{a}{e}\right)^{\frac{\mu ab}{b-a}} (D-t)^{1-\frac{\mu ab}{b-a}} e^{\frac{\mu b(D-t)}{b-a}} + \frac{\beta}{\omega P\kappa} \\
=& \left(\frac{a}{e}\right)^{\frac{\mu ab}{b-a}} (D-t)^{-\frac{\mu ab}{b-a}} e^{\frac{\mu b(D-t)}{b-a}} + \frac{\beta}{\omega P\kappa} > 0.
\end{aligned}$$

$$
\begin{aligned}
l(t) =& \mu\beta e^{-\mu(D-t)} \left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}} \left(\frac{1}{\mu} + t - \left(D - b + \frac{1}{\mu} - g(D-b)\right) e^{-\mu(D-b-t)}\right) \\
& - \left\{1 + \mu e^{-\mu(D-b-t)} \left(g(D-b) - D + b - \frac{1}{\mu}\right)\right\} \left(\omega P\kappa + \beta e^{-\mu(D-t)} \left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}}\right) \\
=& \mu\beta t e^{-\mu(D-t)} \left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}} - \omega P\kappa \left(1 + \mu e^{-\mu(D-b-t)} \left(g(D-b) - D + b - \frac{1}{\mu}\right)\right).
\end{aligned} \tag{40}
$$

As shown in Appendix D, $h(t) > 0$ and $\frac{\mathrm{d}h(t)}{\mathrm{d}t} > 0$ in $t \in [D-b, D-a)$. So if there is a $t^{\ddagger} \in [D-b, D-a)$ such that $l(t^{\ddagger}) \geq 0$, then from (41) we have $p(t^{\ddagger}) \geq 0$, and further from (42) we have $q(t^{\ddagger}) \geq 0$. Together with the fact that $\frac{\mathrm{d}q(t)}{\mathrm{d}t} > 0, h(t) > 0, \frac{\mathrm{d}h(t)}{\mathrm{d}t} > 0$ for $t \in [D-b, D-a)$, we have that $p(t) = h(t)q(t)$ is strictly increasing and always positive for $t \in (t^{\ddagger}, D-a)$. Since $F_S(D-t) > 0$ for $t \in [D-b, D-a)$, from (41) it can be concluded that $l(t) > 0$ for $t \in (t^{\ddagger}, D-a)$.

Since $l(t)$ is continuous in the intervals of $[0, D-b)$ and $[D-b, D-a]$, and

$$
\begin{aligned}
& \lim_{t<D-b,\ t\to(D-b)} l(t) \\
&= \mu\beta(D-b)e^{-\mu b} \left(\frac{b}{a}\right)^{\frac{\mu ab}{b-a}} + \omega P\kappa\mu \left(D - b - g(D-b)\right) \\
&= \omega P\kappa\mu \left(D - b - g(D-b) + \frac{\beta h(D-b)(D-b)}{\omega P\kappa}\right) \\
&= \lim_{t>D-b,\ t\to(D-b)} l(t)
\end{aligned}
$$

(in which the first equality is from (40), the second equality is from (24), and the last equality is from (41) and $F_S(b) = 1$), it can be concluded that $l(t)$ is continuous in $[0, D-a]$.

From (40), we have

$$
l(0) = -\omega P\kappa \left(1 + \mu e^{-\mu(D-b)} \left(g(D-b) - D + b - \frac{1}{\mu}\right)\right).
$$

According to the definition of $g(t)$ in Appendix C, $g(D-b) = \mathbb{E}\left[T_{n_r+1} | T_{n_r+1} \geq D-b\right] \geq D-b+1/\mu$, which means $l(0) < 0$. From (41), it can be seen that $l(D-a) = 0$ due to the fact that $F_S(a) = 0$.

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