Worst-Case Jamming on MIMO Gaussian Channels

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***Abstract*—Worst-case jamming of legitimate communications over multiple-input multiple-output Gaussian channels is studied in this paper. A worst-case scenario with a 'smart' jammer that knows all channels and the transmitter's strategy and is only power limited is considered. It is shown that the simplification of the system model by neglecting the properties of the jamming channel leads to a loss of important insights regarding the effects of the jamming power and jamming channel on optimal jamming strategies of the jammer. Without neglecting the jamming channel, a lower-bound on the rate of legitimate communication subject to jamming is derived, and conditions for this bound to be positive are given. The lower-bound rate can be achieved regardless of the quality of the jamming channel, the power limit of the jammer, and the transmit strategy of the jammer. Moreover, general forms of an optimal jamming strategy, on the basis of which insights into the effect of jamming power and jamming channel are exposed, are given. It is shown that the general forms can lead to closed-form optimal jamming solutions when the power limit of the jammer is larger than a threshold. Subsequently, the scenario in which the effect of jamming dominates the effect of noise (the case of practical interest) is considered, and an optimal jamming strategy is derived in closed-form. Simulation examples demonstrate lower-bound rates, performance of the derived jamming strategy, and an effect of inaccurate channel information on the jamming strategy.**

***Index Terms*—Closed-form solution, lower-bound rate, multiple-input multiple-output Gaussian channel, optimization, worst-case jamming.**

I. Introduction

Reliability and security are major concerns in modern wireless communications [1]- [6]. Due to the rapid development of wireless communications, threats to reliable and secure communications are becoming more prevalent as wireless communication networks of different scales containing devices for different purposes become more common and widespread.

One of major threats to wireless communications is jamming. Jamming aims at degrading the quality of communication or disrupting the information transmission in a communication system by directing energy toward the target receiver in a destructive manner [7]. A jamming attack is particularly effective because it is easy to launch using low-cost and small-sized devices while causing very significant disruption [8]. The threat of jamming has been studied in many works [9]- [15], and one of the relevant research interests in this area is to investigate optimal jamming strategies from the perspective of a jammer [8], [12]–[16]. Such a perspective helps to reveal the worst-case effects of jamming on legitimate communications. In the sequel, the terms of "worst-case jamming" and "optimal jamming" are used interchangeably where "worst-case jamming" is from the perspective of the legitimate communication and "optimal jamming" is from the perspective of the jammer.

The study of worst-case jamming is more complicated when the jammer has multiple antennas. In such cases, the jammer is able to use more subtle jamming strategies by adjusting the jamming signal across its antennas and consequently enhancing the effectiveness of jamming on the target communication. As a result, it is of interest to investigate worst-case jamming in the scenario in which the jammer and the legitimate transceivers all employ multiple antennas and to establish the limits of the jammer in terms of degrading the quality of the legitimate communication in such situations. Related problems have been investigated in the literature [17]- [20]. It is shown in [17] that without knowledge of the target signal or its covariance, the jammer can only use basic strategies of allocating power uniformly or maximizing the total power of the interference at the target receiver. In [18], the transmit strategies of a legitimate transmitter and a jammer on a Gaussian multiple-input multiple-output (MIMO) channel are investigated under a game-theoretic model with a general utility function. It is assumed that the jammer and the legitimate transmitter have the same level of channel state information (CSI), i.e., both uninformed, both with statistical CSI, or both with exact instantaneous CSI. Optimal transmission strategies of the legitimate transmitter and the jammer are represented as solutions to different optimization problems for different types of CSI. The optimal jamming of a jammer on MIMO multiple access and broadcast channels with the covariance of the target signal and all channel information available at the jammer is studied in [19] based on game theory. Some properties of the optimal jamming strategies are characterized through the analysis of the Nash equilibrium of the game. A necessary condition for optimal jamming on MIMO channels with arbitrary inputs when the covariance of the target signal and all channel information are available at the jammer is derived in [20]. For the case of a Gaussian target signal, the solution of optimal jamming is given in closed-form. However, it is derived without considering the

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jamming channel, i.e., the jamming channel is assumed to be equal to an identity matrix. As a result, the system model is oversimplified by implicitly assuming that the received jamming signal at the target receiver is exactly the same as the transmitted jamming signal at the jammer. With a similar assumption, a game between a MIMO radar and a jammer is investigated in [21] and the strategies of both sides are analyzed. The worst-case noise, comprised of both multi-user interference and Gaussian noise, in a multi-user cellular system is investigated in [22]. The authors of [22] study the worst-case noise under constraints on its trace, eigenvalues, or diagonal elements, and the capacity of the considered system is derived by solving corresponding minimax problems on the noise and transmit covariance matrices.

With the objective of providing a general solution without oversimplification of the system model, this work addresses the problem of characterizing the worst-case effect of jamming on MIMO Gaussian channels and finds an optimal jamming strategy of the jammer in the jammer-dominant regime, i.e., where jamming dominates noise at the target receiver. Without the assumption of the jamming channel being equal to an identity matrix, it can be shown that finding the optimal jamming strategy does not reduce to a power allocation problem anymore. Moreover, unlike the case in which the optimal solution can be found in closed-form with the assumption on the jamming channel being equal to an identity matrix, the optimal jamming solution without this assumption may not exist in closed-form for non-diagonal jamming channels depending on the jamming power limit. The main contributions of this work are as follows.[1]

First, we show that there is a lower-bound on the rate that the legitimate communication can achieve regardless of the quality of the jamming channel, the power limit of the jammer, and the transmit strategy of the jammer. We give an expression for the lower-bound rate as well as a condition that assures the lower-bound rate is positive. Moreover, it is shown that a non-zero lower-bound rate is not necessarily a result of the jamming channel having lower rank than (or spanning a subset of) the legitimate channel. It also depends on how the jamming power is directed toward the target signal at the receiver via the jamming channel. This result was not obtained in previous works in which the jamming channel is assumed to be equal to an identity matrix.

Second, we characterize general forms of the optimal jamming strategy. Based on the general forms, it is shown that neglecting the jamming channel leads to a loss of important insight into the jamming strategy, i.e., the jammer should generally allocate more power on weak subchannels in the optimal strategy. Moreover, we show that when the power limit of the jammer is larger than a threshold, the optimal jamming strategy can be obtained in closed-form from the general forms. It includes the scalar solutions in [20] and [21] as special cases in which the jamming channel is equal to an identity matrix. When a closed-form solution is not available, insights into the jamming strategy can still be provided based on the characteristics of the general forms.

[1]Some very preliminary results have been presented in [23].

Third, we focus on the practical scenario in which the effect of jamming dominates the effect of noise and find a closed-form optimal jamming strategy in the jammer-dominant regime. It is shown that there is a unified closed-form solution for the optimal jamming strategy in this regime. Moreover, it is shown that the proposed closed-form solution in the jammer-dominant regime leads to performance very close to optimal even for the scenarios in which the effect of jamming is not dominant.

The rest of the paper is organized as follows. Section II explains the system model considered in this work. The problem of finding a lower-bound rate for the legitimate transceiver and general forms of the optimal jamming strategy is investigated in Section III where closed-form solutions are derived when the power limit of the jammer exceeds a threshold. The scenario in which the effect of jamming is dominant over the effect of noise, i.e., in the so-called jammer-dominant regime, is studied in Section IV and a closed-form solution for the optimal jamming strategy is found. Section V discusses simulation examples confirming the results obtained in the previous sections, and Section VI concludes the paper. Section VII "Appendix" provides proofs for the lemmas and theorems.

II. System Model

A legitimate transmitter with n_{t} antennas sends a signal $\mathbf{s}$ to a receiver with n_{r} antennas. The elements of $\mathbf{s}$ are independent and identically distributed Gaussian random variables with zero means and covariance $\mathbf{Q}_{\mathrm{s}}$. A jammer with n_{z} antennas attempts to jam the legitimate communication by transmitting a jamming signal $\mathbf{z}$ to the receiver. Denote the channel from the legitimate transmitter to the receiver as $\mathbf{H}_{\mathrm{r}}$ (of size $n_{\mathrm{r}} \times n_{\mathrm{t}}$) and the jamming channel from the jammer to the receiver as $\mathbf{H}_{\mathrm{z}}$ (of size $n_{\mathrm{r}} \times n_{\mathrm{z}}$). The case of block fading channels is considered here. Therefore, the channels remain constant during a fading block. In the presence of the jamming signal, the received signal at the receiver can be expressed as

$$\mathbf{y} = \mathbf{H}_{\mathrm{r}}\mathbf{s} + \mathbf{H}_{\mathrm{z}}\mathbf{z} + \mathbf{n}, \tag{1}$$

where $\mathbf{n}$ is Gaussian noise with zero mean and covariance $\sigma^2\mathbf{I}$. Here $\mathbf{I}$ denotes the identity matrix of an appropriate size. Note that given the Gaussian channel and Gaussian target signal, the worst-case jamming signal should also be Gaussian [24]. Denote the covariance of $\mathbf{z}$ as $\mathbf{Q}_{\mathrm{z}}$. Then the information rate of the legitimate communication in the presence of jamming is expressed as

$$R^{\mathrm{J}} = \log|\mathbf{I} + \mathbf{H}_{\mathrm{r}}\mathbf{Q}_{\mathrm{s}}\mathbf{H}_{\mathrm{r}}^{\mathrm{H}}(\mathbf{H}_{\mathrm{z}}\mathbf{Q}_{\mathrm{z}}\mathbf{H}_{\mathrm{z}}^{\mathrm{H}} + \sigma^2\mathbf{I})^{-1}|, \tag{2}$$

where $|\cdot|$ and $(\cdot)^{\mathrm{H}}$ denote the determinant and the Hermitian transpose, respectively. The jammer aims at decreasing the above rate as much as possible given its power limit P_{z}. If the jamming channel $\mathbf{H}_{\mathrm{z}}$ is unknown, the jammer has no better strategy than uniformly allocating its transmission power, i.e., $\mathbf{Q}_{\mathrm{z}} = P_{\mathrm{z}}/n_{\mathrm{z}} \cdot \mathbf{I}$. If the jammer knows $\mathbf{H}_{\mathrm{z}}$ but not $\mathbf{Q}_{\mathrm{s}}$ and/or $\mathbf{H}_{\mathrm{r}}$, it can maximize the jamming power at the receiver, i.e., maximize $\mathrm{Tr}\{\mathbf{H}_{\mathrm{z}}\mathbf{Q}_{\mathrm{z}}\mathbf{H}_{\mathrm{z}}^{\mathrm{H}}\}$ [17], where $\mathrm{Tr}\{\cdot\}$ denotes the trace. In both of the above two cases, the optimal

TABLE I: Table of Notations

$\mathbf{X}$	matrix
$\mathrm{Tr}\{\mathbf{X}\}$	the trace of $\mathbf{X}$
$\lvert\mathbf{X}\rvert$	the determinant of $\mathbf{X}$
$\mathbf{X} \succ \mathbf{0}$	$\mathbf{X}$ is positive definite
$\mathbf{X} \succeq \mathbf{0}$	$\mathbf{X}$ is positive semidefinite
$\mathbf{X}^{\mathrm{H}}$	the Hermitian transpose of $\mathbf{X}$
$\mathbf{U}_{\mathrm{r}}, \mathbf{U}_{\mathrm{z}}$	the unitary matrix consisting of left-singular vectors of $\mathbf{H}_{\mathrm{r}}, \mathbf{H}_{\mathrm{z}}$
$\mathbf{V}_{\mathrm{r}}, \mathbf{V}_{\mathrm{z}}$	the unitary matrix consisting of right-singular vectors of $\mathbf{H}_{\mathrm{r}}, \mathbf{H}_{\mathrm{z}}$
$\mathbf{\Omega}_{\mathrm{r}}, \mathbf{\Omega}_{\mathrm{z}}$	the diagonal matrix consisting of singular values of $\mathbf{H}_{\mathrm{r}}, \mathbf{H}_{\mathrm{z}}$
$\bar{\mathbf{\Omega}}_{\mathrm{r}}, \bar{\mathbf{\Omega}}_{\mathrm{z}}$	the block of $\mathbf{\Omega}_{\mathrm{r}}, \mathbf{\Omega}_{\mathrm{z}}$ consisting of only positive singular values
$\mathbf{U}_{\mathbf{X}}$	the unitary matrix consisting of eigenvectors of $\mathbf{X}$
$\mathbf{\Lambda}_{\mathbf{X}}$	the diagonal matrix consisting of eigenvalues of $\mathbf{X}$
$\bar{\mathbf{\Lambda}}_{\mathbf{X}}$	the block of $\mathbf{\Lambda}_{\mathbf{X}}$ consisting of only positive eigenvalues
$\mathbf{U}_{\mathbf{X}1}$	the columns of $\mathbf{U}_{\mathbf{X}}$ corresponding to $\bar{\mathbf{\Lambda}}_{\mathbf{X}}$
$\mathbf{U}_{\mathbf{X}2}$	the columns of $\mathbf{U}_{\mathbf{X}}$ corresponding to zero eigenvalues

strategy of the jammer is well-known. In this paper, similar to [19], [20], and [21], the jammer is assumed to have the knowledge of $\mathbf{H}_{\mathrm{r}}$, $\mathbf{H}_{\mathrm{z}}$, and $\mathbf{Q}_{\mathrm{s}}$ but not $\mathbf{s}$. For example, the jammer may obtain information on $\mathbf{H}_{\mathrm{r}}$ and $\mathbf{H}_{\mathrm{z}}$ when it is also capable of eavesdropping and the channels are reciprocal. The jammer is not able to perform correlated jamming [25]. However, the jammer can use the available knowledge to find the optimal $\mathbf{Q}_{\mathrm{z}}$ such that the rate (2) is minimized. It should be noted that the worst-case perspective of the study aims at understanding a limit on what can be achieved by jamming and the resulting jamming strategy. Therefore, the practicality of the results lies in the fact that, given a specific problem setup, the obtained results can be used to understand what would be the effect of jamming in the worst-case. It should be noted that the assumption on the availability of $\mathbf{H}_{\mathrm{z}}$ is weaker than the assumption imposed by neglecting the jamming channel as considered in [20] and [21]. In the latter case, the jamming channel has been assumed not only to be known but also to be equal to an identity matrix.

Given the above system model, we are especially interested in the case in which the effect of jamming on the received legitimate signal dominates the effect of the noise. It is worth noting, however, that the noise term in (2) cannot be simply neglected in general due to the fact that the term $\mathbf{H}_{\mathrm{z}}\mathbf{Q}_{\mathrm{z}}\mathbf{H}_{\mathrm{z}}^{\mathrm{H}}$ can be rank-deficient, e.g., when $\mathbf{H}_{\mathrm{z}}$ does not have full row rank. It will also be shown later that the optimal form of $\mathbf{H}_{\mathrm{z}}\mathbf{Q}_{\mathrm{z}}\mathbf{H}_{\mathrm{z}}^{\mathrm{H}}$ is rank-deficient when $\mathbf{H}_{\mathrm{r}}\mathbf{Q}_{\mathrm{s}}\mathbf{H}_{\mathrm{r}}^{\mathrm{H}}$ is rank-deficient. There was no need to address these cases in [20] and [21] where the jamming channel $\mathbf{H}_{\mathrm{z}}$ was assumed to be $\mathbf{I}$. As a result, while we focus on the case in which jamming dominates noise, we do not neglect the noise term $\sigma^2\mathbf{I}$. Instead, we define a jammer-dominant regime which will be explained in detail later and find an optimal jamming strategy in this regime.

The symbols and notations used in this paper are summarized in Table I for clarity of presentation.

III. Lower-bound Rate and General Forms of Optimal Jamming Strategy

In this section, we first investigate the effect of jamming on the legitimate communication with the objective of finding a part of the information rate of the legitimate communication which is unaffected by jamming. This unaffected part of the information rate is of interest because it leads to results on the lower-bound rate of the legitimate communication. Then, we study general forms of the optimal jamming strategy, which are fundamental for finding closed-form solutions of the optimal jamming strategy.

A. Lower-bound rate for legitimate communication

Given the system model, an optimal jamming strategy can be found by solving the following problem[2]:

$$\min_{\mathbf{Q}_{\mathrm{z}}} \quad R^{\mathrm{J}} \tag{3a}$$

$$\mathbf{s.t.} \qquad \mathrm{Tr}\{\mathbf{Q}_{\mathrm{z}}\} \leq P_{\mathrm{z}}. \tag{3b}$$

With only one pair of transceivers, the above problem is a basic jamming problem on a MIMO channel. While the above problem is convex, our objective is to gain insights into the optimal jamming strategy and the corresponding effect on the legitimate communication, which cannot be provided by a numerical solution. We also aim at finding a closed-form solution for the optimal jamming strategy when possible.

Note that problem (3) can be extended to a maxmin problem in which, given the optimal jamming strategy, the legitimate transmitter aims at maximizing its information rate under jamming. Such extension is straightforward since the optimal strategy of the legitimate transmitter is to consider the jamming signal as noise and use waterfilling to obtain its optimal transmit covariance. Therefore, our focus in the sequel will be on deriving the optimal jamming strategy of the jammer.

Denote the singular value decomposition (SVD) of $\mathbf{H}_{\mathrm{z}}$ as $\mathbf{H}_{\mathrm{z}} = \mathbf{U}_{\mathrm{z}}\mathbf{\Omega}_{\mathrm{z}}\mathbf{V}_{\mathrm{z}}^{\mathrm{H}}$ and the rank of $\mathbf{H}_{\mathrm{z}}$ as r_{z}. The matrices $\mathbf{U}_{\mathrm{z}}$, $\mathbf{\Omega}_{\mathrm{z}}$, and $\mathbf{V}_{\mathrm{z}}$ are of sizes $n_{\mathrm{r}} \times n_{\mathrm{r}}$, $n_{\mathrm{r}} \times n_{\mathrm{z}}$, and $n_{\mathrm{z}} \times n_{\mathrm{z}}$, respectively. Denote the SVD of $\mathbf{H}_{\mathrm{r}}$ as $\mathbf{H}_{\mathrm{r}} = \mathbf{U}_{\mathrm{r}}\mathbf{\Omega}_{\mathrm{r}}\mathbf{V}_{\mathrm{r}}^{\mathrm{H}}$. The matrices $\mathbf{U}_{\mathrm{r}}$, $\mathbf{\Omega}_{\mathrm{r}}$, and $\mathbf{V}_{\mathrm{r}}$ are of sizes $n_{\mathrm{r}} \times n_{\mathrm{r}}$, $n_{\mathrm{r}} \times n_{\mathrm{t}}$, and $n_{\mathrm{t}} \times n_{\mathrm{t}}$, respectively. Define

$$\mathbf{B} \triangleq \mathbf{U}_{\mathrm{z}}^{\mathrm{H}}\mathbf{H}_{\mathrm{r}}\mathbf{Q}_{\mathrm{s}}\mathbf{H}_{\mathrm{r}}^{\mathrm{H}}\mathbf{U}_{\mathrm{z}}. \tag{4}$$

Note that $\mathbf{B}$ has the same rank as $\mathbf{H}_{\mathrm{r}}\mathbf{Q}_{\mathrm{s}}\mathbf{H}_{\mathrm{r}}^{\mathrm{H}}$. Using the definition of $\mathbf{B}$ and the SVD of $\mathbf{H}_{\mathrm{z}}$, the objective function in (2) can be rewritten as

$$R^{\mathrm{J}} = \log|\mathbf{I} + \mathbf{B}(\mathbf{\Omega}_{\mathrm{z}}\hat{\mathbf{Q}}_{\mathrm{z}}\mathbf{\Omega}_{\mathrm{z}}^{\mathrm{H}} + \sigma^2\mathbf{I})^{-1}|, \tag{5}$$

where

$$\hat{\mathbf{Q}}_{\mathrm{z}} \triangleq \mathbf{V}_{\mathrm{z}}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}\mathbf{V}_{\mathrm{z}}. \tag{6}$$

Lemma 1: If we denote $\mathbf{B}$ using blocks such that

$$\mathbf{B} = \begin{array}{c c} & \begin{array}{cc} r_{\mathrm{z}} & n_{\mathrm{r}}-r_{\mathrm{z}} \end{array} \\ \begin{array}{c} r_{\mathrm{z}} \\ n_{\mathrm{r}}-r_{\mathrm{z}} \end{array} & \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix} \end{array}, \tag{7}$$

and define

$$\tilde{\mathbf{B}} \triangleq \mathbf{B}_{11} - \mathbf{B}_{12}(\sigma^2\mathbf{I} + \mathbf{B}_{22})^{-1}\mathbf{B}_{21}, \tag{8}$$

[2]The positive semi-definite constraint $\mathbf{Q}_{\mathrm{z}} \succeq 0$ is assumed by default and it is omitted for brevity throughout this paper.

then $\tilde{\mathbf{B}}$ is positive definite (PD) if $\mathbf{B}$ is PD.

Proof: See Subsection VII-A in the Appendix.

Before solving the optimization problem (3), it is essential to express the objective function of problem (3) in a different form so as to reveal the optimal structure of $\mathbf{Q}_\mathrm{z}$. Denote the diagonal matrix $\mathbf{\Omega}_\mathrm{z}$ using blocks as

$$\mathbf{\Omega}_\mathrm{z} \triangleq \begin{array}{c c} & \begin{array}{cc} r_\mathrm{z} & n_\mathrm{z}-r_\mathrm{z} \end{array} \\ \begin{array}{c} r_\mathrm{z} \\ n_\mathrm{r}-r_\mathrm{z} \end{array} & \left[\begin{array}{cc} \bar{\mathbf{\Omega}}_\mathrm{z} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{array}\right] \end{array}, \tag{9}$$

where $\bar{\mathbf{\Omega}}_\mathrm{z}$ is an $r_\mathrm{z} \times r_\mathrm{z}$ diagonal matrix made up of the positive diagonal elements of $\mathbf{\Omega}_\mathrm{z}$, and $\mathbf{0}$ denotes an all-zero matrix of an appropriate size. It can be seen that the allocation of jamming power should be limited to at most r_z dimensions corresponding to the r_z non-zero singular values of $\mathbf{\Omega}_\mathrm{z}$. Indeed, allocating jamming power anywhere else has no effect on the received signal and only leads to jamming power waste. As a result, $\hat{\mathbf{Q}}_\mathrm{z}$ should adopt the following form:

$$\hat{\mathbf{Q}}_\mathrm{z} = \begin{array}{c c} & \begin{array}{cc} r_\mathrm{z} & n_\mathrm{z}-r_\mathrm{z} \end{array} \\ \begin{array}{c} r_\mathrm{z} \\ n_\mathrm{z}-r_\mathrm{z} \end{array} & \left[\begin{array}{cc} \mathbf{Q}'_\mathrm{z} & \mathbf{\Gamma}_\mathrm{z} \\ \mathbf{\Gamma}_\mathrm{z}^\mathrm{H} & \mathbf{0} \end{array}\right] \end{array}, \tag{10}$$

where $\mathbf{Q}'_\mathrm{z}$ and $\mathbf{\Gamma}_\mathrm{z}$ are to be determined. The matrix $\mathbf{\Gamma}_\mathrm{z}$ does not affect the rate of R^J in (5). Therefore, $\mathbf{\Gamma}_\mathrm{z}$ is set to be $\mathbf{0}$ for simplicity and consequently

$$\hat{\mathbf{Q}}_\mathrm{z} = \begin{array}{c c} & \begin{array}{cc} r_\mathrm{z} & n_\mathrm{z}-r_\mathrm{z} \end{array} \\ \begin{array}{c} r_\mathrm{z} \\ n_\mathrm{z}-r_\mathrm{z} \end{array} & \left[\begin{array}{cc} \mathbf{Q}'_\mathrm{z} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{array}\right] \end{array}. \tag{11}$$

Let us define a new matrix $\tilde{\mathbf{\Omega}}_\mathrm{z}$ given as

$$\tilde{\mathbf{\Omega}}_\mathrm{z} \triangleq \begin{array}{c c} & \begin{array}{cc} r_\mathrm{z} & n_\mathrm{r}-r_\mathrm{z} \end{array} \\ \begin{array}{c} r_\mathrm{z} \\ n_\mathrm{r}-r_\mathrm{z} \end{array} & \left[\begin{array}{cc} \bar{\mathbf{\Omega}}_\mathrm{z} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{array}\right] \end{array}. \tag{12}$$

The channel matrix $\tilde{\mathbf{\Omega}}_\mathrm{z}$ has size $n_\mathrm{r} \times n_\mathrm{r}$, which is larger than the size of $\mathbf{\Omega}_\mathrm{z}$ if $n_\mathrm{r} > n_\mathrm{z}$, smaller than the size of $\mathbf{\Omega}_\mathrm{z}$ if $n_\mathrm{r} < n_\mathrm{z}$, and has the same size as $\mathbf{\Omega}_\mathrm{z}$ if $n_\mathrm{r} = n_\mathrm{z}$. Also define the following new jamming covariance matrix $\tilde{\mathbf{Q}}_\mathrm{z}$:

$$\tilde{\mathbf{Q}}_\mathrm{z} \triangleq \begin{array}{c c} & \begin{array}{cc} r_\mathrm{z} & n_\mathrm{r}-r_\mathrm{z} \end{array} \\ \begin{array}{c} r_\mathrm{z} \\ n_\mathrm{r}-r_\mathrm{z} \end{array} & \left[\begin{array}{cc} \mathbf{Q}'_\mathrm{z} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{array}\right] \end{array}, \tag{13}$$

where $\mathbf{Q}'_\mathrm{z}$ is the same as in (10).

With the above definitions of $\tilde{\mathbf{\Omega}}_\mathrm{z}$ and $\tilde{\mathbf{Q}}_\mathrm{z}$, it can be seen that $\mathbf{\Omega}_\mathrm{z}\hat{\mathbf{Q}}_\mathrm{z}\mathbf{\Omega}_\mathrm{z}^\mathrm{H}$ in (5) is equal to $\tilde{\mathbf{\Omega}}_\mathrm{z}\tilde{\mathbf{Q}}_\mathrm{z}\tilde{\mathbf{\Omega}}_\mathrm{z}$ (note that $\tilde{\mathbf{\Omega}}_\mathrm{z}^\mathrm{H} = \tilde{\mathbf{\Omega}}_\mathrm{z}$). As a result, the rate in (5) can be equivalently rewritten as

$$R^\mathrm{J} = \log|\mathbf{I} + \mathbf{B}(\tilde{\mathbf{\Omega}}_\mathrm{z}\tilde{\mathbf{Q}}_\mathrm{z}\tilde{\mathbf{\Omega}}_\mathrm{z} + \sigma^2\mathbf{I})^{-1}|. \tag{14}$$

Therefore, we consider $\tilde{\mathbf{\Omega}}_\mathrm{z}$ and $\tilde{\mathbf{Q}}_\mathrm{z}$ as the equivalent channel matrix and the equivalent jamming covariance matrix to $\mathbf{\Omega}_\mathrm{z}$ and $\hat{\mathbf{Q}}_\mathrm{z}$, respectively. The advantage of solving the optimization problem (3) using the above equivalent form of the objective function is that $\tilde{\mathbf{\Omega}}_\mathrm{z}$ in (14) is always PD and therefore can be extracted from the inverse term, which simplifies the solution finding procedure.

Using the above observations and equations (6) and (11) it can be seen that the optimal form of $\mathbf{Q}_\mathrm{z}$ is

$$\mathbf{Q}_\mathrm{z} = \mathbf{V}_\mathrm{z}\begin{bmatrix} \mathbf{Q}'_\mathrm{z} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}\mathbf{V}_\mathrm{z}^\mathrm{H}, \tag{15}$$

where the two diagonal blocks in the block diagonal matrix have sizes $r_\mathrm{z} \times r_\mathrm{z}$ and $(n_\mathrm{z} - r_\mathrm{z}) \times (n_\mathrm{z} - r_\mathrm{z})$, respectively.

Lemma 2: A lower-bound rate that can be achieved given the transmit strategy of the legitimate transceiver $\mathbf{Q}_\mathrm{s}$ and the legitimate channel $\mathbf{H}_\mathrm{r}$ is

$$R^0 = \log\left|\mathbf{I} + \frac{1}{\sigma^2}\mathbf{B}_{22}\right|, \tag{16}$$

while the minimization of R^J with respect to $\mathbf{Q}_\mathrm{z}$ is equivalent to the minimization of

$$\bar{R}^\mathrm{J} = \log|\mathbf{I} + \bar{\mathbf{\Omega}}_\mathrm{z}^{-1}\tilde{\mathbf{B}}\bar{\mathbf{\Omega}}_\mathrm{z}^{-1}(\mathbf{Q}'_\mathrm{z} + \mathbf{D})^{-1}|, \tag{17}$$

where $\mathbf{D} \triangleq \sigma^2\bar{\mathbf{\Omega}}_\mathrm{z}^{-2}$.

Proof: See Subsection VII-B in the Appendix.

Since $\mathbf{B}_{22} \succeq 0$ (which follows from (7) and the fact that $\mathbf{B} \succeq 0$), it can be seen from Lemma 2 that $R^0 = 0$ only when (1) $n_\mathrm{r} = r_\mathrm{z}$ or (2) $n_\mathrm{r} > r_\mathrm{z}$ and $\mathbf{B}_{22} = 0$.

In the first case, the jammer has at least the same number of antennas as the receiver and the jamming channel has full rank. Therefore, the column space of $\mathbf{H}_\mathrm{z}$ includes the column space of $\mathbf{H}_\mathrm{r}$ as a subset. Thus, the jammer can spread its jamming power on all the data streams (on the eigenchannels of $\mathbf{H}_\mathrm{r}$) of the legitimate communication. As a result, the legitimate communication does not have a positive lower-bound data rate.

In the second case, $\mathbf{B}_{22} = 0$ implies two things. First, the received legitimate signal has a rank no greater than r_z. Second, using (4) and (7), it can be seen that $\mathbf{B}_{22} = 0$ leads to a requirement on how the jamming channel directs the jamming signal to the receiver. Specifically, for R^0 to be 0, the columns of $\mathbf{U}_\mathrm{z}$ corresponding to the zero eigenvalues of $\mathbf{H}_\mathrm{z}$ must belong to the null space of the received signal covariance $\mathbf{H}_\mathrm{r}\mathbf{Q}_\mathrm{s}\mathbf{H}_\mathrm{r}^\mathrm{H}$. Consequently, the jamming power must be directed so that all data streams of the legitimate communication are subject to jamming.

The above two cases can be summarized as a sufficient and necessary condition for R^0 to be 0. Specifically, the lower-bound rate is zero if and only if the span of the jamming channel includes the span of $\mathbf{H}_\mathrm{r}\mathbf{Q}_\mathrm{s}\mathbf{H}_\mathrm{r}^\mathrm{H}$ as a subspace. Otherwise, the jammer cannot jam all the data streams of the legitimate communication and the legitimate transceiver can achieve a positive minimum rate through the data streams which are not jammed. Note that even if the rank of the jamming channel is higher than that of the legitimate channel (and thereby the jammer has more spatial degrees of freedom), the legitimate communication may still achieve a positive lower-bound rate depending on how the jamming signal is directed.

Following the above discussion, conditions for the lower-bound rate R^0 to be positive depending on the rank of $\mathbf{B}$ can be given.

Lemma 3: Denote $r_\mathbf{B}$ as the rank of $\mathbf{B}$. The following results regarding $R^0 > 0$ and $r_\mathbf{B}$ hold:

1) R^0 must be positive if $r_\mathbf{B} > r_\mathrm{z}$.

2) R^0 can be positive as long as $r_{\mathbf{B}} > 0$.

Proof: See Subsection VII-C in the Appendix.

As can be seen from (4) and (16), the lower-bound rate R^0 does not depend on the jamming signal $\mathbf{Q}'_\mathrm{z}$, or either of $\mathbf{\Omega}_\mathrm{z}$ and $\mathbf{V}_\mathrm{z}$ in the jamming channel. The only factors related to jamming that the lower-bound rate depends on are the rank r_z and the matrix $\mathbf{U}_\mathrm{z}$. As a result, given r_z, the rate R^0 does not depend on the specific jamming signal or the channel gain of the jamming channel but rather on how the jamming signal is directed towards the receiver, i.e., the alignment of the jamming signal towards the received legitimate signal. Given the above fact, it is of interest to find the $\mathbf{U}_\mathrm{z}$ that leads to the smallest R^0 given $\mathbf{H}_\mathrm{r}\mathbf{Q}_\mathrm{s}\mathbf{H}_\mathrm{r}^\mathrm{H}$.

Lemma 4: Given $\mathbf{H}_\mathrm{r}\mathbf{Q}_\mathrm{s}\mathbf{H}_\mathrm{r}^\mathrm{H}$, R^0 is minimized when the last[3] $n_\mathrm{r} - r_\mathrm{z}$ columns of $\mathbf{U}_\mathrm{z}$ consist of the eigenvectors corresponding to the smallest $n_\mathrm{r} - r_\mathrm{z}$ eigenvalues of $\mathbf{H}_\mathrm{r}\mathbf{Q}_\mathrm{s}\mathbf{H}_\mathrm{r}^\mathrm{H}$. If the legitimate transmitter uses waterfilling, R^0 is minimized when the last $n_\mathrm{r} - r_\mathrm{z}$ columns of $\mathbf{U}_\mathrm{z}$ consist of the last $n_\mathrm{r} - r_\mathrm{z}$ columns of $\mathbf{U}_\mathrm{r}$.

Proof: See Subsection VII-D in the Appendix.

It should be noted that minimizing R^0 is only minimizing the lower-bound rate. It is different from minimizing the information rate of the legitimate communication under jamming, which we will study next.

B. General forms of optimal jamming strategies

Given the above lemmas, we next solve the problem (3) by finding the optimal $\mathbf{Q}'_\mathrm{z}$ in (15). Denote

$$\mathbf{A} \triangleq \bar{\mathbf{\Omega}}_\mathrm{z}^{-1}\tilde{\mathbf{B}}\bar{\mathbf{\Omega}}_\mathrm{z}^{-1}. \tag{18}$$

The following theorem investigates the optimal form of $\mathbf{Q}'_\mathrm{z}$ when $\mathbf{A}$ is PD .

Theorem 1: In the case in which $\mathbf{A} \succ 0$, the general form of the optimal $\mathbf{Q}'_\mathrm{z}$ for the jamming problem (3) is given by

$$\mathbf{Q}'_\mathrm{z} = \mathbf{A}^{\frac{1}{2}}\left(\mathbf{A}^{-\frac{1}{2}}(\lambda\mathbf{I}+\mathbf{Z})^{-1}\mathbf{A}^{-\frac{1}{2}} + \frac{1}{4}\mathbf{I}\right)^{\frac{1}{2}}\mathbf{A}^{\frac{1}{2}} - \frac{1}{2}\mathbf{A} - \mathbf{D}, \tag{19}$$

where λ is chosen so that $\mathrm{Tr}\{\mathbf{Q}'_\mathrm{z}\} = P_\mathrm{z}$ and $\mathbf{Z}$ is a positive semi-definite (PSD) matrix that satisfies $\mathrm{Tr}\{\mathbf{Z}\mathbf{Q}'_\mathrm{z}\} = \mathbf{0}$.

Proof: See Subsection VII-E in the Appendix.

The parameters λ and $\mathbf{Z}$ in (19) are in fact the Lagrange multipliers associated with the constraints $\mathrm{Tr}\{\mathbf{Q}'_\mathrm{z}\} \leq P_\mathrm{z}$ and $\mathbf{Q}'_\mathrm{z} \succeq 0$, respectively. As mentioned earlier, a special case of the problem (3) that assumes the jamming channel $\mathbf{H}_\mathrm{z}$ to be the identity matrix $\mathbf{I}$ has been investigated in [20] as well as in the game theoretic setup for MIMO radar application [21]. Under this special assumption, $\mathbf{U}_\mathrm{z}$, $\mathbf{\Omega}_\mathrm{z}$, and $\mathbf{V}_Z^\mathrm{H}$ are all equal to $\mathbf{I}$. Consequently, $\mathbf{A}$ and $\bar{\mathbf{\Omega}}_\mathrm{z}$ simplify to $\tilde{\mathbf{B}}$ and $\mathbf{I}$, respectively. With the above simplifications, the problem of finding an optimal jamming strategy reduces to finding an optimal jamming power allocation, and the solution is given in a scalar form.

It should be noted that neglecting the jamming channel leads to a loss of insight into the effect of the jamming channel on the jammer's strategy. Before showing the effect of the jamming channel, we briefly explain the difference in finding the solution with and without considering the jamming channel as follows. Without considering the jamming channel, the problem becomes minimizing the following objective function:

$$\bar{R}^\mathrm{J} = \log|\mathbf{I} + \bar{\mathbf{\Omega}}_\mathrm{z}^{-1}\tilde{\mathbf{B}}\bar{\mathbf{\Omega}}_\mathrm{z}^{-1}(\mathbf{Q}'_\mathrm{z}+\mathbf{I})^{-1}| \tag{20}$$

subject to the jammer's power limit. If we create a new variable $\mathbf{X}$ such that $\mathbf{X} = \mathbf{Q}'_\mathrm{z} + \mathbf{I}$, it is not difficult to obtain a closed-form solution for $\mathbf{X}$ subject to the power constraint $\mathrm{Tr}\{\mathbf{X}\} \leq \mathrm{Tr}\{\mathbf{Q}'_\mathrm{z} + \mathbf{I}\}$. The optimal solution for $\mathbf{Q}'_\mathrm{z}$ is then $\mathbf{X} - \mathbf{I}$ if the result is PSD. When the result is indefinite, we can still obtain a closed-form solution for $\mathbf{Q}'_\mathrm{z}$ from the solution for $\mathbf{X}$ based on *projection*, i.e., projecting the negative eigenvalues of $\mathbf{Q}'_\mathrm{z}$ to zero and reducing the positive eigenvalues accordingly. This is possible because the eigenvalues of $\mathbf{Q}'_\mathrm{z}$ are equal to the eigenvalues of $\mathbf{X}$ minus 1 while the eigenvectors remain the same. However, when the jamming channel is considered and $\mathbf{I}$ in the brackets in (20) is replaced by $\mathbf{D}$, the above-mentioned method is not applicable anymore. Indeed, although $\mathbf{D}$ is a diagonal matrix, it is in general impossible to know the eigenvalues of $\mathbf{X} - \mathbf{D}$ based on a non-diagonal matrix $\mathbf{X}$. Therefore, it may be impossible to find a closed-form solution in general. However, the obtained general form of the optimal jamming strategy in (19) can lead to the optimal closed-form solution in special cases. The following remark provides an example.

Remark 1: In the case in which $\mathbf{A} \succ 0$, there is a threshold value for P_z, denoted by P_z^*, such that when $P_\mathrm{z} > P_\mathrm{z}^*$ the optimal $\mathbf{Q}'_\mathrm{z}$ is a PD matrix given by

$$\mathbf{Q}'_\mathrm{z} = \mathbf{A}^{\frac{1}{2}}\left(\frac{1}{\lambda}\mathbf{A}^{-1} + \frac{1}{4}\mathbf{I}\right)^{\frac{1}{2}}\mathbf{A}^{\frac{1}{2}} - \frac{1}{2}\mathbf{A} - \mathbf{D}. \tag{21}$$

The justification for the expression (21) when $\mathbf{Q}'_\mathrm{z}$ is PD is straightforward. Given the constraints $\mathrm{Tr}\{\mathbf{Z}\mathbf{Q}'_\mathrm{z}\} = \mathbf{0}$ and $\mathbf{Z} \preceq \mathbf{0}$ in Theorem 1, it holds true that $\mathbf{Z} = \mathbf{0}$ if $\mathbf{Q}'_\mathrm{z}$ is PD. Substituting $\mathbf{Z} = \mathbf{0}$ in (19) leads to (21). The threshold P_z^* is the trace of $\mathbf{Q}'_\mathrm{z}$ when the minimum eigenvalue of $\mathbf{Q}'_\mathrm{z}$ in (21) approaches zero. The threshold P_z^* can be determined numerically given a specific problem although it cannot be simply expressed as a function of other parameters. If we search for λ_0 such that the minimum eigenvalue of $\mathbf{Q}'_\mathrm{z}$ in (21) is 0 when λ is equal to λ_0, and becomes positive when λ becomes larger than λ_0, the trace of $\mathbf{Q}'_\mathrm{z}$ with λ substituted by λ_0 is equal to P_z^*.

It should be noted that the solution (21) includes the scalar solutions in [20] and [21] as special cases in which the jamming channel is equal to an identity matrix.

The solution (21) can be used to demonstrate the effect of the jamming channel on the optimal jamming strategy. In order to isolate and reveal this effect of the jamming channel, consider the case in which $\mathbf{H}_\mathrm{r}$, $\mathbf{Q}_\mathrm{s}$, and correspondingly $\tilde{\mathbf{B}}$ are equal to $\mathbf{I}$. As a result, the target signal $\mathbf{H}_\mathrm{r}\mathbf{Q}_\mathrm{s}\mathbf{H}_\mathrm{r}^\mathrm{H}$ at the receiver is an identity matrix, and the effects of the variance of the target signal and of the legitimate communication channel on the jamming strategy are eliminated. In this case, using the

[3]By 'last n columns' in an eigenvector matrix, we refer to the n columns of eigenvectors corresponding to the n smallest eigenvalues.

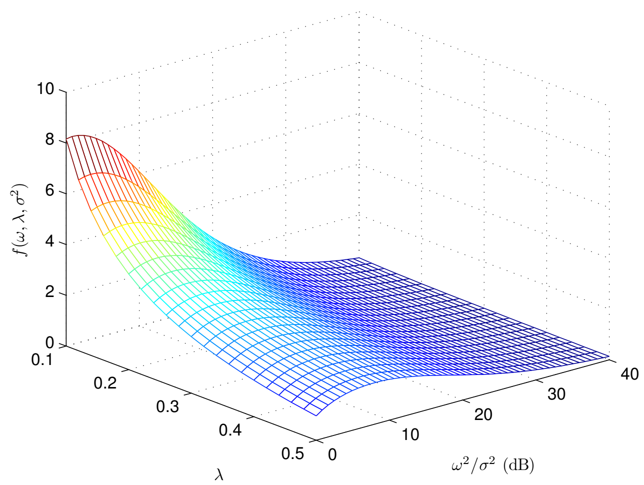

Fig. 1: Illustration of the effect of the jamming channel on the optimal jamming strategy.

definition of $\mathbf{A}$ and $\mathbf{D}$, $\mathbf{Q}'_{\mathrm{z}}$ in (19) becomes

$$\mathbf{Q}'_{\mathrm{z}} = \left(\frac{1}{\lambda}\bar{\mathbf{\Omega}}_{\mathrm{z}}^{-2} + \frac{1}{4}\bar{\mathbf{\Omega}}_{\mathrm{z}}^{-4}\right)^{\frac{1}{2}} - \frac{1}{2}\bar{\mathbf{\Omega}}_{\mathrm{z}}^{-2} - \sigma^2\bar{\mathbf{\Omega}}_{\mathrm{z}}^{-2}. \quad (22)$$

In the above expression, $\mathbf{Q}'_{\mathrm{z}}$ becomes a power allocation matrix. The power allocation across the diagonal elements of $\mathbf{Q}'_{\mathrm{z}}$ depends on $\bar{\mathbf{\Omega}}_{\mathrm{z}}$ and is of interest. Following the form of (22), a function $f(\omega,\lambda,\sigma^2)$ can be defined as follows:

$$f(\omega,\lambda,\sigma^2) = \left(\frac{1}{\lambda}\omega^{-2} + \frac{1}{4}\omega^{-4}\right)^{\frac{1}{2}} - \frac{1}{2}\omega^{-2} - \sigma^2\omega^{-2}. \quad (23)$$

The plot of $f(\omega,\lambda,\sigma^2)$ versus ω^2 (σ^2 is fixed at 0.01) and λ is shown in Fig. 1, which explicitly demonstrates the following effects of jamming channel on the optimal jamming strategy.

First, for a given power budget (fixing λ), the jammer allocates more jamming power on the weak jamming subchannels (corresponding to the small elements of $\bar{\mathbf{\Omega}}_{\mathrm{z}}$) as compared to its power allocated on the strong jamming subchannels (corresponding to the large elements of $\bar{\mathbf{\Omega}}_{\mathrm{z}}$). This can be seen from Fig. 1 because a smaller ω generally leads to a larger $f(\omega,\lambda,\sigma^2)$ for a given λ. Although it seems counter-intuitive that it is optimal for the jammer to use most of its power on weak jamming subchannels, it should be noted that the target is a signal transmitted through a MIMO channel that has multiple data streams. While some streams might be significantly reduced if the jammer allocates most of its power on the strong jamming subchannels, the effect of such a jamming strategy on other streams can be very limited due to the combination of limited remaining jamming power and weak jamming subchannels. Therefore, the jammer takes advantage of the stronger jamming subchannels by achieving a decent jamming effect with limited jamming power, and achieves the overall optimality by allocating the remaining power on the weaker subchannels.

Second, although in general more jamming power is allocated on weak jamming subchannels, the jammer reduces its power allocation on a subchannel when it becomes too weak, reaching a certain threshold. This is intuitive since if the most jamming power is allocated to the worst subchannel, a subchannel with zero gain would be allocated with the largest portion of the jammer's power. This is clearly not optimal. Therefore, as can be seen from Fig. 1, $f(\omega,\lambda,\sigma^2)$ decreases when ω^2/σ^2 becomes too small for a given power budget, or equivalently, for a given λ.

Third, when the power limit of the jammer increases (or equivalently, λ decreases), the majority of the extra power is allocated on weak subchannels. This can also be seen from Fig. 1 by comparing two curves with different values of λ.

The above insights in the optimal jamming strategy obtained from exploring the general form (19), which leads to the solution (21) and consequently (22) and (23), cannot be seen if the jamming channel is neglected.

C. Characterization of $\mathbf{Z}$ and λ

In order to obtain more insights from the general form of the optimal $\mathbf{Q}'_{\mathrm{z}}$ in (19), the properties of $\mathbf{Z}$ and λ are investigated in this subsection.

Theorem 2: $\mathbf{Q}'_{\mathrm{z}}$ in (19) is PSD when

$$\mathbf{Z} \preceq (\mathbf{D}\mathbf{A}^{-1}\mathbf{D} + \mathbf{D})^{-1} - \lambda\mathbf{I}. \quad (24)$$

Moreover, $\mathbf{Q}'_{\mathrm{z}} = \mathbf{0}$ when the equality holds.

Proof: See Subsection VII-F in the Appendix.

Using Theorem 2 and Remark 1, $\mathbf{Z}$ can be easily found for two special cases. The first special case is when P_{z} is sufficiently large such that $\mathbf{Q}'_{\mathrm{z}}$ is PD, and in such case $\mathbf{Z} = \mathbf{0}$. The second special case is when P_{z} approaches zero and as a result $\mathbf{Q}'_{\mathrm{z}}$ approaches $\mathbf{0}$. In this case, $\mathbf{Z} = (\mathbf{D}\mathbf{A}^{-1}\mathbf{D}+\mathbf{D})^{-1} - \lambda\mathbf{I}$. However, for all other possible P_{z} in between, $\mathbf{Z}$ and λ are yet to be characterized.

Denote the ranks of $\mathbf{Z}$ and $\mathbf{Q}'_{\mathrm{z}}$ by $r_{\mathbf{Z}}$ and $r_{\mathbf{Q}}$, respectively, and the eigenvalue decomposition (EVD) of $\mathbf{Z}$ as

$$\mathbf{Z} = \begin{array}{c} \begin{array}{cc} \scriptstyle r_{\mathbf{Z}} & \scriptstyle r_{\mathrm{z}}-r_{\mathbf{Z}} \end{array} \\ \left[\begin{array}{cc} \mathbf{U}_{\mathbf{Z}1} & \mathbf{U}_{\mathbf{Z}2} \end{array}\right] \end{array} \begin{bmatrix} \bar{\mathbf{\Lambda}}_{\mathbf{Z}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{U}_{\mathbf{Z}1}^{\mathrm{H}} \\ \mathbf{U}_{\mathbf{Z}2}^{\mathrm{H}} \end{bmatrix}, \quad (25)$$

and the EVD of $\mathbf{Q}'_{\mathrm{z}}$ in (19) as

$$\mathbf{Q}'_{\mathrm{z}} = \begin{array}{c} \begin{array}{cc} \scriptstyle r_{\mathbf{Q}} & \scriptstyle r_{\mathrm{z}}-r_{\mathbf{Q}} \end{array} \\ \left[\begin{array}{cc} \mathbf{U}_{\mathbf{Q}1} & \mathbf{U}_{\mathbf{Q}2} \end{array}\right] \end{array} \begin{bmatrix} \bar{\mathbf{\Lambda}}_{\mathbf{Q}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{U}_{\mathbf{Q}1}^{\mathrm{H}} \\ \mathbf{U}_{\mathbf{Q}2}^{\mathrm{H}} \end{bmatrix}. \quad (26)$$

The following theorem is in order.

Theorem 3: For all choices of P_{z}, $r_{\mathbf{Z}} + r_{\mathbf{Q}} \leq n_{\mathrm{z}}$ and the $\mathbf{Z}$ in (19) must satisfy the following condition:

$$\bar{\mathbf{\Lambda}}_{\mathbf{Z}} = (\mathbf{U}_{\mathbf{Z}1}^{\mathrm{H}}(\mathbf{D}\mathbf{A}^{-1}\mathbf{D} + \mathbf{D})\mathbf{U}_{\mathbf{Z}1})^{-1} - \lambda\mathbf{I}. \quad (27)$$

Proof: See Subsection VII-G in the Appendix.

Theorem 3 leads to several results that characterizes $\mathbf{Z}$ and λ. An immediate result based on Theorem 3 is given in the following remark.

Remark 2: $\mathbf{Z}$ in (19) always satisfies the condition that $\mathbf{U}_{\mathbf{Z}1}^{\mathrm{H}}(\mathbf{D}\mathbf{A}^{-1}\mathbf{D} + \mathbf{D})\mathbf{U}_{\mathbf{Z}1}$ is diagonal.

Combining Theorems 2 and 3, a further characterization of λ can also be obtained.

Lemma 5: Define λ^* such that $\mathbf{Q}'_{\mathrm{z}}$ in (21) is PD for all $\lambda < \lambda^*$ and indefinite for all $\lambda > \lambda^*$. Denote the maximum and minimum eigenvalues of $\mathbf{D}\mathbf{A}^{-1}\mathbf{D}+\mathbf{D}$ as $\mu_{\Delta}^{\max}$ and $\mu_{\Delta}^{\min}$, respectively. When $\mathbf{Z} \neq \mathbf{0}$, it holds that $1/\mu_{\Delta}^{\max} \leq \lambda^* \leq 1/\mu_{\Delta}^{\min}$.

Proof: See Subsection VII-H in the Appendix.

Summarizing the above results, the corresponding change in $\mathbf{Z}$ and λ versus the jamming power P_{z} is identified. When P_{z} is sufficiently large such that $\mathbf{Q}_{\mathrm{z}}'$ is PD, then $0 < \lambda < \lambda^*$ and $\mathbf{Z} = \mathbf{0}$. As P_{z} decreases, λ increases. Meanwhile, $\bar{\mathbf{\Lambda}}_{\mathbf{Z}}$ in (27) gradually grows larger in size, and as a result the rank of $\mathbf{Z}$ increases while the rank of $\mathbf{Q}_{\mathrm{z}}'$ decreases. Ultimately, when P_{z} approaches zero, λ approaches $1/\mu_{\Delta}^{\min}$ and $\mathbf{Z}$ approaches $(\mathbf{D}\mathbf{A}^{-1}\mathbf{D}+\mathbf{D})^{-1} - \lambda\mathbf{I}$ while $\mathbf{Q}_{\mathrm{z}}'$ approaches $\mathbf{0}$.

It should be noted that λ is different if the jamming channel $\mathbf{H}_{\mathrm{z}}$ is assumed to be $\mathbf{I}$. In that case, $\mathbf{Z}$ is always $\mathbf{0}$ while λ varies from 0 (when the jamming power P_{z} is infinite) to positive infinity (when the jamming power P_{z} is 0). Without considering the jamming channel, the parameter λ is enough for adjusting the jamming strategy, which reduces to a jamming power allocation strategy given a target signal. However, when $\mathbf{H}_{\mathrm{z}}$ is not equal to $\mathbf{I}$, the jamming strategy has to be adjusted according to not only the power limit but also the jamming channel. The matrix $\mathbf{Z}$ can be seen as the parameter to adjust the jamming strategy according to the jamming channel, which explains why $\mathbf{Z}$ must always satisfy (27) (in which $\mathbf{D}$ is a characterization of the jamming channel).

The results in Theorem 2 and 3 can also be used for obtaining an efficient suboptimal jamming solution which requires a search over λ instead of solving semi-definite programming problems. The main idea lies in the choice of $\mathbf{U}_{\mathbf{Z}1}$ in (27) among those which diagonalize $\mathbf{D}\mathbf{A}^{-1}\mathbf{D}+\mathbf{D}$. The details, however, are beyond the scope of this study.

D. The case when $\mathbf{A} \succeq \mathbf{0}$

In the case in which $\mathbf{A}$ is PSD but not PD, assume that the rank of $\mathbf{A}$ is $r_{\mathbf{A}}$. Denote the EVD of $\mathbf{A}$ as $\mathbf{U}_{\mathbf{A}}\mathbf{\Lambda}_{\mathbf{A}}\mathbf{U}_{\mathbf{A}}^{\mathrm{H}}$ and the block expression of $\mathbf{A}$ as

$$\mathbf{A} = \begin{array}{c} \begin{array}{cc} \scriptstyle r_{\mathbf{A}} & \scriptstyle r_{\mathrm{z}}-r_{\mathbf{A}} \end{array} \\ \left[\begin{array}{cc} \mathbf{U}_{\mathbf{A}1} & \mathbf{U}_{\mathbf{A}2} \end{array}\right] \end{array} \begin{bmatrix} \bar{\mathbf{\Lambda}}_{\mathbf{A}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{U}_{\mathbf{A}1}^{\mathrm{H}} \\ \mathbf{U}_{\mathbf{A}2}^{\mathrm{H}} \end{bmatrix}. \quad (28)$$

When $\mathbf{A}$ is PSD but not PD, the problem of minimizing $\bar{R}^{\mathrm{J}}$ subject to the jammer's power constraint is not a strictly convex problem. Therefore, the optimal solution is not necessarily unique and the result in Theorem 1 cannot be directly extended. The following theorem gives a general form of one optimal $\mathbf{Q}_{\mathrm{z}}'$ that minimizes $\bar{R}^{\mathrm{J}}$ when $\mathbf{A}$ is a PSD matrix but not PD.

Theorem 4: When $\mathbf{A}$ is PSD but not PD, a general form of one optimal solution $\mathbf{Q}_{\mathrm{z}}'$ to the jamming problem (3) is given by

$$\begin{aligned}\mathbf{Q}_{\mathrm{z}}' =& \mathbf{U}_{\mathbf{A}1}\Bigg(\bar{\mathbf{\Lambda}}_{\mathbf{A}}^{\frac{1}{2}}\left(\bar{\mathbf{\Lambda}}_{\mathbf{A}}^{-\frac{1}{2}}(\lambda\mathbf{I}+\mathbf{Z})^{-1}\bar{\mathbf{\Lambda}}_{\mathbf{A}}^{-\frac{1}{2}} + \frac{1}{4}\mathbf{I}\right)^{\frac{1}{2}}\bar{\mathbf{\Lambda}}_{\mathbf{A}}^{\frac{1}{2}} \\ &- \frac{1}{2}\bar{\mathbf{\Lambda}}_{\mathbf{A}}\Bigg)\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}} + \mathbf{U}_{\mathbf{A}2}\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{D}\mathbf{U}_{\mathbf{A}2}\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}} - \mathbf{D}, \quad (29)\end{aligned}$$

where λ is chosen so that $\mathrm{Tr}\{\mathbf{Q}_{\mathrm{z}}'\} = P_{\mathrm{z}}$ and $\mathbf{Z}$ is a PSD matrix that satisfies $\mathbf{Z}\mathbf{Q}_{\mathrm{z}}' = \mathbf{0}$.

Proof: See Subsection VII-I in the Appendix.

Note that although the expressions of the general forms (19) and (29) are similar, the expression (29) cannot be directly obtained from the results when $\mathbf{A}$ is a PD matrix. In fact, finding the general form of the solution is more challenging when $\mathbf{A}$ does not have full rank and, as a result, the solution is not unique. Intuitively, the general form given in (29) suggests that the jammer focuses all its jamming power onto the positive eigenchannels corresponding to the positive eigenvalues of $\mathbf{A}$ while avoiding 'spilling' jamming power into the null space of $\mathbf{A}$. The implementation of the above idea in the solution (29) can be verified by observing the fact that $\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}'\mathbf{U}_{\mathbf{A}2} = \mathbf{0}$ with $\mathbf{Q}_{\mathrm{z}}'$ given by (29), and the fact that the expressions for $\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}'\mathbf{U}_{\mathbf{A}1}$ using $\mathbf{Q}_{\mathrm{z}}'$ given by (29) and (19) are the same. The latter fact implies that the allocation of jamming power on the positive eigenvalues of $\mathbf{A}$ in (29) is optimal. In fact, it can be seen that the jamming strategy (29) approaches that given by (19) as the rank of $\mathbf{A}$ increases. Indeed, substituting $\mathbf{U}_{\mathbf{A}1} = \mathbf{U}_{\mathbf{A}}$ and $\mathbf{U}_{\mathbf{A}2} = \mathbf{0}$ into (29) leads to (19).

Remark 3: In the case in which $\mathbf{A}$ is PSD but not PD, there is a threshold value for P_{z} such that when P_{z} exceeds the threshold the optimal $\mathbf{Q}_{\mathrm{z}}'$ is a PD matrix given by

$$\begin{aligned}\mathbf{Q}_{\mathrm{z}}' =& \mathbf{U}_{\mathbf{A}1}\left(\bar{\mathbf{\Lambda}}_{\mathbf{A}}^{\frac{1}{2}}\left(\frac{1}{\lambda}\bar{\mathbf{\Lambda}}_{\mathbf{A}}^{-1} + \frac{1}{4}\mathbf{I}\right)^{\frac{1}{2}}\bar{\mathbf{\Lambda}}_{\mathbf{A}}^{\frac{1}{2}} - \frac{1}{2}\bar{\mathbf{\Lambda}}_{\mathbf{A}}\right)\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}} \\ &+ \mathbf{U}_{\mathbf{A}2}\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{D}\mathbf{U}_{\mathbf{A}2}\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}} - \mathbf{D}. \quad (30)\end{aligned}$$

The justification is similar to the one used for Remark 1 and the threshold value of the power limit can also be found using similar approach to that used before, and thus we omit the details for the sake of brevity.

IV. Optimal Jamming Strategy in the Jammer-Dominant Regime

The results on the general forms of the optimal jamming strategy obtained in Section III lead to closed-form solutions in some special cases, i.e., when the power limit of the jammer is larger than a threshold. Nevertheless, the optimal jamming strategy in general has not been covered yet. This section aims at finding the optimal jamming strategy in closed-form in the general case. The general optimal jamming strategy will be investigated in the scenario in which the effect of jamming on the legitimate communication dominates that of the noise. In order to characterize this scenario and find the optimal jamming strategy, we define a jammer-dominant regime as follows.

Definition 1: The jammer-dominant regime is the collection of all possible combinations ($\mathbf{Q}_{\mathrm{z}}'$, $\mathbf{D}$) such that

$$\mathbf{Q}_{\mathrm{z}}' \succeq 0,\ \mathbf{D} = \sigma^2\bar{\mathbf{\Omega}}_{\mathrm{z}}^{-2} \succ 0 \quad (31)$$

$$\mathbf{U}_{\mathbf{Q}1}^{\mathrm{H}}(\mathbf{Q}_{\mathrm{z}}' - \mathbf{D})\mathbf{U}_{\mathbf{Q}1} \succ 0 \quad (32)$$

$$\mathrm{Tr}\{\mathbf{D}\} \leq \eta\mathrm{Tr}\{\mathbf{Q}_{\mathrm{z}}'\}, \quad (33)$$

where η is a small positive number which characterizes the threshold beyond which jamming dominates noise.

With the above defined jammer-dominant regime, when a given combination of $\mathbf{Q}_{\mathrm{z}}'$ and $\mathbf{D}$ lies in the regime, the effect of noise is considered to be negligible as compared to that of jamming. Accordingly, we consider $\mathbf{U}_{\mathbf{Q}1}^{\mathrm{H}}\mathbf{D}\mathbf{U}_{\mathbf{Q}1}$ negligible as compared to $\mathbf{U}_{\mathbf{Q}1}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}'\mathbf{U}_{\mathbf{Q}1}$ in the jammer-dominant regime. Note that the above assumption is not equivalent to neglecting $\mathbf{D}$ as compared to $\mathbf{Q}_{\mathrm{z}}'$ since the rank of $\mathbf{Q}_{\mathrm{z}}'$ matters in the definition (31) - (33).

Remark 4: The following properties hold for any $\mathbf{Q}_{\mathrm{z}}'$ in the jammer-dominant regime defined by (31) - (33)[4]:

- When $\mathbf{Q}_{\mathrm{z}} \succ 0$, $\mathbf{U}_a^{\mathrm{H}}\mathbf{D}\mathbf{U}_a$ is negligible as compared to $\mathbf{U}_a^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}'\mathbf{U}_a$ for an arbitrary unitary matrix $\mathbf{U}_a$ (with an appropriate size). The reason is that $\mathbf{U}_{\mathbf{Q}1}$ becomes $\mathbf{U}_{\mathbf{Q}}$, when $\mathbf{Q}_{\mathrm{z}} \succ 0$. As $\mathbf{U}_{\mathbf{Q}}^{\mathrm{H}}\mathbf{D}\mathbf{U}_{\mathbf{Q}}$ is negligible as compared to $\mathbf{U}_{\mathbf{Q}}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}'\mathbf{U}_{\mathbf{Q}}$, so is $\mathbf{U}_a^{\mathrm{H}}\mathbf{U}_{\mathbf{Q}}(\mathbf{U}_{\mathbf{Q}}^{\mathrm{H}}\mathbf{D}\mathbf{U}_{\mathbf{Q}})\mathbf{U}_{\mathbf{Q}}^{\mathrm{H}}\mathbf{U}_a$ as compared to $\mathbf{U}_a^{\mathrm{H}}\mathbf{U}_{\mathbf{Q}}(\mathbf{U}_{\mathbf{Q}}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}'\mathbf{U}_{\mathbf{Q}})\mathbf{U}_{\mathbf{Q}}^{\mathrm{H}}\mathbf{U}_a$.
- When $\mathbf{Q}_{\mathrm{z}} \succeq 0$, $\mathbf{U}_b^{\mathrm{H}}\mathbf{U}_{\mathbf{Q}1}^{\mathrm{H}}\mathbf{D}\mathbf{U}_{\mathbf{Q}1}^{\mathrm{H}}\mathbf{U}_b$ is negligible as compared to $\mathbf{U}_b^{\mathrm{H}}\mathbf{U}_{\mathbf{Q}1}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}'\mathbf{U}_{\mathbf{Q}1}^{\mathrm{H}}\mathbf{U}_b$ for an arbitrary unitary matrix $\mathbf{U}_b$ (with an appropriate size).
- For any PSD matrix $\mathbf{C}$ such that $\mathbf{C} \prec \mathbf{D}$ given that ($\mathbf{Q}_{\mathrm{z}}'$, $\mathbf{D}$) lies in the jammer-dominant regime, $\mathbf{U}_{\mathbf{Q}1}^{\mathrm{H}}\mathbf{C}\mathbf{U}_{\mathbf{Q}1}$ is also negligible as compared to $\mathbf{U}_{\mathbf{Q}1}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}'\mathbf{U}_{\mathbf{Q}1}$, i.e., ($\mathbf{Q}_{\mathrm{z}}'$, $\mathbf{C}$) also lies in the jammer-dominant regime.
- Given $\mathbf{Q}_{\mathrm{z}}'$, if $(\mathbf{Q}_{\mathrm{z}}', \mathbf{D}_1)$ and $(\mathbf{Q}_{\mathrm{z}}', \mathbf{D}_2)$ are both in the jammer-dominant regime, then $(\mathbf{Q}_{\mathrm{z}}', \alpha\mathbf{D}_1 + (1-\alpha)\mathbf{D}_2)$ also lies in the jammer-dominant regime for any $\alpha \in [0,1]$. Therefore, the jammer-dominant regime is convex for any given $\mathbf{Q}_{\mathrm{z}}'$.

The conditions (31)-(33) in Definition 1 specify a region that does not depend on a specific problem or system configuration. However, our focus is to investigate the optimal jamming strategy for any given specific problem in which the effect of jamming on the legitimate communication dominates the effect of noise. The following characterizes the problems in which we are interested.

Definition 2: A jammer-dominant problem is a problem in the form of (3) that has an optimal solution in the jammer-dominant regime.[5]

Now the focus is clear, the optimal jamming strategy can be investigated. For a jammer-dominant problem, which has an optimal solution in the jammer-dominant regime, the following theorem gives the optimal solution of the jamming strategy represented by $\mathbf{Q}_{\mathrm{z}}'$.

Theorem 5: In the jammer-dominant regime, an optimal $\mathbf{Q}_{\mathrm{z}}'$ to the jamming problem (3) is given by

$$\mathbf{Q}_{\mathrm{z}}' = \mathbf{U}_{\mathbf{A}1}\left(\left(\frac{1}{\lambda}\bar{\mathbf{\Lambda}}_{\mathbf{A}} + \frac{1}{4}\bar{\mathbf{\Lambda}}_{\mathbf{A}}^2\right)^{\frac{1}{2}} - \frac{1}{2}\bar{\mathbf{\Lambda}}_{\mathbf{A}}\right)\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}, \quad (34)$$

where λ is chosen so that $\mathrm{Tr}\{\mathbf{Q}_{\mathrm{z}}'\} = P_{\mathrm{z}}$.

Proof: See Subsection VII-J in the Appendix.

Unlike the general forms for the optimal jamming strategy in (19) and (29), the $\mathbf{Q}_{\mathrm{z}}'$ given in (34) gives the specific optimal solution in the jammer-dominant regime. Moreover, unlike the optimal solutions (21) and (30) for the special cases when the jammer has a sufficiently large power budget, the optimal solution $\mathbf{Q}_{\mathrm{z}}'$ in (34) applies without extra conditions in the jammer-dominant regime. It also unifies the cases $\mathbf{A} \succ \mathbf{0}$ and $\mathbf{A} \succeq \mathbf{0}$.

Given the solution (34), the condition in (32) becomes

$$\left(\frac{1}{\lambda}\bar{\mathbf{\Lambda}}_{\mathbf{A}} + \frac{1}{4}\bar{\mathbf{\Lambda}}_{\mathbf{A}}^2\right)^{\frac{1}{2}} \succeq \mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{D}\mathbf{U}_{\mathbf{A}1} + \frac{1}{2}\bar{\mathbf{\Lambda}}_{\mathbf{A}}. \quad (35)$$

Note that while a sufficient condition on λ to satisfy (35) can be obtained, an explicit condition in terms of a closed-form threshold of λ cannot be given.

Now the constraints (32)-(33) in Definition 1 can be rewritten as

$$\left(\frac{1}{\lambda}\bar{\mathbf{\Lambda}}_{\mathbf{A}} + \frac{1}{4}\bar{\mathbf{\Lambda}}_{\mathbf{A}}^2\right)^{\frac{1}{2}} \succeq \mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{D}\mathbf{U}_{\mathbf{A}1} + \frac{1}{2}\bar{\mathbf{\Lambda}}_{\mathbf{A}} \quad (36)$$

$$\mathrm{Tr}\{\mathbf{D}\} \leq \eta P_{\mathrm{z}}. \quad (37)$$

Given a specific problem, (37) should be checked first. If (37) is satisfied, the parameter λ in (34) can be found by a bisection search so that the trace of $\mathbf{Q}_{\mathrm{z}}'$ in (34) is equal to P_{z}. Then, the resulting λ can be used to determine whether (36) is satisfied.

It is worth noting that it is possible to extend the results on optimal jamming strategies including (34) to multi-user MIMO jamming. Examples include jamming MIMO multiple access channels and jamming time division multiplexing (TDM) or frequency division multiplexing (FDM) based multi-user MIMO systems. While the extension is straightforward mathematically, it should be noted that the jammer needs the related CSI and the transmit strategy of all target communications for optimizing its strategy for optimal jamming.

It is also possible to extend the results considering multiple jammers. If the jammers are not coordinated with information of their channels and jamming power limits shared, then the best strategy for each of them is to determine its jamming strategy as if it is the only jammer. In the case in which the jammers are coordinated with each other and share their channel and power limit information, they may achieve a jointly optimal jamming strategy. Specifically, the jammers can form a virtual jammer which has a number of antennas equal to the sum of the antennas of all jammers. Then, the jammers can jointly optimize a single jamming covariance matrix for the virtual jammer with power constraints for the diagonal blocks of the jamming matrix. Further details, however, are beyond the scope of this study.

V. Simulations

Example 1: The jammer-dominant regime. In this simulation example, the number of antennas at the legitimate transmitter, the receiver, and the jammer are set as $n_{\mathrm{t}} = 4$, $n_{\mathrm{r}} = 3$, and $n_{\mathrm{z}} = 3$, respectively. The transmission power of the legitimate transmitter, denoted as P_{t}, and the noise power are set such that $P_{\mathrm{t}}/\mathrm{Tr}\{\sigma^2\mathbf{I}\} = 10$ dB. The legitimate transmitter allocates its power based on the waterfilling process. As the nature of jamming is to destructively focus energy towards the target, the jamming power is assumed to be at least of the same level as the legitimate transmission power [9]. Therefore, we vary the power limit of the jammer P_{z} such that $P_{\mathrm{z}}/P_{\mathrm{t}}$ ranges from

[4] Hereafter we use for brevity the expression '$\mathbf{Q}_{\mathrm{z}}'$ is in the jammer-dominant regime' to express that 'the pair ($\mathbf{Q}_{\mathrm{z}}'$, $\mathbf{D}$) is in the jammer-dominant regime'.

[5] The term 'an optimal solution' is used here because the optimal solution can be non-unique.

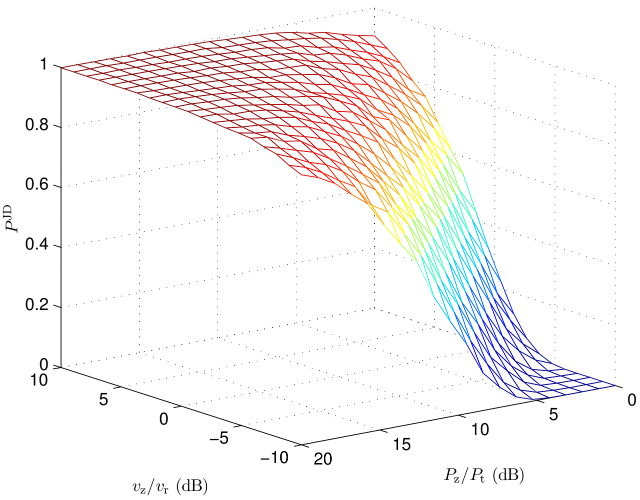

Fig. 2: Probability that the jamming solution given by (34) lies in the jammer-dominant regime versus the jammer power limit and jamming channel, $\eta = 0.1$.

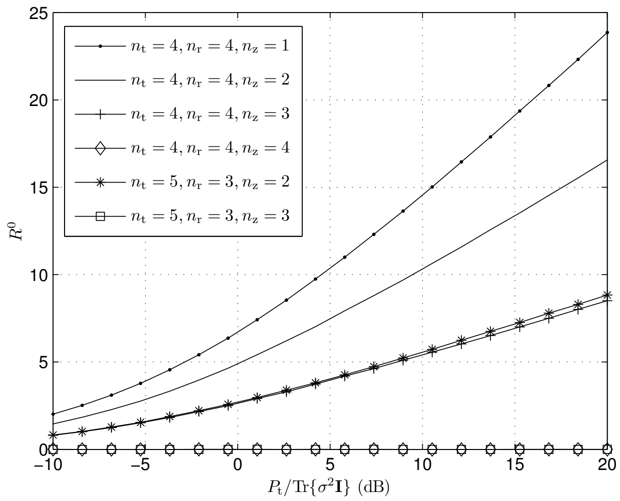

Fig. 3: Lower-bound rate versus $P_{\rm t}/{\rm Tr}\{\sigma^2\mathbf{I}\}$ for different values of $n_{\rm t}$, $n_{\rm r}$, and $n_{\rm z}$.

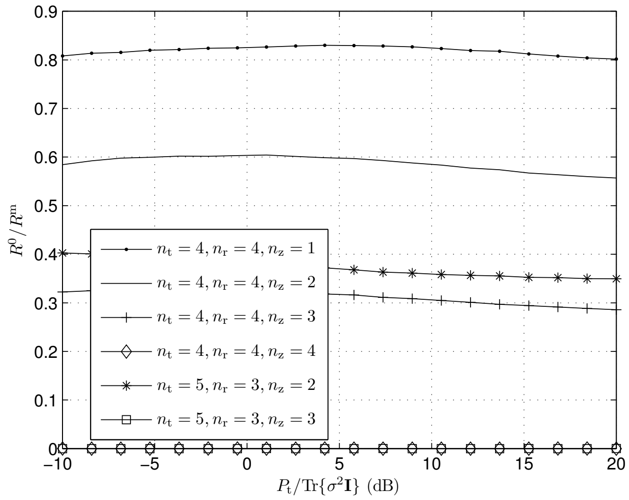

Fig. 4: Ratio of the lower-bound rate to the maximum rate without jamming versus $P_{\rm t}/{\rm Tr}\{\sigma^2\mathbf{I}\}$ for different values of $n_{\rm t}$, $n_{\rm r}$, and $n_{\rm z}$.

0 dB to 20 dB. The elements of the legitimate channel $\mathbf{H}_{\rm r}$ are generated from the complex Gaussian distribution with zero mean and variance $v_{\rm r} = 1$. Note that we consider the case of block fading channels. Different channel realizations correspond to different fading blocks. The elements of the jamming channel $\mathbf{H}_{\rm z}$ are generated from the complex Gaussian distribution with zero mean and variance $v_{\rm z}$. The value of $v_{\rm z}$ varies such that $v_{\rm z}/v_{\rm r}$ varies from -10 dB to 10 dB. The scaling factor η in (33) is set to be 0.1.

Given each combination of $P_{\rm z}$ and $v_{\rm z}$, we use 3000 channel realizations and calculate the probability, denoted by $P^{\rm JD}$, that the solution given by (34) lies in the jammer-dominant regime as defined by (31)-(33). The result is plotted in Fig. 2. It can be seen from the figure that for the case in which the jammer does not have any advantage in either transmission power or channel strength, e.g., $P_{\rm z}/P_{\rm t} = 0$ dB and $v_{\rm z}/v_{\rm r} = -0.5$ dB, there is only a 21% chance that the solution (34) falls into the jammer-dominant regime and thus is optimal. However, when $P_{\rm z}/P_{\rm t}$ increases by 5 dB (or by 9 dB) while $v_{\rm z}/v_{\rm r}$ remains at -0.5 dB, the solution (34) is in the jammer-dominant regime and thus is optimal for 70% (or 87%) of channel realizations. In fact, it can be seen from the figure that for most of the combinations of $P_{\rm z}/P_{\rm t} > 0$ dB and $v_{\rm z}/v_{\rm r} > 0$ dB, the chance for solution (34) to be optimal (i.e., in the jammer-dominant regime) is very high.

Example 2: The lower-bound rate. In this example, we demonstrate the lower-bound rate that can be achieved under jamming versus the power limit of the legitimate transmitter. The transmission power of the legitimate transmitter varies such that $P_{\rm t}/{\rm Tr}\{\sigma^2\mathbf{I}\}$ ranges from -10 dB to 20 dB. The elements of the legitimate channel $\mathbf{H}_{\rm r}$ are generated from the complex Gaussian distribution with zero mean and unit variance. For each combination of $n_{\rm t}$, $n_{\rm r}$, and $n_{\rm z}$ tested, we calculate the lower-bound rate R^0 for 10000 channel realizations and the results are shown in Fig. 3. Moreover, Fig. 4 demonstrates the ratio of R^0 to the maximum rate achievable without jamming (denoted as $R^{\rm m}$) versus $P_{\rm t}/{\rm Tr}\{\sigma^2\mathbf{I}\}$.

It can be seen from Fig. 3 that, when $n_{\rm t} = n_{\rm r} = 4$, increasing $n_{\rm z}$ from 1 to 2 decreases the lower-bound rate by approximately 1/3. The same scope of decrease can be observed when $n_{\rm z}$ is increased from 2 to 3, or from 3 to 4. The reason is that the legitimate communication has $4-1=3$ data streams unaffected by the jammer when $n_{\rm z} = 1$. When $n_{\rm z}$ is increased from 1 to 2, the unaffected data streams reduce by 1/3. Fig. 4 shows that while the lower-bound rate increases with $P_{\rm t}$, the ratio of $R^0/R^{\rm m}$ is almost constant. Therefore, statistically, it is possible to obtain an approximate expected value of the maximum achievable rate that still can be achieved under the worst-case jamming using only the knowledge of $n_{\rm t}$, $n_{\rm r}$, and $n_{\rm z}$.

Example 3: Comparison of jamming strategies. In this example, we demonstrate the information rate of the legitimate

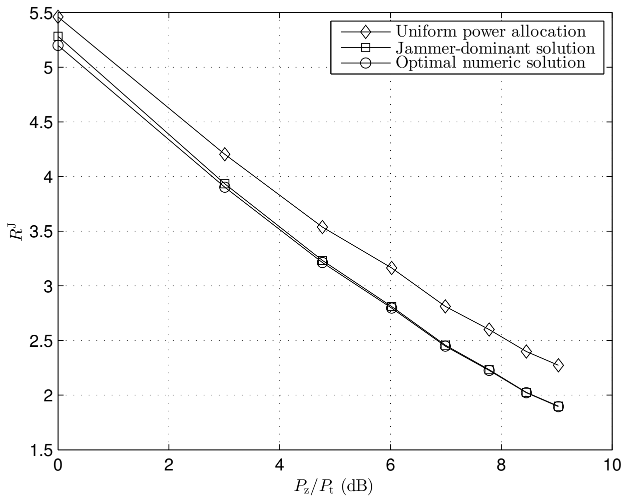

Fig. 5: The information rate of legitimate communication under different jamming strategies.

communication under different jamming strategies. The number of antennas at the legitimate transmitter, the receiver, and the jammer are set as $n_{\rm t} = 4$, $n_{\rm r} = 3$, and $n_{\rm z} = 3$, respectively. The power limits are set such that $P_{\rm t}/{\rm Tr}\{\sigma^2\mathbf{I}\} = 10$ dB while $P_{\rm z}/P_{\rm t}$ ranges from 0 dB to 9 dB. The elements of $\mathbf{H}_{\rm r}$ and $\mathbf{H}_{\rm z}$ are generated from the complex Gaussian distribution with zero mean and unit variance. Given the above setup, we compare the information rate of the legitimate communication when the jammer uses uniform power allocation jamming strategy with that of the proposed jammer-dominant solution (34). In addition, we also get the numerically obtained optimal solution of problem (3) by using the general method based on difference-of-convex functions (DC) programming as proposed in [26]- [28] (note that these papers, which give numerical solutions for optimization problems, are not targeted at jamming problems). Because of the high complexity of obtaining the optimal jamming solution based on DC programming, 600 channel realizations are used for each value of $P_{\rm z}/P_{\rm t}$. The results are plotted in Fig. 5.

In Fig. 5, the proposed solution (34) is in general asymptotically optimal as $P_{\rm z}$ increases. This is because the probability that jamming dominates noise increases with $P_{\rm z}$ and our solution becomes globally optimal in the jammer-dominant regime. When $P_{\rm z}$ is small (e.g., when $P_{\rm z}/P_{\rm t} = 0$ dB), the proposed solution (34) has a performance degradation as compared to the numerically obtained optimal solution. Indeed, recall from Simulation Example 1 that there is only a 21% chance that the regime is the jammer-dominant one when $P_{\rm z}/P_{\rm t} = 0$ dB. However, even with such low $P_{\rm z}$, the performance degradation of the solution (34) as compared to the numerically obtained optimal solution is still not significant, which means the closed-form solution (34) can still achieve a satisfactory performance even outside the jammer-dominant regime. The uniform power allocation is worse than both our proposed solution and the numerically obtained optimal solution.

Example 4: Errors in channel knowledge. In this example, we demonstrate the performance of the proposed jamming strategy when the information about $\mathbf{H}_{\rm r}$ and $\mathbf{H}_{\rm z}$ is inaccurate. Specifically, the jammer's estimate of $\mathbf{H}_{\rm r}$ is modeled as $\sqrt{1-\delta_{\rm r}^2}\mathbf{H}_{\rm r} + \delta_{\rm r}\mathbf{E}_{\rm r}$, where $\delta_{\rm r}$ is the weight of channel estimation error and $\mathbf{E}_{\rm r}$ is a matrix representing the error in the channel estimation. When $\delta_{\rm r} = 0$, the jammer has perfect channel knowledge of $\mathbf{H}_{\rm r}$. In contrast, the jammer's knowledge of $\mathbf{H}_{\rm r}$ is completely erroneous when $\delta_{\rm r} = 1$. Similarly, the jammer's estimate of $\mathbf{H}_{\rm z}$ is modeled as $\sqrt{1-\delta_{\rm z}^2}\mathbf{H}_{\rm z}+\delta_{\rm z}\mathbf{E}_{\rm z}$.

TABLE II: Summary of Results

Lower-bound rate under jamming	(16)
General form of optimal solutions for $\mathbf{A} \succ \mathbf{0}$	(19)
General form of optimal solutions for $\mathbf{A} \succeq \mathbf{0}$	(29)
Characterization of λ and $\mathbf{Z}$ in the general form (19)	(24) and (27)
Closed-form solutions when P_z is sufficiently large	(21) and (30)
Closed-form solution in the jammer-dominant regime	(34)

The case in which the transmitter, receiver, and jammer all have 4 antennas is considered. The power limits are set as $P_{\rm t}/{\rm Tr}\{\sigma^2\mathbf{I}\} = P_{\rm z}/{\rm Tr}\{\sigma^2\mathbf{I}\} = 10$ dB. For each combination of $\delta_{\rm r}$ and $\delta_{\rm z}$, 400 channel realizations of $\mathbf{H}_{\rm r}$ and $\mathbf{H}_{\rm z}$ are used, in which $\mathbf{H}_{\rm r}$ and $\mathbf{H}_{\rm z}$ are generated from the complex Gaussian distribution with zero mean and unit variance. For each channel realization, $\mathbf{E}_{\rm r}$ and $\mathbf{E}_{\rm z}$ are also independently and randomly generated from the complex Gaussian distribution with zero mean and unit variance. For each combination of $\delta_{\rm r}$ and $\delta_{\rm z}$, three rates with three different jamming strategies are calculated, i.e., the rate with the jammer-dominant strategy (34), the rate with the uniform power allocation jamming strategy, and the worst-case rate (assuming perfect channel knowledge) obtained numerically. Note that with channel estimation error, the jammer-dominant strategy is compared with the uniform power allocation jamming strategy as there is no other strategy to be compared with in the scenario of inaccurate channel information. Define $R^{\rm W}$ as the worst-case rate and ΔR as the rate obtained based on the jammer-dominant strategy minus the rate obtained based on the uniform power allocation jamming strategy. Then, the normalized term $\Delta R/R^{\rm W}$ can be used as a measurement of the comparative advantage of the jammer-dominant strategy over the uniform power allocation jamming strategy. The jammer-dominant strategy performs better when $\Delta R/R^{\rm W} < 0$ while the uniform power allocation jamming strategy performs better when $\Delta R/R^{\rm W} > 0$.

It can be seen from Fig. 6 that the jammer-dominant strategy performs better when $\delta_{\rm r}$ and $\delta_{\rm z}$ are less than, approximately, 0.3. Therefore, Fig. 6 demonstrates that the jammer-dominant strategy can be a better choice even if minor channel estimation errors are present. The jammer may adapt its jamming strategy according to its confidence in the channel information accuracy if the confidence can be measured or estimated. In such a case, the jammer-dominant strategy can be used when the confidence of the channel information accuracy is high.

VI. Conclusion

In this paper, worst-case jamming on MIMO Gaussian channels has been studied and conditions for a positive lower-

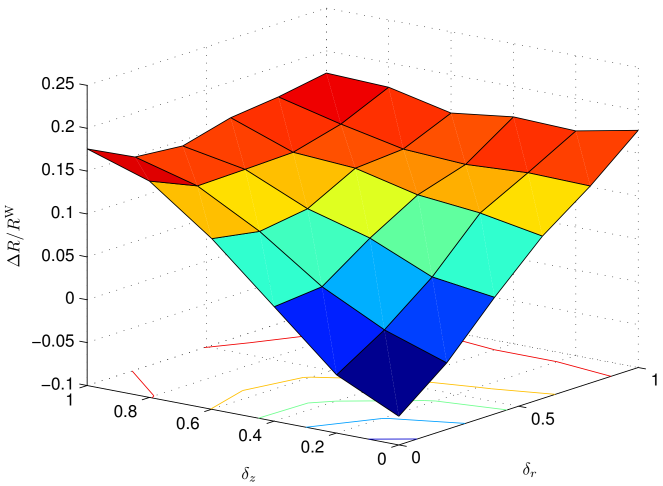

Fig. 6: Performance of the jammer-dominant solution when the channel knowledge has errors.

bound rate for the legitimate communication have been obtained. The main results are summarized in Table II. From the jammer's perspective, it has been shown that the jammer generally allocates more power on weak jamming subchannels according to its optimal jamming strategy. Regarding the jammer-dominant regime, it has been shown that there is a large chance that the optimal jamming strategy falls in this regime when the jammer has at least 5 dB advantage in its transmission power compared to the transmission power of the legitimate transmitter. It has also been demonstrated that the proposed closed-form solution (34) is very close to the general optimal solution (which can be found numerically by solving a corresponding DC programming problem) even when the probability of being in the jammer-dominant regime is very small. Therefore, the proposed closed-form solution, which is optimal for the jammer-dominant regime, can also be used as a general close-to-optimal solution for the jamming strategy on MIMO Gaussian channels, especially considering that the complexity of finding the optimal solution numerically is significantly higher.

VII. Appendix

A. Proof of Lemma 1

If $\mathbf{B}$ is PD, the following matrix:

$$\hat{\mathbf{B}} = \mathbf{B} + \begin{array}{c} \\ r_z \\ n_r - r_z \end{array}\!\!\begin{array}{c} \begin{array}{cc} r_z & n_r - r_z \end{array} \\ \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \sigma^2\mathbf{I} \end{bmatrix} \end{array} \tag{38}$$

and its inverse $\hat{\mathbf{B}}^{-1}$ are also PD, where $\mathbf{0}$ denotes an all-zero matrix of an appropriate size. Given that $\hat{\mathbf{B}}$ is PD, it can be seen that the two blocks on the diagonal of $\hat{\mathbf{B}}$ are both PD. Then, using block matrix inversion [29], it follows that the upper-left block of $\hat{\mathbf{B}}^{-1}$ is $(\mathbf{B}_{11} - \mathbf{B}_{12}(\sigma^2\mathbf{I} + \mathbf{B}_{22})^{-1}\mathbf{B}_{21})^{-1}$, which is the inverse of $\tilde{\mathbf{B}}$. Given that $\hat{\mathbf{B}}^{-1}$ is PD, the upper-left block of $\hat{\mathbf{B}}^{-1}$, i.e., the inverse of $\tilde{\mathbf{B}}$, must also be PD. Therefore, $\tilde{\mathbf{B}}$ is also PD. This proves Lemma 1. ∎

B. Proof of Lemma 2

Using the definitions (7), (8), (11), and (12), the objective function in (5) can be rewritten as

$$\begin{aligned}
R^{\mathrm{J}} &= \log\left|\mathbf{I} + \mathbf{B}(\tilde{\boldsymbol{\Omega}}_{\mathrm{z}}\tilde{\mathbf{Q}}_{\mathrm{z}}\tilde{\boldsymbol{\Omega}}_{\mathrm{z}} + \sigma^2\mathbf{I})^{-1}\right| \\
&= \log\left|\mathbf{I} + \tilde{\boldsymbol{\Omega}}_{\mathrm{z}}^{-1}\mathbf{B}\tilde{\boldsymbol{\Omega}}_{\mathrm{z}}^{-1}(\tilde{\mathbf{Q}}_{\mathrm{z}} + \sigma^2\tilde{\boldsymbol{\Omega}}_{\mathrm{z}}^{-2})^{-1}\right| \\
&= \log\Bigg|\mathbf{I} + \begin{bmatrix} \bar{\boldsymbol{\Omega}}_{\mathrm{z}} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{B}_{11} & \mathbf{B}_{12} \\ \mathbf{B}_{21} & \mathbf{B}_{22} \end{bmatrix} \begin{bmatrix} \bar{\boldsymbol{\Omega}}_{\mathrm{z}} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}^{-1} \\
&\qquad \cdot \left(\begin{bmatrix} \mathbf{Q}'_{\mathrm{z}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} + \sigma^2 \begin{bmatrix} \bar{\boldsymbol{\Omega}}_{\mathrm{z}}^{-2} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}\right)^{-1}\Bigg| \\
&= \log\Bigg|\mathbf{I} + \begin{bmatrix} \bar{\boldsymbol{\Omega}}_{\mathrm{z}}^{-1}\mathbf{B}_{11}\bar{\boldsymbol{\Omega}}_{\mathrm{z}}^{-1} & \bar{\boldsymbol{\Omega}}_{\mathrm{z}}^{-1}\mathbf{B}_{12} \\ \mathbf{B}_{21}\bar{\boldsymbol{\Omega}}_{\mathrm{z}}^{-1} & \mathbf{B}_{22} \end{bmatrix} \\
&\qquad \cdot \begin{bmatrix} (\mathbf{Q}'_{\mathrm{z}} + \sigma^2\bar{\boldsymbol{\Omega}}_{\mathrm{z}}^{-2})^{-1} & \mathbf{0} \\ \mathbf{0} & \frac{1}{\sigma^2}\mathbf{I} \end{bmatrix}\Bigg| \\
&= \log\left|\begin{bmatrix} \mathbf{I} + \bar{\boldsymbol{\Omega}}_{\mathrm{z}}^{-1}\mathbf{B}_{11}\bar{\boldsymbol{\Omega}}_{\mathrm{z}}^{-1}\mathbf{J}^{-1} & \frac{1}{\sigma^2}\bar{\boldsymbol{\Omega}}_{\mathrm{z}}^{-1}\mathbf{B}_{12} \\ \mathbf{B}_{21}\bar{\boldsymbol{\Omega}}_{\mathrm{z}}^{-1}\mathbf{J}^{-1} & \mathbf{I} + \frac{1}{\sigma^2}\mathbf{B}_{22} \end{bmatrix}\right|,
\end{aligned} \tag{39}$$

where in the last step $\mathbf{J} \triangleq \mathbf{Q}'_{\mathrm{z}} + \sigma^2\bar{\boldsymbol{\Omega}}_{\mathrm{z}}^{-2}$.

The rate R^{J} in (39) can be simplified as $R^{\mathrm{J}} = R^0 + \bar{R}^{\mathrm{J}}$ where R^0 given in (16) represents the part of the rate that is not affected by jamming and

$$\begin{aligned}
\bar{R}^{\mathrm{J}} = \log\Big|&\mathbf{I} + \bar{\boldsymbol{\Omega}}_{\mathrm{z}}^{-1}\mathbf{B}_{11}\bar{\boldsymbol{\Omega}}_{\mathrm{z}}^{-1}\mathbf{J}^{-1} \\
&- \frac{1}{\sigma^2}\bar{\boldsymbol{\Omega}}_{\mathrm{z}}^{-1}\mathbf{B}_{12}(\mathbf{I} + \frac{1}{\sigma^2}\mathbf{B}_{22})^{-1}\mathbf{B}_{21}\bar{\boldsymbol{\Omega}}_{\mathrm{z}}^{-1}\mathbf{J}^{-1}\Big|
\end{aligned} \tag{40}$$

is the part of the rate that is affected by jamming.

Therefore, the minimization of R^{J} in (3a) is equivalent to minimizing $\bar{R}^{\mathrm{J}}$. Using the definition of $\tilde{\mathbf{B}}$ in (8), $\bar{R}^{\mathrm{J}}$ can be rewritten as (17). This completes the proof of Lemma 2. ∎

C. Proof of Lemma 3

Proof of statement 1): The dimensions of the null spaces of the matrix $\mathbf{B}$ and the matrix $\mathbf{H}_{\mathrm{r}}\mathbf{Q}_{\mathrm{s}}\mathbf{H}_{\mathrm{r}}^{\mathrm{H}}$, given the definition of $\mathbf{B}$, are both $n_{\mathrm{r}} - r_{\mathbf{B}}$. Note that $\mathbf{B}_{22} = \mathbf{U}_{\mathrm{zm}}^{\mathrm{H}}\mathbf{H}_{\mathrm{r}}\mathbf{Q}_{\mathrm{s}}\mathbf{H}_{\mathrm{r}}^{\mathrm{H}}\mathbf{U}_{\mathrm{zm}}$ where $\mathbf{U}_{\mathrm{zm}}$ consists of the last $n_{\mathrm{r}} - r_{\mathrm{z}}$ columns of $\mathbf{U}_{\mathrm{z}}$. Thus, the rank of $\mathbf{B}_{22}$ is no less than the larger of 0 and $n_{\mathrm{r}} - r_{\mathrm{z}} - (n_{\mathrm{r}} - r_{\mathbf{B}})$. Therefore, the rank of $\mathbf{B}_{22}$ is larger than 0 if $r_{\mathbf{B}} > r_{\mathrm{z}}$. As a result, $R^0 > 0$ if $r_{\mathbf{B}} > r_{\mathrm{z}}$.

Proof of statement 2): As $r_{\mathbf{B}} > 0$, $\mathbf{B}$ has at least one positive eigenvalue. Denote the matrix consisting of the eigenvectors that correspond to the positive eigenvalues of $\mathbf{B}$ as $\mathbf{U}_{\mathbf{B}1}$. Then $\mathbf{B}_{22} = \mathbf{U}_{\mathrm{zm}}^{\mathrm{H}}\mathbf{H}_{\mathrm{r}}\mathbf{Q}_{\mathrm{s}}\mathbf{H}_{\mathrm{r}}^{\mathrm{H}}\mathbf{U}_{\mathrm{zm}}$ has positive eigenvalues as long as $\mathbf{U}_{\mathrm{zm}}^{\mathrm{H}}\mathbf{U}_{\mathbf{B}1} \neq \mathbf{0}$. This inequality holds if any column of $\mathbf{U}_{\mathrm{zm}}$ lies in the space spanned by the columns of $\mathbf{U}_{\mathbf{B}1}$. As long as $\mathbf{B}_{22}$ has at least one positive eigenvalue, we have $R^0 > 0$. Therefore, it is possible that $R^0 > 0$ as long as $r_{\mathbf{B}} > 0$. ∎

D. Proof of Lemma 4

Denote the eigenvalues of $\mathbf{B}_{22}$ as $\mu_1(\mathbf{B}_{22})$, …, $\mu_r(\mathbf{B}_{22})$ (all eigenvalues are sorted in ascending order in this proof only) where r represents the rank of $\mathbf{B}_{22}$. It can be seen from (16) that R^0 is equal to $\sum_{i=1}^{r}(1 + \mu_i(\mathbf{B}_{22})/\sigma^2)$. Note that $\mathbf{B}_{22}$ is a principal submatrix of $\mathbf{U}_{\rm z}^{\rm H}\mathbf{H}_{\rm r}\mathbf{Q}_{\rm s}\mathbf{H}_{\rm r}^{\rm H}\mathbf{U}_{\rm z}$ while the latter has the same eigenvalues as $\mathbf{H}_{\rm r}\mathbf{Q}_{\rm s}\mathbf{H}_{\rm r}^{\rm H}$. Using the result on eigenvalue interlacing for principal submatrices (i.e., the Cauchy interlacing theorem [30]), it can be seen that $\mu_i(\mathbf{B}_{22}) \geq \mu_i(\mathbf{U}_{\rm z}^{\rm H}\mathbf{B}\mathbf{U}_{\rm z}) = \mu_i(\mathbf{H}_{\rm r}\mathbf{Q}_{\rm s}\mathbf{H}_{\rm r}^{\rm H}), \forall i \in \{1, 2, \ldots, r\}$. Therefore, the minimum value that R^0 can achieve is $\sum_{i=1}^{n_{\rm r}-r_{\rm z}}(1+\mu_i(\mathbf{H}_{\rm r}\mathbf{Q}_{\rm s}\mathbf{H}_{\rm r}^{\rm H})/\sigma^2)$. It is not difficult to see that $\mu_i(\mathbf{B}_{22}) = \mu_i(\mathbf{H}_{\rm r}\mathbf{Q}_{\rm s}\mathbf{H}_{\rm r}^{\rm H}), \forall i \in \{1, 2, \ldots, r\}$ and the minimum R^0 is achieved when the last $n_{\rm r} - r_{\rm z}$ columns of $\mathbf{U}_{\rm z}$ consist of the eigenvectors corresponding to the smallest $n_{\rm r} - r_{\rm z}$ eigenvalues of $\mathbf{H}_{\rm r}\mathbf{Q}_{\rm s}\mathbf{H}_{\rm r}^{\rm H}$ given that $\mathbf{B} = \mathbf{U}_{\rm z}^{\rm H}\mathbf{H}_{\rm r}\mathbf{Q}_{\rm s}\mathbf{H}_{\rm r}^{\rm H}\mathbf{U}_{\rm z}$. In the case in which the legitimate transmitter uses waterfilling, the eigenvector matrix of $\mathbf{H}_{\rm r}\mathbf{Q}_{\rm s}\mathbf{H}_{\rm r}^{\rm H}$ is $\mathbf{U}_{\rm r}$ and therefore the eigenvectors corresponding to the smallest $n_{\rm r} - r_{\rm z}$ eigenvalues of $\mathbf{H}_{\rm r}\mathbf{Q}_{\rm s}\mathbf{H}_{\rm r}^{\rm H}$ are given by the last $n_{\rm r} - r_{\rm z}$ columns of $\mathbf{U}_{\rm r}$. ■

E. Proof of Theorem 1

The Lagrangian for the problem of minimizing (17) subject to the power constraint of $\mathbf{Q}'_{\rm z}$ can be written as

$$\begin{aligned}\mathcal{L}(\mathbf{Q}'_{\rm z}, \lambda, \mathbf{Z}) = &\log|\mathbf{A} + \mathbf{Q}'_{\rm z} + \mathbf{D}| - \log|\mathbf{Q}'_{\rm z} + \mathbf{D}| \\ &+ \lambda(\mathrm{Tr}\{\mathbf{Q}'_{\rm z}\} - P_{\rm z}) + \mathrm{Tr}\{\mathbf{Q}'_{\rm z}\mathbf{Z}\}, \qquad (41)\end{aligned}$$

where λ and $\mathbf{Z}$ are the Lagrange multipliers. The Karush-Kuhn-Tucker (KKT) optimality conditions for this problem are then given as

$$\mathrm{Tr}\{\mathbf{Q}'_{\rm z}\} \leq 1,\ \mathbf{Q}'_{\rm z} \succeq 0,\ \lambda \geq 0 \qquad (42a)$$

$$\mathbf{Z} \preceq 0,\ \lambda(\mathrm{Tr}\{\mathbf{Q}'_{\rm z}\} - 1) = 0,\ \mathrm{Tr}\{\mathbf{Q}'_{\rm z}\mathbf{Z}\} = 0 \qquad (42b)$$

$$(\mathbf{Q}'_{\rm z} + \mathbf{D} + \mathbf{A})^{-\rm T} - (\mathbf{Q}'_{\rm z} + \mathbf{D})^{-\rm T} + \lambda\mathbf{I} + \mathbf{Z}^{\rm T} = \mathbf{0}, \qquad (42c)$$

where $(\cdot)^{\rm T}$ denotes transpose. The equation (42c) can be equivalently written as

$$(\mathbf{Q}'_{\rm z} + \mathbf{D} + \mathbf{A})^{-1} = (\mathbf{Q}'_{\rm z} + \mathbf{D})^{-1} - \lambda\mathbf{I} - \mathbf{Z}, \qquad (43)$$

which further indicates that

$$\mathbf{Q}'_{\rm z} + \mathbf{D} + \mathbf{A} = \left((\mathbf{Q}'_{\rm z} + \mathbf{D})^{-1} - (\lambda\mathbf{I} + \mathbf{Z})\right)^{-1}. \qquad (44)$$

Using the matrix inversion lemma [31], the right-hand side of (44) is equivalent to

$$\mathbf{Q}'_{\rm z}+\mathbf{D}+(\mathbf{Q}'_{\rm z}+\mathbf{D})\left((\lambda\mathbf{I}+\mathbf{Z})^{-1}-(\mathbf{Q}'_{\rm z}+\mathbf{D})\right)^{-1}(\mathbf{Q}'_{\rm z}+\mathbf{D}). \qquad (45)$$

Then (44) can be written as

$$\mathbf{A} = (\mathbf{Q}'_{\rm z}+\mathbf{D})\left((\lambda\mathbf{I}+\mathbf{Z})^{-1} - (\mathbf{Q}'_{\rm z}+\mathbf{D})\right)^{-1}(\mathbf{Q}'_{\rm z}+\mathbf{D}), \qquad (46)$$

and equivalently,

$$(\mathbf{Q}'_{\rm z} + \mathbf{D})\mathbf{A}^{-1}(\mathbf{Q}'_{\rm z} + \mathbf{D}) + (\mathbf{Q}'_{\rm z} + \mathbf{D}) = (\lambda\mathbf{I} + \mathbf{Z})^{-1}. \qquad (47)$$

In order to derive the general form of the optimal solution in our case, we rewrite (47) as

$$\begin{aligned}&\left((\mathbf{Q}'_{\rm z} + \mathbf{D})\mathbf{A}^{-\frac{1}{2}} + \frac{1}{2}\mathbf{A}^{\frac{1}{2}}\right)\left(\mathbf{A}^{-\frac{1}{2}}(\mathbf{Q}'_{\rm z} + \mathbf{D}) + \frac{1}{2}\mathbf{A}^{\frac{1}{2}}\right) \\ &= (\lambda\mathbf{I} + \mathbf{Z})^{-1} + \frac{1}{4}\mathbf{A}. \qquad (48)\end{aligned}$$

Meanwhile, by multiplying both sides of equation (48) with $\mathbf{A}^{-\frac{1}{2}}$ we can obtain

$$\left(\mathbf{A}^{-\frac{1}{2}}(\mathbf{Q}'_{\rm z} + \mathbf{D})\mathbf{A}^{-\frac{1}{2}} + \frac{1}{2}\mathbf{I}\right)^2 = \mathbf{A}^{-\frac{1}{2}}(\lambda\mathbf{I} + \mathbf{Z})^{-1}\mathbf{A}^{-\frac{1}{2}} + \frac{1}{4}\mathbf{I}, \qquad (49)$$

which gives the form of optimal $\mathbf{Q}'_{\rm z}$ as follows

$$\mathbf{Q}'_{\rm z} = \mathbf{A}^{\frac{1}{2}}\left(\mathbf{A}^{-\frac{1}{2}}(\lambda\mathbf{I} + \mathbf{Z})^{-1}\mathbf{A}^{-\frac{1}{2}} + \frac{1}{4}\mathbf{I}\right)^{\frac{1}{2}}\mathbf{A}^{\frac{1}{2}} - \frac{1}{2}\mathbf{A} - \mathbf{D}. \qquad (50)$$

This completes the proof of Theorem 1. ■

F. Proof of Theorem 2

Proof: We first prove that $\mathbf{Q}'_{\rm z} = \mathbf{0}$ when $\mathbf{Z} = (\mathbf{D}\mathbf{A}^{-1}\mathbf{D} + \mathbf{D})^{-1} - \lambda\mathbf{I}$. A sufficient and necessary condition for $\mathbf{Q}'_{\rm z}$ in (19) to be $\mathbf{0}$ is

$$\left(\frac{1}{2}\mathbf{I}+\mathbf{A}^{-\frac{1}{2}}\mathbf{D}\mathbf{A}^{-\frac{1}{2}}\right)^{-\frac{1}{2}}\mathbf{A}^{-\frac{1}{2}}\mathbf{Q}'_{\rm z}\mathbf{A}^{-\frac{1}{2}}\left(\frac{1}{2}\mathbf{I}+\mathbf{A}^{-\frac{1}{2}}\mathbf{D}\mathbf{A}^{-\frac{1}{2}}\right)^{-\frac{1}{2}} = \mathbf{0}. \qquad (51)$$

Substituting (19) into the left-hand side of (51), the above condition can be rewritten as

$$\begin{aligned}&\left(\frac{1}{2}\mathbf{I} + \mathbf{A}^{-\frac{1}{2}}\mathbf{D}\mathbf{A}^{-\frac{1}{2}}\right)^{-\frac{1}{2}}\left(\mathbf{A}^{-\frac{1}{2}}(\lambda\mathbf{I} + \mathbf{Z})^{-1}\mathbf{A}^{-\frac{1}{2}} + \frac{1}{4}\mathbf{I}\right)^{\frac{1}{2}} \\ &\cdot\left(\frac{1}{2}\mathbf{I} + \mathbf{A}^{-\frac{1}{2}}\mathbf{D}\mathbf{A}^{-\frac{1}{2}}\right)^{-\frac{1}{2}} = \mathbf{I}, \qquad (52)\end{aligned}$$

which is equivalent to

$$\mathbf{A}^{-\frac{1}{2}}(\lambda\mathbf{I} + \mathbf{Z})^{-1}\mathbf{A}^{-\frac{1}{2}} + \frac{1}{4}\mathbf{I} = \left(\frac{1}{2}\mathbf{I} + \mathbf{A}^{-\frac{1}{2}}\mathbf{D}\mathbf{A}^{-\frac{1}{2}}\right)^2. \qquad (53)$$

The right-hand side of the above equation is equal to

$$\frac{1}{4}\mathbf{I} + \mathbf{A}^{-\frac{1}{2}}\mathbf{D}\mathbf{A}^{-\frac{1}{2}} + \mathbf{A}^{-\frac{1}{2}}\mathbf{D}\mathbf{A}^{-1}\mathbf{D}\mathbf{A}^{-\frac{1}{2}}. \qquad (54)$$

As a result, it must hold that $\mathbf{Z} = (\mathbf{D}\mathbf{A}^{-1}\mathbf{D} + \mathbf{D})^{-1} - \lambda\mathbf{I}$ when $\mathbf{Q}'_{\rm z} = \mathbf{0}$.

Next we prove that $\mathbf{Q}'_{\rm z}$ in (19) is PSD when $\mathbf{Z} \preceq (\mathbf{D}\mathbf{A}^{-1}\mathbf{D} + \mathbf{D})^{-1} - \lambda\mathbf{I}$. When $\mathbf{Z} \preceq (\mathbf{D}\mathbf{A}^{-1}\mathbf{D} + \mathbf{D})^{-1} - \lambda\mathbf{I}$, it holds that

$$\mathbf{A}^{-\frac{1}{2}}(\lambda\mathbf{I} + \mathbf{Z})^{-1}\mathbf{A}^{-\frac{1}{2}} \succeq \mathbf{A}^{-\frac{1}{2}}\mathbf{D}\mathbf{A}^{-1}\mathbf{D}\mathbf{A}^{-\frac{1}{2}} + \mathbf{A}^{-\frac{1}{2}}\mathbf{D}\mathbf{A}^{-\frac{1}{2}}. \qquad (55)$$

The fact that $\mathbf{X}_1 \succeq \mathbf{X}_2$ if and only if $\mathbf{X}_2^{-1} \succeq \mathbf{X}_1^{-1}$ for any pair of PD matrices $\mathbf{X}_1$ and $\mathbf{X}_2$ (see [32]) is used in the above step. The right-hand side of (55) is equal to

$$\left(\frac{1}{2}\mathbf{I} + \mathbf{A}^{-\frac{1}{2}}\mathbf{D}\mathbf{A}^{-\frac{1}{2}}\right)^2 - \frac{1}{4}\mathbf{I}. \qquad (56)$$

Then, (55) can be written as

$$\left(\mathbf{A}^{-\frac{1}{2}}\mathbf{Q}'_{\rm z}\mathbf{A}^{-\frac{1}{2}} + \left(\frac{1}{2}\mathbf{I}+\mathbf{A}^{-\frac{1}{2}}\mathbf{D}\mathbf{A}^{-\frac{1}{2}}\right)\right)^2 \succeq \left(\frac{1}{2}\mathbf{I} + \mathbf{A}^{-\frac{1}{2}}\mathbf{D}\mathbf{A}^{-\frac{1}{2}}\right)^2. \qquad (57)$$

Using the fact that $\mathbf{X}_1^{1/2} \succeq \mathbf{X}_2^{1/2}$ if $\mathbf{X}_1 \succeq \mathbf{X}_2$ for any PD $\mathbf{X}_1$ and PSD $\mathbf{X}_2$ [32], inequality (57) leads to $\mathbf{A}^{-\frac{1}{2}}\mathbf{Q}'_{\rm z}\mathbf{A}^{-\frac{1}{2}} \succeq 0$ and, as a result, $\mathbf{Q}'_{\rm z} \succeq 0$. ■

G. Proof of Theorem 3

Proof: The constraint $\mathrm{Tr}\{\mathbf{Q}_{\mathrm{z}}'\mathbf{Z}\} = 0$ in (42b) indicates that $\mathrm{Tr}\{\mathbf{Q}_{\mathrm{z}}'^{\frac{1}{2}}(-\mathbf{Z})\mathbf{Q}_{\mathrm{z}}'^{\frac{1}{2}}\} = 0$. As $-\mathbf{Z}$ and $\mathbf{Q}_{\mathrm{z}}'$ are both PSD, $\mathbf{Q}_{\mathrm{z}}'^{\frac{1}{2}}(-\mathbf{Z})\mathbf{Q}_{\mathrm{z}}'^{\frac{1}{2}}$ must be PSD. Therefore, the trace constraint implies that $\mathbf{Q}_{\mathrm{z}}'^{\frac{1}{2}}\mathbf{Z}\mathbf{Q}_{\mathrm{z}}'^{\frac{1}{2}} = \mathbf{0}$. Consequently, it can be shown that

$$\mathbf{U}_{\mathbf{Z}1}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}'\mathbf{U}_{\mathbf{Z}1} = \mathbf{U}_{\mathbf{Z}1}^{\mathrm{H}}\mathbf{U}_{\mathbf{Q}1}\bar{\mathbf{\Lambda}}_{\mathbf{Q}}\mathbf{U}_{\mathbf{Q}1}^{\mathrm{H}}\mathbf{U}_{\mathbf{Z}1} = \mathbf{0}. \tag{58}$$

As a result, $\mathbf{Z}$ and $\mathbf{Q}_{\mathrm{z}}'$ satisfy the condition

$$\mathbf{Z}\mathbf{Q}_{\mathrm{z}}' = \mathbf{Q}_{\mathrm{z}}'\mathbf{Z} = \mathbf{0}. \tag{59}$$

It can be seen that $r_{\mathbf{Z}} + r_{\mathbf{Q}} \leq n_{\mathrm{z}}$ from the above equations.

Then, we can obtain the following result from equation (47)

$$\mathbf{Z}(\mathbf{Q}_{\mathrm{z}}'+\mathbf{D})\mathbf{A}^{-1}(\mathbf{Q}_{\mathrm{z}}'+\mathbf{D})\mathbf{Z}+\mathbf{Z}(\mathbf{Q}_{\mathrm{z}}'+\mathbf{D})\mathbf{Z} = \mathbf{Z}(\lambda\mathbf{I}+\mathbf{Z})^{-1}\mathbf{Z}. \tag{60}$$

Using the property that $\mathbf{Z}\mathbf{Q}_{\mathrm{z}}' = \mathbf{Q}_{\mathrm{z}}'\mathbf{Z} = \mathbf{0}$, (60) simplifies to

$$\mathbf{Z}\mathbf{D}\mathbf{A}^{-1}\mathbf{D}\mathbf{Z} + \mathbf{Z}\mathbf{D}\mathbf{Z} = \mathbf{Z}(\lambda\mathbf{I} + \mathbf{Z})^{-1}\mathbf{Z}. \tag{61}$$

Define $\mathbf{N} \triangleq \mathbf{D}\mathbf{A}^{-1}\mathbf{D} + \mathbf{D}$ and note that $\mathbf{N}$ is PD. The above equation (61) can be rewritten as

$$\mathbf{Z}\big(\mathbf{N} - (\lambda\mathbf{I} - \mathbf{Z})^{-1}\big)\mathbf{Z} = \mathbf{0}. \tag{62}$$

Using (25), equation (62) can be rewritten as

$$\begin{bmatrix} \bar{\mathbf{\Lambda}}_{\mathbf{Z}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}\left(\tilde{\mathbf{N}} - \left(\lambda\mathbf{I} + \begin{bmatrix} \bar{\mathbf{\Lambda}}_{\mathbf{Z}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}\right)^{-1}\right)\begin{bmatrix} \bar{\mathbf{\Lambda}}_{\mathbf{Z}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} = \mathbf{0}, \tag{63}$$

where

$$\tilde{\mathbf{N}} = \begin{bmatrix} \mathbf{U}_{\mathbf{Z}1}^{\mathrm{H}} \\ \mathbf{U}_{\mathbf{Z}2}^{\mathrm{H}} \end{bmatrix}\mathbf{N}\begin{bmatrix} \mathbf{U}_{\mathbf{Z}1} & \mathbf{U}_{\mathbf{Z}2} \end{bmatrix}. \tag{64}$$

From (63) and (64), the following result can be obtained:

$$\mathbf{U}_{\mathbf{Z}1}^{\mathrm{H}}\mathbf{N}\mathbf{U}_{\mathbf{Z}1} - (\lambda\mathbf{I} + \mathbf{\Lambda}_{\mathbf{Z}})^{-1} = \mathbf{0}, \tag{65}$$

or equivalently,

$$\mathbf{\Lambda}_{\mathbf{Z}} = (\mathbf{U}_{\mathbf{Z}1}^{\mathrm{H}}\mathbf{N}\mathbf{U}_{\mathbf{Z}1})^{-1} - \lambda\mathbf{I}. \tag{66}$$

Substituting $\mathbf{N}$ back into (66) completes the proof. ■

H. Proof of Lemma 5

Proof: Using (27) in Theorem 3, it can be seen that $\mathbf{Z} \succeq \mathbf{0}$ whenever $\lambda < 1/\mu_{\Delta}^{\max}$, which contradicts the constraint $\mathbf{Z} \preceq \mathbf{0}$ in (42b). Therefore, $\lambda^* \geq 1/\mu_{\Delta}^{\max}$. According to Theorem 2, at the point when λ becomes as small as $1/\mu_{\Delta}^{\min}$ (the same point when $(\mathbf{D}\mathbf{A}^{-1}\mathbf{D} + \mathbf{D})^{-1} - \lambda\mathbf{I}$ becomes negative semi-definite), $\mathbf{Z} = (\mathbf{D}\mathbf{A}^{-1}\mathbf{D} + \mathbf{D})^{-1} - \lambda\mathbf{I}$ leads to the result $\mathbf{Q}_{\mathrm{z}}' = \mathbf{0}$. Therefore, $\lambda^* \leq 1/\mu_{\Delta}^{\min}$. ■

I. Proof of Theorem 4

Given the block representation of $\mathbf{A}$ in (28), the $\bar{R}^{\mathrm{J}}$ in (17) can be rewritten as

$$\begin{aligned}
\bar{R}^{\mathrm{J}} &= \log\left|\mathbf{I} + \begin{bmatrix} \mathbf{U}_{\mathbf{A}1} & \mathbf{U}_{\mathbf{A}2} \end{bmatrix}\begin{bmatrix} \bar{\mathbf{\Lambda}}_{\mathbf{A}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \mathbf{U}_{\mathbf{A}1}^{\mathrm{H}} \\ \mathbf{U}_{\mathbf{A}2}^{\mathrm{H}} \end{bmatrix}\mathbf{Q}_{\mathrm{z}}''^{-1}\right| \\
&= \log\left|\mathbf{I} + \begin{bmatrix} \bar{\mathbf{\Lambda}}_{\mathbf{A}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}\left(\begin{bmatrix} \mathbf{U}_{\mathbf{A}1}^{\mathrm{H}} \\ \mathbf{U}_{\mathbf{A}2}^{\mathrm{H}} \end{bmatrix}\mathbf{Q}_{\mathrm{z}}''\begin{bmatrix} \mathbf{U}_{\mathbf{A}1} & \mathbf{U}_{\mathbf{A}2} \end{bmatrix}\right)^{-1}\right| \\
&= \log\left|\mathbf{I} + \begin{bmatrix} \bar{\mathbf{\Lambda}}_{\mathbf{A}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}''\mathbf{U}_{\mathbf{A}1} & \mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}''\mathbf{U}_{\mathbf{A}2} \\ \mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}''\mathbf{U}_{\mathbf{A}1} & \mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}''\mathbf{U}_{\mathbf{A}2} \end{bmatrix}^{-1}\right| \\
&= \log\left|\mathbf{I} + \bar{\mathbf{\Lambda}}_{\mathbf{A}}\mathbf{F}_1^{-1}\right|,
\end{aligned} \tag{67}$$

where $\mathbf{Q}_{\mathrm{z}}'' \triangleq \mathbf{Q}_{\mathrm{z}}' + \mathbf{D}$, and the result on block matrix inversion [29] is used in the last equality, in which

$$\mathbf{F}_1 \triangleq \mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\left(\mathbf{Q}_{\mathrm{z}}'' - \mathbf{Q}_{\mathrm{z}}''\mathbf{U}_{\mathbf{A}2}(\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}''\mathbf{U}_{\mathbf{A}2})^{-1}\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}''\right)\mathbf{U}_{\mathbf{A}1}. \tag{68}$$

Since $\bar{\mathbf{\Lambda}}_{\mathbf{A}}$ in (67) is PD, the result (19) can be used here. Applying the result (19), the optimal $\mathbf{F}_1$ is given by

$$\mathbf{F}_1 = \bar{\mathbf{\Lambda}}_{\mathbf{A}}^{\frac{1}{2}}\left(\bar{\mathbf{\Lambda}}_{\mathbf{A}}^{-\frac{1}{2}}(\lambda\mathbf{I} + \mathbf{Z})\bar{\mathbf{\Lambda}}_{\mathbf{A}}^{-\frac{1}{2}} + \frac{1}{4}\mathbf{I}\right)^{\frac{1}{2}}\bar{\mathbf{\Lambda}}_{\mathbf{A}}^{\frac{1}{2}} - \frac{1}{2}\bar{\mathbf{\Lambda}}_{\mathbf{A}}. \tag{69}$$

It should be noted that the variable λ is determined by the trace of $\mathbf{F}_1$, which satisfies

$$\mathrm{Tr}\{\mathbf{F}_1\} \leq \mathrm{Tr}\{\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}(\mathbf{Q}_{\mathrm{z}}' + \mathbf{D})\mathbf{U}_{\mathbf{A}1}\} \leq P_{\mathrm{z}} + \mathrm{Tr}\{\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{D}\mathbf{U}_{\mathbf{A}1}\}. \tag{70}$$

Considering the fact that

$$\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\left(\mathbf{Q}_{\mathrm{z}}'' - \mathbf{Q}_{\mathrm{z}}''\mathbf{U}_{\mathbf{A}2}(\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}''\mathbf{U}_{\mathbf{A}2})^{-1}\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}''\right)\mathbf{U}_{\mathbf{A}2} = \mathbf{0} \tag{71}$$

$$\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\left(\mathbf{Q}_{\mathrm{z}}'' - \mathbf{Q}_{\mathrm{z}}''\mathbf{U}_{\mathbf{A}2}(\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}''\mathbf{U}_{\mathbf{A}2})^{-1}\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}''\right)\mathbf{U}_{\mathbf{A}2} = \mathbf{0} \tag{72}$$

$$\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\left(\mathbf{Q}_{\mathrm{z}}'' - \mathbf{Q}_{\mathrm{z}}''\mathbf{U}_{\mathbf{A}2}(\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}''\mathbf{U}_{\mathbf{A}2})^{-1}\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}''\right)\mathbf{U}_{\mathbf{A}1} = \mathbf{0}, \tag{73}$$

it leads to the following result

$$\mathbf{Q}_{\mathrm{z}}'' - \mathbf{Q}_{\mathrm{z}}''\mathbf{U}_{\mathbf{A}2}(\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}''\mathbf{U}_{\mathbf{A}2})^{-1}\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}'' = \mathbf{U}_{\mathbf{A}1}\mathbf{F}_1\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}. \tag{74}$$

The equation (74) can be further rewritten as

$$\begin{aligned}
&\mathbf{Q}_{\mathrm{z}}''^{-1} - \mathbf{U}_{\mathbf{A}2}(\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}''\mathbf{U}_{\mathbf{A}2})^{-1}\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}} \\
&= \mathbf{Q}_{\mathrm{z}}''^{-1}\mathbf{U}_{\mathbf{A}1}\mathbf{F}_1\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}''^{-1},
\end{aligned} \tag{75}$$

which implies that

$$\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}''^{-1}\mathbf{U}_{\mathbf{A}1} = \mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}''^{-1}\mathbf{U}_{\mathbf{A}1}\mathbf{F}_1\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}''^{-1}\mathbf{U}_{\mathbf{A}1}. \tag{76}$$

As a result, it holds that

$$\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}''^{-1}\mathbf{U}_{\mathbf{A}1}(\mathbf{F}_1\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}''^{-1}\mathbf{U}_{\mathbf{A}1} - \mathbf{I}) = \mathbf{0}. \tag{77}$$

Since $\mathbf{F}_1$ and $\mathbf{Q}_{\mathrm{z}}''$ are both PD matrices, $\mathbf{Q}_{\mathrm{z}}''$ must satisfy

$$\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}''^{-1}\mathbf{U}_{\mathbf{A}1} = \mathbf{F}_1^{-1}, \tag{78}$$

which is equivalent to

$$\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}(\mathbf{Q}_{\mathrm{z}}'+\mathbf{D})^{-1}\mathbf{U}_{\mathbf{A}1}=\mathbf{F}_1^{-1}. \tag{79}$$

Note that the above constraint (79) does not specify a unique $\mathbf{Q}_{\mathrm{z}}'$. The reason is that the problem of minimizing $\bar{R}^{\mathrm{J}}$ is not strictly convex when $\mathbf{A}$ does not have full rank. It is not difficult to see from the above derivations that all $\mathbf{Q}_{\mathrm{z}}'$ satisfying the following two conditions are optimal. The first condition is that (79) must be satisfied and the second is that the trace constraint (70) must be satisfied with equality, i.e.,

$$\mathrm{Tr}\{\mathbf{F}_1\}=\mathrm{Tr}\{\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}(\mathbf{Q}_{\mathrm{z}}'+\mathbf{D})\mathbf{U}_{\mathbf{A}1}\}=P_{\mathrm{z}}+\mathrm{Tr}\{\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{D}\mathbf{U}_{\mathbf{A}1}\}.$$

One solution satisfying both of the above conditions is

$$\mathbf{Q}_{\mathrm{z}}'=\mathbf{U}_{\mathbf{A}1}\mathbf{F}_1\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}+\mathbf{U}_{\mathbf{A}2}\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{D}\mathbf{U}_{\mathbf{A}2}\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}-\mathbf{D}. \tag{80}$$

Indeed, with the above $\mathbf{Q}_{\mathrm{z}}'$, $\mathbf{Q}_{\mathrm{z}}'+\mathbf{D}$ can be rewritten as

$$\mathbf{Q}_{\mathrm{z}}'+\mathbf{D}=\mathbf{U}_{\mathbf{A}}\begin{bmatrix}\mathbf{F}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{D}\mathbf{U}_{\mathbf{A}2}\end{bmatrix}\mathbf{U}_{\mathbf{A}}^{\mathrm{H}}, \tag{81}$$

and therefore, (79) is satisfied. Meanwhile, it can be seen that when $\mathrm{Tr}\{\mathbf{Q}_{\mathrm{z}}'\}=P_{\mathrm{z}}$, it holds that

$$\begin{aligned}\mathrm{Tr}\{\mathbf{F}_1\}&=\mathrm{Tr}\{\mathbf{U}_{\mathbf{A}1}\mathbf{F}_1\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\}\\ &=\mathrm{Tr}\{\mathbf{Q}_{\mathrm{z}}'-\mathbf{U}_{\mathbf{A}2}\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{D}\mathbf{U}_{\mathbf{A}2}\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}+\mathbf{D}\}\\ &=P_{\mathrm{z}}+\mathrm{Tr}\{\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{D}\mathbf{U}_{\mathbf{A}1}\}.\end{aligned} \tag{82}$$

Therefore, $\mathbf{Q}_{\mathrm{z}}'$ given by (80) satisfy both conditions. Substituting (69) into (80) leads to the expression (29). ■

J. Proof of Theorem 5

We prove Theorem 5 in two steps. As $\mathbf{Q}_{\mathrm{z}}'$ is either PD or PSD (but not PD), we consider the two cases separately. We first assume that the optimal $\mathbf{Q}_{\mathrm{z}}'$ is PD, and then obtain the optimal solution accordingly and verify that the resulting $\mathbf{Q}_{\mathrm{z}}'$ is PD. Then, we assume that the optimal $\mathbf{Q}_{\mathrm{z}}'$ is PSD but not PD, and then obtain the optimal solution accordingly and verify that the resulting $\mathbf{Q}_{\mathrm{z}}'$ is PSD but not PD.

Assume that $\mathbf{Q}_{\mathrm{z}}'$ is PD. It immediately follows that $\mathbf{A}\succ 0$. Indeed, given that

$$\bar{R}^{\mathrm{J}}=\log\left|\mathbf{I}+\mathbf{\Lambda}_{\mathbf{A}}\left(\mathbf{U}_{\mathbf{A}}^{\mathrm{H}}(\mathbf{Q}_{\mathrm{z}}'+\mathbf{D})\mathbf{U}_{\mathbf{A}}\right)^{-1}\right| \tag{83}$$

and that $\mathbf{U}_{\mathbf{A}}^{\mathrm{H}}\mathbf{D}\mathbf{U}_{\mathbf{A}}$ is negligible as compared to $\mathbf{U}_{\mathbf{A}}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}'\mathbf{U}_{\mathbf{A}}$ in the jammer-dominant regime when $\mathbf{Q}_{\mathrm{z}}'\succ 0$ (as shown in Remark 4), $\bar{R}^{\mathrm{J}}$ becomes

$$\bar{R}^{\mathrm{J}}=\log\left|\mathbf{I}+\mathbf{\Lambda}_{\mathbf{A}}\left(\mathbf{U}_{\mathbf{A}}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}'\mathbf{U}_{\mathbf{A}}\right)^{-1}\right|. \tag{84}$$

As a result, it can be shown that a full-rank matrix $\mathbf{Q}_{\mathrm{z}}'$ cannot be optimal if $\mathbf{A}$ is not PD. Then, using the same method used while proving Theorem 1, it can be shown that the matrix $\mathbf{Q}_{\mathrm{z}}'$ minimizing the above $\bar{R}^{\mathrm{J}}$ is given by (34). Note that in this case $\mathbf{\Lambda}_{\mathbf{A}}=\bar{\mathbf{\Lambda}}_{\mathbf{A}}$ and $\mathbf{U}_{\mathbf{A}}=\mathbf{U}_{\mathbf{A}1}$ since $\mathbf{A}\succ 0$, and as a result $\mathbf{Q}_{\mathrm{z}}'$ in (34) is PD, which is in accordance with the assumption. In fact, it can also be shown that the optimal $\mathbf{Q}_{\mathrm{z}}'$ is PD when $\mathbf{A}\succ 0$ in the jammer-dominant regime.

Now assume that $\mathbf{Q}_{\mathrm{z}}'$ is PSD but not PD. In such case, $\mathbf{A}$ is also PSD (but not PD) and $\bar{R}^{\mathrm{J}}$ can be written as

$$\bar{R}^{\mathrm{J}}=\log\left|\mathbf{I}+\begin{bmatrix}\bar{\mathbf{\Lambda}}_{\mathbf{A}} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\\ \mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\end{bmatrix}\left(\mathbf{Q}_{\mathrm{z}}'+\mathbf{D}\right)^{-1}\left[\mathbf{U}_{\mathbf{A}1}\ \mathbf{U}_{\mathbf{A}2}\right]\right|. \tag{85}$$

As shown in Theorem 4, the jammer should only allocate jamming power onto the positive eigenvalues of $\mathbf{A}$. Thus, the columns of $\mathbf{U}_{\mathbf{A}2}$ must be in the null space of the optimal $\mathbf{Q}_{\mathrm{z}}'$. As a result, the optimal $\mathbf{Q}_{\mathrm{z}}'$ satisfies $\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}'\mathbf{U}_{\mathbf{A}2}=\mathbf{0}$, $\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}'\mathbf{U}_{\mathbf{A}2}=\mathbf{0}$, and $\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}'\mathbf{U}_{\mathbf{A}1}=\mathbf{0}$. As a result, $\bar{R}^{\mathrm{J}}$ becomes

$$\begin{aligned}\bar{R}^{\mathrm{J}}&=\log\left|\mathbf{I}+\begin{bmatrix}\bar{\mathbf{\Lambda}}_{\mathbf{A}} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}(\mathbf{Q}_{\mathrm{z}}'+\mathbf{D})\mathbf{U}_{\mathbf{A}1} & \mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{D}\mathbf{U}_{\mathbf{A}2}\\ \mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{D}\mathbf{U}_{\mathbf{A}1} & \mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{D}\mathbf{U}_{\mathbf{A}1}\end{bmatrix}^{-1}\right|\\ &=\log\left|\mathbf{I}+\bar{\mathbf{\Lambda}}_{\mathbf{A}}\mathbf{M}^{-1}\right|,\end{aligned} \tag{86}$$

in which

$$\mathbf{M}\triangleq\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\left(\mathbf{Q}_{\mathrm{z}}'+\mathbf{D}-\mathbf{D}\mathbf{U}_{\mathbf{A}2}(\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{D}\mathbf{U}_{\mathbf{A}2})^{-1}\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{D}\right)\mathbf{U}_{\mathbf{A}1}. \tag{87}$$

The optimal form for $\mathbf{M}$ is given as

$$\mathbf{M}=\left(\frac{1}{\xi}\bar{\mathbf{\Lambda}}_{\mathbf{A}}+\frac{1}{4}\bar{\mathbf{\Lambda}}_{\mathbf{A}}^2\right)^{\frac{1}{2}}-\frac{1}{2}\bar{\mathbf{\Lambda}}_{\mathbf{A}}, \tag{88}$$

where ξ is determined by the trace of $\mathbf{M}$ such that the trace of the right-hand side of (88) is equal to $\mathrm{Tr}\{\mathbf{M}\}$. Therefore, if there exists $\mathbf{Q}_{\mathrm{z}}'$ subject to $\mathrm{Tr}\{\mathbf{Q}_{\mathrm{z}}'\}\le P_{\mathrm{z}}$ such that the resulting $\mathbf{M}$ can achieve (88) and $\mathrm{Tr}\{\mathbf{M}\}$ is maximized, this $\mathbf{Q}_{\mathrm{z}}'$ is optimal. Note that the problem of finding an optimal $\mathbf{Q}_{\mathrm{z}}'$ is not strictly convex in this case. As a result, the optimal solution might not be unique. Recall that the columns of $\mathbf{U}_{\mathbf{A}2}$ must be in the null space of an optimal $\mathbf{Q}_{\mathrm{z}}'$. As a result, the rank of an optimal $\mathbf{Q}_{\mathrm{z}}'$ must be less than or equal to that of $\mathbf{A}$. First, we check if there exists an optimal $\mathbf{Q}_{\mathrm{z}}'$ with a rank equal to the rank of $\mathbf{A}$. Note that when $\mathbf{Q}_{\mathrm{z}}'$ has a rank equal to that of $\mathbf{A}$, all columns of $\mathbf{U}_{\mathbf{A}1}$ lie in the column space of $\mathbf{Q}_{\mathrm{z}}'$. Therefore, $(\mathbf{Q}_{\mathrm{z}}',\mathbf{D})$ lies in the jammer-dominant regime. According to Remark 4 (Property 2), it can be seen that $\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{D}\mathbf{U}_{\mathbf{A}1}$ is negligible compared to $\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}'\mathbf{U}_{\mathbf{A}1}$. Define

$$\mathbf{M}_1\triangleq\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\left(\mathbf{D}-\mathbf{D}\mathbf{U}_{\mathbf{A}2}(\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{D}\mathbf{U}_{\mathbf{A}2})^{-1}\mathbf{U}_{\mathbf{A}2}^{\mathrm{H}}\mathbf{D}\right)\mathbf{U}_{\mathbf{A}1}. \tag{89}$$

The matrix $\mathbf{M}_1$ is PSD because it can be shown using block matrix inversion that

$$\mathbf{M}_1=(\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{D}^{-1}\mathbf{U}_{\mathbf{A}1})^{-1}. \tag{90}$$

Therefore, we have

$$0\preceq\mathbf{M}_1\preceq\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{D}\mathbf{U}_{\mathbf{A}1}, \tag{91}$$

where the second inequality is obtained from the definition (89). Since $\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{D}\mathbf{U}_{\mathbf{A}1}$ is negligible as compared to $\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}'\mathbf{U}_{\mathbf{A}1}$, so is $\mathbf{M}_1$ according to Remark 4 (Property 3). Hence, $\mathbf{M}$ in (87) becomes $\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}'\mathbf{U}_{\mathbf{A}1}$ since $\mathbf{M}=\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}'\mathbf{U}_{\mathbf{A}1}+\mathbf{M}_1$. Therefore, $\bar{R}^{\mathrm{J}}$ can be rewritten as

$$\bar{R}^{\mathrm{J}}=\log\left|\mathbf{I}+\bar{\mathbf{\Lambda}}_{\mathbf{A}}(\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{Q}_{\mathrm{z}}'\mathbf{U}_{\mathbf{A}1})^{-1}\right| \tag{92}$$

and the solution is (34). It can be seen that the solution of $\mathbf{Q}'_{\mathrm{z}}$ in (34) leads to $\mathbf{M}$ in (88) as $\mathbf{M}$ becomes $\mathbf{U}_{\mathbf{A}1}^{\mathrm{H}}\mathbf{Q}'_{\mathrm{z}}\mathbf{U}_{\mathbf{A}1}$ in the jammer-dominant regime. $\mathbf{Q}'_{\mathrm{z}}$ in (34) also maximizes the trace of $\mathbf{M}$ in (87). Moreover, $\mathbf{Q}'_{\mathrm{z}}$ in (34) has the same rank as $\mathbf{A}$ as we assumed. Therefore, (34) is an optimal solution when $\mathbf{Q}'_{\mathrm{z}}$ is PSD but not PD.

Combining the above two parts completes the proof of Theorem 5. ■

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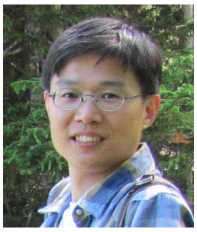

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