Cognitive Radio with Imperfect Spectrum Sensing: The Optimal Set of Channels to Sense

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***Abstract*— Opportunistic channel sensing and access problem of a secondary user with multiple potential primary channels is investigated. The secondary user can sense a limited number of channels, and channel sensing is imperfect. If the secondary user can access all channels sensed free, it is proved that the secondary user should sense the channels with the largest rewards, where the reward of a channel is the reward that the secondary user can acquire if it senses the channel and accesses the channel if the channel is sensed free. If the secondary user can access only a limited number of sensed-free channels, in general it may not be optimal to sense the channels with the largest rewards. However and interestingly, for some special cases (for example, when all the channels have the same detection probability), simple rules are given for the optimal selection of channels to sense. For the general case, methods are given to reduce the searching complexity for the optimal set of channels to sense.**

***Index Terms*— Cognitive radio, opportunistic access, spectrum sensing.**

I. Introduction

In a cognitive radio network, secondary users need to first detect possible primary activities, usually by spectrum sensing, and then access the spectrum if no primary activities are detected. When there are multiple potential primary spectrum bands (called *primary channels*) for a secondary user, the secondary user needs to decide which channel(s) to sense and access, and how they are sensed and accessed. In [1], at the beginning of a time slot, a secondary user sequentially senses the channels one after another, until a free channel or a free channel with good channel quality is found. Then the secondary user transmits in the channel within the remaining duration of the slot. The optimal order for sensing the channels is derived. In [2], sensing order when the channel gain information is known is studied. In [3], sensing order is jointly designed with sensing strategy (to specify when to stop sensing and start secondary transmission) and power allocation, to maximize energy efficiency. In [4], a secondary user senses one channel or senses multiple channels simultaneously at the beginning of a time slot, and accesses sensed-free channel(s) in the remaining duration of the slot. The channel sensing and access problem is formulated as a multi-armed bandit problem. Sensing time optimization is investigated in [5] for a single-channel case and in [6] for a multiple-channel case. Aggregated opportunistic throughput is maximized in [7].

In the above existing works, it is assumed that the secondary user can access all sensed-free channels. In this research, we consider a system when a secondary user simultaneously senses a limited number of channels (e.g., by wideband spectrum sensing technique discussed in [8]) at the beginning of a time slot and uses the remaining duration in the slot for data transmission. Different from existing works, spectrum sensing is imperfect, and the secondary user can only access up to a limited number of sensed-free channels in a slot.[1] We aim at deciding which channels to sense so that the secondary user can gain the maximal reward. We find that, when the secondary user can access all sensed-free channels, the secondary user should sense the channels that have the largest rewards (the definition of reward of a channel is to be given in Section II). However and interestingly, if the secondary user can only access up to a limited number of sensed-free channels at a time, it may not be optimal to sense the channels with the largest rewards, and thus, exhaustive search may be needed to find the optimal set of channels to sense. Some simple rules are given for the optimal selection of channel set to sense in some special cases. And a property is also given for the general case, which helps to simplify the search for the optimal channel set to sense.

II. System Model

Consider a secondary user with N potential primary channels, denoted as Channel 1, Channel 2, ..., Channel N. Similar to [1], [2], [4], [5], time is partitioned into slots, each with fixed duration T. Each slot is further divided into a sensing period with duration τ and a data transmission period with duration $(T-\tau)$. The secondary user can sense M $(\leq N)$ channels simultaneously in the sensing period, and subsequently in the data transmission period it can access up to K $(\leq M)$ channels that are sensed free. To protect primary users, the secondary user is not permitted to access channels sensed busy. Since sensing is imperfect, for sensing of Channel i $(i=1,2,...,N)$, let P_d^i denote the detection probability (i.e., probability of detecting primary activities that do exist), and P_f^i denote the false alarm probability (i.e., the probability of mistakenly estimating presence of primary activities that actually do not exist).

At each slot, say Slot j, Channel i $(i=1,2,...,N)$ is free with probability θ_i. Let $S_i(j)=1$ and $S_i(j)=0$ denote that Channel i is free and busy, respectively; and if Channel i is sensed, let $X_i(j)=1$ and $X_i(j)=0$ denote that Channel i is sensed to be free and busy, respectively. The probability of Channel i being sensed free is denoted $f(\theta_i)=\theta_i(1-P_f^i)+(1-\theta_i)(1-P_d^i)=\theta_i(P_d^i-P_f^i)+1-P_d^i$.

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[1]For example, in a voice conversation, the secondary user may only have limited packets to send during a time period. As another example, as shown in [9], due to energy constraint, the secondary user may not be able to access all sensed-free channels.

In a slot, if the secondary user accesses a channel that is sensed free, it can transmit B bits in the data transmission period. Define *reward* as the *successfully* transmitted bits in a slot. So if there is a missed detection of primary activities in an accessed channel, then the reward is 0. For a channel, say Channel i, define its reward as the expected reward the secondary user can acquire if the secondary user senses Channel i and accesses it if it is sensed free. Also define *conditional reward* of Channel i as the expected reward to access Channel i conditioned on that Channel i is sensed free. So for Channel i, its reward is given as $B\theta_i(1-P_f^i)$, and its conditional reward is given as $BE[S_i(j)|X_i(j)=1] = \frac{B\theta_i(1-P_f^i)}{f(\theta_i)}$, where $E[\cdot]$ denotes expectation.

Since the secondary user does not sense all the channels, and may not access all channels sensed free, the secondary user has two decisions: which channels to sense, and which sensed-free channels to access. For the second decision, it is apparent that: if the number of channels sensed free is not more than K, then all channels sensed free are accessed; otherwise, the secondary user should access the K channels with the K largest conditional rewards. Therefore, in this research, we focus on the first decision of the secondary user: which M channels to sense. Our objective is to maximize the expected reward of the secondary user in a slot (say Slot j), given as:

$$\max_{\mathcal{M}\subseteq\mathcal{N}} R_{\mathcal{M}} \triangleq E\Big[B \max_{\mathcal{K}\subseteq\mathcal{I}_{\mathcal{M}}} \sum_{i\in\mathcal{K}} E\left[S_i(j)|X_i(j)=1\right]\Big] \quad (1)$$

where $\mathcal{N} = \{1,2,...,N\}$, $\mathcal{M}$ denotes the set of channels to sense, $\mathcal{I}_{\mathcal{M}}$ is the set of channels that are sensed free if channels in $\mathcal{M}$ are sensed, $\mathcal{K}$ denotes the set of channels to access, $R_{\mathcal{M}}$ denotes the *reward of* $\mathcal{M}$, defined as expected reward of the secondary user if it senses the channels in set $\mathcal{M}$ and accesses up to K sensed-free channels with the largest conditional rewards. In (1), the outer expectation is for $\mathcal{I}_{\mathcal{M}}$, while the inner expectation is for S_i, $i\in\mathcal{K}$. For (1) and subsequent equations, we have $|\mathcal{M}| = M$, and $|\mathcal{K}| \le K$, where $|\cdot|$ means the cardinality of a set.

III. Optimal Selection of Channels to Sense

We consider two cases: full channel access with $K = M$ (i.e., the secondary user accesses all channels that are sensed free), and partial channel access with $K < M$.

A. Full Channel Access ($K = M$)

Full channel access also means $\mathcal{K} = \mathcal{I}_{\mathcal{M}}$. Then we have the following theorem for the optimal set of channels to sense.

Theorem 1: The optimal set of channels to sense, denoted as $\mathcal{M}^*$, consists of M channels with the M largest values of $\theta_i(1-P_f^i)$, $i\in\mathcal{N}$.

Proof: Since $\mathcal{K} = \mathcal{I}_{\mathcal{M}}$, problem in (1) is equivalent to

$$\begin{aligned}
&\max_{\mathcal{M}\subseteq\mathcal{N}} E\Big[B \sum_{i\in\mathcal{I}_{\mathcal{M}}} E\left[S_i(j)|X_i(j)=1\right]\Big] \\
=& \max_{\mathcal{M}\subseteq\mathcal{N}} B \sum_{i\in\mathcal{M}} \text{Prob}(i\in\mathcal{I}_{\mathcal{M}}) E\left[S_i(j)|X_i(j)=1\right] \\
=& \max_{\mathcal{M}\subseteq\mathcal{N}} B \sum_{i\in\mathcal{M}} f(\theta_i)\cdot\frac{\theta_i(1-P_f^i)}{f(\theta_i)} \\
=& \max_{\mathcal{M}\subseteq\mathcal{N}} B \sum_{i\in\mathcal{M}} \theta_i(1-P_f^i)
\end{aligned}$$

where Prob$(\cdot)$ means probability of an event.

Therefore, to maximize the expected reward of the secondary user, the secondary user should sense the M channels with the M largest values of $\theta_i(1-P_f^i)$, $i\in\mathcal{N}$. ■

Since $B\theta_i(1-P_f^i)$ is reward of Channel i, Theorem 1 is intuitive: to sense the M channels with the M largest rewards.

B. Partial Channel Access ($K < M$)

For partial channel access, our first question is: does an intuitive rule as that in Theorem 1 exist? Unfortunately, for partial channel access, it may not be optimal to sense the M channels with the M largest rewards. Here is an example. Let $N = 5$, $M = 4$, $K = 1$, and $B = 1$. $(\theta_1,\theta_2,...,\theta_5) = (0.83, 0.47, 0.34, 0.39, 0.51)$, $(P_d^1, P_d^2,...,P_d^5) = (0.7, 0.6, 0.55, 0.65, 0.9)$, $(P_f^1, P_f^2,...,P_f^5) = (0.4, 0.2, 0.15, 0.3, 0.5)$. By choosing the $M = 4$ channels with largest $\theta_i(1-P_f^i)$, we have a set $\mathcal{M} = \{1,2,3,4\}$, and the secondary user's expected reward in a slot is 0.746. However, by exhaustive search, the optimal set of channels to sense is $\mathcal{M} = \{1,2,3,5\}$, with which the secondary user's expected reward in a slot is 0.768. Therefore, although the intuitive rule (i.e., selecting the channels with largest rewards) is optimal for full channel access, it may not be optimal for partial channel access. The reason is: in partial channel access, the secondary user may not access a channel that is sensed free.

Since the intuitive rule is not optimal in general for partial channel access, it seems that exhaustive search may be needed to find the optimal set of channels to sense. However, interestingly, in some special cases, some simple rules exist, as shown in Section III-B.1 and III-B.2, while in the general case, the searching complexity for the optimal channel set to sense can be reduced according to a property, as shown in Section III-B.3.

1) With Homogeneous Sensing: Here homogeneous sensing means all the channels have the same detection probability (i.e., $P_d^i = P_d$, $i\in\mathcal{N}$) and the same false alarm probability (i.e., $P_f^i = P_f$, $i\in\mathcal{N}$). Without loss of generality, we assume $\theta_1 > \theta_2 > ... > \theta_N$ in Section III-B.1. We have the following theorem.

Theorem 2: With homogeneous sensing, the optimal set of channels to sense is $\{1,2,...,M\}$.

Proof:

We use proof by contradiction. Assume that the optimal set of channels to sense, $\mathcal{M}^*$, is not $\{1,2,...,M\}$. Denote $\mathcal{M}^*$ as $\mathcal{M}^* = \{n_1,n_2,...,n_M\}$ with $n_1 < n_2 < ... < n_M$. It means $\theta_{n_1} > \theta_{n_2} > ... > \theta_{n_M}$. Note that, with homogeneous sensing, if a channel has a larger free probability θ_i, it also has a larger conditional reward.

Since $\mathcal{M}^*$ is not $\{1,2,..,M\}$, there exists a channel, denoted Channel $l\in\{1,2,...,M\}$, such that $l\notin\mathcal{M}^*$. Then l is smaller than at least one element in $\mathcal{M}^*$, and thus, there exists $k\in\{1,2,...,M\}$ such that $n_{k-1} < l < n_k$.[2] It also means $\theta_{n_{k-1}} > \theta_l > \theta_{n_k}$.

Now we derive an expression for $R_{\mathcal{M}^*}$, the reward of $\mathcal{M}^*$. Consider sensing of the $(M-1)$ channels in $\mathcal{M}^*\backslash\{n_k\}$. Define the set of channels sensed free, $\mathcal{I}_{\mathcal{M}^*\backslash\{n_k\}}$, as the *sensing*

[2] If $k = 1$, then we have $l < n_1$, which can be treated similarly.

result, and denote the set of all 2^{M-1} possible sensing result realizations as $\mathbb{U}$. Further, we have $\mathbb{U} = \mathbb{U}_1 \cup \mathbb{U}_2$, where $\mathbb{U}_1$ is the set of sensing result realizations in which the number of sensed-free channels among Channels $n_1, n_2, ..., n_{k-1}$ is less than K, and $\mathbb{U}_2$ is the set of sensing result realizations in which the number of sensed-free channels among Channels $n_1, n_2, ..., n_{k-1}$ is equal to or more than K. Then the reward of $\mathcal{M}^*$ is given as

$$\begin{aligned} &R_{\mathcal{M}^*} \\ &= f(\theta_{n_k})\Big(\sum_{\mathcal{U}\in\mathbb{U}_1} \text{Prob}(\mathcal{I}_{\mathcal{M}^*\backslash\{n_k\}} = \mathcal{U})\Big(\tfrac{B\theta_{n_k}(1-P_f)}{f(\theta_{n_k})} + r_{\mathcal{U}}^{K-1}\Big) \\ &\quad + \sum_{\mathcal{U}\in\mathbb{U}_2} \text{Prob}(\mathcal{I}_{\mathcal{M}^*\backslash\{n_k\}} = \mathcal{U}) r_{\mathcal{U}}^{K}\Big) \\ &\quad + \Big(1 - f(\theta_{n_k})\Big)\Big(\sum_{\mathcal{U}\in\mathbb{U}_1} \text{Prob}(\mathcal{I}_{\mathcal{M}^*\backslash\{n_k\}} = \mathcal{U}) r_{\mathcal{U}}^{K} \\ &\quad + \sum_{\mathcal{U}\in\mathbb{U}_2} \text{Prob}(\mathcal{I}_{\mathcal{M}^*\backslash\{n_k\}} = \mathcal{U}) r_{\mathcal{U}}^{K}\Big) \\ &= \sum_{\mathcal{U}\in\mathbb{U}_1} \text{Prob}(\mathcal{I}_{\mathcal{M}^*\backslash\{n_k\}} = \mathcal{U})\Big(\theta_{n_k}\big((1-P_f)B \\ &\quad - (P_d - P_f)(r_{\mathcal{U}}^{K} - r_{\mathcal{U}}^{K-1})\big) - (1-P_d)(r_{\mathcal{U}}^{K} - r_{\mathcal{U}}^{K-1})\Big) \\ &\quad + \sum_{\mathcal{U}\in\mathbb{U}} \text{Prob}(\mathcal{I}_{\mathcal{M}^*\backslash\{n_k\}} = \mathcal{U}) r_{\mathcal{U}}^{K} \end{aligned}$$

where $\mathcal{U}$ is a sensing result realization of sensing channels in $\mathcal{M}^*\backslash\{n_k\}$, $r_{\mathcal{U}}^{K-1}$ and $r_{\mathcal{U}}^{K}$ are the rewards by accessing up to $(K-1)$ and K channels in $\mathcal{U}$, respectively, that have the largest conditional rewards.

In $\mathcal{M}^*$, if we replace Channel n_k with Channel l, we get set $\{n_1, n_2, ..., n_{k-1}, l, n_{k+1}, ..., n_M\}$. Similarly, its reward is given as

$$\begin{aligned} &R_{\{n_1,n_2,...,n_{k-1},l,n_{k+1},...,n_M\}} \\ &= \sum_{\mathcal{U}\in\mathbb{U}_1} \text{Prob}(\mathcal{I}_{\mathcal{M}^*\backslash\{n_k\}} = \mathcal{U})\Big(\theta_l\big((1-P_f)B \\ &\quad - (P_d - P_f)(r_{\mathcal{U}}^{K} - r_{\mathcal{U}}^{K-1})\big) - (1-P_d)(r_{\mathcal{U}}^{K} - r_{\mathcal{U}}^{K-1})\Big) \\ &\quad + \sum_{\mathcal{U}\in\mathbb{U}} \text{Prob}(\mathcal{I}_{\mathcal{M}^*\backslash\{n_k\}} = \mathcal{U}) r_{\mathcal{U}}^{K}. \end{aligned}$$

Then the difference of the rewards of $\mathcal{M}^*$ and $\{n_1, n_2, ..., n_{k-1}, l, n_{k+1}, ..., n_M\}$ is given as

$$\begin{aligned} &R_{\{n_1,n_2,...,n_{k-1},l,n_{k+1},...,n_M\}} - R_{\mathcal{M}^*} \\ &= (\theta_l - \theta_{n_k})(1-P_f)B \sum_{\mathcal{U}\in\mathbb{U}_1} \Big[\text{Prob}(\mathcal{I}_{\mathcal{M}^*\backslash\{n_k\}} = \mathcal{U}) \\ &\qquad \cdot \Big(1 - \frac{P_d - P_f}{(1-P_f)B}(r_{\mathcal{U}}^{K} - r_{\mathcal{U}}^{K-1})\Big)\Big] > 0 \quad (2) \end{aligned}$$

where the inequality comes from $\theta_l > \theta_{n_k}$ and the following fact. According to the definition of $r_{\mathcal{U}}^{K}$ and $r_{\mathcal{U}}^{K-1}$, their difference is no more than the conditional reward of Channel 1 (which has the largest conditional reward), which means:

$$\begin{aligned} r_{\mathcal{U}}^{K} - r_{\mathcal{U}}^{K-1} &\le BE[S_1(j)|X_1(j) = 1] \\ &= \tfrac{B\theta_1(1-P_f)}{\theta_1(P_d - P_f) + 1 - P_d} < \tfrac{B(1-P_f)}{P_d - P_f}. \end{aligned} \quad (3)$$

Inequality (2) contradicts the assumption that $\mathcal{M}^*$ is optimal. ∎

2) With Common Detection Probability: Now we consider a special case when all the channels have a common detection probability (i.e., $P_d^i = P_d, i \in \mathcal{N}$)[3] but have different false alarm probabilities. Without loss of generality, we assume $\theta_1 > \theta_2 > ... > \theta_N$ in Section III-B.2.

Theorem 3: When all the channels have a common detection probability, if both $\theta_1 > \theta_2 > ... > \theta_N$ and $\theta_1(1-P_f^1) > \theta_2(1-P_f^2) > ... > \theta_N(1-P_f^N)$ are satisfied, the optimal set of channels to sense is $\{1, 2, ..., M\}$.

Proof: From $\theta_1 > \theta_2 > ... > \theta_N$ and $\theta_1(1-P_f^1) > \theta_2(1-P_f^2) > ... > \theta_N(1-P_f^N)$, we have

$$\begin{aligned} &E[S_1(j)|X_1(j) = 1] > E[S_2(j)|X_2(j) = 1] > \\ &\qquad ... > E[S_N(j)|X_N(j) = 1]. \quad (4) \end{aligned}$$

Then the optimality of $\{1, 2, ..., M\}$ can be proved similarly to the proof of Theorem 2. ∎

Theorem 2 indicates that for homogeneous sensing, the secondary user should sense the M channels with the M largest free probabilities. Theorem 3 indicates that, for a case when only detection probabilities are common while false alarm probabilities are different, if adding factor $(1-P_f^i)$ does not affect the ordering of the free probabilities of the channels, then the secondary user should still sense the M channels with the M largest free probabilities.

Next, for determining of the optimal $\mathcal{M}$, we have a definition of *preferred channel* as follows. Channel a is said to be preferred to Channel b if the following condition is satisfied: if Channel b is in the optimal $\mathcal{M}$, then Channel a should be also in the optimal $\mathcal{M}$.

Theorem 4: When all the channels have a common detection probability, for any pair of channels, a channel is preferred to the other channel if it has both larger free probability (i.e., θ_i) and larger reward (i.e., $B\theta_i(1-P_f^i)$) than those of the other channel, respectively.

Proof: We use proof by contradiction. For Channels i_1 and i_2, assume $\theta_{i_1} > \theta_{i_2}$ and $\theta_{i_1}(1-P_f^{i_1}) > \theta_{i_2}(1-P_f^{i_2})$. Denote $\mathcal{M}^*$ as the optimal set of channels to sense, and $i_1 \notin \mathcal{M}^*$, $i_2 \in \mathcal{M}^*$. Denote conditional rewards of Channels i_1 and i_2 as $y_1 = \frac{B\theta_{i_1}(1-P_f^{i_1})}{\theta_{i_1}(1-P_f^{i_1}) + (1-\theta_{i_1})(1-P_d)}$ and $y_2 = \frac{B\theta_{i_2}(1-P_f^{i_2})}{\theta_{i_2}(1-P_f^{i_2}) + (1-\theta_{i_2})(1-P_d)}$, respectively. Then $y_1 > y_2$.

In $\mathcal{M}^*$, if we replace Channel i_2 by Channel i_1, we get set $\mathcal{M}^\dagger$.

For sensing of the $(M-1)$ channels in $\mathcal{M}^*\backslash\{i_2\}$, denote $\mathcal{U}$ as a sensing result realization (i.e., the set of sensed-free channels). We partition $\mathcal{U}$ into three subsets: $\mathcal{U}_1$ includes the sensed-free channels whose conditional rewards are larger than y_1, $\mathcal{U}_2$ includes the sensed-free channels whose conditional rewards are less than or equal to y_1 and larger than y_2, and $\mathcal{U}_3$ includes the sensed-free channels whose conditional rewards are less than or equal to y_2.

When the sensing result of the $(M-1)$ channels in $\mathcal{M}^*\backslash\{i_2\}$ is fixed as $\mathcal{U}$, denote the reward of $\mathcal{M}^*$ and $\mathcal{M}^\dagger$ as $R_{\mathcal{U}}^*$ and $R_{\mathcal{U}}^\dagger$, respectively. Next, we derive expressions of $R_{\mathcal{U}}^*$ and $R_{\mathcal{U}}^\dagger$. Let $r_{\mathcal{U}}^{K-1}$ and $r_{\mathcal{U}}^{K}$ be the rewards by accessing up to $(K-1)$ channels and K channels in $\mathcal{U}$, respectively, that have the largest conditional rewards. We have the following three possible scenarios for $\mathcal{U}$.

- Scenario with $|\mathcal{U}_1| \ge K$: We have $R_{\mathcal{U}}^* = R_{\mathcal{U}}^\dagger = r_{\mathcal{U}}^{K}$.

[3] As an example, if it is required that the detection probability in each channel is above a common threshold so as to protect primary users, then the secondary user may set its detection probability in each channel as the common threshold value.

- Scenario with $|\mathcal{U}_1| < K$ and $|\mathcal{U}_1| + |\mathcal{U}_2| \geq K$: We have $R^*_{\mathcal{U}} = r^K_{\mathcal{U}}$, $R^\dagger_{\mathcal{U}} = f(\theta_{i_1})(r^{K-1}_{\mathcal{U}} + y_1) + (1 - f(\theta_{i_1}))r^K_{\mathcal{U}}$. Then we have $R^\dagger_{\mathcal{U}} - R^*_{\mathcal{U}} = f(\theta_{i_1})\big(y_1 - (r^K_{\mathcal{U}} - r^{K-1}_{\mathcal{U}})\big) \geq 0$.
- Scenario with $|\mathcal{U}_1| + |\mathcal{U}_2| < K$: We have

$$\begin{aligned} R^*_{\mathcal{U}} &= f(\theta_{i_2})(r^{K-1}_{\mathcal{U}} + y_2) + (1 - f(\theta_{i_2}))r^K_{\mathcal{U}} \\ &= f(\theta_{i_2})\big(y_2 - (r^K_{\mathcal{U}} - r^{K-1}_{\mathcal{U}})\big) + r^K_{\mathcal{U}}, \\ R^\dagger_{\mathcal{U}} &= f(\theta_{i_1})(r^{K-1}_{\mathcal{U}} + y_1) + (1 - f(\theta_{i_1}))r^K_{\mathcal{U}} \\ &= f(\theta_{i_1})\big(y_1 - (r^K_{\mathcal{U}} - r^{K-1}_{\mathcal{U}})\big) + r^K_{\mathcal{U}}. \end{aligned} \tag{5}$$

  If $f(\theta_{i_1}) \geq f(\theta_{i_2})$, then we have $R^\dagger_{\mathcal{U}} > R^*_{\mathcal{U}}$ since $y_1 > y_2$, $y_1 - (r^K_{\mathcal{U}} - r^{K-1}_{\mathcal{U}}) > 0$, and $y_2 - (r^K_{\mathcal{U}} - r^{K-1}_{\mathcal{U}}) \geq 0$. If $f(\theta_{i_1}) < f(\theta_{i_2})$, then (5) can be rewritten as $R^*_{\mathcal{U}} = B\theta_{i_2}(1 - P^{i_2}_f) - f(\theta_{i_2})(r^K_{\mathcal{U}} - r^{K-1}_{\mathcal{U}}) + r^K_{\mathcal{U}}$, and $R^\dagger_{\mathcal{U}} = B\theta_{i_1}(1 - P^{i_1}_f) - f(\theta_{i_1})(r^K_{\mathcal{U}} - r^{K-1}_{\mathcal{U}}) + r^K_{\mathcal{U}}$, which also lead to $R^\dagger_{\mathcal{U}} > R^*_{\mathcal{U}}$ since $\theta_{i_1}(1 - P^{i_1}_f) > \theta_{i_2}(1 - P^{i_2}_f)$ and $r^K_{\mathcal{U}} - r^{K-1}_{\mathcal{U}} \geq 0$.

The probability of the last scenario is nonzero. Therefore, averaged on all possible $\mathcal{U}$'s, the reward of $\mathcal{M}^\dagger$ is larger than the reward of $\mathcal{M}^*$, which contradicts the assumption that $\mathcal{M}^*$ is the optimal set of channels to sense. ■

3) Property for the General Case: For the general case with neither common detection probability nor common false alarm probability, we have the following theorem, whose proof is similar to that of Theorem 4, and is omitted.

Theorem 5: For any pair of channels, a channel is preferred to the other channel if it has both larger sensed-free probability $f(\theta_i)$ and larger conditional reward than those of the other channel, respectively.

Theorem 5 can be used to reduce the searching complexity for the optimal $\mathcal{M}$ in the general case. Based on Theorem 5, the following corollaries can be straightforwardly proved.

Corollary 1: If $(n_1, n_2, ..., n_N)$ is a permutation of $(1, 2, ..., N)$, and if conditions $f(\theta_{n_1}) > f(\theta_{n_2}) > ... > f(\theta_{n_N})$ and $\frac{\theta_{n_1}(1-P^{n_1}_f)}{f(\theta_{n_1})} > \frac{\theta_{n_2}(1-P^{n_2}_f)}{f(\theta_{n_2})} > ... > \frac{\theta_{n_N}(1-P^{n_N}_f)}{f(\theta_{n_N})}$ are satisfied, the optimal sensing channels, denoted as $\mathcal{M}^*$, is $\{n_1, n_2, ..., n_M\}$.

Corollary 2: If $(n_1, n_2, ..., n_N)$ is a permutation of $(1, 2, ..., N)$, with $f(\theta_{n_1}) > f(\theta_{n_2}) > ... > f(\theta_{n_N})$, and if there exists $k \in \{1, 2, ..., M\}$ such that $\frac{\theta_{n_1}(1-P^{n_1}_f)}{f(\theta_{n_1})} > \frac{\theta_{n_2}(1-P^{n_2}_f)}{f(\theta_{n_2})} > ... > \frac{\theta_{n_k}(1-P^{n_k}_f)}{f(\theta_{n_k})} > \max\left\{\frac{\theta_{n_{k+1}}(1-P^{n_{k+1}}_f)}{f(\theta_{n_{k+1}})}, \frac{\theta_{n_{k+2}}(1-P^{n_{k+2}}_f)}{f(\theta_{n_{k+2}})}, ..., \frac{\theta_{n_N}(1-P^{n_N}_f)}{f(\theta_{n_N})}\right\}$, then $\{n_1, n_2, ..., n_k\}$ is a subset of the optimal $\mathcal{M}$.

IV. Performance Evaluation

Next we show numerical results to demonstrate the impact of the selection of channels to sense. Consider 4 channels with channel free probabilities $(\theta_1, \theta_2, \theta_3, \theta_4) = (0.650, 0.727, 0.852, 0.918)$. Three cases are investigated: homogeneous case with $P_d = 0.7$ and $P_f = 0.3$; common detection probability case with $P_d = 0.7$ and $(P^1_f, P^2_f, P^3_f, P^4_f) = (0.1, 0.28, 0.39, 0.43)$; and general case with $(P^1_d, P^2_d, P^3_d, P^4_d) = (0.8, 0.8, 0.8, 0.95)$ and $(P^1_f, P^2_f, P^3_f, P^4_f) = (0.1, 0.28, 0.39, 0.43)$. The secondary user can sense two channels and access one channel. Fig. 1 shows the reward of different set of channels to sense in the three cases. The reward of the intuitive rule (i.e., the two channels with the two largest $\theta_i(1 - P^i_f)$ are sensed) is also indicated. It can be seen that the intuitive rule is optimal in the homogeneous case, and not optimal in the other two cases.

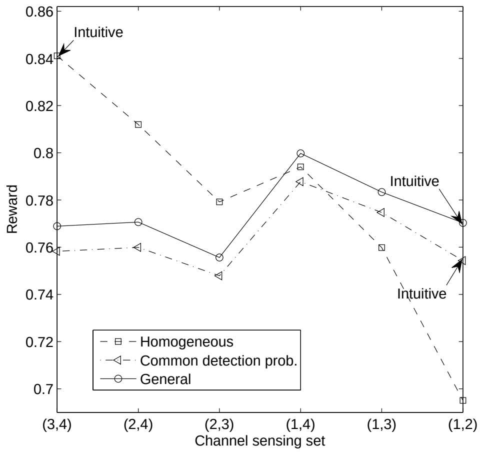

Fig. 1. Reward of the set of channels to sense

V. Conclusion

In this research, we have found some interesting results for the optimal set of channels to be sensed by a secondary user. When the secondary user can utilize all sensed-free channels, the intuitive rule is optimal. However, this intuitive rule is not optimal in general when the secondary user can only access up to a limited number of sensed-free channels. Interestingly, we have found some simple rules for the optimal set of channels to sense in some special cases. And for the general case, we have provided a guideline to reduce the searching complexity for the optimal channel set to sense.

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