Distributed Opportunistic Channel Access in Wireless Relay Networks

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***Abstract*— In this paper, the problem of distributed opportunistic channel access in wireless relaying is investigated. A relay network with multiple source-destination pairs and multiple relays is considered. All source nodes contend through a random access procedure. A winner source may give up its transmission opportunity if its link quality is poor. In this research, we apply the optimal stopping theory to analyze when a winner source should give up its transmission opportunity. By assuming the winner source has channel state information (CSI) of links from itself to relays and from relays to its destination, the existence of an optimal stopping strategy is rigorously proved. The optimal stopping strategy has a pure-threshold structure. The case when a winner source does not have CSI of links from relays to its destination is also studied. Two stopping problems exist, one in the main layer (for channel access of sources), and the other in the sub-layer (for channel access of relays). An intuitive stopping strategy, where the main layer (for the first hop) and sub-layer (for the second hop) maximize their throughput respectively, is derived. The intuitive stopping strategy is shown to be non-optimal. An optimal stopping strategy is then derived theoretically. In either the intuitive stopping strategy or the optimal stopping strategy, the main-layer stopping rule has a pure-threshold structure, while the sub-layer stopping rule has a threshold determined by the channel realization in the preceding first-hop transmission. Our research reveals that multi-user (including multi-source and multi-relay) diversity and time diversity can be utilized in a relay network by our proposed strategies. The effectiveness of the strategies is validated by numerical and simulation results.**

Keywords – Relay, opportunistic channel access, optimal stopping.

I. Introduction

Opportunistic channel access, in which a user with poor channel quality gives up its channel access opportunity to other users with good channel conditions, has received much attention in the literature [1], particularly in centralized networks. A central controller can collect the channel state information (CSI) of the users, and schedule only those users with the best channel conditions. On the other hand, the research on distributed opportunistic channel access is still in its infancy. Without a central controller, it is hard for a user to decide when to give up its transmission opportunity. An intuitive way is to categorize the channel of a user into two states: good state when the channel gain is above a threshold; and bad state otherwise. Then a user gives up its channel access opportunity when its channel is bad. Apparently the multi-user diversity (i.e., different users experience different channel gains) and time diversity (i.e., a user experiences different channel gain when time varies) are not fully utilized by the intuitive method.

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This problem was addressed recently in [2], by means of optimal stopping. The major idea is to let all the users contend for channel access. It is found that, 1) if the winner in a contention has an achievable (transmission) rate smaller than a threshold (which can be obtained numerically), it is optimal for the winner to give up its transmission opportunity and all users continue to contend; and 2) if the winner in a contention has an achievable rate larger than the threshold, it is optimal for the winner to *stop* here, i.e., to utilize the transmission opportunity and transmit its data. The beautiful part of the work is in the *pure-threshold strategy*, which is easy to implement. As extensions to the work in [2], interference channel which can tolerate multiple users transmitting is considered in [3] where more than one node can share the channel simultaneously, and delay constraints are considered for real-time service in [4]. Pure-threshold strategies are also derived in [3], [4].

In this paper, we investigate distributed opportunistic channel access in a relay network, since wireless relaying has recently attracted a lot of research interests [5]–[9]. We consider multiple source-destination pairs aided by multiple relays. Since transmission between each source-destination pair involves two hops: from source to relays and from relays to the destination, the problem of opportunistic channel access in a relay network is quite different from those in a single-hop network (e.g., in references [2]–[4]), and is challenging as multi-source diversity, multi-relay diversity, and time diversity should be all exploited. Two cases are considered: *Case I with full CSI at a winner source* where a winner source in a contention has CSI of links from itself to all relays and from all relays to its destination, and *Case II with partial CSI at a winner source* where a winner source only has CSI of links from itself to all relays. In Case I, it is found that a pure-threshold strategy exists to optimize the average system throughput. There are two stopping problems in Case II, one in the main layer (for channel access of sources) and the other in the sub-layer (for channel access of relays). An intuitive strategy is proposed, which is shown to be non-optimal. We also theoretically derive an optimal strategy for Case II. In either the intuitive strategy or the optimal strategy, the first-hop stopping rule has a pure-threshold structure, while the second-hop stopping rule has a threshold determined by channel gain realization in the preceding first-hop transmission.

II. Case I: with Full CSI at a Winner Source

A. System Model

Consider K source-destination pairs aided by L relays. For transmission from a source to its destination, there is no direct link, and only one relay is selected to help with amplify-and-forward (AF) mode. The transmission power of a source and a relay is P_s and P_r, respectively. Channel reciprocity in terms of channel gain is assumed, and we denote the channel gain from the ith source to the jth relay (and vice versa) as f_{ij}, and the channel gain from the jth relay to the ith destination (and vice versa) as g_{ji}. Assume f_{ij} and g_{ji} follow a complex Gaussian distribution with mean being zero and variance being σ_f^2 and σ_g^2, respectively. Noise is assume to be Gaussian with unit variance. For source-to-destination transmission, say from the ith source to its destination aided by the jth relay, the maximal rate that can be achieved in AF mode is

$$\log_2\left(1+\frac{P_sP_r|f_{ij}|^2|g_{ji}|^2}{1+P_s|f_{ij}|^2+P_r|g_{ji}|^2}\right) \tag{1}$$

and the data transmission time from the source to the relay and from the relay to the destination are both $\frac{\tau_d}{2}$.

Channel contention of the sources is as follows. At the beginning of a time slot with duration δ, each source independently contends for the channel by sending a request-to-send (RTS) packet with probability p_0. There are three possible outcomes:

- If there is no source transmitting RTS in the time slot (with probability $(1-p_0)^K$), then all the sources continue to contend in the next time slot;
- If there are two or more sources transmitting RTS (with probability $1-(1-p_0)^K-Kp_0(1-p_0)^{K-1}$), a collision happens, and then in the next time slot after the RTS transmission all sources continue to contend;
- If there is only one source, say Source i, transmitting RTS (with probability $Kp_0(1-p_0)^{K-1}$), then Source i is called *winner* of the contention. By reception of the RTS, each relay can estimate CSI between Source i and itself. Then the first relay transmits an RTS to Destination i, and Destination i replies with a CTS, which can be received by all relays. By reception of the CTS from Destination i, each relay can estimate its CSI with Destination i. Then all relays send a CTS to Source i in turn. In the CTS from a relay to Source i, CSI of the relay with Source i and with Destination i is included. After reception of the CTSs, Source i knows CSI from itself to all relays and from all relays to its destination. Then Source i has two options: 1) Source i selects the relay that renders its maximal source-to-destination rate, i.e., Source i selects Relay $j^*=\arg\max\limits_{j\in\{1,\ldots,L\}}\left\{\log_2\left(1+\frac{P_sP_r|f_{ij}|^2|g_{ji}|^2}{1+P_s|f_{ij}|^2+P_r|g_{ji}|^2}\right)\right\}$ and transmits its packet to Relay j^* within duration $\frac{\tau_d}{2}$, then Relay j^* forwards the packet to Destination i within duration $\frac{\tau_d}{2}$; or 2) Source i gives up its transmission opportunity, and other sources can detect an idle slot after the RTS and CTS exchanges among Source i, all relays, and Destination i (i.e., that idle slot tells other sources that Source i gives up its transmission opportunity). After that a new contention is started among all the sources.

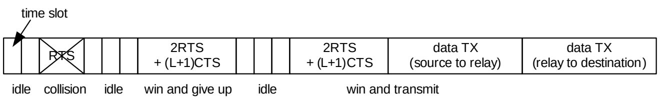

Fig. 1. An example of channel contention of sources

An example of the channel contention procedure is shown in Fig. 1. In the example, no source transmits RTS in the first two time slots. Then two or more sources transmit, which results in a collision. After three idle slots, one winner appears. However, it gives up its transmission opportunity. Then after three more idle slots (the first is used to indicate the previous winner gives up, and the other two are for two new contentions), another winner appears. After exchange of 2 RTSs and $(L+1)$ CTSs, the winner transmits its data to its selected relay and the relay forwards the data to the winner's destination.

B. Optimal Stopping Strategy

Define an *observation* as the process of channel contention among the sources until a successful contention (i.e., a winner source appears). In an observation, the number of contentions follows a geometric distribution with parameter $Kp_0(1-p_0)^{K-1}$. Among all the contentions in an observation, the last contention is successful with duration (excluding data transmission) $2\tau_{RTS}+(L+1)\tau_{CTS}$ where τ_{RTS} and τ_{CTS} are duration of an RTS and CTS, respectively, and any other contention is either an idle slot (with duration δ) or a collision (with duration τ_{RTS}). The mean of the duration of an observation is then given as $\tau_o=2\tau_{RTS}+(L+1)\tau_{CTS}+\frac{(1-p_0)^K}{Kp_0(1-p_0)^{K-1}}\cdot\delta+\frac{1-(1-p_0)^K-Kp_0(1-p_0)^{K-1}}{Kp_0(1-p_0)^{K-1}}\cdot\tau_{RTS}$.

After each observation, the winner source decides whether to continue a new observation (i.e., a new contention is started) or to stop (i.e., the winner source transmits its data). In the nth observation, let $s(n)$ denote the winner source. Then the observed information in the nth observation is $X(n):=\{s(n),f_{s(n)1}(n),...,f_{s(n)L}(n),g_{1s(n)}(n),...,g_{Ls(n)}(n)\}$. Here f and g with index (n) means the channel gain realizations at the end of the nth observation. For the nth observation, the reward Y_n is the total traffic volume that can be sent if the winner source transmits its data, which is a function of $X(n)$, and the cost T_n is the total waiting time from the first observation until the nth observation plus the data transmission time. If it is decided to stop at the Nth observation, then the average system throughput is $\frac{Y_N}{T_N}$. In the sequel, capital N is called the *stopping time*. And our objective is to find the optimal stopping time (also called optimal stopping strategy), N^*, which attains average system throughput $\sup\limits_{N\geq 0}\frac{E[Y_N]}{E[T_N]}$. Here $E[\cdot]$ means expectation. According to [10, Chapter 6], this maximal-expected-return problem can be equivalently transformed into a standard form with its reward being $(Y_N-\lambda^*T_N)$. In particular, to get N^*, we need to find an optimal strategy to reach maximal expected reward $V^*(\lambda^*)=\sup\limits_{N\geq 0}\{E[Y_N]-\lambda^*E[T_N]\}$ where λ^* satisfies $\sup\limits_{N\geq 0}\{E[Y_N]-\lambda^*E[T_N]\}=0$. Here λ^* is actually the maximal system throughput in our problem. This transformation method will be used when we solve the optimal stopping problems in our research, as shown in the sequel.

To formulate our research problem as an optimal stopping problem, in the nth observation, the reward is $Y_n=\frac{\tau_d}{2}R_n$ with the spent time denoted as $T_n=\sum\limits_{l=1}^{n}t_l+\tau_d$ where R_n is the achievable rate of the winner source in the nth observation via the best relay, given as

$$R_n=\sum_{i=1}^{K}I(\![s(n)=i]\!)\max_{j\in\{1,\ldots,L\}}\left\{\log_2\left(1+\frac{P_sP_r|f_{ij}(n)|^2|g_{ji}(n)|^2}{1+P_s|f_{ij}(n)|^2+P_r|g_{ji}(n)|^2}\right)\right\} \tag{2}$$

$I(\cdot)$ means an indicator function, and t_l is the time spent in the lth observation with mean being τ_o. For finding a strategy N^* to achieve maximal average system throughput $\frac{E[Y_N]}{E[T_N]}$, it is equivalent [10] to design a strategy which attains

$$V^*(\lambda^*)=\sup_{N\geq 0}\left\{\frac{\tau_d}{2}E[R_N]-\lambda^*E\left[\tau_d+\sum_{l=1}^{n}t_l\right]\right\} \tag{3}$$

where λ^* satisfies $V^*(\lambda^*) = 0$.

Before deriving an optimal stopping strategy N^*, two conditions should be checked which guarantee the existence of an optimal stopping strategy. Here λ can be viewed as the system throughput, while λ^* has the physical meaning of maximal system throughput.

Lemma 1: The first condition is satisfied as

$$E\left[\sup_n \left\{\frac{\tau_d}{2}R_n - \lambda\left(\tau_d + \sum_{l=1}^{n} t_l\right)\right\}\right] < \infty.$$

Proof: See Appendix I. ■

Lemma 2: The second condition is also satisfied, namely $\limsup_{n\to\infty}\left\{\frac{\tau_d}{2}R_n - \lambda\left(\tau_d + \sum_{l=1}^{n} t_l\right)\right\} = -\infty \;\; a.s.$

Proof: See Appendix II. ■

Based on Lemmas 1 and 2, the existence of an optimal stopping strategy is guaranteed.

Theorem 1: An optimal stopping strategy which achieves maximal system throughput $\sup_{N\geq 0}\frac{E[Y_N]}{E[T_N]}$ is given as: $N^* = \min\{n \geq 1 : R_n \geq 2\lambda^*\}$ where λ^* is the solution of the equation $E\left[\max\left\{\frac{\tau_d}{2}R_n - \lambda\tau_d, 0\right\}\right] = \lambda\tau_o$.

Proof: See Appendix III. ■

With threshold $2\lambda^*$ as a fixed value, our derived strategy N^* has a pure-threshold structure and achieves the maximal system throughput $\lambda^* = \frac{E[Y_{N^*}]}{E[T_{N^*}]}$. And as the solution of the equation $E\left[\max\left\{\frac{\tau_d}{2}R_n - \lambda\tau_d, 0\right\}\right] = \lambda\tau_o$, the maximal system throughput λ^* always uniquely exists. The proof is similar to that of Proposition 3.1 in [2], and thus, is omitted. The uniqueness of λ^* is consistent with its physical meaning as the maximal system throughput.

With $\{R_n\}_{n=1,\ldots,\infty}$ i.i.d. and pure-threshold structure of N^*, the stopping time denoted N determined by the optimal stopping strategy N^* follows a geometric distribution with $\text{Prob}(N = n) = F_{R_n}(2\lambda^*)^{n-1}(1 - F_{R_n}(2\lambda^*))$ where $F_{R_n}(\cdot)$ means cumulative distribution function (CDF) of R_n given in (2). Let R_{N^*} denote the achievable rate when the winner source stops. It has the CDF as $F_{R_{N^*}}(x) = I(x \geq 2\lambda^*)\frac{F_{R_n}(x) - F_{R_n}(2\lambda^*)}{1 - F_{R_n}(2\lambda^*)}$.

With the stopping time N determined by the strategy N^* geometrically distributed, the expectation of the stopping time $E[N] = \frac{1}{1 - F_{R_n}(2\lambda^*)}$ is finite. According to Wald Theorem [10] we have $E[T_N] = E[t_l]E[N] + \tau_d = \frac{\tau_o}{1 - F_{R_n}(2\lambda^*)} + \tau_d$.

In addition, the pure-threshold structure largely simplifies implementation. In details, after the nth successful channel contention, Source $s(n)$ wins the channel and calculates its achievable transmission rate R_n (which is via the best relay). If $R_n \geq 2\lambda^*$, Source $s(n)$ transmits to the best relay and the best relay helps forward to Destination $s(n)$; otherwise, Source $s(n)$ gives up the transmission opportunity and re-contends for channel access with the other $(K-1)$ sources again. In this way, the maximal average system throughput λ^* can be achieved.

Note that the value of λ^* can be calculated off-line. And the following iterative algorithm can be used to calculate λ^*:

$$\lambda_{k+1} = \lambda_k + \alpha \cdot \left\{E\left[\max\left(\frac{\tau_d}{2}R_n - \lambda_k\tau_d, 0\right)\right] - \lambda_k\tau_o\right\} \quad (4)$$

where λ_0 is a non-negative initial value and α is step size such that $\epsilon \leq \alpha \leq \frac{2-\epsilon}{\tau_o + \tau_d}$ where $\epsilon > 0$ can be arbitrarily selected.

Theorem 2: The sequence $\{\lambda_k\}$ generated by the iterative algorithm converges to λ^*.

Proof: See Appendix IV. ■

III. Case II: with Partial CSI at a Winner Source

A. System Model

In the previous section, the winner source in each observation has CSI of links from itself to all relays and from all relays to its destination. Next we consider a more practical case that the winner source in each observation has only CSI of links from itself to all relays. Since the winner source does not have CSI in the second hop, relay is not selected by the winner source. Rather, there is another channel access contention among the relays, with details as follows.

The channel contention of sources is similar to that in Section II. The difference is as follows: if there is only one source, say Source i, transmitting RTS in a contention, there is no information exchange between relays and Destination i. So Source i has only its CSI to the L relays (obtained from the L CTSs from the relays). And if Source i decides to stop, it broadcasts its packet to all relays, and then all relays start to contend for channel access, as follows. At the beginning of a time slot, each relay independently transmits an RTS with probability p_1. If no relay transmits RTS, or two or more relays transmit, then a new contention of relays is started subsequently. If only one relay, say Relay j, transmits RTS (in which information of Destination i is included), then Destination i estimates its channel gain g_{ji} with Relay j and replies with a CTS with channel gain information g_{ji} included. Then Relay j can decide 1) to stop (i.e., to forward its received packet to Destination i, and then a new source contention is started), or 2) to give up its transmission opportunity and then a new contention of relays is started.

The channel access is actually a bi-layer stopping problem: the main layer for access of sources, and the sub-layer for access of relays. In either layer, still define an observation as the process until a successful winner appears. So in the main layer, the winner source in the nth observation, denote $s(n)$, decides whether to stop based on its observed information $\{s(n), f_{s(n)1}(n), ..., f_{s(n)L}(n)\}$. In the sub-layer, the winner relay in the mth observation, denote $s(m)$, decides whether to stop based on its observed information $\{s(m), g_{s(m)s(n)}(m)\}$ and channel gain realization $f_{s(n)s(m)}(n)$ in the preceding first-hop transmission. Recall that information of $f_{s(n)j}(n)(j = 1, 2, ..., L)$ is already obtained by Relay j when Source $s(n)$ broadcasts to relays in the first hop.

Similar to Section II, the mean of duration of an observation in the main layer and the sub-layer are $\tau_o^s = \tau_{RTS} + L\tau_{CTS} + \frac{(1-p_0)^K}{Kp_0(1-p_0)^{K-1}} \cdot \delta + \frac{1-(1-p_0)^K - Kp_0(1-p_0)^{K-1}}{Kp_0(1-p_0)^{K-1}} \cdot \tau_{RTS}$ and $\tau_o^r = \tau_{RTS} + \tau_{CTS} + \frac{(1-p_1)^L}{Lp_1(1-p_1)^{L-1}} \cdot \delta + \frac{1-(1-p_1)^L - Lp_1(1-p_1)^{L-1}}{Lp_1(1-p_1)^{L-1}} \cdot \tau_{RTS}$, respectively. In this paper, superscript 's' and 'r' stand for source (first hop) and relays (second hop), respectively.

A winner source does not have CSI of links in the second hop (from relays to destinations). Rather, statistical information (e.g., channel gain distribution) of channel gains in the second hop is assumed to be available. Therefore, in the main layer, the reward (which is the source-to-destination data volume) in the nth observation is the expected reward in the sub-layer. On the other hand, in the sub-layer, the stopping problem should be conditioned on channel gain realization of the preceding first-hop transmission.

In the main layer, let n and N denote the observation index and stopping time, respectively. And in the sub-layer, let m and M denote the observation index and stopping time, respectively. We use $E_1[\cdot]$ and $E_2[\cdot]$ to present expectations on the main layer and sub-layer, respectively.

B. Intuitive Stopping strategy

An intuitive method to solve the bi-layer stopping problem is to let the sub-layer and main layer apply optimal stopping theory to maximize sub-layer and main-layer throughput, respectively.

We first consider the sub-layer. The relays already know channel gain realization $\mathcal{F} = \{f_{s(n)1}(n), ..., f_{s(n)L}(n)\}$ in the preceding first-hop transmission.[1] Then in the mth observation, the achievable rate of the winner relay, $s(m)$, is

$$R_m = \sum_{j=1}^{L} I([s(m) = j]) \log_2\left(1 + \frac{P_s P_r |f_{s(n)j}(n)|^2 |g_{js(n)}(m)|^2}{1 + P_s |f_{s(n)j}(n)|^2 + P_r |g_{js(n)}(m)|^2}\right) \tag{5}$$

The reward in the mth observation is $Y_m = \frac{\tau_d}{2} R_m$. The cost is the total waiting time until the mth observation plus the data transmission time in the second hop: $T_m = \sum_{l=1}^{m} t_l^r + \frac{\tau_d}{2}$, where t_l^r is the time used in the lth observation. Then we need to find an optimal stopping rule M^* in the sub-layer to attain the maximal $\lambda^* = \sup_{M \geq 0} \frac{E_2[Y_M|\mathcal{F}]}{E_2[T_M|\mathcal{F}]}$.

In the main layer, define T_n as the total waiting time until the nth observation plus the data transmission time in the first hop: $T_n = \sum_{l=1}^{n} t_l^s + \frac{\tau_d}{2}$, where t_l^s is the time used in the lth observation. If the stopping time is N, then the reward is $E_2[Y_{M^*}|\mathcal{F}]$, and the waiting time is $E_2[T_{M^*}|\mathcal{F}] + T_N$. Then we need to find an optimal stopping rule N^* to attain $\sup_{N \geq 0} \frac{E_1[E_2[Y_{M^*}|\mathcal{F}]]}{E_1[E_2[T_{M^*}|\mathcal{F}] + T_N]}$.

For the sub-layer optimal stopping problem, we have the following theorem.

Theorem 3: Conditioned on $\mathcal{F}$, a sub-layer optimal stopping rule achieving the maximal sub-layer throughput $\lambda^* = \sup_{M \geq 0} \frac{E_2[Y_M|\mathcal{F}]}{E_2[T_M|\mathcal{F}]}$ is given as $M^* = \min\{m \geq 1 : R_m \geq \lambda^*\}$ where λ^* is the unique solution of the equation $E_2[\max(R_m - \lambda, 0)|\mathcal{F}] = \frac{2\lambda\tau_o^r}{\tau_d}$ and always exists.

Proof: See Appendix V. ■

Define $F_{R_m}(\cdot)$ as the CDF of R_m given in (5). The sub-layer optimal stopping rule has the following property.

Corollary 1: Conditioned on $\mathcal{F}$, we have finite λ^*, $E_2[T_{M^*}|\mathcal{F}] = \frac{\tau_o^r}{1 - F_{R_m}(\lambda^*)} + \frac{\tau_d}{2}$ and $E_2[Y_{M^*}|\mathcal{F}] = \frac{\lambda^* \tau_o^r}{1 - F_{R_m}(\lambda^*)} + \frac{\lambda^* \tau_d}{2}$.

Proof: See Appendix VI. ■

Based on the acquired strategy M^* for the sub-layer stopping problem, a main-layer optimal stopping rule which achieves maximal system throughput is given in the following theorem.

Theorem 4: An optimal stopping rule for the main-layer problem is of the form $N^* = \min\{n \geq 1 : R_n^1 - \gamma^* R_n^2 \geq \gamma^* \frac{\tau_d}{2}\}$ where γ^* satisfies equation $E_1[\max\{R_n^1 - \gamma R_n^2 - \gamma\frac{\tau_d}{2}, 0\}] = \gamma\tau_o^s$, and R_n^1 and R_n^2 are given as: $R_n^1 = \lambda^* E_2[T_{M^*}|\mathcal{F}]$ and $R_n^2 = E_2[T_{M^*}|\mathcal{F}]$.[2]

Proof: See Appendix VII. ■

Note that here γ^* is actually the maximal main-layer system throughput.

[1] Note that it means Relay j knows $f_{s(n)j}(n)$, $j = 1, 2, ..., L$.

[2] Note that here M^* is the optimal stopping rule of the sub-layer conditioned on $\mathcal{F}$, and λ^* is the corresponding maximal throughput in the sub-layer stopping problem. Therefore, R_n^1 and R_n^2 are functions of $\mathcal{F}$.

From Theorem 3 and 4, it can be seen that, the intuitive optimal stopping strategy $\{N^*, M^*\}$ with $M^* = \min\{m \geq 1 : R_m \geq \lambda^*\}$ and $N^* = \min\{n \geq 1 : R_n^1 - \gamma^* R_n^2 \geq \gamma^* \frac{\tau_d}{2}\}$ has semi-pure-threshold structure. In details, with sub-layer stopping rule M^*, its threshold is not a fixed value, but depends on channel gain realization $\mathcal{F}$ in the preceding first-hop transmission. Different from M^*, the main-layer stopping rule N^* has a fixed-valued threshold $\gamma^* \frac{\tau_d}{2}$.

The intuitive stopping strategy can be implemented as follows.

For channel access of sources, upon a successful contention in the nth observation, the winner source, $s(n)$, has the information of its channel gains $\mathcal{F} = \{f_{s(n)1}(n), ..., f_{s(n)L}(n)\}$. Source $s(n)$ can calculate R_n^1 and R_n^2 by solving the sub-layer optimal stopping problem conditioned on $\mathcal{F}$. During the calculation of R_n^1 and R_n^2, Source $s(n)$ needs to calculate λ^*, which is the threshold of the sub-layer optimal stopping rule conditioned on $\mathcal{F}$. In the main-layer stopping rule, γ^* is a fixed value satisfying $E_1[\max\{R_n^1 - \gamma R_n^2 - \gamma\frac{\tau_d}{2}, 0\}] = \gamma\tau_o^s$.

- If $R_n^1 - \gamma^* R_n^2 < \gamma^* \frac{\tau_d}{2}$, Source $s(n)$ gives up its transmission opportunity and re-contend with other sources.
- If $R_n^1 - \gamma^* R_n^2 \geq \gamma^* \frac{\tau_d}{2}$, Source $s(n)$ broadcasts its data and the value of λ^* to all relays, and the channel contention of relays starts. Upon a successful contention in the mth observation, the winner relay, $s(m)$, which has information of $f_{s(n)s(m)}(n)$ in the preceding first-hop transmission, calculates its source-to-destination rate R_m. If $R_m < \lambda^*$, Relay $s(m)$ gives up its transmission opportunity, and re-contends with other relays. Otherwise, Relay $s(m)$ forwards its received data (from Source $s(n)$) to Destination $s(n)$, and the source-to-destination transmission process for the packet from Source $s(n)$ is complete, and all sources start a new contention.

Note that, the threshold in the main layer γ^* (for simplicity of presentation, the constant factor $\frac{\tau_d}{2}$ is omitted) can be calculated off-line, while the threshold λ^* in the sub-layer depends on the channel gain realization $\mathcal{F}$ in the preceding first-hop transmission, and thus, should be calculated online at Source $s(n)$, who knows $\mathcal{F}$. The following iterative algorithm can be used to calculate γ^* and λ^*.

To calculate λ^*, we have

$$\lambda_{l+1} = \lambda_l + \alpha_\lambda \cdot \left\{E_2[\max(R_m - \lambda_l, 0)|\mathcal{F}] - \frac{2\lambda_l\tau_o^r}{\tau_d}\right\} \tag{6}$$

where step size α_λ satisfies $\epsilon \leq \alpha_\lambda \leq \frac{\tau_d(2-\epsilon)}{2\tau_o^r + \tau_d}$ for a fixed positive ϵ.

For main-layer problem, to calculate γ^*, we have

$$\gamma_{k+1} = \gamma_k + \alpha_\gamma \cdot \left\{E_1\left[\max\left(R_n^1 - \gamma_k R_n^2 - \gamma_k\frac{\tau_d}{2}, 0\right)\right] - \gamma_k\tau_o^s\right\} \tag{7}$$

where step size α_γ satisfies $\epsilon \leq \alpha_\gamma \leq \frac{2(2-\epsilon)}{2E_1[R_n^2] + \tau_d + 2\tau_o^s}$ for a fixed positive ϵ.

Theorem 5: The sequence $\{\gamma_k\}$ generated by the iterative algorithm converges to γ^*.

Proof: See Appendix VIII. ■

Since the calculation of γ^* involves the calculation of λ^* conditioned on $\mathcal{F}$, convergence of $\{\gamma_k\}$ to γ^* also guarantees convergence of $\{\lambda_l\}$ to λ^*.

C. Non-optimality of Intuitive Stopping strategy

The intuitive stopping strategy $\{N^*, M^*\}$ first maximizes sub-layer system throughput and then maximizes that of main-layer system. It is interesting to notice that the intuitive stopping strategy is not optimal, as follows.

The expected system throughput can be expressed as $\frac{E_1\left[\lambda^* E_2[T_{M^*}|\mathcal{F}]\right]}{E_1[E_2[T_{M^*}|\mathcal{F}]+T_{N^*}]}$ in the intuitive stopping strategy. The sub-layer stopping rule M^* maximizes λ^*. Considering the term T_{N^*} in the expression of the expected system throughput, the sub-layer stopping rule M^*, which maximizes λ^*, may not maximize $\frac{E_1\left[\lambda^* E_2[T_{M^*}|\mathcal{F}]\right]}{E_1[E_2[T_{M^*}|\mathcal{F}]+T_{N^*}]}$.

D. Optimal Stopping strategy

Next we derive an optimal stopping strategy for the sub-layer and main layer.

For $\gamma \geq 0$ and a particular stopping rule in the sub-layer (which is conditioned on $\mathcal{F}$) denoted M, the maximal average reward achieved by main-layer optimal stopping rule can be expressed as:

$$V^*(\gamma) := \sup_{N\geq 0} \left\{ E_1 \left[E_2[Y_M|\mathcal{F}] - \gamma(E_2[T_M|\mathcal{F}] + T_N) \right] \right\} \quad (8)$$

which is equivalent to

$$V^*(\gamma) := \sup_{N\geq 0} \left\{ E_1 \left[E_2 \left[Y_M - \gamma T_M|\mathcal{F}\right] - \gamma T_N \right] \right\}. \quad (9)$$

In the expression of (9), the sub-layer affects only the term $E_2\left[Y_M - \gamma T_M|\mathcal{F}\right]$. Therefore, to increase the maximal system throughput γ^*, we need to increase $V^*(\gamma)$ (this is because $V^*(\gamma)$ is a decreasing function of γ, and γ^* is the root of $V^*(\gamma) = 0$). And to achieve the largest $V^*(\gamma)$, the sub-layer should maximize $E_2\left[Y_M - \gamma T_M|\mathcal{F}\right]$. Based on this, we have the following theorem for the sub-layer. Here we use $W^*(\gamma)$ to denote the maximal reward $\sup_{M\geq 0} E_2\left[Y_M - \gamma T_M|\mathcal{F}\right]$ in the sub-layer.

Theorem 6: For fixed $\gamma \geq 0$, an optimal stopping rule $M^*(\gamma)$ for maximizing $E_2\left[Y_M - \gamma T_M|\mathcal{F}\right]$ is of the form $M^*(\gamma) = \min\left\{m \geq 1 : \frac{\tau_d}{2} R_m \geq W^*(\gamma) + \frac{\tau_d}{2}\gamma\right\}$ where $W^*(\gamma)$ satisfies

$$E_2\left[\max\left(\frac{\tau_d}{2}R_m - \frac{\tau_d}{2}\gamma, W^*(\gamma)\right)\middle|\mathcal{F}\right] = W^*(\gamma) + \gamma\tau_o^r. \quad (10)$$

Proof: See Appendix IX. ■

Although Theorem 6 is for any particular value of γ, it is desired the sub-layer stopping rule is corresponding to the maximal system throughput γ^*. How to obtain the value of γ^* will be discussed in the main-layer stopping rule, as follows.

Theorem 7: With the sub-layer system following the strategy $M^*(\gamma^*)$, an optimal strategy to maximize the average system throughput is given as $N^* = \min\left\{n \geq 1 : W^*(\gamma^*) \geq \frac{\tau_d}{2}\gamma^*\right\}$ where γ^* satisfies $E_1\left[\max(W^*(\gamma) - \frac{\tau_d}{2}\gamma, 0)\right] = \gamma\tau_o^s$.

Proof: See Appendix X. ■

Overall, we can see that the optimal stopping strategy $\{N^*, M^*\}$ has the form of $M(\gamma^*) = \min\{m \geq 1 : \frac{\tau_d}{2} R_m \geq W^*(\gamma^*) + \frac{\tau_d}{2}\gamma^*\}$ and $N^* = \{n \geq 1 : W^*(\gamma^*) \geq \frac{\tau_d}{2}\gamma^*\}$, which achieves average system throughput maximum γ^*. Here γ^* is a fixed value satisfying $E_1\left[\max\left(W^*(\gamma) - \frac{\tau_d}{2}\gamma, 0\right)\right] = \gamma\tau_o^s$ where $W^*(\gamma)$ is an unique root of $E_2\left[\max\left(\frac{\tau_d}{2}R_m - \frac{\tau_d}{2}\gamma, W^*(\gamma)\right)|\mathcal{F}\right] = W^*(\gamma) + \gamma\tau_o^r$.

Note that the optimal stopping strategy $\{N^*, M^*\}$ has also semi-pure-threshold structure, as in the main layer the threshold $\frac{\tau_d}{2}\gamma^*$ is a fixed value, while in the sub-layer the threshold $W^*(\gamma^*) + \frac{\tau_d}{2}\gamma^*$ is conditioned on the channel gain realization in the preceding first-hop transmission.

The optimal stopping strategy can be carried out as follows. For channel access of sources, upon a successful contention in the nth observation, the winner source, $s(n)$, has the information of its channel gains $\mathcal{F} = \{f_{s(n)1}(n), ..., f_{s(n)L}(n)\}$. Source $s(n)$ can calculate $W^*(\gamma^*)$ by solving the sub-layer optimal stopping problem conditioned on $\mathcal{F}$.

- If $W^*(\gamma^*) < \frac{\tau_d}{2}\gamma^*$, Source $s(n)$ gives up its transmission opportunity and re-contend with other sources.
- If $W^*(\gamma^*) \geq \frac{\tau_d}{2}\gamma^*$, Source $s(n)$ broadcasts its data and also the value of $W^*(\gamma^*) + \frac{\tau_d}{2}\gamma^*$ to all relays, and channel contention of relays starts. Upon a successful contention in the mth observation, the winner relay, $s(m)$, who has information of $f_{s(n)s(m)}(n)$ in the preceding first-hop transmission, calculates its source-to-destination rate R_m. If $\frac{\tau_d}{2}R_m < W^*(\gamma^*) + \frac{\tau_d}{2}\gamma^*$, Relay $s(m)$ gives up its transmission opportunity, and re-contends with other relays; otherwise, Relay $s(m)$ forwards its received data (from Source $s(n)$ in the preceding first-hop transmission) to Destination $s(n)$, and the source-to-destination transmission process for the packet from Source $s(n)$ is complete, and all sources start a new contention.

Similar to the intuitive stopping strategy, the threshold in the main layer γ^* (with the constant factor $\frac{\tau_d}{2}$ omitted) can be calculated off-line, while the threshold $W^*(\gamma^*)$ (with the constant $\frac{\tau_d}{2}\gamma^*$ omitted) is dependent on $\mathcal{F}$, and thus, should be calculated online at Source $s(n)$, who knows $\mathcal{F}$. The following iterative algorithm can be used to calculate γ^* and $W^*(\gamma^*)$.

In the main layer, iterative algorithm is given below: $\gamma_{k+1} = \gamma_k + \alpha_\gamma\left(E_1\left[\max(W^*(\gamma_k) - \frac{\tau_d}{2}\gamma_k, 0)\right] - \gamma_k\tau_o^s\right)$ where step size α_γ satisfies $\epsilon \leq \alpha_\gamma \leq \frac{2-\epsilon}{\tau_d + \tau_o^s + \tau_o^r \cdot E_1\left[\frac{1}{1-F_{R_m}(2W^*(0)/\tau_d)}\right]}$ for a fixed positive ϵ.

For each iteration of main layer, $W^*(\gamma_k)$ can be calculate below:

$$\begin{aligned} W_{l+1}(\gamma_k) &= W_l(\gamma_k) \\ &+ \alpha_W\left(E_2\left[\max\left(\tfrac{\tau_d}{2}R_m - \tfrac{\tau_d}{2}\gamma_k - W_l(\gamma_k), 0\right)|\mathcal{F}\right] - \gamma_k\tau_o^r\right) \end{aligned}$$

where step size α_W satisfies $\epsilon \leq \alpha_W \leq 2 - \epsilon$ for a fixed positive ϵ.

Theorem 8: The sequence $\{\gamma_k\}$ generated by the iterative algorithm converges to γ^*.

Its proof is similar to that of Theorem 5, with details omitted.

IV. Performance Evaluation

We use computer simulation to validate our analysis. Consider 5 sources and 4 relays in our network. Channels from sources to relays experience i.i.d. Rayleigh fading while channels from relays to destinations also experience i.i.d. Rayleigh fading. The channel contention parameters are set as: $p_0 = p_1 = 0.3$, $\delta = 20\ \mu$s, $\tau_{RTS} = \tau_{CTS} = 40\ \mu$s, $\tau_d = 2$ ms. Consider the scenario that the average received signal-to-noise ratio (SNR) in the first and the second hops are the same. When the average SNR varies from 0.5 to 10, Fig. 2 shows the numerically calculated (shown as "analytical" in Fig. 2) and simulated (shown as "sim" in Fig. 2) system throughput of Case I, Case II with intuitive stopping strategy, and Case II with optimal stopping strategy. It can be seen that the analytical and simulation results match well with each other, which confirms the accuracy of the analysis of our three strategies.

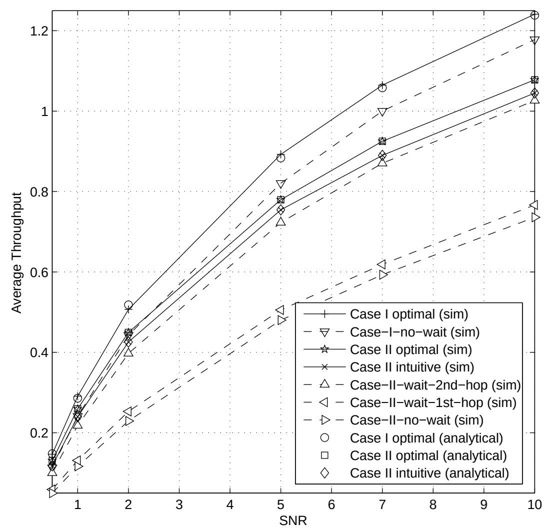

Fig. 2. Comparison of analytical and simulation results of our three strategies.

Next we perform comparison with alternative strategies. In particular, we consider four alternative strategies: 1) *Case-I-no-wait* strategy: a winner source has full CSI, and always transmits (i.e., always stop and does not wait); 2) *Case-II-no-wait* strategy: a winner source has partial CSI, and a winner source or relay always transmits; 3) *Case-II-wait-1st-hop* strategy: a winner source has partial CSI, and a winner source applies optimal stopping rule, while a winner relay always transmits; 4) *Case-II-wait-2nd-hop* strategy: a winner source has partial CSI, and a winner source always transmits while a winner relay applies optimal stopping rule.

Fig. 2 also shows a comparison of our three strategies and the four alternative strategies. It can be seen that our optimal strategy in Case I and the Case-I-no-wait strategy have better performance than others. This is because of the full CSI at a winner source. The optimal stopping strategy exploits the time diversity of sources (by deciding whether to stop or not) and multi-user diversity of relays (by selecting the best relay). On the other hand, Case-I-no-wait strategy exploits only the multi-user diversity of relays, and therefore, has worse performance than the optimal strategy.

In Case II, among the five strategies, our intuitive strategy and our optimal strategy are the best, with the former having some performance loss compared with the latter, as expected. For the two alternative strategies with a stopping rule applied in one hop, i.e., Case-II-wait-1st-hop and Case-II-wait-2nd-hop, they have a big performance gap, and Case-II-wait-1st-hop strategy is close to the Case-II-no-wait strategy (the worst strategy) while Case-II-wait-2nd-hop strategy is close to our intuitive strategy. The reason is as follows. In Case-II-wait-1st-hop strategy, the threshold in the stopping rule (which is in the first hop) is based on only statistical information of second-hop channels. On the other hand, in Case-II-wait-2nd-hop strategy, the threshold in the stopping rule (which is in the second hop) can be determined based on exact CSI in the first hop. Compared with statistical channel gain information, the exact CSI can help select the *best* threshold.

V. Conclusion

In a wireless relay network, the sources and relays all experience fading. It is desired to exploit the multi-source diversity, multi-relay diversity, and time diversity. To achieve this, opportunistic channel access is needed, which is investigated in our research in a distributed structure. For the two considered cases (with a winner source having or not having CSI of the second hop), we derive optimal stopping strategies for opportunistic channel access. Further research may include the cases with quantized CSI and with quality-of-service constraints.

Appendix I
Proof of Lemma 1

The mean of achievable transmission rate at the nth observation is

$$E[R_n]=\sum_{i=1}^{K}\frac{1}{K}E\left[\max_{j\in\{1,\dots,L\}}\left\{\log_2\left(1+\frac{P_sP_r|f_{ij}(n)|^2|g_{ji}(n)|^2}{1+P_s|f_{ij}(n)|^2+P_r|g_{ji}(n)|^2}\right)\right\}\right].$$

Since f_{ij} and g_{ji} follow complex Gaussian distribution with mean being zero and variance being σ_f^2 and σ_g^2, respectively, we have $E[|f_{ij}|^2]=\sigma_f^2$and $E[|g_{ji}|^2]=\sigma_g^2$. Then we have

$$E[R_n]<\sum_{i=1}^{K}\frac{1}{K}E\left[\sum_{j=1}^{L}\log_2\left(1+\frac{P_sP_r|f_{ij}(n)|^2|g_{ji}(n)|^2}{1+P_s|f_{ij}(n)|^2+P_r|g_{ji}(n)|^2}\right)\right]$$
$$\overset{(a)}{\le}\sum_{i=1}^{K}\frac{1}{K}\sum_{j=1}^{L}\frac{1}{\ln 2}E(P_s|f_{ij}|^2)E(P_r|g_{ji}|^2)$$
$$=\frac{1}{\ln 2}LP_sP_r\sigma_f^2\sigma_g^2<\infty \tag{11}$$
$$E[R_n^2]<\sum_{i=1}^{K}\frac{1}{K}E\left[\sum_{j=1}^{L}\log_2^2\left(1+\frac{P_sP_r|f_{ij}(n)|^2|g_{ji}(n)|^2}{1+P_s|f_{ij}(n)|^2+P_r|g_{ji}(n)|^2}\right)\right]$$
$$\overset{(b)}{\le}\sum_{i=1}^{K}\frac{1}{K}\sum_{j=1}^{L}\frac{1}{(\ln 2)^2}E[P_s^2|f_{ij}|^4]E[P_r^2|g_{ji}|^4]$$
$$=\frac{4}{(\ln 2)^2}LP_s^2P_r^2\sigma_f^4\sigma_g^4<\infty \tag{12}$$

where (a) and (b) come from the fact that for $x,y\ge 0$, we have$\log_2\left(1+\frac{xy}{1+x+y}\right)\le\frac{\frac{xy}{1+x+y}}{\ln 2}\le\frac{xy}{\ln 2}$. Based on [10], from $E[R_n]<\infty$, we have $\sup_n\{\frac{\tau_d}{2}R_n-nc\}<\infty\ a.s.$; from $E[R_n^2]<\infty$, we have $E\left[\sup_n\{\frac{\tau_d}{2}R_n-nc\}\right]<\infty$. By decomposition similar to (43) in [2], the first condition for existence of an optimal stopping strategy can be proved.

Appendix II
Proof of Lemma 2

Using a similar method to that in [2], for $0<\varepsilon<\tau_o$, we have the following decomposition

$$\frac{\tau_d}{2}R_n-\lambda\left(\tau_d+\sum_{l=1}^{n}t_l\right)$$
$$=\left[\frac{\tau_d}{2}R_n-n\lambda\left(\tau_o-\varepsilon\right)-\tau_d\lambda\right]+\left[\lambda\sum_{l=1}^{n}\left(\tau_o-\varepsilon-t_l\right)\right]. \tag{13}$$

From [10, Theorem 4.1], $\tau_o-\varepsilon>0$, and (12), we have

$$\lim_{n\to\infty}\left[\frac{\tau_d}{2}R_n-n\lambda\left(\tau_o-\varepsilon\right)\right]=-\infty\ a.s. \tag{14}$$

Next we focus on the second component on the right-hand side of (13). Using [10, Theorem 4.2], when $E\left[\tau_o-\varepsilon-t_l\right]<$

0 holds, $E\left[\sup_{n\geq 0}\sum_{l=1}^{n}(\tau_o-\varepsilon-t_l)\right] < \infty$ if and only if $E\left[(\tau_o-\varepsilon-t_l)^+\right]^2<\infty$, where $(\tau_o-\varepsilon-t_l)^+=\max(\tau_o-\varepsilon-t_l,0)$. We have $E\left[\tau_o-\varepsilon-t_l\right] = \tau_o-\varepsilon-\tau_o = -\varepsilon < 0$, and $E\left[(\tau_o-\varepsilon-t_l)^+\right]^2 \leq E\left[\tau_o-\varepsilon-t_l\right]^2 = (\tau_o-\varepsilon)^2 - 2(\tau_o-\varepsilon)\tau_o + E[t_l^2] < \infty$ (since $E[t_l^2]$ can be shown to be finite). As a result, we have

$$E\left[\limsup_{n\to\infty}\left\{\sum_{l=1}^{n}(\tau_o-\varepsilon-t_l)\right\}\right]\leq E\left[\sup_{n\geq 0}\left\{\sum_{l=1}^{n}(\tau_o-\varepsilon-t_l)\right\}\right]<\infty$$

which leads to

$$\limsup_{n\to\infty}\left\{\sum_{l=1}^{n}(\tau_o-\varepsilon-t_l)\right\}<\infty\ a.s. \tag{15}$$

From (13), (14), and (15), we have

$$\limsup_{n\to\infty}\left\{\frac{\tau_d}{2}R_n-\lambda\left(\tau_d+\sum_{l=1}^{n}t_l\right)\right\}=-\infty\ a.s.$$

Appendix III
Proof of Theorem 1

Recall that to maximize throughput $\frac{E[Y_N]}{E[T_N]}$, we need to achieve $V^*(\lambda^*)=\sup_{N\geq 0}\{E[Y_N]-\lambda^*E[T_N]\}$ where λ^* satisfies $V^*(\lambda^*)=0$. Here λ^* is actually maximal average throughput. Therefore we need to know expression of $V^*(\lambda)$.

For $\lambda\geq 0$, the stopping strategy which achieves maximal reward $V^*(\lambda)$ can be described as $N^*=\min\{n\geq 1:\frac{\tau_d}{2}R_n-\lambda\tau_d\geq V^*(\lambda)\}$, where $V^*(\lambda)$ is determined by optimality equation

$$V_n^*=\max\left\{\frac{\tau_d}{2}R_n-\lambda\tau_d-\lambda\sum_{l=1}^{n}t_l,E[V_{n+1}^*|X(1),...,X(n)]\right\}. \tag{16}$$

Here V_n^* represents expected reward if the winner source at the nth observation does not stop and the optimal stopping strategy is followed starting from the $(n+1)$th observation. Since $V_n^*=V^*(\lambda)-\lambda\sum_{l=1}^{n-1}t_l$, after taking expectation over both sides of (16) we have:

$$E\left[V^*(\lambda)-\lambda\sum_{l=1}^{n-1}t_l\right]=E\left[\max\left\{\frac{\tau_d}{2}R_n-\lambda\tau_d-\lambda\sum_{l=1}^{n}t_l,V^*(\lambda)-\lambda\sum_{l=1}^{n}t_l\right\}\right]$$

which leads to $V^*(\lambda)=E\left[\max\left\{\frac{\tau_d}{2}R_n-\lambda\tau_d,V^*(\lambda)\right\}-\lambda t_n\right]$.

Setting $V^*(\lambda^*)=0$, the maximal throughput λ^* satisfies

$$E\left[\max\left\{\frac{\tau_d}{2}R_n-\lambda^*\tau_d,0\right\}\right]=\lambda^*E\left[t_n\right]=\lambda^*\tau_o. \tag{17}$$

And an optimal stopping strategy which maximizes throughput is of form

$$N^*=\min\left\{n\geq 1:\frac{\tau_d}{2}R_n-\lambda^*\tau_d\geq V^*(\lambda^*)\right\}=\min\{n\geq 1:R_n\geq 2\lambda^*\}.$$

Appendix IV
Proof of Theorem 2

By Proposition 1.2.3 in [11], we take $\bigtriangledown f(\lambda)=\lambda\tau_o-E\left\{\max\left\{\frac{\tau_d}{2}R_n-\lambda\tau_d,0\right\}\right\}$ where $\bigtriangledown f(\cdot)$ means gradient of function $f(\cdot)$. Then there is one unique solution satisfying $\bigtriangledown f(\lambda)=0$, which is λ^* in our optimal stopping problem.

For the Lipschitz continuity condition, we have $|\bigtriangledown f(x)-\bigtriangledown f(y)| = \left|x\tau_o-E\left[\max\left\{\frac{\tau_d}{2}R_n-x\tau_d,0\right\}\right]-y\tau_o+E\left[\max\left\{\frac{\tau_d}{2}R_n-y\tau_d,0\right\}\right]\right|\leq(\tau_o+\tau_d)\,|x-y|=C|x-y|$ where $C=\tau_o+\tau_d$. This means the Lipschitz continuity condition in Proposition 1.2.3 in [11] is satisfied.

Define directions d_k as the steepest descent direction

$$d_k\triangleq-\bigtriangledown f(\lambda_k)=E\left\{\max\left\{\frac{\tau_d}{2}R_n-\lambda_k\tau_d,0\right\}\right\}-\lambda_k\tau_o.$$

It can be proved that $\{d_k\}$ is gradient related.

Then based on Proposition 1.2.3 in [11], a generated sequence $\{\lambda_k\}$ by

$$\lambda_{k+1}=\lambda_k+\alpha_k d_k\ \text{ when }\ \epsilon\leq\alpha\leq(2-\epsilon)\cdot\frac{1}{C}=\frac{2-\epsilon}{\tau_o+\tau_d} \tag{18}$$

converges to the stationary point of f, which is λ^*.

Iteration form (18) is actually the iteration form (4).

Appendix V
Proof of Theorem 3

We first prove the finiteness of $E[R_m^2]$.

$$\begin{aligned}E_2[R_m^2|\mathcal{F}]&=E_2\Big[\sum_{j=1}^{L}I\left([s(m)=j]\right)\\&\cdot\log_2^2\left(1+\frac{P_sP_r|f_{s(n)j}(n)|^2|g_{js(n)}(m)|^2}{1+P_s|f_{s(n)j}(n)|^2+P_r|g_{js(n)}(m)|^2}\right)|\mathcal{F}\Big]\\&=\sum_{j=1}^{L}\frac{1}{L}E_2\left[\log_2^2\left(1+\frac{P_sP_r|f_{s(n)j}(n)|^2|g_{js(n)}(m)|^2}{1+P_s|f_{s(n)j}(n)|^2+P_r|g_{js(n)}(m)|^2}\right)|\mathcal{F}\right]\\&\overset{(c)}{\leq}\sum_{j=1}^{L}\frac{1}{L}\frac{1}{(\ln 2)^2}P_r^2E[|g_{js(n)}|^4]=\sum_{j=1}^{L}\frac{1}{L}\frac{2}{(\ln 2)^2}P_r^2\sigma_g^4<\infty\end{aligned} \tag{19}$$

where (c) comes from the fact that for $x,y\geq 0$, we have

$$\log_2\left(1+\frac{xy}{1+x+y}\right)\leq\frac{\frac{xy}{1+x+y}}{\ln 2}\leq\frac{y}{\ln 2}. \tag{20}$$

With the finite property of $E_2[R_m^2|\mathcal{F}]$, we have

$$E_2[R_m|\mathcal{F}]<\infty. \tag{21}$$

Then similar to proofs of Lemmas 1 and 2, the existence conditions of an optimal stopping rule in the sub-layer can be proved. With the reward as $\frac{\tau_d}{2}R_m-\lambda\frac{\tau_d}{2}-\lambda\sum_{l=1}^{m}t_l^r$, by following a similar way to that in proof of Theorem 1, we can obtain an optimal stopping rule for the sub-layer as the form: $M^*=\min\{m\geq 1:R_m\geq\lambda^*\}$ where λ^* satisfies the equality $E_2\left[\max\left(R_m-\lambda,0\right)|\mathcal{F}\right]=\frac{2\lambda\tau_o^r}{\tau_d}$. And the existence and uniqueness of λ^* can be straightforwardly proved.

Appendix VI
Proof of Corollary 1

$E_2\left[\max\left(R_m-\lambda,0\right)|\mathcal{F}\right]$ is a decreasing function from $+\infty$ to 0 with respect to λ, and $\frac{\lambda\tau_o^r}{\tau_d}$ linearly increases with respect to λ. Hence, the uniqueness and non-negativeness of the root λ^* are guaranteed, since λ^* is the root of $E_2\left[\max\left(R_m-\lambda,0\right)|\mathcal{F}\right]=\frac{2\lambda\tau_o^r}{\tau_d}$. Further, we have $\frac{2\lambda^*\tau_o^r}{\tau_d}=E_2[\max(R_m-\lambda^*,0)\,|\mathcal{F}]\leq E_2[R_m|\mathcal{F}]\overset{\text{from (21)}}{<}\infty$ which leads to $\lambda^*<\infty$.

Stopping time M in the sub-layer is geometrically distributed. Then according to Wald Theorem [10], $E_2[T_{M^*}|\mathcal{F}]=\frac{\tau_o^r}{1-F_{R_m}(\lambda^*)}+\frac{\tau_d}{2}$. Also, we have $E_2[Y_{M^*}|\mathcal{F}]=\frac{\lambda^*\tau_o^r}{1-F_{R_m}(\lambda^*)}+\frac{\lambda^*\tau_d}{2}$.

Appendix VII
Proof of Theorem 4

Recall that to maximize throughput $\frac{E_1[\lambda^* E_2[T_{M^*}|\mathcal{F}]]}{E_1[E_2[T_{M^*}|\mathcal{F}]+T_{N^*}]}$, we need to achieve

$$V^*(\gamma^*)=\sup_{N\geq 0}\left\{E_1\left[\lambda^* E_2[T_{M^*}|\mathcal{F}]-\gamma^*\left(E_2[T_{M^*}|\mathcal{F}]+\frac{\tau_d}{2}+\sum_{l=1}^{N}t_l^s\right)\right]\right\}$$

where γ^* satisfies $V^*(\gamma^*)=0$. To derive an optimal stopping rule, we first need to calculate $V^*(\gamma)$.

For $\gamma \geq 0$, an optimal stopping rule $N^*(\gamma) = \min\{n\geq 1: R_n^1-\gamma\frac{\tau_d}{2}-\gamma R_n^2 \geq V^*(\gamma)\}$ exists (the proof is omitted) and achieves $V^*(\gamma)$ that satisfies the equation $E_1\left[V^*(\gamma)-\gamma\sum_{l=1}^{n-1}t_l^s\right] = E_1\left[\max\left\{R_n^1-\gamma\frac{\tau_d}{2}-\gamma R_n^2-\gamma\sum_{l=1}^{n}t_l^s, V^*(\gamma)-\gamma\sum_{l=1}^{n}t_l^s\right\}\right]$, which leads to $V^*(\gamma)=E_1\left[\max\{R_n^1-\gamma\frac{\tau_d}{2}-\gamma R_n^2, V^*(\gamma)\}-\gamma t_n^s\right]$.

Setting $V^*(\gamma^*)=0$, the maximal throughput γ^* satisfies

$$E_1\left[\max\left\{R_n^1-\gamma^* R_n^2-\gamma^*\frac{\tau_d}{2}, 0\right\}\right]=\tau_o^s. \quad (22)$$

And an optimal stopping rule which achieves γ^* is

$$N^*=\min\left\{n\geq 1: R_n^1-\gamma^* R_n^2 \geq \gamma^*\frac{\tau_d}{2}\right\}.$$

Appendix VIII
Proof of Theorem 5

Similar to proof of Theorem 2, Lipschitz continuity conditions in the sub-layer and main layer are derived as follows.

In the sub-layer problem, we have:

$$\left|\frac{2x\tau_o^r}{\tau_d}-E_2[\max(R_m-x,0)\,|\mathcal{F}]-\frac{2y\tau_o^r}{\tau_d}+E_2[\max(R_m-y,0)|\mathcal{F}]\right| \leq \left(\frac{2\tau_o^r}{\tau_d}+1\right)|x-y|.$$

Step-size α_λ is fixed, which satisfies $\epsilon \leq \alpha_\lambda \leq \frac{\tau_d(2-\epsilon)}{2\tau_o^r+\tau_d}$.

In the main-layer problem, we have:

$$\begin{aligned}&\left|x\tau_o^s - E_1\left[\max\left(R_n^1-xR_n^2-x\frac{\tau_d}{2},0\right)\right]-y\tau_o^s\right.\\&\left.+E_1\left[\max\left(R_n^1-yR_n^2-y\frac{\tau_d}{2},0\right)\right]\right|\\&\leq\left(\tau_o^s+E_1[R_n^2]+\frac{\tau_d}{2}\right)|x-y|.\end{aligned}$$

Step-size α_γ is fixed, which satisfies $\epsilon\leq\alpha_\gamma \leq \frac{2(2-\epsilon)}{2E_1[R_n^2]+\tau_d+2\tau_o^s}$.

Appendix IX
Proof of Theorem 6

Similar to proof of (19), we have $E_2[(R_m)^2] < \infty$, which guarantees existence of an optimal stopping rule. To achieve maximal reward $W^*(\gamma) = \sup_{M\geq 0}\{E_2\left[Y_M-\gamma T_M|\mathcal{F}\right]\}$, an optimal stopping rule takes the form $M^*(\gamma) = \min\left\{m\geq 1: \frac{\tau_d}{2}R_m \geq W^*(\gamma)+\frac{\tau_d}{2}\gamma\right\}$ where $W^*(\gamma)$ satisfies the equation $E_2\left[\max\left(\frac{\tau_d}{2}R_m-\frac{\tau_d}{2}\gamma, W^*(\gamma)\right)\middle|\mathcal{F}\right] = W^*(\gamma)+\gamma\tau_o^r$. By rearranging terms, we have

$$E_2\left[\max\left(\frac{\tau_d}{2}R_m-\frac{\tau_d}{2}\gamma-W^*(\gamma),0\right)\middle|\mathcal{F}\right]=\gamma\tau_o^r. \quad (23)$$

Since the left hand side of (23) continuously decreases from ∞ to 0 with $W^*(\gamma)$, while the right hand side is a constant, a finite unique solution $W^*(\gamma)$ always exists.

Appendix X
Proof of Theorem 7

Recall that to maximize throughput $\frac{E_1[E_2[Y_{M^*}|\mathcal{F}]]}{E_1[E_2[T_{M^*}|\mathcal{F}]+T_{N^*}]}$, we need to achieve $V^*(\gamma^*)=\sup_{N\geq 0}\left\{E_1\left[W^*(\gamma^*)-\gamma^*\left(\frac{\tau_d}{2}+\sum_{l=1}^{N}t_l^s\right)\right]\right\}$ where γ^* satisfies $V^*(\gamma^*)=0$.

To derive an optimal stopping rule, we first need to calculate $V^*(\gamma)$. For $\gamma \geq 0$, an optimal stopping rule N^* to achieve $V^*(\gamma)$ exists which is proved as follows.

Similar to (19), conditioned on $\mathcal{F}$, we have

$$\begin{aligned}R_m &\leq \max_{j\in\{1,\ldots,L\}}\left\{\log_2\left(1+\frac{P_sP_r|f_{s(n)j}(n)|^2|g_{js(n)}(m)|^2}{1+P_s|f_{s(n)j}(n)|^2+P_r|g_{js(n)}(m)|^2}\right)\right\}\\&\leq \frac{P_s}{\ln 2}\max_{j\in\{1,\ldots,L\}}|f_{s(n)j}(n)|^2. \quad (24)\end{aligned}$$

From (23), we have $W^*(\gamma) < \frac{\tau_d}{2}\cdot\frac{P_s}{\ln 2}\max_{j\in\{1,\ldots,L\}}|f_{s(n)j}(n)|^2$, which leads to $E_1[(W^*(\gamma))^2]<\infty$ by integrating $(W^*(\gamma))^2$ over joint PDF of $\{|f_{s(n)1}(n)|^2,\ldots,|f_{s(n)L}(n)|^2\}$ which are independently and exponentially distributed i.i.d. random variables.

Similar to proofs of Lemmas 1 and 2, $E_1\left[W^*(\gamma)^2\right] < \infty$ and $E_1[(t_l^s)^2] < \infty$ guarantee existence of an optimal stopping rule. By using optimal stopping rule $N^*(\gamma)=\min\{n\geq 1: W^*(\gamma)-\frac{\tau_d}{2}\gamma\geq V^*(\gamma)\}$, we can achieve $V^*(\gamma)$ which satisfies the equation as $E_1\left[\max(W^*(\gamma)-\frac{\tau_d}{2}\gamma, V^*(\gamma))\right] = V^*(\gamma)+\gamma\tau_o^s$. Setting $V^*(\gamma)=0$, the maximal throughput γ^* satisfies $E_1\left[\max(W^*(\gamma^*)-\frac{\tau_d}{2}\gamma^*,0)\right]=\gamma^*\tau_o^s$. And an optimal stopping rule which maximizes the throughput is $N^* = \min\left\{n\geq 1: W^*(\gamma^*)\geq \frac{\tau_d}{2}\gamma^*\right\}$.

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