Average Rate Maximization in Relay Networks over Slow Fading Channels

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***Abstract*— In this paper, the average rate maximization problem given transmission power constraints in a decode-and-forward (DF) relay network over slow fading channels is studied. For the problem, either long-term or short-term power constraint is imposed on the source and relays. In each combination of the power constraints at the source and relays, the original optimization problem is decomposed into sub-problems, each corresponding to a specific channel gain realization. For each sub-problem, a fast algorithm with closed-form solutions is provided. The case with additional peak power spectrum density constraints is also investigated. Numerical results are presented to demonstrate the effectiveness of the algorithms.**

I. Introduction

Recently relay communications have emerged as a solution to improve transmission reliability and rate, where relays help forward signal from sources to destinations [1]–[3]. Due to limited resources of the sources and relays, power allocation in relay networks is a critical issue, and has received much attention. For power allocation over relay networks, bit error rate (BER)/outage probability, network capacity, and received signal-to-noise ratio (SNR) are popularly used performance measures, as follows.

- BER/outage probability: In [4], under the assumption of perfect channel state information (CSI), power allocation strategies under three scenarios are studied to minimize the BER. Optimal power allocation strategies are given in [5] for minimizing the outage probability in relay networks with perfect CSI, limited CSI, or no CSI. With the objective of minimizing the outage probability, optimal power allocation strategies are presented in [6] for multi-hop relay networks. A two-hop relay network with one or more relays is considered in [7], and power allocation strategies are proposed, which are shown to minimize both the system symbol error rate and outage probability.
- Network capacity: The sum capacity of multiple source-destination pairs assisted with relays is investigated in [8]. Capacity bounds are studied in [9] for a multi-channel relay network with one relay, and power and/or time/bandwidth allocation is also discussed. For a relay network with one relay, outage capacity is studied in [10], with focus on the upper bounds and lower bounds. With a quality-of-service constraint on delay, the throughput of a relay network is maximized in [11], based on cross-layer resource allocation.
- Received SNR: In [12], by implementing amplify-and-forward (AF) transmission mode, a scenario of a source-destination pair with one and two relays is studied for a distributed space-time coding (DSTC) system, with aim at maximizing the SNR at the output of the DSTC system. Both decode-and-forward (DF) and AF transmission modes are considered in [13] for a source-destination pair with one or more relays, and the received SNR at the destination is maximized. Beamforming and power allocation problems are jointly investigated in [14] to maximize the received SNR at the destination. The research in [15] aims at minimizing the total transmission power with a target outage probability for the received SNR.

In aforementioned research efforts on relay networks, it can be seen that, when there are multiple relays, the multiple relays will forward the same information (received from the source) to the destination. This can increase the link reliability.

In a practical wireless communication system, the channel between a transceiver pair experiences fading. Therefore, *ergodic capacity*, the expectation of channel capacity over all the channel fading states, has attracted research attention recently in resource allocation problems. The ergodic capacity region together with resource allocation problems in multiple-access channel is studied in [16]. Ergodic capacity and resource allocation strategies in broadcast channels are given in [17]. The ergodic capacity and power allocation problems are studied in [18] under both multiple-access channel and broadcast channel in cognitive radio networks.

Note that ergodic capacity is defined for fast fading channels only. When slow fading channel is considered, outage capacity is a commonly used performance metric [19]. However, in our paper, we are interested in a setup that is different from the traditional slow fading model. In our model, in addition to the slow fading, we assume that the transmitter has the CSI (the fact that the channel is in slow fading makes this assumption reasonable). As pointed out in many recent works (e.g. [20]), the presence of CSI at transmitter makes the average rate a reasonable alternative performance metric for the slow fading channel. The transmit rate can be matched to the fading (e.g., by adaptive modulation) when timely CSI is available at the transmitter in slow fading. In this case the fading-induced outages can be essentially eliminated and an average rate can be achieved, which is defined as the expectation of rate over a sufficiently long interval and coincides with the ergodic capacity (i.e., expectation over fading distribution) when the fading process is ergodic. In our research, we target at maximizing the average rate in relay networks over slow fading channels.

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It should be noticed that with the CSI at the transmitter, there is no need to use codeword spanning over a large number of fading blocks (but rather, spanning over a single fading block), as the average rate is just a mathematical average of the realized instant rates in every fading block. Without using the long-spanning codeword, delay is not a serious problem anymore.

In this work, our focus is on maximizing the average rate of a source-destination pair aided by multiple relays in a slow fading environment. Since the transmitter can be adaptive to the fading state when considering the average rate in slow fading, link outage can be essentially eliminated and link reliability is not a big problem, and thus, it may not be necessary for all the relays to forward the same information to the destination. On the other hand, to increase the average rate, it is better to let different relays forward different information to destination. Therefore, we consider the case that the traffic from the source is split into multiple streams, each of which goes through a relay. Then it is essential how to assign the source traffic to the multiple relays and how the multiple relays relay the assigned traffic. A good solution is to use orthogonal frequency division multiple access (OFDMA) technology [21]–[25] to assign each relay a portion of the available bandwidth of the target source-destination pair, by means of subcarrier allocation[1]. Since different relays experience different fading, some relays may have good channel conditions while others may have bad channel quality. Then the relays with good channels should be assigned more bandwidth portions, while relays with bad channels should have less or no bandwidth portions. This means uneven traffic splitting is expected. Further, since the source or a relay has power constraint (short-term or long-term), it is also critical how those nodes allocate power for assigned bandwidth portions to maximize the average rate and to meet their short-term or long-term power constraints. In other words, power and bandwidth allocation should be jointly considered. So, two major differences of our research problem from existing work for wireless relaying are: first, we consider maximization of average rate, and the source traffic is split over multiple relays; and second, we use joint power and bandwidth allocation.

In the literature, research on ergodic capacity/average rate of relay networks is still in its infancy, which mainly focuses on upper and lower bounds. Upper and lower bounds of the ergodic capacity in a multiple-input multiple-output (MIMO) relay network are studied in [26], while upper bounds of the ergodic capacity of a multi-hop relay network are analyzed in [27]. On the other hand, the research problem of joint bandwidth and power allocation has been raised very recently for relay networks. In [28], joint power and bandwidth allocation is studied in a wireless network with multiple source-destination pairs, each aided by one or no relay to maximize the sum capacity and minimum capacity of multiple users and minimize the total power consumption. In [29], power and bandwidth allocation is considered in a relay network with one source-destination pair aided by multiple relays to maximize the capacity. The major differences of our work from [28] and [29] are: 1) we consider average rate maximization, and 2) source traffic is split into multiple streams over multiple relays in our work.

In this paper, with the goal as maximizing the average rate by jointly optimizing transmission power and bandwidth allocation strategies, we investigate the DF relaying case with one source, one destination, and multiple relays. Four different power constraint scenarios are considered. The research problem in each scenario is decomposed into sub-problems, each corresponding to a specific channel gain realization. Although each sub-problem can be solved by traditional methods, iterative calculations are needed (with no closed-form solutions), and the computational complexity involved may not be desired. In this work, as our major contribution, we investigate properties of the sub-problems, and provide fast algorithms with closed-form solutions for them. The rest of the paper is organized as follows. In Section II the system model is given and the average rate maximization problem is formulated. For different scenarios of constraints, the average rate maximization problem is solved in Section III, and fast algorithms with closed-form solutions are presented accordingly. Implementations issues are discussed in Section IV. Further discussion for the case with peak power spectrum density constraints is given in Section V. Numerical results are presented in Section VI, followed by conclusion remarks in Section VII.

II. System Model and Problem Formulation

Consider a target source-destination pair in a wireless network: source S and destination D. The source-destination pair is assigned (by the resource allocation coordinator for the whole network) a portion of the bandwidth of the network (e.g., by means of subcarrier allocation in OFDMA). To facilitate the fast algorithms proposed in this paper, it is suggested that the bandwidth assigned for a source-destination pair be within the coherence bandwidth. Since we consider only a particular source-destination pair in a wireless network (e.g., WiMAX, LTE) that may include a large number of source-destination pairs, the assigned bandwidth for the target source-destination pair is only a small portion of the total bandwidth of the network. Therefore, it is reasonable to assume that the bandwidth for the source-destination pair is smaller than the coherence bandwidth, and the channel gains (from the source to relays and from relays to the destination) are flat within the bandwidth for the source-destination pair. [2]

The source-destination pair is assisted by N relays R_i's ($i \in \mathcal{N}$, where $\mathcal{N} = \{1, 2, ..., N\}$). The DF method is employed by relays. Denote the channel gain between the source S and Relay R_i as g_i, and the channel gain between Relay R_i and the destination D as h_i, which, as mentioned above, are flat within the assigned bandwidth for the source-destination pair.

[1]Note that OFDMA is a core technique in broadband wireless networks (e.g., IEEE 802.16 WiMAX, and Long Term Evolution (LTE) proposal in 3GPP). Therefore, our work can be implemented easily in those networks. In particular, if a source-destination pair is assigned a number of subcarriers in an OFDMA network, those subcarriers can be further distributed to the multiple relays.

[2]If the assigned bandwidth is not within the coherence bandwidth (for example, the assigned bandwidth is big, or the assigned subcarriers are not consecutive), a new optimization problem can be formulated. If the assigned bandwidth consists of several flat-fading sub-bands each with a number of subcarriers, the new problem is still convex, and thus, can be solved numerically by traditional methods. However, unlike the case when assigned bandwidth is within the coherence bandwidth in our paper, no fast algorithms with closed-form solutions can be found. On the other hand, if the source-destination pair is assigned a number of subcarriers each with different channel gain, the new problem is a mixed-integer problem, and is NP-hard.

All the channel gains are supposed to be block-fading, which means the channel gains keep stable within each transmission block. At the beginning of each fading block, the source can measure channel gains g_i's and h_i's ($i \in \mathcal{N}$), which can be implemented by a training scheme, detailed in Section IV-A. For simplicity of presentation, the bandwidth for the source-destination pair is normalized to be 1. Relay R_i is assigned a portion, denoted as x_i, of the bandwidth, referred to as *sub-band* i, for transmission from the source to R_i and from R_i to the destination. The source transmits different information to different relays. After decoding the information received from the source, each relay forwards the decoded information to the destination over its assigned sub-band.

Denote the transmission power of the source on sub-band i as P_i, and the transmission power of Relay R_i as Q_i. By normalizing the power spectrum density of background noise as 1, the channel capacity on the path $S \rightarrow R_i \rightarrow D$, or say Link i, can be expressed as

$$C_i = \min\left(x_i \ln\left(1+\frac{P_i g_i}{x_i}\right),\ x_i \ln\left(1+\frac{Q_i h_i}{x_i}\right)\right). \quad (1)$$

Then the total channel capacity from the source S to the destination D is

$$C = \sum_{i=1}^{N} C_i. \quad (2)$$

Channel gains g_i and h_i are assumed to be ergodic processes over fading blocks. Denote the joint probability density function (PDF) of the channel gains as $f(\boldsymbol{g}, \boldsymbol{h})$, where $\boldsymbol{g} = (g_1, ..., g_N)$ and $\boldsymbol{h} = (h_1, ..., h_N)$. It is reasonable to further assume that g_i's and h_i's are independent from each other [30]. In this case, power and bandwidth allocation should be based on the instantaneous channel states. Denote the power allocation policy to be P_i and Q_i, and the bandwidth allocation strategy to be x_i. Note that P_i, Q_i and x_i are all dependent on the fading state $\boldsymbol{g}$ and $\boldsymbol{h}$, which are omitted in the notations of P_i, Q_i and x_i for the sake of simplicity. Then the expected rate from the source S to the destination D, referred to as the *average rate*, is

$$C_e = \mathbb{E}\left[\sum_{i=1}^{N} \min\left(x_i \ln\left(1+\frac{P_i g_i}{x_i}\right),\right.\right.$$
$$\left.\left. x_i \ln\left(1+\frac{Q_i h_i}{x_i}\right)\right)\right] \quad (3)$$

where $\mathbb{E}$ means expectation for $\boldsymbol{g}$ and $\boldsymbol{h}$. And we have

$$P_i \geq 0,\ \forall i \in \mathcal{N} \quad (4)$$

$$Q_i \geq 0,\ \forall i \in \mathcal{N} \quad (5)$$

$$x_i \geq 0,\ \forall i \in \mathcal{N};\quad \sum_{i=1}^{N} x_i \leq 1. \quad (6)$$

In addition, the transmission power at the source and relays is limited. The instantaneous source power may be bounded by a pre-specified value P^{ST}, referred to as *short-term source-power constraint*, given as

$$\sum_{i=1}^{N} P_i \leq P^{ST}, \quad (7)$$

or the average source power may be bounded by a pre-specified value P^{LT}, referred to as *long-term source-power constraint*, given as

$$\mathbb{E}\left[\sum_{i=1}^{N} P_i\right] \leq P^{LT}. \quad (8)$$

Similarly, the short-term relay-power constraint bounds the instantaneous power at Relay R_i by Q_i^{ST}, given as

$$Q_i \leq Q_i^{ST},\ \forall i \in \mathcal{N} \quad (9)$$

while the long-term relay-power constraint bounds the average power at Relay R_i by Q_i^{LT}, given as

$$\mathbb{E}\left[Q_i\right] \leq Q_i^{LT},\ \forall i \in \mathcal{N}. \quad (10)$$

And a group of optimization problems can be formulated in the following form.[3]

Problem P1 :

$$\begin{array}{ll} \max\limits_{\{P_i\},\{Q_i\},\{x_i\}} & \mathbb{E}\Big[\sum\limits_{i=1}^{N} \min\Big(x_i \ln\Big(1+\frac{P_i g_i}{x_i}\Big), \\ & \quad x_i \ln\Big(1+\frac{Q_i h_i}{x_i}\Big)\Big)\Big] \\ \text{s.t.} & (4), (5) \text{ and } (6), \\ & (7) \text{ or } (8)\ , \\ & (9) \text{ or } (10)\ . \end{array} \quad (11)$$

We need to derive the power and bandwidth allocation P_i's, Q_i's and x_i's with respect to every specific channel realization $\boldsymbol{g}$ and $\boldsymbol{h}$.

Note that for Link i, when the channel capacity from the source to Relay R_i is equal to that from Relay R_i to the destination, we have $x_i \ln\left(1+\frac{P_i g_i}{x_i}\right) = x_i \ln\left(1+\frac{Q_i h_i}{x_i}\right)$, or equivalently

$$P_i g_i = Q_i h_i. \quad (12)$$

Suppose in an optimal solution of Problem P1 we have P_i^* and Q_i^* for Link i. We claim that P_i^* and Q_i^* must satisfy equation (12). The claim is reasonable for the following reason. Suppose P_i^* and Q_i^* do not satisfy equation (12), for example, $P_i^* g_i < Q_i^* h_i$. Then, if we change Q_i^* to $P_i^* g_i / h_i$ (which is less than Q_i^*), we can reach the same utility value of Problem P1 with lower power from Relay R_i to the destination.

With the above claim, the short-term and long-term transmission power constraints of Q_i in (9) and (10) become

$$\frac{P_i g_i}{h_i} \leq Q_i^{ST}, \forall i \in \mathcal{N}, \quad (13)$$

and

$$\mathbb{E}\left[\frac{P_i g_i}{h_i}\right] \leq Q_i^{LT}, \forall i \in \mathcal{N} \quad (14)$$

respectively.

Therefore, Problem P1 is equivalent to Problem P2 shown on the next page, which is convex (the proof is given in Appendix I).

[3] Note that here we assume the source (or relay) power constraint is either short-term or long-term, not both, and our target is on fast algorithms with closed-form solutions. With both short-term and long-term power constraints at either the source or relays, no fast algorithms with closed-form solutions can be found, and traditional methods such as sub-gradient methods and interior-point algorithms may be used to solve the problems numerically.

$$\begin{array}{lll}
 & \textbf{Problem P2}: & \\
\max\limits_{\{P_i\},\{x_i\}} & \mathbb{E}\left[\sum\limits_{i=1}^{N} x_i \ln\left(1+\frac{P_i g_i}{x_i}\right)\right] & \\
\text{s.t.} & P_i \geq 0, x_i \geq 0, \forall i \in \mathcal{N}; \quad \sum\limits_{i=1}^{N} x_i \leq 1 & \\
 & \sum\limits_{i=1}^{N} P_i \leq P^{ST} \text{ (short-term)} \quad \text{or} \quad \mathbb{E}\left[\sum\limits_{i=1}^{N} P_i\right] \leq P^{LT} \text{ (long-term)} & \\
 & \frac{P_i g_i}{h_i} \leq Q_i^{ST} \text{ (short-term)} \quad \text{or} \quad \mathbb{E}\left[\frac{P_i g_i}{h_i}\right] \leq Q_i^{LT} \text{ (long-term)}, \quad \forall i \in \mathcal{N}. &
\end{array} \tag{15}$$

The resource allocation is conducted at the source S. In the following two sections, we will focus on how to solve the resource allocation problem and how to implement the resource allocation, respectively.

III. Average Rate Maximization

In this section, Problem P2 with four scenarios of power constraints is studied sequentially. In each scenario, optimal bandwidth and power allocation strategy are derived.

A. Scenario with Long-term Source-power and Long-term Relay-power Constraints

In this scenario, the Lagrangian of Problem P2 can be written as

$$\begin{aligned}
&\mathcal{L}(\{P_i\},\{x_i\},\{\mu_i\},\lambda) \\
= \ &\mathbb{E}\left[\sum_{i=1}^{N} x_i \ln\left(1+\frac{P_i g_i}{x_i}\right)\right] - \lambda\left(\mathbb{E}\left[\sum_{i=1}^{N} P_i\right] - P^{LT}\right) \\
&- \sum_{i=1}^{N} \mu_i \left(\mathbb{E}\left[\frac{P_i g_i}{h_i}\right] - Q_i^{LT}\right)
\end{aligned} \tag{16}$$

where μ_i and λ are non-negative Lagrange multipliers corresponding to the constrains $\mathbb{E}\left[\frac{P_i g_i}{h_i}\right] \leq Q_i^{LT}$ and $\mathbb{E}\left[\sum_{i=1}^{N} P_i\right] \leq P^{LT}$, respectively. Then the Lagrange dual function is given as

$$\begin{aligned}
\mathcal{D}(\lambda,\{\mu_i\}) = &\max_{\{P_i\},\ \{x_i\}} \mathcal{L}(\{P_i\},\{x_i\},\{\mu_i\},\lambda) \\
\text{s.t.} \quad &P_i \geq 0, x_i \geq 0,\ \forall i \in \mathcal{N}; \quad \sum_{i=1}^{N} x_i \leq 1.
\end{aligned} \tag{17}$$

Since Problem P2 in this scenario is convex, and strictly feasible points are available in this problem, which satisfies the Slater's condition [31], the minimum of the duality function

$$\min_{\lambda\geq 0,\ \mu_i\geq 0,\ i\in\mathcal{N}} \mathcal{D}(\lambda,\{\mu_i\}) \tag{18}$$

is guaranteed to achieve the optimal utility of Problem P2. Therefore, optimizing Problem P2 is equivalent to minimizing the duality function $\mathcal{D}(\lambda,\{\mu_i\})$. To converge to the minimum of $\mathcal{D}(\lambda,\{\mu_i\})$, the Lagrange multipliers λ and μ_i's can be updated by resorting to the sub-gradient method [32] as

$$\lambda(t+1) = \left(\lambda(t) - a(t)\left(P^{LT} - \mathbb{E}\left[\sum_{i=1}^{N} P_i(t)\right]\right)\right)^{+} \tag{19}$$

and

$$\mu_i(t+1) = \left(\mu_i(t) - a(t)\left(Q_i^{LT} - \mathbb{E}\left[\frac{P_i(t) g_i}{h_i}\right]\right)\right)^{+}, \quad \forall i \in \mathcal{N} \tag{20}$$

where $P_i(t)$ is the optimal power allocation solution of the dual function in (17) at the t^{th} iteration, $a(t)$ is the positive step size at the t^{th} iteration, and $(x)^{+} = \max(x,0)$.

In other words, Problem P2 is decomposed into two levels. In the higher level, λ and μ_i's are updated iteratively as in (19) and (20). In the lower level, the Lagrange dual function in (17) is obtained for each iteration.

The update of λ and μ_i's in (19) and (20) will be discussed in Section IV-B. So next we focus on how to obtain the optimal solution of the dual function in (17),[4] i.e., how to obtain $\mathcal{D}(\lambda,\{\mu_i\})$ for given λ and μ_i's.

It can be seen that the optimization problem in (17) can be solved by investigating the sub-problem as follows for every realization of channel gains $\boldsymbol{g}$ and $\boldsymbol{h}$.

$$\begin{aligned}
\max_{\{P_i\},\ \{x_i\}} \quad &\sum_{i=1}^{N} x_i \ln\left(1+\frac{P_i g_i}{x_i}\right) - \lambda\sum_{i=1}^{N} P_i - \sum_{i=1}^{N} \mu_i \frac{P_i g_i}{h_i} \\
\text{s.t.} \quad &P_i \geq 0, x_i \geq 0,\ \forall i \in \mathcal{N}; \quad \textstyle\sum_{i=1}^{N} x_i \leq 1.
\end{aligned} \tag{21}$$

Note that for the ease of presentation, the constant terms λP^{LT} and $\sum_{i=1}^{N} \mu_i Q_i^{LT}$ in the objective function are discarded.

The sub-problem in (21) is a convex problem, and traditionally, can be solved by numerical optimization methods, such as sub-gradient method and interior-point algorithm [33]. However, the traditional methods require iterative calculations and can only numerically achieve the optimal solution (i.e., no closed-form solution is achieved). In this paper, we will not use the numerical methods to solve the sub-problem in (21). Rather, we will use KKT conditions to analyze special properties of the convex sub-problem in (21). Based on the special properties, we give a fast algorithm with a closed-form optimal solution for the sub-problem, which avoids high computational complexity.

The sub-problem in (21) satisfies the Slater's condition. Then, the KKT condition which serves as a sufficient and necessary condition for the optimal solution can be listed as

[4] Note that the method in Section IV-B also applies for the scenarios discussed in Section III-B, III-C, and III-D. So in those three subsections, we also focus only on the lower-level problems.

follows [31].

$$\ln\left(1+\frac{P_i g_i}{x_i}\right)-\frac{P_i g_i}{x_i+P_i g_i}-\Gamma^*+\delta_i^*=0,\ \forall i\in\mathcal{N} \quad (22a)$$

$$\frac{x_i g_i}{x_i+g_i P_i}-\lambda-\frac{\mu_i g_i}{h_i}+\Delta_i^*=0,\ \forall i\in\mathcal{N} \quad (22b)$$

$$\Delta_i^* P_i=0, \delta_i^* x_i=0,\ \forall i\in\mathcal{N};\ \Gamma^*\left(\sum_{i=1}^{N} x_i-1\right)=0 \quad (22c)$$

$$P_i\geq 0, x_i\geq 0,\ \forall i\in\mathcal{N};\ \sum_{i=1}^{N} x_i\leq 1. \quad (22d)$$

in which Δ_i^*, δ_i^* and Γ^* are non-negative Lagrange multipliers associated with constraints $P_i\geq 0$, $x_i\geq 0$, and $\sum_{i=1}^{N} x_i\leq 1$, respectively. In this paper, when superscript $*$ is used for a Lagrange multiplier, it means the Lagrange multiplier is associated with a sub-problem for a realization of $(\boldsymbol{g},\boldsymbol{h})$ (to distinguish from the Lagrange multipliers associated with the original optimization problem P2). Equations (22a) and (22b) are obtained by setting the derivative of the Lagrangian of the sub-problem in (21) with respect to x_i and P_i as 0, respectively. The following lemma is in order for the KKT condition (22a)-(22d).

Lemma 1: If $\mathcal{A}=\{j|P_j>0, x_j>0\}\neq\emptyset$ (null set), then $|\mathcal{A}|\leq 1$.

Proof: Define $\mathrm{SNR}_i=\frac{P_i g_i}{x_i}$. Equations (22a) and (22b) can be rewritten as

$$\ln(1+\mathrm{SNR}_i)-\frac{\mathrm{SNR}_i}{1+\mathrm{SNR}_i}=\Gamma^*-\delta_i^*,\quad \forall i\in\mathcal{N} \quad (23a)$$

$$\frac{1}{1+\mathrm{SNR}_i}=\frac{\lambda}{g_i}+\frac{\mu_i}{h_i}-\frac{\Delta_i^*}{g_i},\quad \forall i\in\mathcal{N}. \quad (23b)$$

Suppose $i^\dagger\in\mathcal{A}$. From (22c) it can be seen that $\delta_{i^\dagger}^*$ and $\Delta_{i^\dagger}^*$ should be zero. Define $S(x)\triangleq\ln(1+x)-\frac{x}{1+x}$, which is a monotonic increasing non-negative function of non-negative x. Since $\mathrm{SNR}_{i^\dagger}=\frac{P_{i^\dagger}g_{i^\dagger}}{x_{i^\dagger}}\in(0,+\infty)$ and $\delta_{i^\dagger}^*=0$, from (23a) we have

$$\Gamma^*=S(\mathrm{SNR}_{i^\dagger})+\delta_{i^\dagger}^*=S(\mathrm{SNR}_{i^\dagger})\in(0,+\infty).$$

When λ and $\mu_{i^\dagger}$ are nonzero (this case is general in the updating procedure of λ and $\mu_{i^\dagger}$), after dividing the left- and right-hand sides of (23a) by the left- and right-hand sides of (23b), respectively, for $i^\dagger$, we get

$$(1+\mathrm{SNR}_{i^\dagger})\ln(1+\mathrm{SNR}_{i^\dagger})-\mathrm{SNR}_{i^\dagger}=\frac{\Gamma^*}{\frac{\lambda}{g_{i^\dagger}}+\frac{\mu_{i^\dagger}}{h_{i^\dagger}}}. \quad (24)$$

Define $T(x)\triangleq(1+x)\ln(1+x)-x$, which is a monotonic increasing non-negative function of non-negative x. From (23a) and (24), we have

$$S\left(T^{-1}\left(\frac{\Gamma^*}{\frac{\lambda}{g_{i^\dagger}}+\frac{\mu_{i^\dagger}}{h_{i^\dagger}}}\right)\right)=\Gamma^* \quad (25)$$

and $S(T^{-1}(\cdot))$ is a monotonic increasing function. So if there exists $i^\ddagger(\neq i^\dagger)\in\mathcal{A}$, it should also satisfy equation (25), which leads to

$$\frac{\Gamma^*}{\frac{\lambda}{g_{i^\dagger}}+\frac{\mu_{i^\dagger}}{h_{i^\dagger}}}=\frac{\Gamma^*}{\frac{\lambda}{g_{i^\ddagger}}+\frac{\mu_{i^\ddagger}}{h_{i^\ddagger}}}. \quad (26)$$

Note that $g_{i^\dagger}$, $h_{i^\dagger}$, $g_{i^\ddagger}$ and $h_{i^\ddagger}$ are independent channel gains, while λ, $\mu_{i^\dagger}$ and $\mu_{i^\ddagger}$ are fixed for the problem in (21). Therefore, the probability that equation (26) holds is zero. So there is at most one element in set $\mathcal{A}$ almost surely, i.e., $|\mathcal{A}|\leq 1$. This completes the proof.[5] ■

Note that $\mathcal{A}$ is actually the set of selected relays, i.e., with non-zero power and bandwidth assignment. Therefore, Lemma 1 indicates that either of the two following cases happens: i) no relay is assigned bandwidth and power (e.g., when the channels are poor, it may be better not to transmit, since the power constraints are in a long-term scale); ii) only one relay is selected.

From Lemma 1, it is clear that, if $i\in\mathcal{A}$, then i is unique and $x_i=1$ since the objective function of the sub-problem in (21) is an increasing function with respect to x_i. Substituting $x_i=1$ in equation (22b), we have

$$P_i=\frac{h_i}{\lambda h_i+\mu_i g_i}-\frac{1}{g_i} \quad (27)$$

and the corresponding achieved utility of the sub-problem in (21) is $\frac{\lambda}{g_i}+\frac{\mu_i}{h_i}-\ln\left(\frac{\lambda}{g_i}+\frac{\mu_i}{h_i}\right)-1$. Since $P_i>0$, we have $\frac{\lambda}{g_i}+\frac{\mu_i}{h_i}<1$, based on (27). The above achieved utility can be proved to be always positive when $\frac{\lambda}{g_i}+\frac{\mu_i}{h_i}>0$. On the other hand, the utility, i.e., $\frac{\lambda}{g_i}+\frac{\mu_i}{h_i}-\ln\left(\frac{\lambda}{g_i}+\frac{\mu_i}{h_i}\right)-1$, decreases if $\frac{\lambda}{g_i}+\frac{\mu_i}{h_i}$ increases within range $(0,1)$. Therefore, if there is no relay that satisfies $\frac{\lambda}{g_i}+\frac{\mu_i}{h_i}<1$, then no relay is selected; otherwise, the relay with the minimum value of $\frac{\lambda}{g_i}+\frac{\mu_i}{h_i}$ is selected.

Then the optimal solution for the sub-problem in (21) can be obtained in the procedure as follows.

Algorithm 1 Searching procedure for the optimal solution of the sub-problem in (21).

1: Define set $\mathcal{I}\triangleq\{i|\frac{\lambda}{g_i}+\frac{\mu_i}{h_i}<1\}$.
2: **if** $\mathcal{I}=\emptyset$ **then**
3: The maximal utility is 0, with $P_i=0$ and $x_i=0$, $\forall i\in\mathcal{N}$.
4: **else**
5: Find $i^*=\arg\min_{i\in\mathcal{I}}\left(\frac{\lambda}{g_i}+\frac{\mu_i}{h_i}\right)$.
Output the optimal index i^*, with optimal bandwidth and power allocation strategy $x_{i^*}=1$ and $P_{i^*}=\frac{h_{i^*}}{\lambda h_{i^*}+\mu_{i^*}g_{i^*}}-\frac{1}{g_{i^*}}$, respectively, and the maximal utility $\left(\frac{\lambda}{g_{i^*}}+\frac{\mu_{i^*}}{h_{i^*}}-\ln(\frac{\lambda}{g_{i^*}}+\frac{\mu_{i^*}}{h_{i^*}})-1\right)$.

It can be seen that the complexity of Algorithm 1 is $O(N)$.

Remark: When the source- and relay-power constraints are both long-term, if no relay is with good channels at one moment, then it is reasonable not to select any relay at this moment, to save power for moments when relays are with good channels. And if there are relays with good channels, then it is good to select the relay with the best channels. Since each relay is associated with two channels (from the source and to the destination), Algorithm 1 indicates a metric to measure the overall quality of the two channels with a relay, i.e., $\frac{\lambda}{g_i}+\frac{\mu_i}{h_i}$. This metric is reasonable since it includes

[5]Note that a similar proof method is adopted in [34].

the channel gains of the two channels. Interestingly, when the measure is below a fixed value 1, the relay is considered to be with overall good channels.

B. Scenario with Short-term Source-power and Long-term Relay-power Constraints

Similar to Section III-A, Problem P2 in this scenario (with short-term source-power and long-term relay-power constraints) can be solved by investigating the following sub-problem for each realization of $(\boldsymbol{g}, \boldsymbol{h})$

$$\max_{\{P_i\},\ \{x_i\}} \quad \sum_{i=1}^{N} x_i \ln\left(1+\frac{P_i g_i}{x_i}\right) - \sum_{i=1}^{N} \mu_i \frac{P_i g_i}{h_i}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} P_i \le P^{ST} \tag{28a}$$

$$P_i \ge 0, x_i \ge 0,\ \forall i \in \mathcal{N};\ \sum_{i=1}^{N} x_i \le 1 \tag{28b}$$

which is a convex problem satisfying Slater's condition. To analyze this sub-problem, the KKT condition is listed as follows.

$$\ln\left(1+\frac{P_i g_i}{x_i}\right) - \frac{P_i g_i}{x_i + P_i g_i} - \Gamma^* + \delta_i^* = 0,\ \forall i \in \mathcal{N} \tag{29a}$$

$$\frac{x_i g_i}{x_i + g_i P_i} - \lambda^* - \frac{\mu_i g_i}{h_i} + \Delta_i^* = 0,\ \forall i \in \mathcal{N} \tag{29b}$$

$$\lambda^* \left(\sum_{i=1}^{N} P_i - P^{ST}\right) = 0 \tag{29c}$$

$$\Delta_i^* P_i = 0, \delta_i^* x_i = 0,\ \forall i \in \mathcal{N};\ \Gamma^* \left(\sum_{i=1}^{N} x_i - 1\right) = 0 \tag{29d}$$

$$\text{Constraints (28a) – (28b)} \tag{29e}$$

where λ^*, Δ_i^*, δ_i^*, and Γ^* are non-negative Lagrange multipliers associated with constraints $\sum_{i=1}^{N} P_i \le P^{ST}$, $P_i \ge 0$, $x_i \ge 0$, and $\sum_{i=1}^{N} x_i \le 1$, respectively.

For the KKT condition, the following lemmas are in order.

Lemma 2: If $\mathcal{A} \triangleq \{j | P_j > 0, x_j > 0\} \ne \emptyset$,

- when the constraint $\sum_{i=1}^{N} P_i \le P^{ST}$ in (28a) is inactive[6], $|\mathcal{A}| = 1$.
- when the constraint $\sum_{i=1}^{N} P_i \le P^{ST}$ in (28a) is active, $|\mathcal{A}| \le 2$.

Proof: Please refer to Appendix II. ■

Lemma 3: For $\forall i, j \in \mathcal{A}$, $\text{SNR}_i = \text{SNR}_j$.

Proof: Please refer to Appendix III. ■

Lemma 2 and Lemma 3 indicate that, in the scenario, at most two relays are selected. And the SNR for all selected relays should be the same.

Define a new set $\mathcal{B} \triangleq \{j | P_j = 0, x_j = 0\}$. Note that set $\mathcal{B}$ includes relays that are not selected. Therefore, $\mathcal{A} \bigcup \mathcal{B} = \{1, ..., N\}$.[7] Considering the conclusion of Lemma 3 and constraint $\sum_{i=1}^{N} x_i \le 1$, the common SNR indicated by Lemma 3 is shown to be $\text{SNR} = \sum_{i \in \mathcal{A}} g_i P_i$. Then the utility of the sub-problem in (28) is

$$\begin{aligned} & \sum_{i=1}^{N} x_i \ln\left(1+\frac{P_i g_i}{x_i}\right) - \sum_{i=1}^{N} \mu_i \frac{P_i g_i}{h_i} \\ = & \sum_{i \in \mathcal{A}} x_i \ln(1+\text{SNR}) - \sum_{i=1}^{N} \mu_i \frac{P_i g_i}{h_i} \\ = & \ln\left(1 + \sum_{i \in \mathcal{A}} g_i P_i\right) - \sum_{i=1}^{N} \mu_i \frac{P_i g_i}{h_i} \\ = & \ln\left(1 + \sum_{i=1}^{N} g_i P_i\right) - \sum_{i=1}^{N} \mu_i \frac{P_i g_i}{h_i} \end{aligned} \tag{30}$$

where the second equality is because $\sum_{i \in \mathcal{A}} x_i = 1$ and the third equality is because $P_i = 0$ for $\forall i \in \mathcal{B}$ and $\mathcal{A} \bigcup \mathcal{B} = \mathcal{N}$. Then the sub-problem in (28) is equivalent to

$$\max_{\{P_i\},\ \{x_i\}} \quad \ln\left(1 + \sum_{i=1}^{N} P_i g_i\right) - \sum_{i=1}^{N} \mu_i \frac{P_i g_i}{h_i}$$

$$\text{s.t.} \quad \sum_{i=1}^{N} P_i \le P^{ST} \tag{31a}$$

$$P_i \ge 0, \forall i \in \mathcal{N}. \tag{31b}$$

Recall that at most two relays are selected.

- When there are two selected relays, there are totally $\binom{N}{2} = \frac{N(N-1)}{2}$ possible combinations of the two relays. In a specific combination, denote the two selected relays as R_i and R_j. The problem in (31) is re-written as

$$\begin{aligned} \max_{P_i, P_j} \quad & \ln(1 + P_i g_i + P_j g_j) - \left(\mu_i \frac{P_i g_i}{h_i} + \mu_j \frac{P_j g_j}{h_j}\right) \\ \text{s.t.} \quad & P_i \ge 0, P_j \ge 0, \\ & P_i + P_j = P^{ST} \end{aligned}$$

  the closed-form solution of which can be obtained straightforwardly based on corresponding KKT condition, and is omitted here.
- When there is only one selected relay, there are totally N possible cases for the selected relay. For each specific case, denote the selected relay as R_i. Set the derivative of the utility function of the problem in (31) as zero. Then, we can get the power allocation for the link from the source to Relay R_i, given as

$$P_i = \left[\frac{1}{g_i}\left(\frac{h_i}{\mu_i} - 1\right)\right]_0^{P^{ST}}$$

  where $[x]_a^b$ is defined as $\max(\min(b, x), a)$. Note that when $P_i = 0$, it means that no relay is selected.

Comparing the $\frac{N(N-1)}{2} + N$ optimal utilities for the above cases, we can obtain the maximal utility of problem in (31) (the largest among the $N + \frac{N(N-1)}{2}$ optimal utilities) and the associated power allocation strategy. Note that when all the $N + \frac{N(N-1)}{2}$ optimal utilities are non-positive, then no relay is selected. The computational complexity is $O(N^2)$.

[6] At the optimal point of the problem (say in (28)), if the equality in constraint $\sum_{i=1}^{N} P_i \le P^{ST}$ holds (i.e., $\sum_{i=1}^{N} P_i = P^{ST}$), we say the constraint is active; otherwise, we say it is inactive.

[7] Note that cases of "$P_j > 0, x_j = 0$" and "$P_j = 0, x_j > 0$" are omitted because neither of them achieves a utility more than that in the case "$P_j = 0, x_j = 0$" for the sub-problem in (28).

For bandwidth allocation strategy, when there is only one selected relay (say Relay R_i) in the above optimal power allocation, it can be seen that $x_i = 1$, as the utility function in (28) is an increasing function of x_i. When there are two selected relays (say Relays R_i and R_j), we have $x_i + x_j = 1$ (since the utility function in (28) is an increasing function of x_i and x_j), which leads to $\text{SNR}_i = \text{SNR}_j = P_i g_i + P_j g_j$ (from Lemma 3), and further we have $x_i = \frac{P_i g_i}{\text{SNR}_i} = \frac{P_i g_i}{P_i g_i + P_j g_j}$, and $x_j = \frac{P_j g_j}{\text{SNR}_j} = \frac{P_j g_j}{P_i g_i + P_j g_j}$.

Remark: Comparing with Section III-A, the scenario in Section III-B has short-term source-power constraint, which reduces the flexibility in power and bandwidth allocation. The reduced flexibility is the main reason that at most one relay is selected in the scenario in Section III-A, while two relays may be selected in the scenario in Section III-B. The reduced flexibility is also the reason that, with short-term source-power constraint, we do not have that simple metric as in Algorithm 1 to determine which relay(s) to be selected.

C. Scenario with Long-term Source-power and Short-term Relay-power Constraints

In this scenario, the sub-problem for every realization of $(\boldsymbol{g}, \boldsymbol{h})$ is

$$
\begin{aligned}
\max_{\{P_i\},\ \{x_i\}} \quad & \sum_{i=1}^{N} x_i \ln\left(1 + \frac{P_i g_i}{x_i}\right) - \lambda \sum_{i=1}^{N} P_i \\
\text{s.t.} \quad & \frac{P_i g_i}{h_i} \le Q_i^{ST},\ \forall i \in \mathcal{N} && \text{(32a)} \\
& P_i \ge 0, x_i \ge 0,\ \forall i \in \mathcal{N};\ \sum_{i=1}^{N} x_i \le 1. && \text{(32b)}
\end{aligned}
$$

The sub-problem in (32) is a convex problem satisfying Slater's condition. The KKT condition is

$$
\begin{aligned}
\ln\left(1 + \frac{P_i g_i}{x_i}\right) - \frac{P_i g_i}{x_i + P_i g_i} - \Gamma^* + \delta_i^* = 0,\ \forall i \in \mathcal{N} && \text{(33a)} \\
\frac{x_i g_i}{x_i + g_i P_i} - \lambda - \frac{\mu_i^* g_i}{h_i} + \Delta_i^* = 0,\ \forall i \in \mathcal{N} && \text{(33b)} \\
\mu_i^* \left(\frac{P_i g_i}{h_i} - Q_i^{ST}\right) = 0,\ \forall i \in \mathcal{N} && \text{(33c)} \\
\Delta_i^* P_i = 0, \delta_i^* x_i = 0,\ \forall i \in \mathcal{N};\ \Gamma^* \left(\sum_{i=1}^{N} x_i - 1\right) = 0 && \text{(33d)} \\
\text{Constraints (32a) – (32b)} && \text{(33e)}
\end{aligned}
$$

where μ_i^*, Δ_i^*, δ_i^*, and Γ^* are non-negative Lagrange multipliers associated with constraints $\frac{P_i g_i}{h_i} \le Q_i^{ST}$, $P_i \ge 0$, $x_i \ge 0$, and $\sum_{i=1}^{N} x_i \le 1$, respectively.

Define $\mathcal{A} \triangleq \{j | P_j > 0,\ x_j > 0\} = \mathcal{A}_1 \cup \mathcal{A}_2$, where $\mathcal{A}_1 \triangleq \left\{ j | P_j = \frac{Q_j^{ST} h_j}{g_j}, x_j > 0 \right\}$ and $\mathcal{A}_2 \triangleq \left\{ j | 0 < P_j < \frac{Q_j^{ST} h_j}{g_j}, x_j > 0 \right\}$. Note that $\mathcal{A}$ includes the selected relays. From (12) it can be seen that $\mathcal{A}_1$ includes the selected relays with power (for its hop to destination) being the maximal allowed power, while $\mathcal{A}_2$ includes the selected relays with power (for its hop to destination) less than the maximal allowed power. By analyzing the KKT condition in (33), a series of lemmas can be expected as follows.

Lemma 4: When $\mathcal{A} \neq \emptyset$, $|\mathcal{A}_2| \le 1$.

The proof is similar to that of Lemma 1, and is omitted here. Note that in the proof, we have $\mu_i^* = 0,\ i \in \mathcal{A}_2$ (based on (33c)).

Lemma 5: For $\forall i, j \in \mathcal{A}$, $\text{SNR}_i = \text{SNR}_j$.

Proof: Please refer to the proof of Lemma 3. ■

Lemma 6: For $\forall i \in \mathcal{A}_1$ and $\forall j \in \mathcal{A}_2$, $g_i \ge g_j$.

Proof: Please refer to Appendix IV. ■

Lemmas 4–6 indicate that, among all the selected relays, at most one relay transmits over its link to the destination with power less than the maximal allowed power, while all other selected relays use the maximal allowed power; and the relay with less than maximal allowed power has less channel gain in the first hop (from the source to the relay) than other selected relays. All selected relays have the same SNR.

With the aid of Lemma 5 and by following the same procedure in Section III-B, the sub-problem in (32) is equivalent to the following optimization problem

$$
\begin{aligned}
\max_{\{P_i\}} \quad & \ln\left(1 + \sum_{i=1}^{N} P_i g_i\right) - \lambda \sum_{i=1}^{N} P_i \\
\text{s.t.} \quad & 0 \le P_i \le \frac{Q_i^{ST} h_i}{g_i}, \forall i \in \mathcal{N}.
\end{aligned} \tag{34}
$$

For the solution of the problem in (34), the following lemma is in order.

Lemma 7: For $\forall k \in \mathcal{B}$ and $\forall j \in \mathcal{A}$, $g_k \le g_j$.

Proof: Please refer to Appendix V. ■

From Lemmas 4, 6, and 7, it is clear that, in the optimal solution of the problem in (34), the set of all the links (i.e., $\{1, ..., N\}$) can be partitioned into three sub-sets, $\mathcal{A}_1$ (in which each relay uses its maximal allowed power), $\mathcal{A}_2$ (where $|\mathcal{A}_2| \le 1$, in which the relay, if exists, uses less than its maximal allowed power) and $\mathcal{B}$ (which included relays that are not selected); and for $\forall i \in \mathcal{A}_1, j \in \mathcal{A}_2, k \in \mathcal{B}$, we have $g_i \ge g_j \ge g_k$. If we sort the N links in descending order of the channel gains g_i's, i.e., $g_{s_1} \ge g_{s_2} \ge .... \ge g_{s_N}$, where $(s_1, ..., s_N)$ is a permutation of $(1, ..., N)$, then there are $N+1$ possible cases for the optimal solution of the problem in (34): in Case j, there are $j-1$ channels in set $\mathcal{A}_1$, detailed as follows.

- Case 1: $\mathcal{A}_1 = \emptyset$, $s_1 \in \mathcal{A}_2 \cup \mathcal{B}$, and $s_2, s_3, ..., s_N \in \mathcal{B}$, which means $P_{s_1} \in [0, \frac{Q_{s_1}^{ST} h_{s_1}}{g_{s_1}})$, and $P_{s_2} = P_{s_3} = ... = P_{s_N} = 0$.
- Case 2: $\mathcal{A}_1 = \{s_1\}$, $s_2 \in \mathcal{A}_2 \cup \mathcal{B}$, and $s_3, s_4, ..., s_N \in \mathcal{B}$, which means $P_{s_1} = \frac{Q_{s_1}^{ST} h_{s_1}}{g_{s_1}}$, $P_{s_2} \in [0, \frac{Q_{s_2}^{ST} h_{s_2}}{g_{s_2}})$, and $P_{s_3} = P_{s_4} = ... = P_{s_N} = 0$.

$\vdots$

- Case j: $\mathcal{A}_1 = \{s_1, s_2, ..., s_{j-1}\}$, $s_j \in \mathcal{A}_2 \cup \mathcal{B}$, and $s_{j+1}, s_{j+2}, ..., s_N \in \mathcal{B}$, which means $P_{s_i} = \frac{Q_{s_i}^{ST} h_{s_i}}{g_{s_i}}$ for $i = 1, 2, ..., j-1$, $P_{s_j} \in [0, \frac{Q_{s_j}^{ST} h_{s_j}}{g_{s_j}})$, and $P_{s_{j+1}} = P_{s_{j+2}} = ... = P_{s_N} = 0$.

$\vdots$

- Case $N+1$: $\mathcal{A}_1 = \{s_1, s_2, ..., s_N\}$, which means $P_{s_i} = \frac{Q_{s_i}^{ST} h_{s_i}}{g_{s_i}}$ for $i = 1, ..., N$.

Therefore, to solve the problem in (34), we need to search the $N+1$ cases only. We can first find the optimal[8] solutions in Cases 1, 2, ..., and $N+1$, respectively. Then the optimal solution with the largest objective function among the $N+1$ cases will be the global optimal solution for problem in (34).

Next we show how to find the optimal solutions for the $N+1$ cases.

In Case $j \leq N$, the power allocation values for all links (except Link s_j) are known (for $i < j$, Link s_i uses its maximal allowed power in the second hop; for $i > j$, Link s_j is not assigned power). Denote the power allocation value for Link s_j in Case j as φ_j, the optimal value of which is to be determined as follows.

For the problem in (34), the achieved utility in Case j is given as

$$\ln\left(1+\sum_{i=1}^{j-1} Q_{s_i}^{ST} h_{s_i} + \varphi_j g_{s_j}\right) - \lambda\left(\sum_{i=1}^{j-1} \frac{Q_{s_i}^{ST} h_{s_i}}{g_{s_i}} + \varphi_j\right). \tag{35}$$

The optimal φ_j can be obtained by setting the derivative of (35) to zero, given as

$$\frac{g_{s_j}}{1+\sum\limits_{i=1}^{j-1} Q_{s_i}^{ST} h_{s_i} + \varphi_j g_{s_j}} - \lambda = 0. \tag{36}$$

Generally after optimal utility values in the $N+1$ cases are obtained, the largest one is the optimal utility of the problem in (34).

Actually it may not be necessary to search all the $N+1$ cases. To demonstrate this, we take a look at (36) first, for which we have the following three observations for the optimal φ_j, denoted φ_j^*.

- When $1+\sum\limits_{i=1}^{j-1} Q_{s_i}^{ST} h_{s_i} < \frac{g_{s_j}}{\lambda} < 1+\sum\limits_{i=1}^{j} Q_{s_i}^{ST} h_{s_i}$, we have
$$\varphi_j^* = \frac{1}{g_{s_j}}\left(\frac{g_{s_j}}{\lambda} - 1 - \sum_{i=1}^{j-1} Q_{s_i}^{ST} h_{s_i}\right) \in \left(0, \frac{Q_{s_j}^{ST} h_{s_j}}{g_{s_j}}\right).$$
- When $\frac{g_{s_j}}{\lambda} \leq 1+\sum\limits_{i=1}^{j-1} Q_{s_i}^{ST} h_{s_i}$, the solution for (36) is $\frac{1}{g_{s_j}}\left(\frac{g_{s_j}}{\lambda} - 1 - \sum\limits_{i=1}^{j-1} Q_{s_i}^{ST} h_{s_i}\right) \leq 0$. Since the feasible region of φ_j is $[0, \frac{Q_{s_j}^{ST} h_{s_j}}{g_{s_j}})$, the optimal solution for Case j is $\varphi_j^* = 0$.
- When $\frac{g_{s_j}}{\lambda} \geq 1+\sum\limits_{i=1}^{j} Q_{s_i}^{ST} h_{s_i}$, the solution for (36) is $\frac{1}{g_{s_j}}\left(\frac{g_{s_j}}{\lambda} - 1 - \sum\limits_{i=1}^{j-1} Q_{s_i}^{ST} h_{s_i}\right) \geq \frac{Q_{s_j}^{ST} h_{s_j}}{g_{s_j}}$. However, the feasible region of φ_j is $[0, \frac{Q_{s_j}^{ST} h_{s_j}}{g_{s_j}})$. It can be seen that the maximal utility in the feasible region in Case j is less than the utility when $\varphi_j = \frac{Q_{s_j}^{ST} h_{s_j}}{g_{s_j}}$. The utility in the latter case is not greater than the maximal utility in Case $j+1$. Therefore, we can virtually set $\varphi_j^* = \frac{Q_{s_j}^{ST} h_{s_j}}{g_{s_j}}$.

[8]Note that this optimal solution is limited to a particular case, not global optimal solution.

Define $j^\dagger = \arg\min_j \left(\varphi_j^* \in [0, \frac{Q_{s_j}^{ST} h_{s_j}}{g_{s_j}})\right)$. The following lemma is in order.

Lemma 8: The maximal utility in Case $j^\dagger$ is the maximal utility of problem in (34).

Proof: Please refer to Appendix VI. ■

Lemma 8 tells that, to obtain the optimal solution for problem (34), we need only search until the first case (say Case j) such that φ_j^* is not equal to $\frac{Q_{s_j}^{ST} h_{s_j}}{g_{s_j}}$. Accordingly, the optimal solution of the problem in (34) can be searched as follows.

Algorithm 2 Searching procedure for the optimal solution of the problem in (34).

1: Sort the N links in descending order of the channel gains g_i's, i.e., $g_{s_1} \geq g_{s_2} \geq .... \geq g_{s_N}$.
2: Find $j^\dagger = \arg\min_j \left(\varphi_j^* < \frac{Q_{s_j}^{ST} h_{s_j}}{g_{s_j}}\right)$
3: Output the optimal power configuration: $P_{s_j} = \frac{Q_{s_j}^{ST} h_{s_j}}{g_{s_j}}$ for $0 < j < j^\dagger$, $P_{s_j} = \frac{1}{g_{s_j}}\left(\frac{g_{s_j}}{\lambda} - 1 - \sum\limits_{i=1}^{j-1} Q_{s_i}^{ST} h_{s_i}\right)$ for $j = j^\dagger$, and $P_{s_j} = 0$ for $j > j^\dagger$. The maximal utility is $\ln\left(1+\sum\limits_{i=1}^{j^\dagger} P_{s_i} g_{s_i}\right) - \lambda \sum\limits_{i=1}^{j^\dagger} P_{s_i}$. The bandwidth configuration is $x_{s_j} = \frac{P_{s_j} g_{s_j}}{\sum\limits_{i=1}^{j^\dagger} P_{s_i} g_{s_i}}$, $j \in \mathcal{N}$.

In Step 3 of the algorithm, the bandwidth configuration is from the fact that, for $\forall j \in \mathcal{A}$, $\text{SNR}_j = \sum\limits_{i \in \mathcal{A}} g_i P_i$.

It can be seen that the complexity of the algorithm is $O(N)$.

Remark: Compared to the scenario in Section III-A, the short-term source-power constraint in Section III-B changes the upper bound of the number of selected relays from one to two, while the short-term relay-power constraint in Section III-C makes all selected relays except one transmit to the destination with their maximal allowed power levels. The difference of the impact of short-term source-power constraint and short-term relay-power constraint lies in the fact that the source power is used to transmit to all selected relays while the relay power is used to transmit to the destination only. Interestingly, with short-term relay-power constraints, relays with larger channel gains g_i's are selected. Note that it does not mean the relay selection is independent of h_i's, since h_i's affect how many relays are selected.

D. Scenario with Short-term Source-power and Short-term Relay-power Constraints

Similar to previous subsections, the following sub-problem is expected to be solved for every realization of $(\boldsymbol{g}, \boldsymbol{h})$ in this scenario.

$$\max_{\{P_i\},\ \{x_i\}} \quad \sum_{i=1}^{N} x_i \ln\left(1+\frac{P_i g_i}{x_i}\right)$$

$$\text{s.t.} \quad \frac{P_i g_i}{h_i} \le Q^{ST}, \forall i \in \mathcal{N} \tag{37a}$$

$$\sum_{i=1}^{N} P_i \le P^{ST}, \tag{37b}$$

$$P_i \ge 0, x_i \ge 0,\ \forall i \in \mathcal{N};\quad \sum_{i=1}^{N} x_i \le 1. \tag{37c}$$

The KKT condition is written as follows.

$$\ln(1+\frac{P_i g_i}{x_i}) - \frac{P_i g_i}{x_i + P_i g_i} - \Gamma^* + \delta_i^* = 0,\ \forall i \in \mathcal{N} \tag{38a}$$

$$\frac{x_i g_i}{x_i + g_i P_i} - \lambda^* - \frac{\mu_i^* g_i}{h_i} + \Delta_i^* = 0,\ \forall i \in \mathcal{N} \tag{38b}$$

$$\lambda^* \left(\sum_{i=1}^{N} P_i - P^{ST}\right) = 0 \tag{38c}$$

$$\mu_i^* \left(\frac{P_i g_i}{h_i} - Q_i^{ST}\right) = 0,\ \forall i \in \mathcal{N} \tag{38d}$$

$$\Delta_i^* P_i = 0, \delta_i^* x_i = 0,\ \forall i \in \mathcal{N};\ \Gamma^* \left(\sum_{i=1}^{N} x_i - 1\right) = 0 \tag{38e}$$

$$\text{Constraints (37a) – (37c)} \tag{38f}$$

where μ_i^*, λ^*, Δ_i^*, δ_i^*, and Γ^* are non-negative Lagrange multipliers associated with constraints $\frac{P_i g_i}{h_i} \le Q^{ST}$, $\sum_{i=1}^{N} P_i \le P^{ST}$, $P_i \ge 0$, $x_i \ge 0$, and $\sum_{i=1}^{N} x_i \le 1$, respectively.

Define $\mathcal{A} = \{j|P_j > 0, x_j > 0\} = \mathcal{A}_1 \bigcup \mathcal{A}_2$ where $\mathcal{A}_1 = \{j|P_j = \frac{Q_j^{ST} h_j}{g_j}, x_j > 0\}$ and $\mathcal{A}_2 = \{j|0 < P_j < \frac{Q_j^{ST} h_j}{g_j}, x_j > 0\}$. Define $\mathcal{B} = \{j|P_j = 0, x_j = 0\}$. So $\mathcal{A}$, $\mathcal{A}_1$, $\mathcal{A}_2$, and $\mathcal{B}$ have the same physical meanings as in Section III-C.

To solve the sub-problem in (37), the following lemmas are in order.

Lemma 9:

- When the constraint in (37b) is inactive, $|\mathcal{A}_2| = 0$.
- When the constraint in (37b) is active, the associated Lagrange multiplier $\lambda^* > 0$, and $|\mathcal{A}_2| \le 1$.

Proof: Please refer to Appendix VII. ■

Similar to Lemmas 5-7, we have the following lemmas (with proofs omitted).

Lemma 10: For $\forall i, j \in \mathcal{A}, \mathrm{SNR}_i = \mathrm{SNR}_j$.

Lemma 11: For $\forall i \in \mathcal{A}_1$ and $\forall j \in \mathcal{A}_2$, $g_i \ge g_j$.

Lemma 12: For $\forall j \in \mathcal{A}$ and $\forall k \in \mathcal{B}$, $g_j \ge g_k$.

Similar to Section III-C, the sub-problem in (37) can be reduced (according to Lemma 10) to the following problem

$$\max_{\{P_i\}} \quad \ln\left(1+\sum_{i=1}^{N} P_i g_i\right)$$

$$\text{s.t.} \quad 0 \le P_i \le \frac{Q_i^{ST} h_i}{g_i},\ \forall i \in \mathcal{N} \tag{39a}$$

$$\sum_{i=1}^{N} P_i \le P^{ST}. \tag{39b}$$

It can be easily proved that the problem in (39) can be solved as follows (the proof is omitted). First sort all the links, $\{1, ..., N\}$, based on the descending order of channel gains g_i's such that $g_{s_1} \ge g_{s_2} \ge ... \ge g_{s_N}$. Denote $i^* = \arg\min_i \sum_{j=1}^{i} \frac{Q_{s_j}^{ST} h_{s_j}}{g_{s_j}} > P^{ST}$.[9] Then the solution for the problem in (39) is: $P_{s_i} = \frac{Q_{s_i}^{ST} h_{s_i}}{g_{s_i}}, i = 1, ..., i^*-1$; $P_{s_{i^*}} = P^{ST} - \sum_{i=1}^{i^*-1} \frac{Q_{s_i}^{ST} h_{s_i}}{g_{s_i}}$; and $P_{s_i} = 0, i = i^*+1, i^*+2, ..., N$; $x_{s_i} = \frac{P_{s_i} g_{s_i}}{\sum_{i=1}^{i^*} P_{s_i} g_{s_i}}, i \in \mathcal{N}$. The computational complexity is $O(N)$.

Remark: Similar to Section III-C, relays with larger g_i's are selected, and all selected relays except one use the maximal allowed power level to transmit to the destination. The difference is that the short-term source-power constraint in Section III-D makes the power allocation simpler, i.e., the short-term source power is assigned to relays according to the descending order of g_i's, until all the short-term source power is used up. The simplicity is because, with short-term source-power constraint, when the channels are not good at a moment, we do not need to save source power for other moments with good channels.

IV. Implementation Issues

A. Training for Channel Gains

Information of g_i's and h_i's can be obtained by training. In the literature, training schemes have already been studied in relay networks [35], [36]. In our research, the training scheme can be designed as follows. First, each relay sends (in turn) a pilot to the source. Then the source can estimate g_i's. Note that here we assume channel reciprocity. Then, the destination broadcasts a pilot to all the relays, and after that, each relay amplifies its received signal (from the destination) and sends in turn to the source. Since the source already has the information of g_i's, it can estimate h_i's based on the received signals from the relays. It can be seen that the overhead is $2N+1$, linear to the number of relays, which is usually considered acceptable in relay networks [35], [36].

B. Online Calculation of Resource Allocation

Take the scenario in Section III-A as an example. To update λ and μ_i's as in (19) and (20), $\left(P^{LT} - \mathbb{E}\left[\sum_{i=1}^{N} P_i(t)\right]\right)$ and $\left(Q_i^{LT} - \mathbb{E}\left[\frac{P_i(t) g_i}{h_i}\right]\right)$ should be calculated, in which the two expectation terms require the resource allocation solution of the sub-problem in (21) for any possible realization of channel gains g_i's and h_i's. Thus, the computational complexity is high. In addition, PDF of the channel gains $f(\boldsymbol{g}, \boldsymbol{h})$ is required to be known in advance. To address these problems, an online (fading block-by-fading block) calculation method of resource allocation is adopted, similar to that in [37].

Step 1: Initialize $\lambda(0)$ and $\mu_i(0)$ $(i \in \mathcal{N})$.

Step 2: At the beginning of the t^{th} fading block, the source S measures channel gains $g_i(t)$'s and $h_i(t)$'s $(i \in \mathcal{N})$, solves sub-problem (21) for that particular channel gain realization

[9] Note that if $\sum_{j=1}^{N} \frac{Q_{s_j}^{ST} h_{s_j}}{g_{s_j}} < P^{ST}$, then $P_{s_i} = \frac{Q_{s_i}^{ST} h_{s_i}}{g_{s_i}}, i = 1, ..., N$.

and gets solutions $P_i(t)$, broadcasts resource allocation decision to all relays, and updates Lagrange multipliers

$$\lambda(t+1)=\left(\lambda(t)-a(t)\left(P^{LT}-\sum_{i=1}^{N}P_i(t)\right)\right)^{+} \quad (40)$$

$$\mu_i(t+1)=\left(\mu_i(t)-a(t)\left(Q_i^{LT}-\frac{P_i(t)g_i(t)}{h_i(t)}\right)\right)^{+}, \quad \forall i\in\mathcal{N}. \quad (41)$$

It is proved in [37] that $\lambda(t)$ and $\mu_i(t)$'s will converge to the optimal λ and μ_i's values with probability 1, under the conditions 1) $a(t)\geq 0, \sum_{t=0}^{+\infty}a(t)=+\infty$, and $\sum_{t=0}^{+\infty}a(t)^2<+\infty$, and 2) $(P^{LT}-\sum_{i=1}^{N}P_i(t))$ and $(Q_i^{LT}-\frac{P_i(t)g_i(t)}{h_i(t)})$ are bounded.

Compared with (19) and (20), expectation terms do not exist in (40) and (41), thus leading to much less complexity. In addition, the new updating method also works when the channel statistics information $f(\boldsymbol{g},\boldsymbol{h})$ is unknown, because 1) the impact of $f(\boldsymbol{g},\boldsymbol{h})$ is represented by the realizations of the channel gains g_i's and h_i's ($i\in\mathcal{N}$) in continuous fading blocks, and 2) the realizations of channel gains can be measured at the beginning of the fading blocks.

Note that the updating method in (40) and (41) is called *stochastic sub-gradient method*, broadly used for solving stochastic optimization problems. More details for this method can be found in references [37], [38] and [39].

V. The Case with Peak Power Spectrum Density Constraints

In this section, we consider the case when additional constraints of peak power spectrum density (PSD) are imposed on the source and relays. Denote the peak power spectrum density constraints for the source and relay i as P^{PEAK} and Q_i^{PEAK}, respectively. Then, we have

$$\frac{P_i}{x_i}\leq P^{\text{PEAK}},\ \forall i\in\mathcal{N} \quad (42)$$

and

$$\frac{Q_i}{x_i}\leq Q_i^{\text{PEAK}},\ \forall i\in\mathcal{N}. \quad (43)$$

Since Q_i can be replaced with $\frac{P_ig_i}{h_i}$ by equation (12), the constraints (42) and (43) can be combined to be

$$\frac{P_i}{x_i}\leq U_i^{\text{PEAK}},\ \forall i\in\mathcal{N} \quad (44)$$

where $U_i^{\text{PEAK}}=\min\left(P^{\text{PEAK}},\frac{Q_i^{\text{PEAK}}h_i}{g_i}\right)$. Then the average rate maximization problem in this paper with additional peak power spectrum density constraints can be formulated as Problem P2 with additional constraints (44). Again, the problem can be decomposed into two levels, and next we focus on the sub-problems in the lower level. We still consider the four scenarios as in Section III. Fast algorithms with closed-form solutions may exist in scenarios with long-term source-power and long-term relay-power constraints and with short-term source-power and long-term relay-power constraints, as shown in the following two subsections, respectively. However, for the other two scenarios, fast algorithms with closed-form solutions cannot be found, and thus, traditional methods (such as sub-gradient method and interior-point algorithm) may have to be resorted to.

A. Scenario with Long-term Source-power and Long-term Relay-power Constraints

In this scenario, a sub-problem as in (21) with additional constraints in (44) is supposed to be solved. Similar to Section III, we define $\mathcal{A}\triangleq\{j|P_j>0,x_j>0\}$ and $\mathcal{B}\triangleq\{j|P_j=0,x_j=0\}$. It can be seen that $\mathcal{A}\cup\mathcal{B}=\{1,...,N\}$. Denote $\mathcal{A}=\mathcal{C}\cup\mathcal{D}$, where $\mathcal{C}\triangleq\left\{j|0<\frac{P_j}{x_j}<U_j^{\text{PEAK}},x_j>0\right\}$ (i.e., the set of selected relays with transmit power spectrum density over the first hop less than the peak value) and $\mathcal{D}\triangleq\left\{j|\frac{P_j}{x_j}=U_j^{\text{PEAK}},x_j>0\right\}$ (i.e., the set of selected relays with transmit power spectrum density over the first hop equal to the peak value). It can be proved using the similar method to that in Lemma 1 that

Lemma 13: If $\mathcal{C}\neq\emptyset$, then $|\mathcal{C}|\leq 1$.

Further, we have the following two lemmas.

Lemma 14: If $\mathcal{D}\neq\emptyset$, then $|\mathcal{D}|\leq 1$.

Proof: For $j\in\mathcal{D}$, $P_j=x_jU_j^{\text{PEAK}}$. The sub-problem can be rewritten as

$$\begin{aligned}\max_{\{P_i\},\ \{x_i\}}\quad & \sum_{i\in\mathcal{C}}x_i\ln\left(1+\frac{P_ig_i}{x_i}\right)-\lambda\sum_{i\in\mathcal{C}}P_i- \\ & \sum_{i\in\mathcal{C}}\mu_i\frac{P_ig_i}{h_i}+\sum_{j\in\mathcal{D}}x_j\Bigg(\ln\left(1+U_j^{\text{PEAK}}g_j\right) \\ & -\lambda U_j^{\text{PEAK}}-\mu_j\frac{U_j^{\text{PEAK}}g_j}{h_j}\Bigg) \\ \text{s.t.}\quad & \sum_{i\in\mathcal{C}}x_i+\sum_{j\in\mathcal{D}}x_j\leq 1.\end{aligned} \quad (45)$$

When $\mathcal{D}\neq\emptyset$,

$$\max_{j\in\mathcal{D}}\left(\ln\left(1+U_j^{\text{PEAK}}g_j\right)-\lambda U_j^{\text{PEAK}}-\mu_j\frac{U_j^{\text{PEAK}}g_j}{h_j}\right)>0.$$

Define

$$j^*=\arg\max_{j}\left(\ln\left(1+U_j^{\text{PEAK}}g_j\right)-\lambda U_j^{\text{PEAK}}-\mu_j\frac{U_j^{\text{PEAK}}g_j}{h_j}\right).$$

If $|\mathcal{D}|>1$, then it will achieve a higher objective function of (45) if the bandwidth assignments for links in $|\mathcal{D}|$ are instead all assigned to Link j^* only. Therefore, in the optimal solution, we have $|\mathcal{D}|=1$. ■

Similar to proof of Lemma 14, we can prove the following lemma.

Lemma 15: $|\mathcal{A}|\leq 1$.

According to Lemma 15, at most one relay is selected. Suppose the only selected relay, if exists, is R_i. To maximize the objective function in (21), x_i should be set as 1. By setting the derivative of the objective function in (21) with respect to P_i as 0, P_i can be obtained as $\left(\frac{h_i}{\lambda h_i+\mu_ig_i}-\frac{1}{g_i}\right)$. Considering the constraint $P_i\geq 0$ and $\frac{P_i}{x_i}\leq U_i^{\text{PEAK}}$ further, the optimal power configuration should be $P_i=\left[\frac{h_i}{\lambda h_i+\mu_ig_i}-\frac{1}{g_i}\right]_0^{U_i^{\text{PEAK}}}$.

In summary, the optimal solution of the problem in (21) with additional constraints in (44) can be obtained as follows.

Algorithm 3 Searching procedure for the optimal solution of the problem in (21) with additional constraints in (44).

1: Define $V_i = \ln\left(1+\left[\frac{h_i}{\lambda h_i+\mu_i g_i}-\frac{1}{g_i}\right]_0^{U_i^{\text{PEAK}}} g_i\right) - \lambda\left[\frac{h_i}{\lambda h_i+\mu_i g_i}-\frac{1}{g_i}\right]_0^{U_i^{\text{PEAK}}} - \mu_i \frac{\left[\frac{h_i}{\lambda h_i+\mu_i g_i}-\frac{1}{g_i}\right]_0^{U_i^{\text{PEAK}}} g_i}{h_i}$.
2: **if** $\max_i V_i \le 0$ **then**
3: The maximal utility is 0, with $P_i = 0$ and $x_i = 0$, $\forall i \in \mathcal{N}$.
4: **else**
5: Find $i^* = \arg\max_i V_i$.
Output the optimal index i^*, with optimal bandwidth and power allocation strategy $x_{i^*} = 1$ and $P_{i^*} = \left[\frac{h_{i^*}}{\lambda h_{i^*}+\mu_{i^*} g_{i^*}}-\frac{1}{g_{i^*}}\right]_0^{U_{i^*}^{\text{PEAK}}}$, respectively, and the maximal utility V_{i^*}.

It can be seen that the complexity of the algorithm is $O(N)$.

B. Scenario with Short-term Source-power and Long-term Relay-power Constraints

In this scenario, a sub-problem as in (28) with additional constraints (44) is going to be solved. For the sub-problem, still define $\mathcal{A} \triangleq \{j|P_j > 0, x_j > 0\}$ and $\mathcal{A} = \mathcal{C} \cup \mathcal{D}$, where $\mathcal{C} \triangleq \left\{j|0 < \frac{P_j}{x_j} < U_i^{\text{PEAK}}, x_j > 0\right\}$ and $\mathcal{D} \triangleq \left\{j|\frac{P_j}{x_j} = U_i^{\text{PEAK}}, x_j > 0\right\}$. The following lemmas can be expected.

Lemma 16: If $\mathcal{C} \neq \emptyset$,

- when the constraint $\sum_{i=1}^{N} P_i \le P^{ST}$ in (28a) is inactive, $|\mathcal{C}| = 1$.
- when the constraint $\sum_{i=1}^{N} P_i \le P^{ST}$ in (28a) is active, $|\mathcal{C}| \le 2$.

Lemma 17: If $\mathcal{D} \neq \emptyset$, then $|\mathcal{D}| \le 1$.

Lemma 18: When the constraint $\sum_{i=1}^{N} P_i \le P^{ST}$ in (28a) is inactive, $|\mathcal{A}| \le 1$.

The proofs of the three lemmas are similar to those of Lemmas 2, 14, and 15, respectively, and thus, are omitted.

According to Lemma 18, when the constraint $\sum_{i=1}^{N} P_i \le P^{ST}$ is inactive, at most one relay is selected. Suppose the only selected relay, if exists, is R_i. Similar to Section V-A, the optimal bandwidth and power configuration is $x_i = 1$ and $P_i = \left[\frac{1}{g_i}\left(\frac{h_i}{\mu_i}-1\right)\right]_0^{U_i^{\text{PEAK}}}$.

In summary, when $\sum_{i=1}^{N} P_i \le P^{ST}$ is inactive (e.g., a sufficient condition for this is $\max_i(U_i^{\text{peak}}) < P^{ST}$), the optimal solution of the sub-problem in (28) with additional constraints in (44) can be obtained as follows.

Algorithm 4 Searching procedure for the optimal solution of the sub-problem in (28) with additional constraints in (44) (when $\sum_{i=1}^{N} P_i \le P^{ST}$ is inactive).

1: Define $Z_i = \ln\left(1+\left[\frac{1}{g_i}\left(\frac{h_i}{\mu_i}-1\right)\right]_0^{U_i^{\text{PEAK}}} g_i\right) - \mu_i \frac{\left[\frac{1}{g_i}\left(\frac{h_i}{\mu_i}-1\right)\right]_0^{U_i^{\text{PEAK}}} g_i}{h_i}$.
2: **if** $\max_i Z_i \le 0$ **then**
3: The maximal utility is 0, with $P_i = 0$ and $x_i = 0$, $\forall i \in \mathcal{N}$.
4: **else**
5: Find $i^* = \arg\max_i Z_i$.
Output the optimal index i^*, with optimal bandwidth and power allocation strategy $x_{i^*} = 1$ and $P_{i^*} = \left[\frac{1}{g_{i^*}}\left(\frac{h_{i^*}}{\mu_{i^*}}-1\right)\right]_0^{U_{i^*}^{\text{PEAK}}}$, respectively, and the maximal utility Z_{i^*}.

It can be seen that the complexity of the algorithm is $O(N)$. When $\sum_{i=1}^{N} P_i \le P^{ST}$ is active, fast algorithms with closed-form solutions cannot be found, and therefore, traditional methods (such as sub-gradient method and interior-point algorithm) may have to be resorted to.

Table I summarizes the number of selected relays and algorithm complexity for the six scenarios discussed in Section III and Section V, denoted as LT-LT, ST-LT, LT-ST, ST-ST, LT-LT (PK) and ST-LT (PK), respectively. Here the LT/ST before the hyphen means long-term/short-term source-power constraint, while LT/ST after the hyphen means long-term/short-term relay-power constraint, and PK means that peak power spectrum density constraints are imposed.

VI. Numerical Results

In this section, numerical results are given to show the performance of the algorithms for the four scenarios, LT-LT, ST-LT, LT-ST and ST-ST, discussed in Section III. Peak power spectrum density constraints are not considered. The channel gains g_i's, h_i's ($i = 1, ..., N$) are independent and exponentially distributed (i.e., Rayleigh fading channels) with mean being 1. The channel gain distributions are supposed to be unknown by any node. So the stochastic sub-gradient method introduced in Section IV is used to update Lagrange multipliers λ and μ_i's iteratively over fading blocks. All the initial values of Lagrange multipliers are set as 1. The step size $a(t)$ in (40) and (41) is selected as $a(t) = \frac{1}{t+200}$.

A. Convergence of Lagrange multipliers

In this subsection, the convergence of Lagrange multipliers λ and μ_i's is illustrated (note that for the original optimization problem, short-term power constraints do not have Lagrange multipliers). In this numerical example, the long-term and short-term power constraint of the source, P^{LT} and P^{ST}, are set to have a value equal to the number of relays. For example, when there are 4 relays, both P^{LT} and P^{ST} are set as 4. The long-term and short-term power constraint of every relay, Q_i^{LT} and Q_i^{ST}, are set as 1.

Fig. 1, Fig. 2 and Fig. 3 show the updating iterations of the Lagrange multipliers as in (40) and (41) in different scenarios with 4 relays, 8 relays and 16 relays, respectively. Note that

TABLE I
NUMBER OF SELECTED RELAYS AND ALGORITHM COMPLEXITY

Scenario	Number of selected relays	Algorithm complexity
LT-LT	0 or 1	$O(N)$
ST-LT	$0, 1,$ or 2	$O(N^2)$
LT-ST	$0, 1, ..., N-1,$ or N	$O(N)$
ST-ST	$1, ..., N-1,$ or N	$O(N)$
LT-LT (PK)	0 or 1	$O(N)$
ST-LT (PK) (with $\sum_{i=1}^{N} P_i \leq P^{ST}$ inactive)	0 or 1	$O(N)$

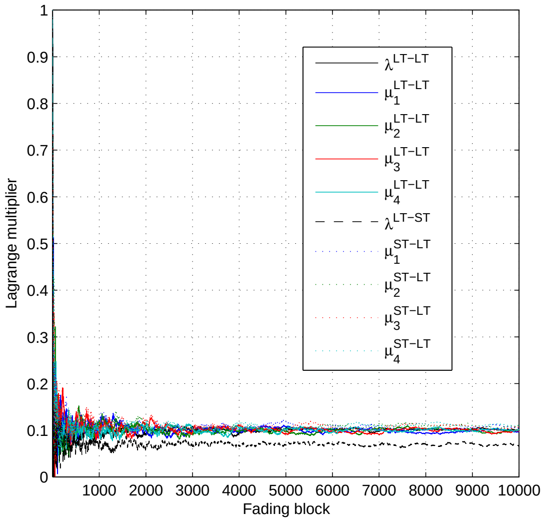

Fig. 1. Convergence of Lagrange multipliers for the average rate maximization problem with 4 relays.

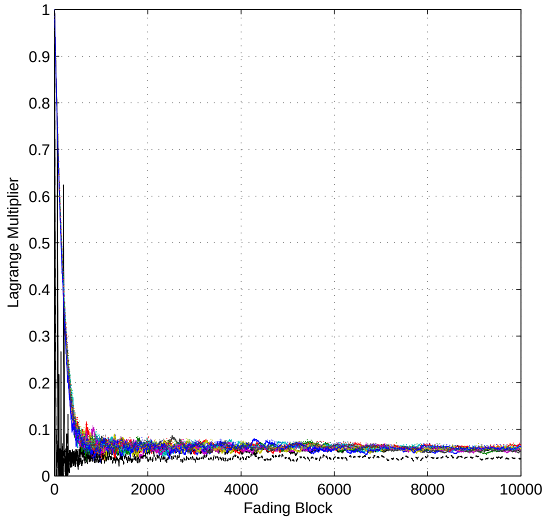

Fig. 2. Convergence of Lagrange multipliers for the average rate maximization problem with 8 relays.

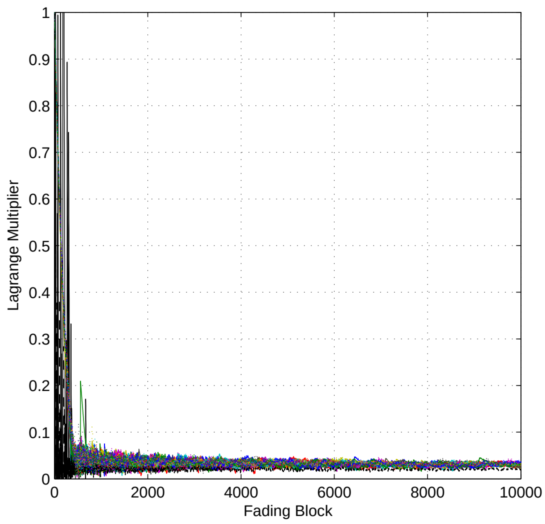

Fig. 3. Convergence of Lagrange multipliers for the average rate maximization problem with 16 relays.

in Fig. 2 and Fig. 3, legends are not illustrated due to the large number of Lagrange multipliers. It can be seen from the figures that all the Lagrange multipliers converge within a few thousand fading blocks. In particular, all the Lagrange multipliers in Fig. 1, Fig. 2 and Fig. 3 converge to nonzero values, which means that the associated long-term constraints are all active. It can be also seen from the three figures that the number of relays has negligible impact on the convergence speed.

B. Average rates

Consider 4 relays. When the source-power constraint (either long-term or short-term) is fixed as 1, Fig. 4 shows the average rate, where the horizontal axis means the (long-term or short-term) relay-power constraint. Here all relays have the same value of power constraint in each scenario. When the relay-power constraint (either long-term or short-term) is fixed as 0.25, Fig. 5 shows the average rate, where the horizontal axis means the (long-term of short-term) source-power constraint. It can be seen that, with the same amount of power constraints, the LT-LT scenario always has the largest average rate, while the ST-ST scenario has the smallest average rate. This is because a long-term power constraint allows more flexibility than a short-term power constraint does, and therefore, leads to more efficient resource allocation.

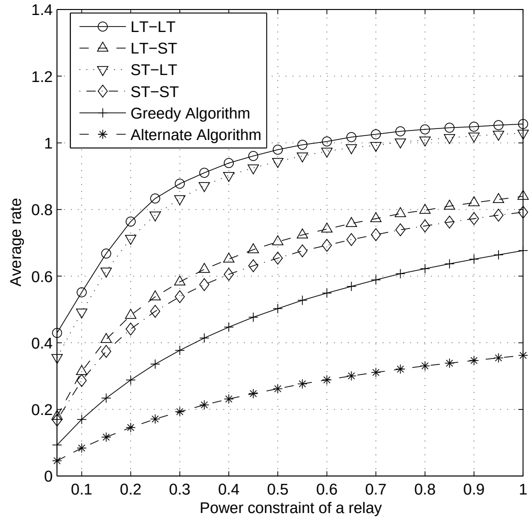

Fig. 4. Average rate vs. relay-power constraint (the source-power constraint is 1).

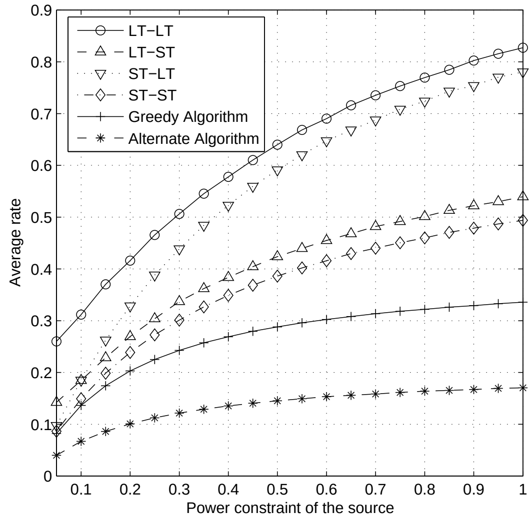

Fig. 5. Average rate vs. source-power constraint (relay-power constraint is 0.25).

As a comparison, two other resource allocation algorithms are considered: greedy algorithm and alternate algorithm. In either algorithm, both source and relays have short-term power constraints only, and in each fading block, only one relay is allowed to relay using the whole available bandwidth and its maximal allowed power. And the source uses its maximal allowed power. The difference of the greedy and alternate algorithms is: in the greedy algorithm, in a fading block only the relay which can render the maximal source-to-destination transmission rate is permitted to relay, while in the alternate algorithm, all the relays take turn to relay in fading blocks. The simulation results are also shown in Fig. 4 and Fig. 5. It can be seen that our proposed algorithms outperform the greedy algorithm and alternate algorithm. The is because our proposed algorithms take advantage of joint power and bandwidth allocation.

C. Range and variance of instant rates

Consider 4 relays. When the source-power constraint (either long-term or short-term) is fixed as 1, Table II shows the range and variance of instant rates (achieved in every fading block) in the four power constraint scenarios. Here all relays have the same value of power constraint. It can be seen that the maximal instant rate and the variance in LT-LT scenario are the largest among the four scenarios. This is because the source-power and relay-power constraints are both for long-term, resulting in the most flexibility in resource allocation. When either source-power constraint or relay-power constraint becomes short-term, the maximal instant rate and variance are reduced, but still larger than those in ST-ST. When at least one power constraint is for long-term (i.e., in LT-LT, ST-LT, or LT-ST scenario), it can be seen that the minimal instant rate is zero or almost zero. This is because when the channels are with poor quality, it is better not to select any relay, to save long-term power for moments when channels are good. On the other hand, in ST-ST scenario, the minimal instant rate is also close to zero. This is because although at any moment we have relay(s) selected, if the channels are extremely poor, the achievable instant rate is very small.

Long-term constraint can help to achieve a higher average rate, while short-term constraint is able to guarantee a smaller fluctuation of instant rates. Therefore, long-term constraints may be better for applications whose major performance metric is average rate (such as data transfer), while short-term constraints may be favored by applications that enjoy small rate fluctuation (such as voice applications).

D. Average number of selected relays

Still consider 4 relays. When the source-power constraint (either long-term or short-term) is fixed as 1, Fig. 6 shows the average number of selected relays when the relay-power constraint varies. Here all relays have the same value of power constraint, which can be as small as 10^{-3} and as large as 1000.

First consider LT-LT scenario and ST-LT scenario. When relay-power constraint is very small, the relay-power constraint is the “bottleneck” constraint. And thus, the two scenarios have similar performance (because the difference in the two scenarios is in source-power constraint), i.e., only when the channels are very good, a relay is selected. Therefore, the average number of selected relays is small. When the relay-power constraint increases, the possibility of selecting relay(s) increases, and thus, the average number of relays increases. Since the number of selected relays in LT-LT and ST-LT scenarios is bounded by one and two, respectively, the average number of relays in ST-LT scenario is larger than that in LT-LT scenario.

Similarly, the LT-ST and ST-ST scenarios have similar performance when the relay-power constraint is very small. Take ST-ST scenario as an example. According to the resource allocation algorithm in Section III-D, relays are selected according to the channel gains g_i's until the source power is used up. Since the source-power constraint is much larger than the relay-power constraint, all four relays should be selected, and thus, the average number of selected relays is 4 as shown in

TABLE II
RANGE AND VARIANCE OF INSTANT RATES

	Power constraint of a relay	0.5	1	1.5	2
Range	LT-LT	[0, 2.82]	[0, 2.86]	[0, 2.84]	[0, 2.84]
	ST-LT	[0, 2.4]	[0.02, 2.4]	[0.07, 2.4]	[0.08, 2.4]
	LT-ST	[0, 2.05]	[0, 2.33]	[0, 2.4]	[0, 2.63]
	ST-ST	[0.03, 1.76]	[0.05, 2.07]	[0.03, 2.27]	[0.05, 2.37]
Variance	LT-LT	0.25	0.29	0.28	0.28
	ST-LT	0.13	0.14	0.14	0.14
	LT-ST	0.1	0.14	0.17	0.19
	ST-ST	0.05	0.07	0.08	0.09

Fig. 6. When the relay-power constraint increases, the source power may not be sufficient to support all the four relays, and thus, the average number of selected relays tends to decrease.

When the relay-power constraint is very large, the source-power constraint is the "bottleneck" constraint. So ST-ST and ST-LT scenarios have similar performance. Take ST-ST as an example. Since the source-power constraint is much less than the relay-power constraint, according to the resource allocation algorithm in Section III-D, only the relay with the largest g_i's is selected. Therefore, the average number of relays converges to 1. Similarly, LT-LT and LT-ST scenario have similar performance when the relay-power constraint is very large. Take LT-ST as an example. When there are relays selected, according to Algorithm 2, it is very likely that at most one relay is selected. This is because, if there are two or more selected relays, at least one relay should transmit with its (very large) maximal allowed power, which the very limited source power is very likely not sufficient to support, according to (12). Consider the possibility that no relay is selected when channel gains are poor, the average number of selected relays converges to a value less than 1.

From Fig. 6, it can also be seen that LT constraint allows fewer relays to be selected compared with ST constraint, because LT constraint has more flexibility in resource allocation, and thus it can "wait" until the moments when there are good channels.

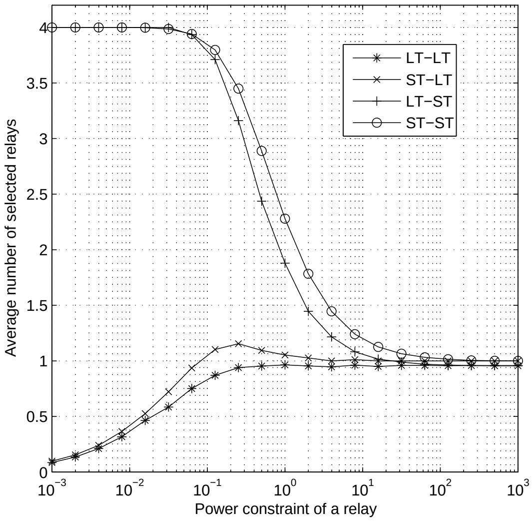

Fig. 6. Average number of selected relays vs. relay-power constraint.

E. Impact of Imperfect CSI

Consider 4 relays with long-term source-power and long-term relay-power constraints. Note that similar results are observed for the other three power constraint scenarios. Fig. 7 (when source-power constraint is 1) and Fig. 8 (when relay-power constraint is 0.25) illustrate the impact of imperfect CSI on achieved average rate, respectively. Similar to reference [40], the measured CSI (channel gains g_i's and h_i's) is disturbed by additive zero mean Gaussian random variable with variance σ^2.

The achieved average rate with perfect CSI, imperfect CSI with $\sigma = 0.1$ and $\sigma = 0.3$ are compared. It can be seen that with larger σ, the achieved average rate is lower.

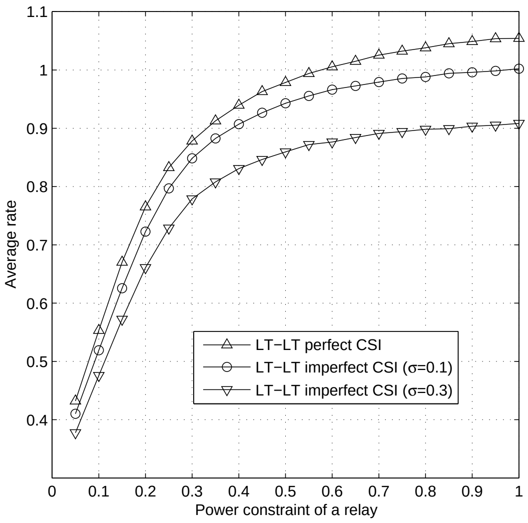

Fig. 7. Average rate vs. relay-power constraint with imperfect CSI.

VII. Conclusion

In this paper, the average rate maximization problem given transmission power constraints in DF relaying has been studied. With a consideration of either long-term or short-term power constraint imposed on the source and relays, the average rate maximization under four scenarios of power constraints

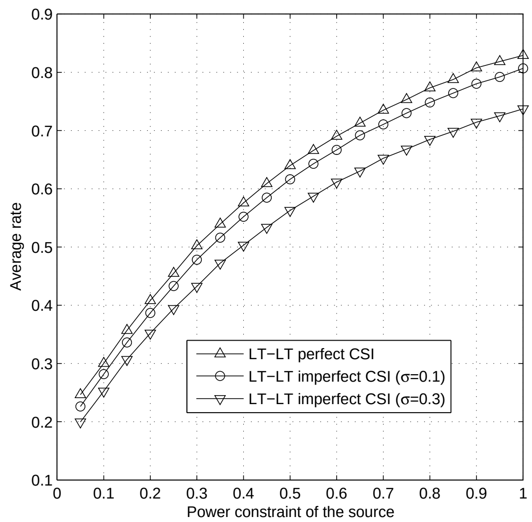

Fig. 8. Average rate vs. source-power constraint with imperfect CSI.

at the source and relays is investigated. In each scenario, the average rate maximization problem is decomposed to sub-problems, each corresponding to a realization of channel gains. For each sub-problem, special properties are found and a fast algorithm with closed-form resource allocation solution is given. This research should provide helpful insight to the design of relay networks over slow fading channels.

Appendix I
Convexity Proof of Problem P2

To prove convexity of Problem P2, we need to prove that the objective function is concave with respect to $\{P_i\}$ and $\{x_i\}$. Since the objection function in Problem P2 is separable with link index i, it is equivalent to prove that function $x\ln\left(1+\frac{ph}{x}\right)$ is concave with respect to p and x. We investigate the Hessian matrix of function $x\ln\left(1+\frac{ph}{x}\right)$, $\mathbb{H}$, which can be obtained as

$$\mathbb{H}=\frac{h^2}{x^3(1+\frac{hp}{x})^2}\begin{bmatrix}-p^2 & px\\ px & -x^2\end{bmatrix}$$

with eigenvalues values $-\frac{h^2(x^2+p^2)}{x^3(1+\frac{hp}{x})^2}$ and 0. With all the eigenvalues not larger than 0, it can be seen that matrix $\mathbb{H}$ is a semi-negative definite matrix. Therefore, the function $x\ln\left(1+\frac{ph}{x}\right)$ is concave. This completes the proof.

Appendix II
Proof of Lemma 2

A similar proof method to that in Lemma 1 is used.

We use proof by contradiction. When the constraint in (28a) is inactive, suppose $|\mathcal{A}|>1$, and $i^\dagger, i^\ddagger\in\mathcal{A}$. Similar to (26), we have

$$\frac{\Gamma^*}{\frac{\lambda^*}{g_{i^\dagger}}+\frac{\mu_{i^\dagger}}{h_{i^\dagger}}}=\frac{\Gamma^*}{\frac{\lambda^*}{g_{i^\ddagger}}+\frac{\mu_{i^\ddagger}}{h_{i^\ddagger}}} \tag{46}$$

and $\Gamma^*>0$.

When the constraint in (28a) is inactive, $\lambda^*=0$ according to (29c). Note that $\mu_{i^\dagger}$ and $\mu_{i^\ddagger}$ are fixed for the problem in (29). In this case, equation (46) holds with probability zero since $h_{i^\dagger}$ and $h_{i^\ddagger}$ are independent random variables.

When the constraint in (28a) is active, $\lambda^*\geq 0$. Suppose $|\mathcal{A}|>2$, and $i^\dagger, i^\ddagger, i'\in\mathcal{A}$. We have

$$\frac{\Gamma^*}{\frac{\lambda^*}{g_{i^\dagger}}+\frac{\mu_{i^\dagger}}{h_{i^\dagger}}}=\frac{\Gamma^*}{\frac{\lambda^*}{g_{i^\ddagger}}+\frac{\mu_{i^\ddagger}}{h_{i^\ddagger}}}=\frac{\Gamma^*}{\frac{\lambda^*}{g_{i'}}+\frac{\mu_{i'}}{h_{i'}}}. \tag{47}$$

Note that $\mu_{i^\dagger}$, $\mu_{i^\ddagger}$, and $\mu_{i'}$ are fixed for the problem in (29). It can be seen that equation (47) holds with probability zero since $g_{i^\dagger}$, $g_{i^\ddagger}$, $g_{i'}$, $h_{i^\dagger}$, $h_{i^\ddagger}$, and $h_{i'}$ are independent random variables.

This completes the proof.

Appendix III
Proof of Lemma 3

According to (29d), for $\forall i\in\mathcal{A}$, we have $\delta_i^*=0$. Since $\mathrm{SNR}_i=\frac{P_i g_i}{x_i}$, equation (29a) can be written as

$$\ln(1+\mathrm{SNR}_i)-\frac{\mathrm{SNR}_i}{1+\mathrm{SNR}_i}=\Gamma^*. \tag{48}$$

The left-hand side of (48) is in the form of $S(x)$ (defined in Section III-A), which is a monotonic increasing function. If $i,j\in\mathcal{A}$, then both i and j satisfy (48), which means $\mathrm{SNR}_i=\mathrm{SNR}_j$.

Appendix IV
Proof of Lemma 6

For $\forall i\in\mathcal{A}_1$ and $\forall j\in\mathcal{A}_2$, it can be seen that $\Delta_i^*=0$ and $\Delta_j^*=0$ according to equation (33d) and $\mu_j^*=0$ according to (33c). Therefore, for i and j, equation (33b) can be rewritten as

$$\frac{g_i}{1+\mathrm{SNR}_i}=\lambda+\frac{\mu_i^* g_i}{h_i} \tag{49}$$

and

$$\frac{g_j}{1+\mathrm{SNR}_j}=\lambda \tag{50}$$

respectively. Together with the conclusion of Lemma 5 and the fact that $\mu_i^*\geq 0$, it is apparent that $g_i\geq g_j$ from equations (49) and (50).

Appendix V
Proof of Lemma 7

We use proof by contradiction. For ease of presentation, assume in the optimal solution of the problem in (34) we have $1\in\mathcal{B}$ and $2\in\mathcal{A}$ and $g_1>g_2$. Then $P_1=0$, and $P_2\in(0,\frac{Q_2^{ST}h_2}{g_2}]$. In the optimal solution, the total utility achieved on Links 1 and 2 is

$$\ln(1+P_2 g_2)-\lambda P_2. \tag{51}$$

It can be seen that there exist $P_1^\dagger$ and $P_2^\dagger$ such that $0<P_1^\dagger\leq\frac{Q_1^{ST}h_1}{g_1}$, $0\leq P_2^\dagger<P_2$, and $P_1^\dagger+P_2^\dagger=P_2$. If the power on Links 1 and 2 is $P_1^\dagger$ and $P_2^\dagger$, respectively, the total achieved utility on Links 1 and 2 is

$$\ln\left(1+P_1^\dagger g_1+P_2^\dagger g_2\right)-\lambda P_2 \tag{52}$$

which is definitely larger than the utility in (51) since $g_1>g_2$.

This completes the proof.

APPENDIX VI
PROOF OF LEMMA 8

Before the proof, we have two properties for φ_j^*: If $\varphi_j^* = 0$, then $\varphi_{j+1}^* = 0$; If $0 < \varphi_j^* < \frac{Q_{s_j}^{ST} h_{s_j}}{g_{s_j}}$, then $\varphi_{j+1}^* = 0$. This is because, according to the three observations of (36), we have

$$\begin{aligned}
&\varphi_j^* = 0 \\
&\Leftrightarrow \tfrac{g_{s_j}}{\lambda} \le 1 + \sum_{i=1}^{j-1} Q_{s_i}^{ST} h_{s_i} \\
&\Rightarrow \tfrac{g_{s_j}}{\lambda} \le 1 + \sum_{i=1}^{j} Q_{s_i}^{ST} h_{s_i} \\
&\Rightarrow \tfrac{g_{s_{j+1}}}{\lambda} \le 1 + \sum_{i=1}^{j} Q_{s_i}^{ST} h_{s_i} \\
&\Leftrightarrow \varphi_{j+1}^* = 0.
\end{aligned} \tag{53}$$

$$\begin{aligned}
&0 < \varphi_j^* < \frac{Q_{s_j}^{ST} h_{s_j}}{g_{s_j}} \\
&\Rightarrow \tfrac{g_{s_j}}{\lambda} < 1 + \sum_{i=1}^{j} Q_{s_i}^{ST} h_{s_i} \\
&\Rightarrow \tfrac{g_{s_{j+1}}}{\lambda} < 1 + \sum_{i=1}^{j} Q_{s_i}^{ST} h_{s_i} \\
&\Rightarrow \varphi_{j+1}^* = 0.
\end{aligned} \tag{54}$$

Then it can be concluded that, for Case $j < j^\dagger$, we have $\varphi^* = \frac{Q_{s_j}^{ST} h_{s_j}}{g_{s_j}}$. Based on the third observations for (36), it can be seen that, for Case $j < j^\dagger$, the maximal utility in Case j is less than the maximal utility in Case $j+1$. Therefore, the maximal utility in Case $j^\dagger$ is larger than the maximal utility in any prior case.

Next we prove that the maximal utility in Case $j^\dagger$ is larger than the maximal utility in Case $j^\dagger+1$. Here we assume $\varphi_{j^\dagger}^* \in (0, \frac{Q_{s_{j^\dagger}}^{ST} h_{s_{j^\dagger}}}{g_{s_{j^\dagger}}})$, as a similar proof can apply when $\varphi_{j^\dagger}^* = 0$.

From the observations for (36) and the fact that $\varphi_{j^\dagger}^* \in (0, \frac{Q_{s_{j^\dagger}}^{ST} h_{s_{j^\dagger}}}{g_{s_{j^\dagger}}})$, it can be seen that $\varphi_{j^\dagger}^*$ is the solution of (36) in Case $j^\dagger$. It also means the maximal utility in Case $j^\dagger$ is larger than the utility when $P_{s_i} = \frac{Q_{s_i}^{ST} h_{s_i}}{g_{s_i}}$ for $i = 1, ..., j^\dagger - 1$, $P_{s_{j^\dagger}} = \frac{Q_{s_{j^\dagger}}^{ST} h_{s_{j^\dagger}}}{g_{s_{j^\dagger}}}$, and $P_{s_{j^\dagger+1}} = P_{s_{j^\dagger+2}} = ... = P_{s_N} = 0$. The latter utility is exactly the utility of the optimal solution in Case $j^\dagger + 1$ (which is because, $\varphi_{j^\dagger+1}^* = 0$ from the first property of φ_j^*).

Now we prove that the maximal utility in Case $j^\dagger + 1$ is larger than the maximal utility in Case $j^\dagger + 2$. Note that the optimal solution in Case $j^\dagger + 1$ is $\varphi_{j^\dagger+1}^* = 0$. This means the solution for (36) in Case $j^\dagger + 1$ is a negative value. In the optimal solution in Case $j^\dagger + 1$ (i.e., $P_{s_{j^\dagger+1}} = \varphi_{j^\dagger+1}^* = 0$), if the value of $P_{s_{j^\dagger+1}}$ increases from $\varphi_{j^\dagger+1}^* = 0$ to $\frac{Q_{s_{j^\dagger+1}}^{ST} h_{s_{j^\dagger+1}}}{g_{s_{j^\dagger+1}}}$, the utility function decreases. On the other hand, in the optimal solution in Case $j^\dagger + 1$, when the value of $P_{s_{j^\dagger+1}}$ is changed to $\frac{Q_{s_{j^\dagger+1}}^{ST} h_{s_{j^\dagger+1}}}{g_{s_{j^\dagger+1}}}$, it leads to the optimal solution in Case $j^\dagger + 2$ (according to the second property of φ_j^*). Therefore, the maximal utility in Case $j^\dagger + 1$ is larger than the maximal utility in Case $j^\dagger + 2$.

By induction, it can be proved that when $j > j^\dagger$, the maximal utility in Case j is larger than the maximal utility in Case $j + 1$. It follows that the maximal utility in Case $j^\dagger$ is larger than the maximal utility in any subsequent case.

APPENDIX VII
PROOF OF LEMMA 9

We first prove $|\mathcal{A}_2| = 0$ when the constraint in (37b) is inactive. We use proof by contradiction. Assume $|\mathcal{A}_2| \ge 1$, i.e., there exists a link, denoted Link i, such that $P_i < \frac{Q_i^{ST} h_i}{g_i}$. It can be seen that the utility function of the sub-problem in (37) can be further improved if we increase P_i by the smaller value between $\left(P^{ST} - \sum_{i=1}^{N} P_i\right) (> 0)$ and $\left(\frac{Q_i^{ST} h_i}{g_i} - P_i\right) (> 0)$, since the utility function of the sub-problem in (37) is an increasing function with respect to P_i. Therefore, when constraint in (37b) is inactive, $\mathcal{A}_2 = \emptyset$.

Next we prove $\lambda^* > 0$ when the constraint in (37b) is active. Here we first show that there exists a link, denoted Link i, such that $0 < P_i < \frac{Q_i^{ST} h_i}{g_i}$. We use proof by contradiction. Assume for any link, say Link i, we have either $P_i = 0$ or $P_i = \frac{Q_i^{ST} h_i}{g_i}$. Denote the number of links with zero power assignment as K. Without loss of generality, assume Links 1, ..., K are with zero power assignment. Therefore, we have

$$\begin{aligned}
&P_j y_j = Q^{ST}, \quad j = K+1, K+2, ..., N \\
&\sum_{j=K+1}^{N} P_j = P^{ST}
\end{aligned} \tag{55}$$

where $y_j = \frac{g_j}{h_j}$. The probability for (55) to hold is zero, since g_j's and h_j's are independent random variables. Therefore, there exists a link, denoted Link i, such that $0 < P_i < \frac{Q_i^{ST} h_i}{g_i}$. Together with (38d) and (38e), we have $\mu_i^* = 0$ and $\Delta_i^* = 0$.

Assume $\lambda^* = 0$. Since $\mu_i^* = 0$ and $\Delta_i^* = 0$, we have $x_i = 0$ according to (38b). This contradicts the fact that Link i is with positive power and bandwidth assignment. Therefore, we have $\lambda^* > 0$.

Then, we prove $|\mathcal{A}_2| \le 1$ when the constraint in (37b) is active. The proof is similar to that of Lemma 4, and is omitted here.

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REFERENCES

[1] J. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.

[2] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inform. Theory*, vol. 51, no. 9, pp. 3037-3063, Sep. 2005.

[3] B. Wang, J. Zhang, and L. Zheng, "Achievable rates and scaling laws of power-constrained wireless sensory relay networks," *IEEE Trans. Inform. Theory*, vol. 52, no. 9, pp. 4084-4104, Sep. 2006.

[4] M. M. Fareed and M. Uysal, "BER-optimized power allocation for fading relay channels," *IEEE Trans. Wireless Commun.*, vol. 7, no. 6, pp. 2350-2359, June 2008.

[5] N. Ahmed, M. A. (Amir) Khojastepour, A. Sabharwal, and B. Aazhang, "Outage minimization with limited feedback for the fading relay channel," *IEEE Trans. Commun.*, vol. 54, no. 4, pp. 659-669, Apr. 2006.

[6] M. O. Hasna and M.-S. Alouini, "Optimal power allocation for relayed transmissions over Rayleigh-fading channels," *IEEE Trans. Wireless Commun.*, vol. 3, no. 6, pp. 1999-2004, Nov. 2004.

[7] Y. Zhao, R. Adve, and T. J. Lim, "Improving amplify-and-forward relay networks: Optimal power allocation versus selection," *IEEE Trans. Wireless Commun.*, vol. 6, no. 8, pp. 3114-3123, Aug. 2007.

[8] S. Serbetli and A. Yener, "Relay assissted F/TDMA Ad Hoc networks: Node classification, power allocation and relaying strategies," *IEEE Trans. Commun.*, vol. 56, no. 6, pp. 937-947, June 2008.

[9] Y. Liang, V. V. Veeravalli, and H. V. Poor, "Resource allocation for wireless fading relay channels: Max-min solution," *IEEE Trans. Inform. Theory*, vol. 53, no. 10, pp. 3432-3453, Oct. 2007.

[10] A. Hϕst-Madsen and J. Zhang, "Capacity bounds and power allocation for wireless relay channels," *IEEE Trans. Inform. Theory*, vol. 51, no. 6, pp. 2020-2040, June 2005.

[11] J. Tang and X. Zhang, "Cross-layer resource allocation over wireless relay networks for quality of service provisioning," *IEEE J. Select. Areas Commun.*, vol. 25, no. 4, pp. 645-656, May 2007.

[12] P. A. Anghel and M. Kaveh, "On the performance of distributed space-time coding systems with one and two non-regenerative relays," *IEEE Trans. Wireless Commun.*, vol. 5, no. 3, pp. 682-692, Mar. 2006.

[13] Y. Li, B. Vucetic, Z. Zhou, and M. Dohler, "Distributed adaptive power allocation for wireless relay networks," *IEEE Trans. Wireless Commun.*, vol. 6, no. 3, pp. 948-958, Mar. 2007.

[14] Y. Jing and H. Jafarkhani, "Network beamforming using relays with perfect channel information," *IEEE Trans. Inform. Theory*, vol. 55, no. 6, pp. 2499-2417, June 2009.

[15] M. Chen, S. Serbetli, and A. Yener, "Distributed power allocation strategies for parallel relay networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 2, pp. 552-561, Feb. 2008.

[16] L. Lai and H. El Gamal, "The water-filling game in fading multiple-access channels," *IEEE Trans. Inform. Theory*, vol. 54, no. 5, pp. 2110-2122, May 2008.

[17] L. Li and A. J. Goldsmish, "Capacity and optimal resource allocation for fading broadcast channels-Part I: Ergodic capacity," *IEEE Trans. Inform. Theory*, vol. 47, no. 3, pp. 1083-1102, Mar. 2001.

[18] R. Zhang, S. Cui, and Y.-C. Liang, "On ergodic sum capacity of fading cognitive multiple-access and broadcast channels," *IEEE Trans. Inform. Theory*, vol. 55, no. 11, pp. 5161-5178, Nov. 2009.

[19] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge University Press, 2005.

[20] N. Jindal and A. Lozana, "Fading models and metrics for contemporary wireless systems," in *Proc. 44th Asilomar Conference on Signals, Systems, and Computers*, pp. 625-629, Nov. 2010.

[21] H. Yin and S. Alamouti, "OFDMA: A broadband wireless access technology," in *Proc. IEEE Sarnoff Symposium*, Mar. 2006.

[22] H.-W. Lee and S. Chong, "Downlink resource allocation in multi-carrier systems: frequency-selective vs. equal power allocation," *IEEE Trans. Wireless Commun.*, vol. 7, no. 10, pp. 3738-3747, Oct. 2008.

[23] Z. Shen, J. G. Andrews, and B. L. Evans, "Adaptive resource allocation in multiuser OFDM systems with proportional rate constraints," *IEEE Trans. Wireless Commun.*, vol. 4, no. 6, pp. 2726-2737, June 2005.

[24] J. Huang, V. G. Subramanian, R. Agrawal, and R. Berry, "Joint scheduling and resource allocation in uplink OFDM systems for broadband wireless access networks," *IEEE J. Select. Areas Commun.*, vol. 27, no. 2, pp. 226–234, Feb. 2009.

[25] C. Y. Ng and C. W. Sung, "Low complexity subcarrier and power allocation for utility maximization in uplink OFDMA systems," *IEEE Trans. Wireless Commun.*, vol. 7, no. 5, pp. 1667-1675, May 2008.

[26] B. Wang, J. Zhang, and A. Hϕst-Madsen, "On the capacity of MIMO relay channels," *IEEE Trans. Inform. Theory*, vol. 51, no. 1, pp. 29-43, Jan. 2005.

[27] G. Farhadi and N. C. Beaulieu, "On the ergodic capacity of multi-hop wireless relaying systems," *IEEE Trans. Wireless Commun.*, vol. 8, no. 5, pp. 2286-2291, May 2009.

[28] X. Gong, S. A. Vorobyov, and C. Tellambura, "Joint bandwidth and power allocation in wireless multi-user decode-and-forward relay networks," in *Proc. 35th IEEE Int. Conf. Acoustics, Speech, and Signal Processing (ICASSP'10)*, pp. 2498-2501, Mar. 2010.

[29] I. Maric and R. D. Yates, "Bandwidth and power allocation for cooperative strategies in Gaussian relay networks," *IEEE Trans. Inform. Theory*, vol. 56, no. 4, pp. 1880-1889, Apr. 2010.

[30] G. Farhadi, and N. C. Beaulieu, "On the ergodic capacity of wireless relaying systems over rayleigh fading channels," *IEEE Trans. Wireless Commun.*, vol. 7, no. 11, pp. 4462-4467, Nov. 2008.

[31] S. P. Boyd and L. Vandenberghe, *Convex Optimization.* Cambridge University Press, 2004.

[32] D. P. Bertsekas, A. Nedic, and A. E. Ozdaglar, *Convex Analysis and Optimization.* Athena Scientific, 2003.

[33] J. Nocedal and S. J. Wright, *Numerical Optimization*. Springer: New York, 1999.

[34] J. Acharya and R. D. Yates, "Dynamic spectrum allocation for uplink users with heterogeneous utilities," *IEEE Trans. Wireless Commun.*, vol. 8, no. 3, pp. 1405-1413, Mar. 2009.

[35] S. Sun and Y. Jing, "Channel training and estimation in distributed space-time coded relay networks with multiple transmit/receive antennas," in *Proc. IEEE Wireless Communications and Networking Conference (WCNC)*, April 2010.

[36] F. Gao, T. Cui and A. Nallanathan, "On channel estination and optimal training design for amplify and forward relay networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 5, pp. 1907-1916, May 2010.

[37] J.-W. Lee, R. R. Mazumdar, and N. B. Shroff, "Opportunistic power scheduling for dynamic multi-server wireless systems," *IEEE Trans. Wireless Commun.*, vol. 5, no. 6, pp. 1506-1515, June 2006.

[38] Y. Song, C. Zhang, and Y. Fang, "Harnessing traffic uncertainties in wireless mesh networks - a stochastic optimization approach", *ACM Mobile Networks and Applications (MONET)*, vol. 14, no. 2, pp. 124-133, Feb. 2009.

[39] X. Wang and N. Gao, "Stochastic resource allocation over fading multiple access and broadcast channels", *IEEE Trans. Inform. Theory*, vol. 56, no. 5, pp. 2382-2391, May 2010.

[40] S. Han, S. Ahn, E. Oh, and D. Hong, "Effect of channel-estimation error on BER performance in cooperative transmission," *IEEE Trans. Veh. Technol.*, vol. 58, no. 4, pp. 2083-2088, Nov. 2009.

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